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Multivariate geostatistical estimation using minimum spatial cross-correlation factors (Case study: Cubuk Andesite quarry, Ankara, Turkey)

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Abstract
The quality properties of andesite (Unit Volume Weight, Uniaxial Compression Strength, Los500, etc.) are required to determine the exploitable blocks and their sequence of extraction. However, the number of samples that can be taken and analyzed is restricted, and thus the quality properties should be estimated at unknown locations. Cokriging has been traditionally used in the estimation of spatially cross-correlated variables. However, it can face unsolvable matrices in its algorithm. An alternative to cokriging is to transform variables into spatially orthogonal factors, and then to apply kriging to them. Independent Component Analysis (ICA) is one of the methods that can be used to generate these factors. However, ICA is applicable to zero lag distance so that using methods with distance parameter in their algorithms would be advantageous. In this work, Minimum Spatial Cross-correlation (MSC) was applied to six mechanical properties of Cubuk andesite quarry located in Ankara, Turkey, in order to transform them into approximately orthogonal factors at several lag distances. The factors were estimated at 1544 (5 m × 5 m) regular grid points using the kriging method, and the results were back-transformed into the original data space. The efficiency of the MSC-kriging was compared with Independent Component kriging (IC-kriging) and cokriging through cross-validation test. All methods were unbiased but the MSC-kriging outperformed the IC-kriging and cokriging because of having the lowest mean errors and the highest correlation coefficients between the estimated and the observed values. The estimation results were used to determine the most profitable blocks and the optimum direction of extraction.

Keywords: *Spatial Cross-Correlation, Kriging, Variogram, Building Stone.*

1. Introduction

Selective extraction of natural building stone, which maximizes the profit and minimizes the environmental effects of mining operation, can be done considering its important physical and mechanical properties such as uniaxial compression strength, porosity, and tensile strength. At the planning stage, due to its expenses, a limited number of samples can be tested so that the researchers have used different methods to estimate the required values of unsampled locations.

Taboada et al. [1] have used geostatistical techniques to estimate the quality of slate deposits as a function of depth. They developed a quality index that indicated the percentage of material that could be extracted. Taboada et al. [2] have used fuzzy kriging in resource evaluation of a granite deposit, and Tutmez and Tercan [3] have applied fuzzy modeling to the spatial estimation of some mechanical properties of rocks. Fuzzy kriging can account for the fact that a block may contain different qualities and that the definition of qualities in the field is subject to uncertainty. In this method, it is necessary to define membership functions to represent uncertainty in the quality variables under consideration. Ayalew et al. [4] and Exadaktylos and Stavropoulou [5] have used the kriging method to determine the spatial variability of rock quality designation and rock mass parameters. Saavedra et al. [6] have used a compositional kriging technique to determine the value of quality attributes in a granite deposit, and Taboada et al. [7] have applied it in determining the spatial distribution and the volume of four commercial quartz grades, namely silicon metal, ferrosilicon, aggregate, and kaolin in a quartz seam.

Kriging is appropriate for univariate studies, and in the presence of spatially cross-correlated variables, it would be better to apply multivariate estimation methods such as the traditional cokriging approach. However, in the cokriging method, the semi-variogram analysis is tedious due to simultaneously modeling p auto-variograms and $p\times(p-1)/2$ cross-variograms to guarantee the positive definiteness of covariance matrices, where p is the number of variables [8-9]. Although the linear model of coregionalization guarantees the positive definiteness of matrices, its imposed restrictions may result in poor variogram fitting that deprives cokriging from some of its possible advantages over the kriging system [10-11]. In order to guarantee the positive definiteness of matrices of the cokriging system, and also to reduce the number of auxiliary variables, Taboada et al. [12] and Martinez et al. [13] have used the Principal Component Analysis (PCA) to find a factor that approximately represents the properties of all original variables due to the highest variance it has. Then they used this factor as an auxiliary variable in the cokriging method.

There are also other works for transforming spatially cross-correlated random variables into a set of orthogonal factors that could be separately estimated using univariate techniques, and the results could be back-transformed into the original data space [14-17]. The Stepwise Conditional Transformation (SCT) is one of these orthogonalization methods that aims to produce normally distributed uncorrelated scores at zero lag distance. Although SCT has the advantage of producing Gaussian factors, it suffers from ordering issues associated with the transformation sequence of variables [18]. The next method is Min/Max Autocorrelation Factors (MAF), which has been used in several research works such as Tercan and Ozcelik [19], Desbarats [20], Vargas-Guzmán and Dimitrakopoulos [21], Rondon [22], Sohrabian and Ozcelik [23], Shakiba [24], and De Freitas Silva and Dimitrakopoulos [25] to produce uncorrelated factors at two lag distances. Some other works such as Ruessink et al. [26], Nielsen [27], Liu et al. [28], and Musafer and Thompson [29] take advantage of non-linear PCA that is capable of removing non-linear relationships.

Sohrabian and Ozcelik [30] have introduced Independent Components Analysis (ICA) to generate independent factors from some mechanical attributes of an andesite quarry. It has been used in several works such as Tercan and Sohrabian [31], Boluwade and Madramootoo [32], and Minniakhmetov and Dimitrakopoulos [33].

The general purpose of orthogonalization techniques is to develop algorithms that produce spatially independent factors. However, in most real datasets, it is practically impossible to generate factors that are orthogonal at all lag distances [34]. Therefore, methods that look for factors with approximate orthogonality at several lag distances have gained popularity. Xie and Myers [35] have suggested a version of simultaneous diagonalization that minimizes the cross-variogram models. The main drawback of this approach is its smoothing feature imposed by model variogram utilization. A method proposed by Cardoso and Souloumiac [36] and Cardoso and Souloumiac [37] has replaced a high-dimensional minimization problem with a set of simple problems in 2-D sub-spaces, and consequently, applies Cholesky decomposition in each sub-space. Despite being fast and convenient, this method is applicable to positive definite matrices, and it would not be operable in the presence of non-invertible sub-space matrices [38]. Another algorithm introduced by Joho and Rahbar [39] and used in Joho [40] applies the Newton method to minimize a second-order Taylor series approximation of a cost function that contains off-diagonal elements of matrices that should be orthogonalized simultaneously. This method, which is mathematically difficult, has a significant handicap of substituting the cost function with its approximation. Mueller and Ferreira [41] and Tichavsky and Yeredor [42] have proposed an efficient diagonalization method named uniformly weighted exhaustive diagonalisation with Gauss iterations (U-WEDGE), which has a good convergence speed but is complicated and practicable to positive definite matrices.

Sohrabian and Tercan [43] have introduced the Minimum Spatial Cross-correlation (MSC) method, which uses the same sub-spaces presented in [36]. However, instead of running the Chokesky decomposition, MSC applies the gradient descent method in the minimization process of a univariate cost function. Then it is simple and can be applied in the approximate orthogonalization of any kind of matrices including those with non-invertible sub-spaces.

In this work, we applied the MSC method to produce factors that were approximately orthogonal at several lag distances. The data consisted of six spatially cross-correlated mechanical attributes of an andesite quarry, located in Ankara, Turkey. Variograms of the generated factors were analyzed, and the parameters obtained were used to estimate each factor, separately, using the kriging method. The method's efficiency was compared with the Independent Components kriging (IC-kriging) and cokriging using their cross-validation results. Then the same procedure was executed to predict the unknown values of six mechanical attributes of 1544 $5m \times 5m$ blocks. Then the estimations were used to classify the andesite blocks as exploitable and non-exploitable.

This paper is structured as what follows. The second section presents a factor approach for multivariate geostatistical estimation. The third section briefly explains the ICA and MSC decomposition methods. The fourth section presents a case study including the quarry and data description, factor generation and efficiency test of the MSC-kriging, and evaluation of the Cubuk andesite quarry. At last, a conclusion is presented.

2. Factor approach for multivariate geostatistical estimation

Suppose that there are p stationary random variables that are isotopically sampled at x data locations. These variables can be shown in the matrix form as follows:

$$\mathbf{Z}(x) = \left[Z_1(x), \quad ..., \quad Z_p(x) \right] \tag{1}$$

where each element of $\mathbf{Z}$ includes one of the variables. The variogram matrix of these variables can be written as:

$$2\mathbf{\Gamma_Z}(h) = E\left\{ [\mathbf{Z}(x) - \mathbf{Z}(x+h)][\mathbf{Z}(x) - \mathbf{Z}(x+h)]^T \right\} \tag{2}$$

where $2\mathbf{\Gamma_Z}(h)$ represents a variogram matrix at lag distance h, E is the expectation, and T is transposition. The diagonal and off-diagonal elements of this $p \times p$ matrix present direct and cross-variograms of variables at each lag distance h, respectively. When $h \to \infty$, the variogram matrix equals variance-covariance matrix $\mathbf{B}$.

If there is a linear transformation $\mathbf{W}$ that transforms the given variables into factors $\mathbf{F}(x) = \left[F_1(x), \quad ..., \quad F_p(x) \right]$ whose cross-variograms are 0 for all lags, then the factors can be independently estimated, and the estimations can be back-transformed into the original data space using $\mathbf{W}^{-1}$.

In our previous work [23], we applied the fast ICA algorithm [44] to generate transformation matrix $\mathbf{W}$ and factors that were independent at zero lag distance. In this work, we used the MSC method to find the appropriate $\mathbf{W}$ matrix that gives factors with the lowest possible cross-correlations at several lag distances. A flow chart of the estimation process using orthogonalized factors is shown in Figure 1.

3. Decomposition methods

We assumed that before running the ICA and MSC algorithms, the multivariate data were whitened with PCA. Without whitening, it was necessary to find an arbitrary transformation matrix with p^2 parameters. However, after whitening, the number of parameters reduced to $p \times (p-1)/2$ and then it could be said that whitening decreased the complexity and solved half of the problem [43-44].

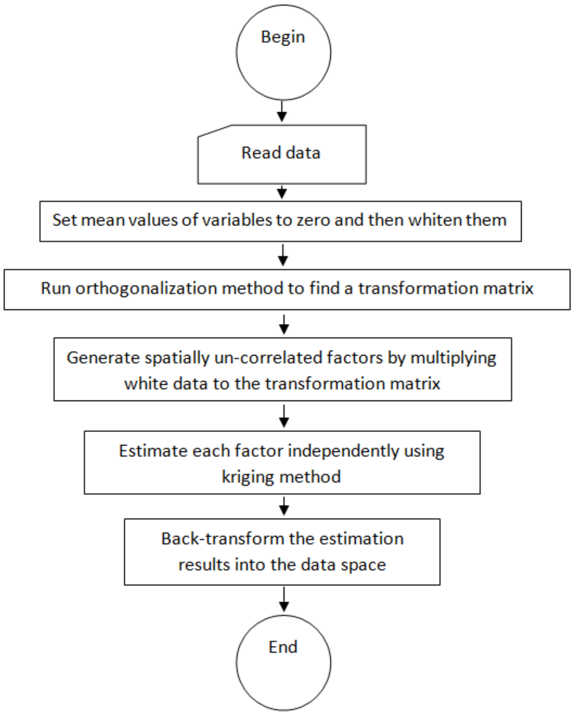


Figure 1. Flow chart of estimation process using orthogonalized factors.

3.1. Independent component analysis (ICA)

ICA, which tries to find mutually independent factors from a linear combination of original variables, is among the blind source separation methods. In blind source separation problems, the transformation and the resulting matrices are unknown, and the number of equations is quite smaller than the number of unknowns. For this reason, statistical properties of the factors should be considered as criteria. For example, the negentropy of independent factors is the highest, and then it can be iteratively maximized. Therefore, ICA uses negentropy in its algorithm searching for factors with super-Gaussian or hyper-Gaussian distributions. In the presence of one normally distributed underlying factor, ICA finds all the non-Gaussian components, and then the Gaussian factor would be automatically explored. For two or more normally distributed independent factors, ICA loses its efficiency, and it cannot find all the independent directions so that some of the resulting components would be a mixture of the normally distributed factors. Independent components can be achieved using several ICA algorithms. Due to its accuracy and speed of convergence [44], we used the FastICA algorithm with deflationary orthogonalizaiton, as follows:

1. Centralize the data to zero mean.
2. Whiten the centralized data by generating the principal components and setting their variances to 1 to obtain $\mathbf{Z}$.
3. Choose c, the number of ICs to be estimated. Set the counter $p \leftarrow 1$.

In our work, c is equal to the number of variables.

4. Choose a randomly generated initial vector of unit norm for W_p, which is the *p*th column of the transformation matrix $\mathbf{W}$.
5. let $W_p \leftarrow E\{zg(zW_p)\} - E\{g'(zW_p)\}W_p$, where g represents the hyperbolic tangent function for smoothing, and z is the rows of data matrix $\mathbf{Z}$ (expectation is taken with respect to z).
6. Do the following orthogonalization:

$$W_p \leftarrow W_p - \sum_{j=1}^{p-1} (W_p^T W_j) W_j \tag{3}$$

7. let $W_p \leftarrow W_p / norm(W_p)$.
8. If the inner product of two consequent W_p is greater than an assigned value, go back to step 5.
9. Set $p \leftarrow p+1$ if $p \le c$, go back to step 4.

3.2. Minimum spatial cross-correlation method

In this section, a brief explanation of the MSC method and its algorithm are presented. Theory of the MSC method is presented in details in [45].

The MSC method is capable of handing linear relationships, and not the non-linear ones. The average distance of adjacent samples can be selected as the optimum lag distance. The number of lags can be chosen by dividing the maximum range of auto/cross-variograms to the lag distance. The MSC factors are a linear combination of the original variables. These factors can be generated by finding an appropriate orthogonal transformation matrix $\mathbf{W}$. In the MSC method, the $p \times p$ minimization problem is replaced by $p \times (p-1)/2$ 2D problems, which are easier to handle. At first, we presented the problem in one of the 2D spaces consisting of two variables, and after that, we generalized it to several variables. Assume that there are two spatially cross-correlated variables with a scatter plot shown in Figure 2. The variogram matrix of these variables at lag distance h_m is:

$$\mathbf{\Gamma_Z}(h_m) = \begin{bmatrix} \gamma_{11}(h_m) & \gamma_{12}(h_m) \\ \gamma_{21}(h_m) & \gamma_{22}(h_m) \end{bmatrix} \quad (4)$$

m is the number of lags that should be regarded. Now the objective is to find a 2×2 transformation matrix $\mathbf{W}$.

$$\mathbf{W} = \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} \\ \sin\theta_{12} & \cos\theta_{12} \end{bmatrix} \quad (5)$$

which generates the F_1 and F_2 factors with minimum sum of absolute cross-variograms over several lags. θ_{12} is the angle that the direction of the first component, F_1, makes with the horizontal axis in a 2D space (Figure 2). The variogram matrix of the produced factors can be written as follows:

$$\mathbf{\Gamma_F}(h_m) = \begin{bmatrix} \cos\theta_{12} & \sin\theta_{12} \\ -\sin\theta_{12} & \cos\theta_{12} \end{bmatrix} \begin{bmatrix} \gamma_{11}(h_m) & \gamma_{12}(h_m) \\ \gamma_{21}(h_m) & \gamma_{22}(h_m) \end{bmatrix} \begin{bmatrix} \cos\theta_{12} & -\sin\theta_{12} \\ \sin\theta_{12} & \cos\theta_{12} \end{bmatrix} \quad (6)$$

$$= \begin{bmatrix} \cos^2\theta_{12}\gamma_{11}(h_m) + \sin^2\theta_{12}\gamma_{22}(h_m) + 2\cos\theta_{12}\sin\theta_{12}\gamma_{12}(h_m) & \cos\theta_{12}\sin\theta_{12}(\gamma_{22}(h_m) - \gamma_{11}(h_m)) + (\cos^2\theta_{12} - \sin^2\theta_{12})\gamma_{12}(h_m) \\ \cos\theta_{12}\sin\theta_{12}(\gamma_{22}(h_m) - \gamma_{11}(h_m)) + (\cos^2\theta_{12} - \sin^2\theta_{12})\gamma_{12}(h_m) & \cos^2\theta_{12}\gamma_{22}(h_m) + \sin^2\theta_{12}\gamma_{11}(h_m) - 2\cos\theta_{12}\sin\theta_{12}\gamma_{12}(h_m) \end{bmatrix}$$

Then for each 2-D case, the objective function that contains the off-diagonal element of variogram matrix of the new factors can be written as follows:

$$\varphi(\theta_{ij}) = \sum_{m=1}^{l}[\gamma_{F_iF_j}(h_m)]^2 = \sum_{m=1}^{l}[\cos\theta_{ij}\sin\theta_{ij}(\gamma_{ii}(h_m) - \gamma_{jj}(h_m)) + (\cos^2\theta_{ij} - \sin^2\theta_{ij})\gamma_{ij}(h_m)]^2 \quad (7)$$

where i and j show the variables that are considered at each step. $\gamma_{F_iF_j}(h_m)$ represents the cross-variogram of the factors F_i and F_j.

Algorithm of the MSC method can be shown as follows:

1. Centralize the data to zero mean.
2. Whiten the centralized data to obtain $\mathbf{Z}$.
3. Give the number of variables p and the number of lags m that should be considered.
4. Choose random initial values for θ_{ij}, $i = 1,..., p-1$ and $j = i+1,..., p$.
5. Minimize the spatial cross-correlation of each pair of variables i and j. Spatial cross-correlation of each pair of variables is a function of θ_{ij}, and has a global minimum that repeats every 1.57 radians (Figure 2). Therefore, it can be easily minimized using the gradient descent algorithm with the step size of ζ, as follows:

$$\theta_{ij}^{t} = \theta_{ij}^{t-1} - \zeta \left.\frac{\partial\varphi(\theta_{ij})}{\partial\theta_{ij}}\right|_{\theta_{ij} = \theta_{ij}^{t-1}}$$

$$\frac{\partial\varphi(\theta_{ij})}{\partial\theta_{ij}} = \frac{\partial\sum_{m=1}^{l}(\gamma_{F_iF_j}(h_m))^2}{\partial\theta_{ij}} = \sum_{m=1}^{l}[(\cos^3\theta_{ij}\sin\theta_{ij} - \sin^3\theta_{ij}\cos\theta_{ij})(2K_m^2 - 8\gamma_{ij}^2(h_m)) + 2K_m(\cos^4\theta_{ij} + \sin^4\theta_{ij} - 6\times\cos^2\theta_{ij}\sin^2\theta_{ij})\gamma_{ij}(h_m)] \quad (8)$$

Where $K_m = \gamma_{jj}(h_m) - \gamma_{ii}(h_m)$.

6. Find the total $p \times p$ transformation matrix, $\mathbf{W}$:

$$\mathbf{W}_{p\times p}=\prod_{i=1}^{p-1}\prod_{j=i+1}^{p}\mathbf{A}_{p\times p}(\theta_{ij})=\mathbf{A}_{p\times p}(\theta_{12})\times\mathbf{A}_{p\times p}(\theta_{13})\times\ldots\times\mathbf{A}_{p\times p}(\theta_{p-1p})=$$
$$\begin{bmatrix}\cos\theta_{12} & -\sin\theta_{12} & 0 & \dots & & \\ \sin\theta_{12} & \cos\theta_{12} & 0 & \dots & & \\ 0 & 0 & 1 & \dots & & \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & & \\ 0 & 0 & 0 & \dots & & \end{bmatrix}\times\begin{bmatrix} & & & & \dots & \\ & & & & \dots & \\ & & & & \dots & \\ & & & & \dots & \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ & & & & \dots & \end{bmatrix}\times\ldots\times \tag{9}$$
$$\begin{bmatrix}1 & 0 & \dots & & 0 & 0 \\ 0 & 1 & \dots & & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & \dots & & 0 & 0 \\ 0 & 0 & \dots & & \mathrm{)s}\,\theta_{p-1p} & -\sin\theta_{p-1p} \\ 0 & 0 & \dots & & \mathrm{n}\,\theta_{p-1p} & \cos\theta_{p-1p}\end{bmatrix}$$

Cardoso and Souloumiac [36] have presented a similar way of approximately orthogonalizing several matrices using 2D sub-spaces. In their method, the Cholesky decomposition, which is a decomposition of a Hermitian, positive-definite matrix into the product of the eigenvalues, and eigenvector matrices, is used. It is rare but possible to face non-invertible covariance matrices in sub-spaces while the total $p\times p$ covariance matrix is positive definite. Then the MSC method has the advantage of being applicable to all kinds of matrices, and it does not have the imposed restriction of the Cholesky decomposition and is free of any assumption about the distribution type of the produced factors. Although the MSC method converges to the absolute minimum without being trapped in the possible local minima of $p-D$ space $(p>2)$, this method is a little slow and requires an appropriate choice of the step size. Another issue is that the MSC method can be applied to the isotopically sampled data. If some variables are unsample at some locations, MSC can be implemented based on the complete-case analysis using the isotopic sub-space of the data [16]. However, this approach reduces the number of samples, and results in the loss of information. Another approach is to apply the imputation methods to complete data by assigning the missing observations. Imputation approaches vary from the simplest one that takes an average of the nearby values to the complicated cases of multiple imputation, which can be found in Little and Rubin [46].

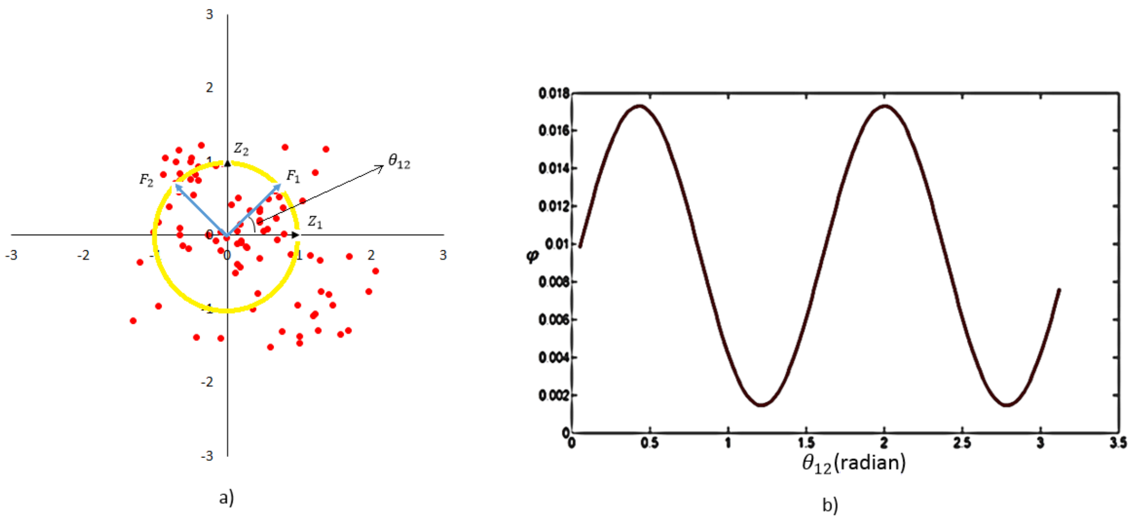


Figure 2. a) Direction of original variables and produced factors, b) φ shown as a function of θ_{12}.

4. Case study

4.1. Quarry and data descriptions

The studied area is an andesite quarry located in the western side of Menekse Hill, 3 km south of Susuz village in the Cubuk district, and 60 km NE of Ankara, the capital of Turkey. The location map of the studied area is given in Figure 3. It occurs in the Tertiary Mamak Formation, which consists of volcanic units such as andesite, dacite, rhyolitic lava, and tuff. Lava flow orientation is between N30–35E and 26–45SE in the centre of the quarry and on the northern part. In the southern part, a significant difference is observed in lava flow orientation. The strike is between E-W and N63W. The dip is between 37° and 55° towards the north and northeast [47].

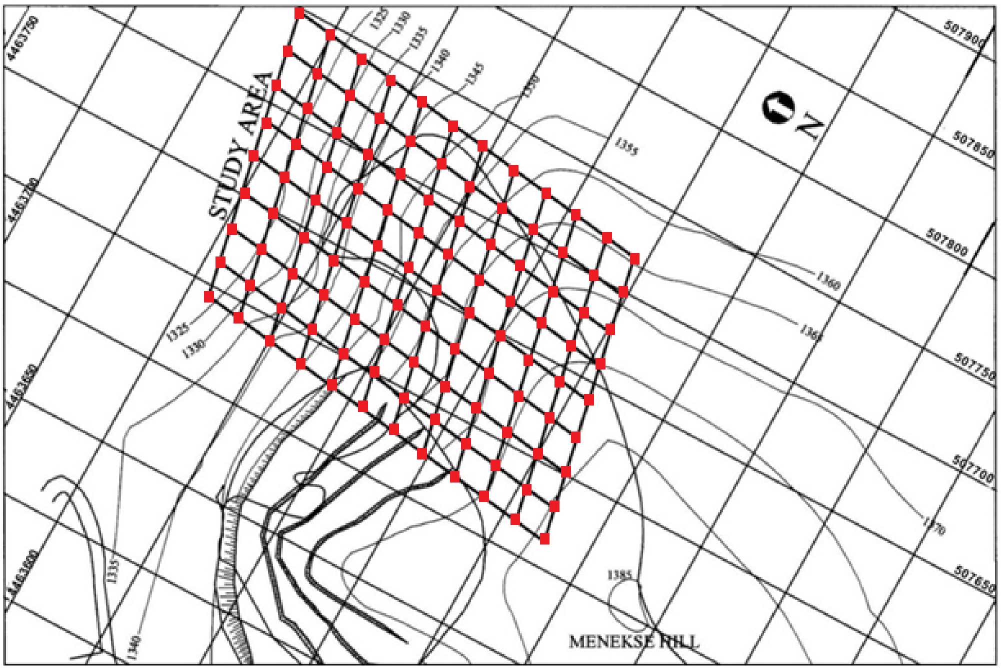


Figure 3. Location map of studied area and regular grid of size 20 m × 20 m used for sampling. Sample locations are shown with red squares. UTM coordinate system was used in this work, and 36 UTM is valid for this area.

In this work, 108 $(20cm \times 20cm \times 20cm)$ rock samples were collected in a $20m \times 20m$ regular grid shown in Figure 3. Then 5 cores were taken from each of these samples and tested for Unit Volume Weight (UW), Water Absorption capacity by mass (WA), Uniaxial Compression Strength (UCS), Tensile Strength (TS), Los500, and Porosity (P) based on the guidelines of ISRM [48]. After discarding the outliers, for each sample location, the amount of attributes was calculated by taking the average of the remained values. Regarding this procedure and also applying a regular sampling grid with an acceptable density over the studied area, it can be said that the samples are representative. The summary statistics of variables and TS 10835 standards [49], which are used to classify andesite as facing and building stones, are presented in Table 1. The scatter plots of variables, demonstrated in Figure 4, do not show a considerable clustering issue.

Using the following equation, the spatial cross-correlations of variables can be calculated at different lag distances, h.

$$r_{ij}^{h} = \frac{\gamma_{ij}^{h}}{\sqrt{\gamma_{ii}^{h} \times \gamma_{jj}^{h}}} \tag{10}$$

Cross-correlation value, r_{ij}^{h}, is the ratio of the experimental cross-variogram of the ith and jth variables to their perfect spatial cross-variogram value, and can vary between -1 and 1 [50-51]. Spatially uncorrelated variables have r values equal to zero at all lag distances. Graphs of the spatial cross-correlations of attributes were calculated and shown in Figure 5. Moreover, the cross-correlation matrix of variables is presented in Table 2. All variables were spatially cross-correlated so that they should be transformed into spatially uncorrelated factors using the orthogonalization algorithms. Then each factor can be estimated through the univariate estimation methods such as kriging, and the results can be back-transformed into the original data space.

Table 1. Summary statistics of data and estimations together with TS 10835 standards (TSE, 1993) used to classify andesite as facing and building stones.

	Unit Weight (UW)				Water Absorption capacity by mass (WA)				Uniaxial Compression Strength (UCS)				Los500				Porosity (P)				Tensile Strength (TS)			
	Min	Max	Mean	σ^2	Min	Max	Mean	σ^2	Min	Max	Mean	σ^2	Min	Max	Mean	σ^2	Min	Max	Mean	σ^2	Min	Max	Mean	σ^2
Data	2.17	2.72	2.62	0.007	0.16	0.80	0.41	0.013	25	131.25	79.15	588	11.20	16.20	14.31	1.48	0.81	4.41	1.96	0.25	5.09	13.45	8.76	5.30
IC-kriging results	2.35	2.71	2.62	-	0.29	0.65	0.41	-	37.33	114.10	79.32	-	12.15	16.04	14.32	-	1.40	3.33	1.96	-	5.47	12.01	8.78	-
MSC-kriging results	2.39	2.73	2.62	-	0.30	0.65	0.41	-	16.02	107.35	79.26	-	12.82	17.30	14.32	-	1.47	2.85	1.96	-	3.07	11.21	8.77	-
TS10835 standards	>2.55 (g/cm^3)				< 0.7 (%)				> 60 (MPa)				< 15.10 (%)				< 2 (%)				> 7 (MPa)			

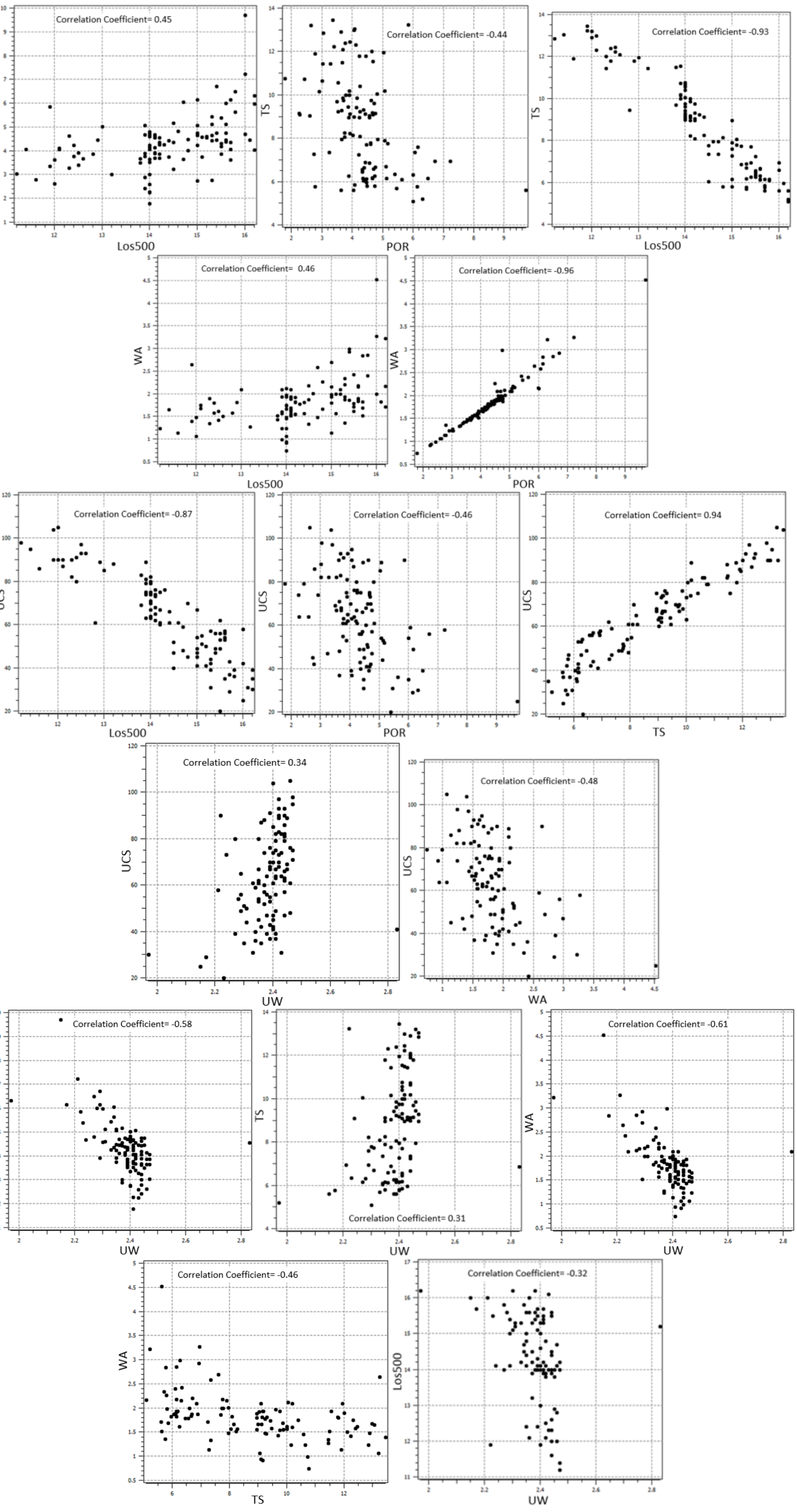


Figure 4. Scatter plots of variables.

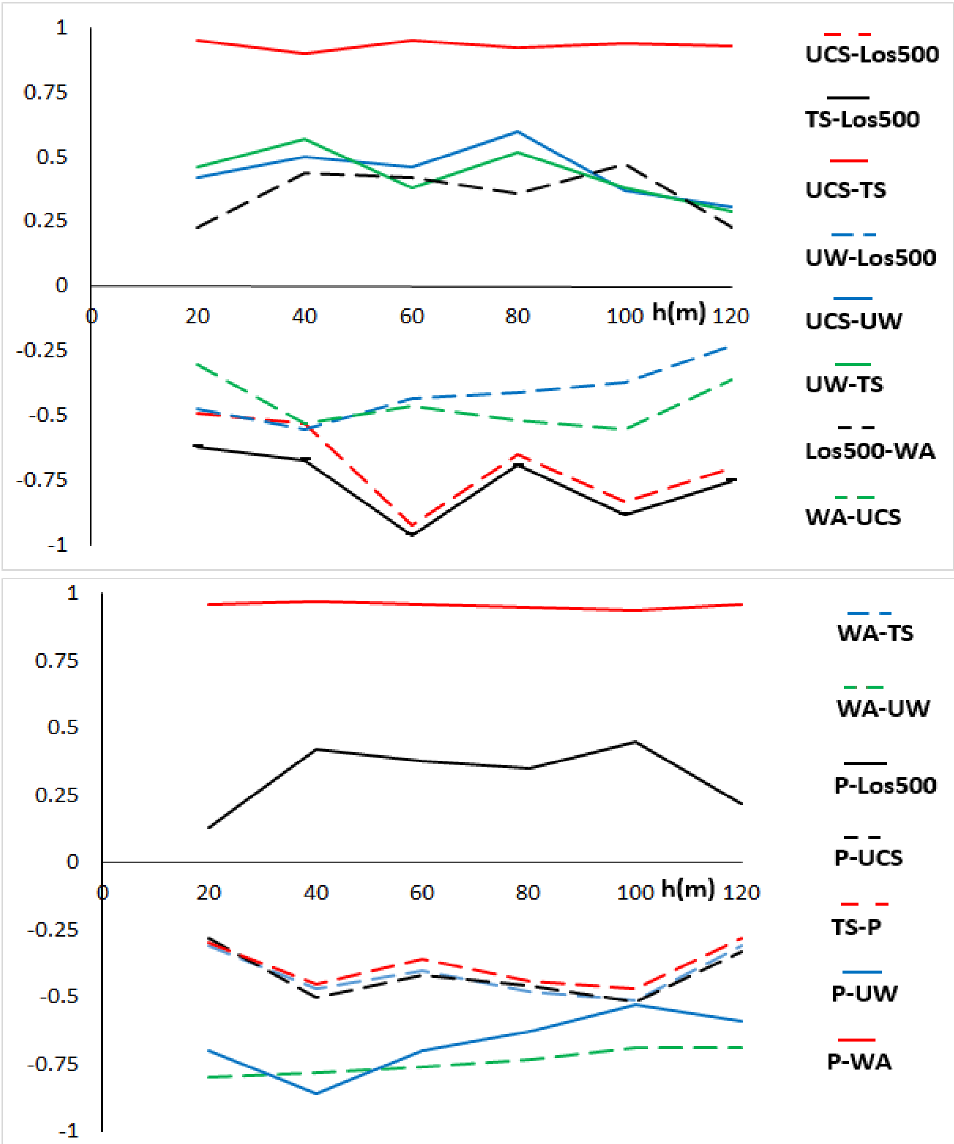


Figure 5. Graphs of spatial cross-correlations of variables. For being clear, the pair of variables are shown in two different graphs. UCS-Los500 denotes the pair of UCS and Los500.

Table 2. Matrix of variables' correlation coefficients.

	POR	Los500	TS	UCS	WA	UW
POR	1					
Los500	0.45	1				
TS	-0.44	-0.93	1			
UCS	-0.46	-0.87	0.94	1		
WA	-0.96	0.46	-0.46	-0.48	1	
UW	-0.58	-0.32	0.31	0.34	-0.61	1

4.2. Factor generation and efficiency test of MSC-kriging

Prior to running the MSC algorithm, the data was centered to zero mean and whitened using PCA to reduce the complexity of the problem to be solved and also to restrict the norm of vectors W to 1. Lag distance selection, which is the first important step, was done considering the auto/cross-variograms of the variables. The maximum range of variograms and the minimum sampling distance were equal to 100 m and 20 m, respectively. Therefore, we chose 5 as the number of lags, and 20 m, 40 m, 60 m, 80 m, and 100 m as distances that would be approximately orthogonalized. Considering the mentioned lag distances, the final transformation matrix including the whitening process and the MSC step was calculated as below:

$$\mathbf{W} = \begin{bmatrix} 2.562 & -8.158 & -1.710 & -11.206 & -4.969 & 11.724 \\ 0.249 & 7.187 & 0.850 & -0.811 & -1.404 & 0.501 \\ -0.118 & 0.000 & 0.048 & -0.011 & 0.015 & -0.005 \\ 1.724 & -0.234 & 0.614 & -0.101 & -0.023 & -0.151 \\ 1.099 & -0.470 & 2.035 & 0.106 & 0.482 & 0.391 \\ -1.836 & -35.84 & -2.242 & -1.496 & -5.317 & 0.403 \end{bmatrix}$$

The MSC factors were generated by multiplying this matrix by the zero centered original data matrix. Cross-variograms of the MSC factors and those of IC components calculated by Sohrabian and Ozcelik [30] are presented in Figure 6. Cross-variograms of the MSC factors, varying between -0.19 and 0.11, are in a tighter interval than those of the IC factors, which lie between -0.21 and 0.23. This demonstrates the efficiency of MSC over ICA in producing spatially uncorrelated factors. For the MSC factors, the largest remnant cross-correlations are at 40 *m,* and then come 20 *m* and 60 *m* lags.

While cross-variograms of the MSC factors have small cross-correlations, their auto-variograms that show a high degree of spatial dependency can be modeled using standard theoretical models. Except for MSC6, which has erratic variogram values at 60 m and 80 m, all variograms are appropriately modeled (Figure 7) regarding the cross-validation results. Unlike the principal components, the MSC and ICA factors do not have any order-related importance, and all of them are equally informative. The model parameters of the MSC factors including the nugget effects, contributions, and variogram ranges are presented in Table 3. The MSC4 and MSC6 with 69 m and 38 m have, consequently, the highest and the lowest variogram ranges. Nugget effects of the factors vary between 0.25 and 0.55.

After producing the MSC factors and analyzing their variograms, a cross-validation test using 108 samples was performed for the efficiency comparison of the MSC-kriging, IC-kriging, and cokriging. Cross-validations were carried out by temporarily removing each sample value from the data and estimating its values using the remained samples. Cokriging was performed using all variables with the variogram models fitted regarding the Cauchy–Schwarz inequality. Figures 8 and 9 present the auto- and cross-variograms of variables together with the fitted models. All variograms were modeled using a nugget effect and one spherical structure with a range of 75 m. The model variogram parameters are given in Table 4. The minimum and maximum number of samples used in the estimations were chosen to be 3 and 17, respectively. For the MSC-kriging and the IC-kriging, the estimated values were back-transformed into the original data space. Then the correlation coefficients of the estimated and observed values, the Mean Errors (ME), and the minimum and maximum errors of the estimations were calculated (Table 5). Consequently, ME and the correlation coefficient of a perfect estimator would be 0 and 1. The mean values of estimations are very close for all the estimation methods, and there is no considerable difference among them so that they are not shown here. Since the MSC-kriging has the lowest ME values, it is the best unbiased estimator; and after that comes the IC-kriging. The highest correlation coefficients between the estimated and observed values are for the MSC-kriging, and the cokriging has the lowest ones. The MSC-kriging and the IC-kriging are more efficient than Cokriging. Thus in the next sub-section, evaluation of the andesite quarry will be done using these methods.

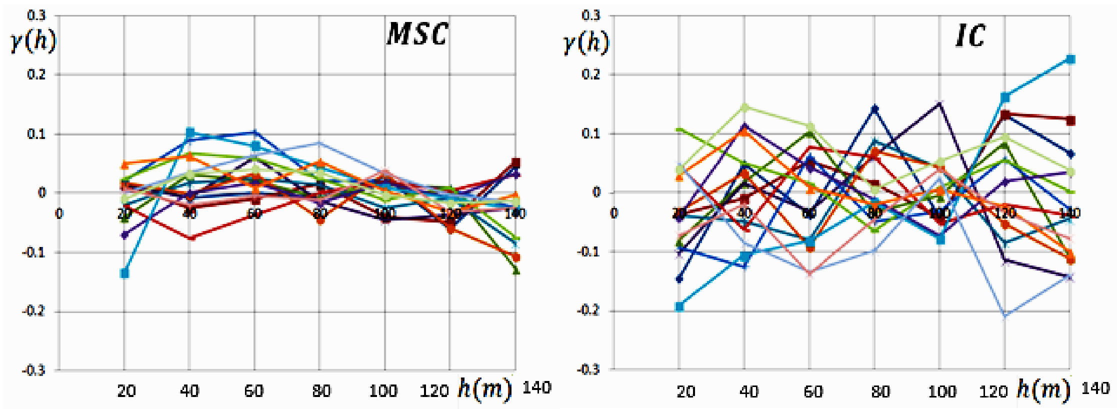


Figure 6. Cross-variograms of MSC and IC factors. Each color represents one of the fifteen cross-variograms of factors. For simplicity and also preventing ambiguity, legends are not shown.

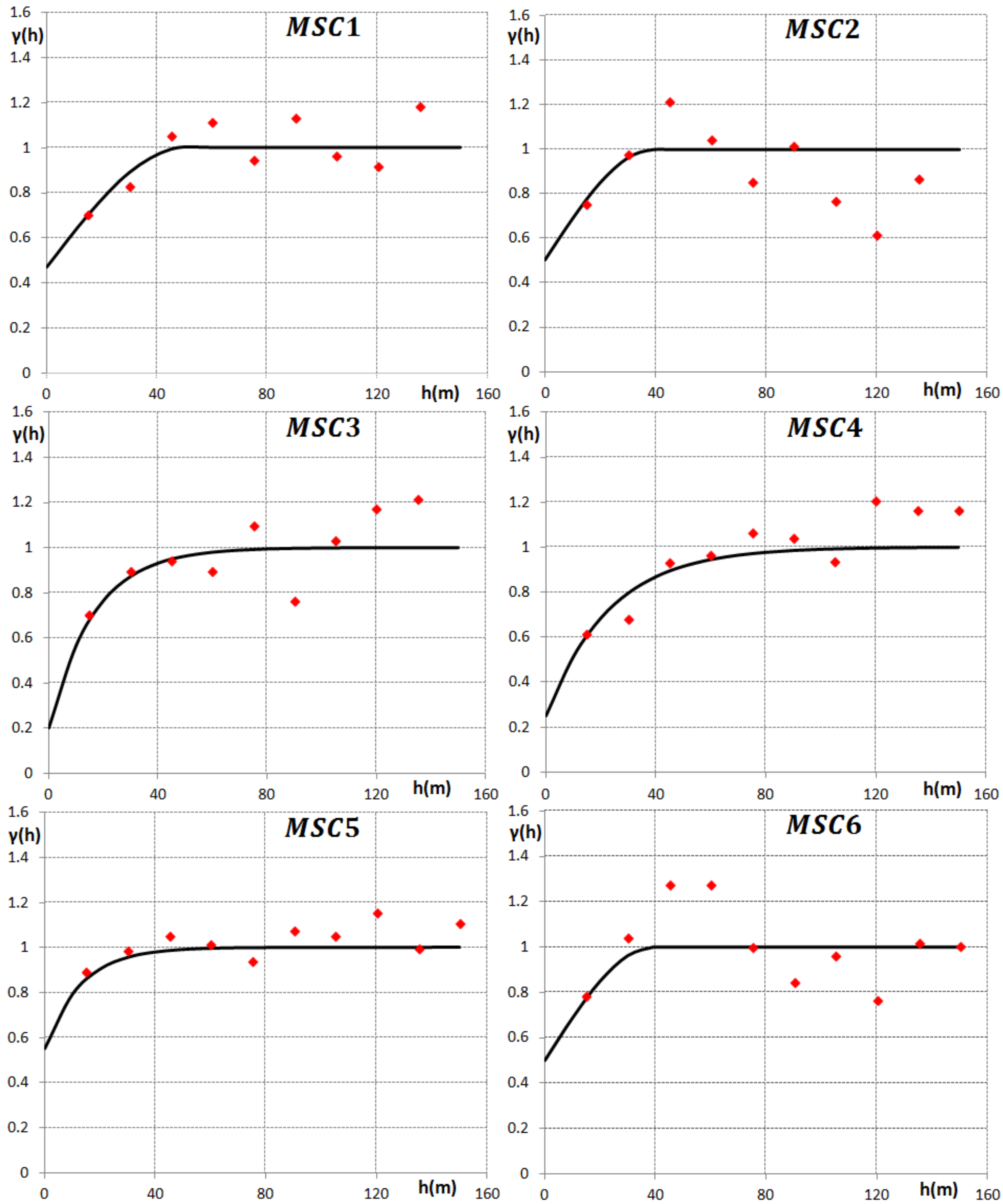


Figure 7. Auto-variograms of MSC factors together with fitted models. Variograms of MSC1, MSC2, and MSC6 are fitted by spherical model. Exponential model is used for MSC3, MSC4, and MSC5.

Table 3. Model variogram parameters fitted to MSC factors.

Factor	Variogram model	Nugget effect	Contribution	Variogram range (*m*)
MSC1	Spherical	0.47	0.53	50
MSC2	Spherical	0.50	0.50	39
MSC3	Exponential	0.20	0.80	49
MSC4	Exponential	0.25	0.75	69
MSC5	Exponential	0.55	0.45	39
MSC6	Spherical	0.50	0.50	38

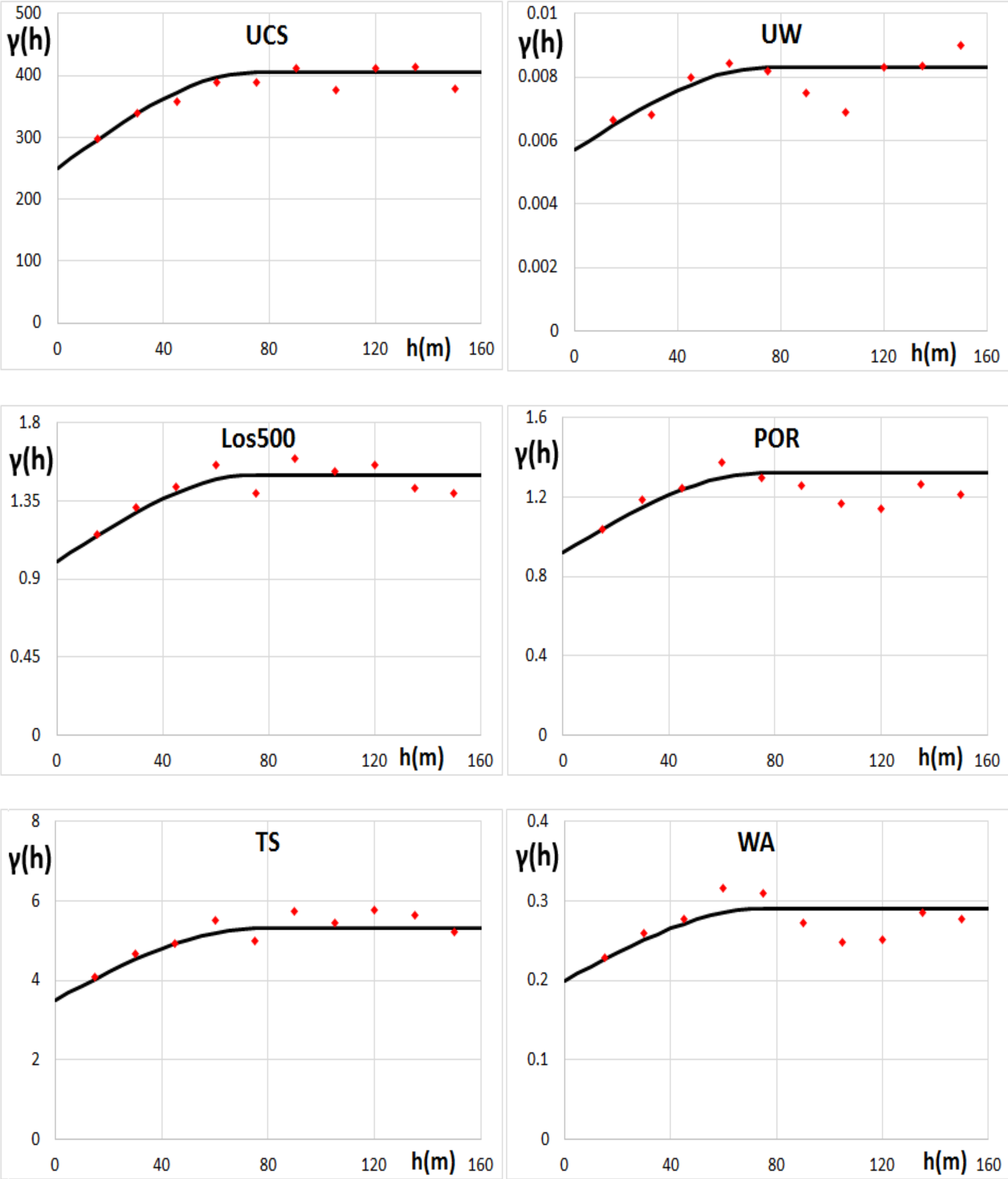


Figure 8. Auto-variograms of data (red dots) together with fitted models (black solid line).

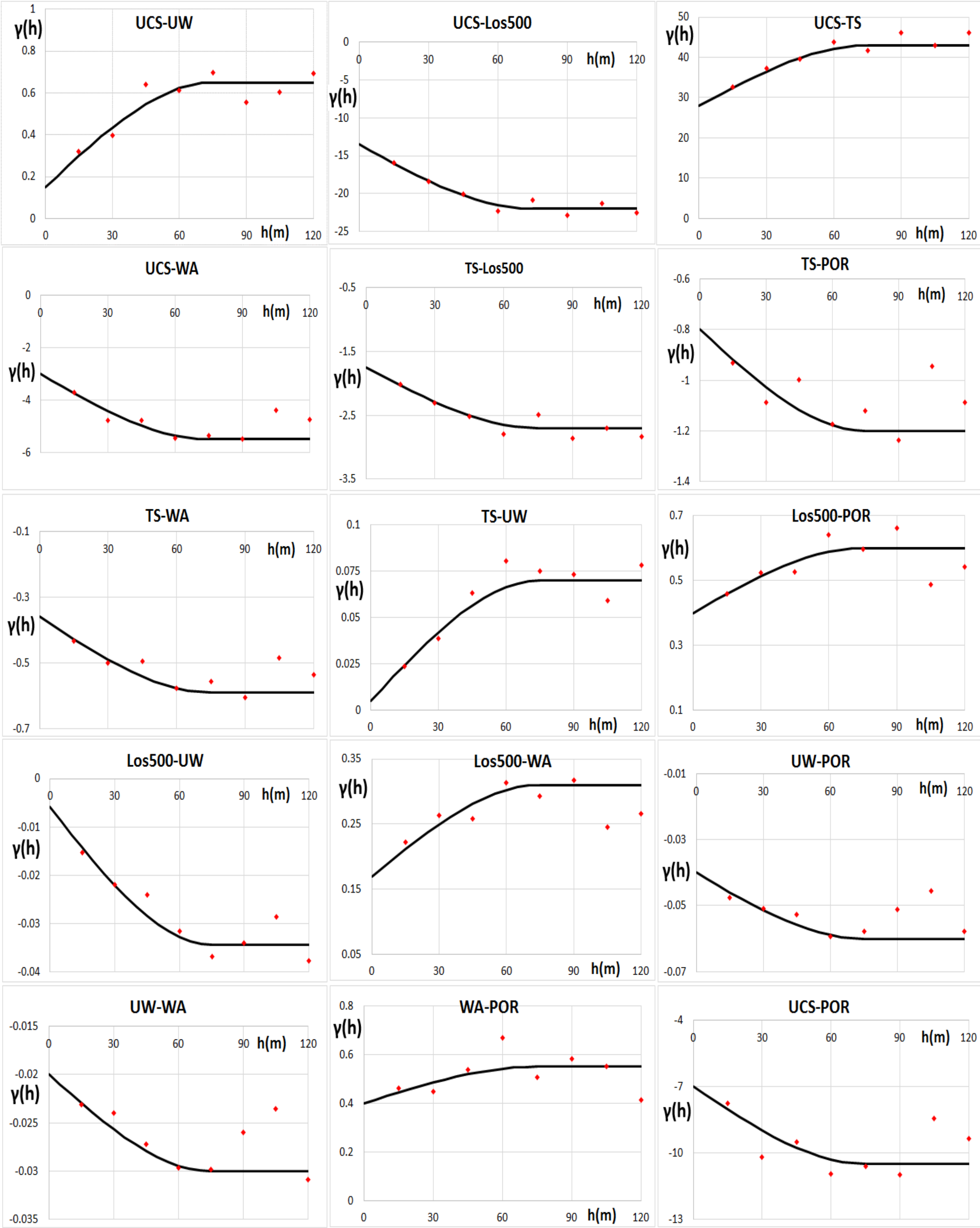


Figure 9. Cross-variograms of data (red dots) together with fitted model (solid black line).

Table 4. Model variogram parameters used in cokriging estimations. All variograms are of spherical type with a range of 75 m.

Pair of variables	C0*	C1*	Pair of variables	C0	C1	Pair of variables	C0	C1
UCS-UCS	250	155	UCS-TS	28	15	TS-WA	-0.36	-0.23
TS-TS	3.5	1.8	UCS-Los500	-13.5	-8.5	TS-POR	-0.8	-0.4
Los500-Los500	1	0.48	UCS-UW	0.15	0.5	Los500-UW	-0.005	-0.029
UW-UW	0.0057	0.0026	UCS-WA	-3	-2.5	Los500-WA	0.17	0.14
WA-WA	0.9	0.09	UCS-POR	-7	-3.5	Los500-POR	0.40	0.20
POR-POR	0.92	0.38	TS-Los500	-1.75	-0.90	UW-WA	-0.02	-0.01
WA-POR	0.4	0.15	TS-UW	0.005	0.065	UW-POR	-0.04	-0.02

*C0 and C1 are nugget effect and contribution, respectively.

Table 5. Comparison of cross-validation results of MSC-kriging, IC-kriging, and Cokriging.

	Correlation coefficient between estimated and observed values						Mean error					
Variables	**UCS (MPa)**	**TS (MPa)**	**UW (g/cm^3)**	**WA (%)**	**P (%)**	**Los500**	**UCS (MPa)**	**TS (MPa)**	**UW (g/cm^3)**	**WA (%)**	**P (%)**	**Los500**
MSC-kriging	0.56	0.57	0.45	0.31	0.32	0.58	0.17	0.045	0.0009	-0.0039	-0.008	-0.0095
IC-kriging	0.53	0.51	0.41	0.27	0.22	0.52	0.64	0.074	0.0005	-0.0005	-0.001	-0.03
Cokriging	0.52	0.45	0.39	0.16	0.18	0.48	0.64	0.06	0.0016	-0.0032	-0.012	-0.02

	Minimum error value						Maximum error value					
Variables	**UCS (MPa)**	**TS (MPa)**	**UW (g/cm^3)**	**WA (%)**	**P (%)**	**Los500**	**UCS (MPa)**	**TS (MPa)**	**UW (g/cm^3)**	**WA (%)**	**P (%)**	**Los500**
MSC-kriging	-60.97	-5.29	-0.138	-0.38	-2.27	-2.42	53.85	4.28	0.42	0.32	1.17	2.74
IC-kriging	-42.6	-4.81	-0.129	-0.41	-2.42	-2.65	44.19	4.94	0.42	0.29	1.24	2.83
Cokriging	-40.62	-3.95	-0.188	-0.35	-2.20	-2.89	62.2	5.15	0.42	0.26	1.22	2.22

4.3. Evaluation of Cubuk andesite quarry

The MSC factors have negligible cross-variogram values at all lag distances (Figure 6) so that they can be estimated separately using the kriging method. 1544 blocks of equal sizes (5 m × 5 m) were considered in the prediction process. The sample dimension is very small in comparison to that of the blocks, and the estimation of attributes at a specific scale different from the sample support would cause support relative problems. In order to solve this problem, the blocks can be divided into the supports of finite size [52-53]. In this way, an enormous number of sub-divisions would be emerged, which may be intense to solve. Isaaks and Srivastava [9] and Journel and Huijbregts [10], respectively, have suggested 16 and 36 as the number of sub-divisions in a 2D space. Thus in this work, we divided each 2D block into 25 sub-divisions and estimated the attributes of interest at each sub-division's center, and then took their average to obtain the block estimations. After predicting the MSC block estimations, the results obtained were back-transformed into the data space and compared to those of the IC-kriging. A comparison was made by checking the summary statistics of the results including the minimum, maximum, and mean values (Table 1), and also drawing the quantile plots of the estimations against those of the data (Figure 10). The quantile plots indicate a better reproduction of the data distributions through the MSC-kriging.

After back-transformation, the TS 10835 standards of Table 1 were used to classify andesite as the facing and building stones. Figure 11 shows the interpolated maps of variables. Based upon the estimated values for variables, the blocks were classified as exploitable and non-exploitable in Figure 12. In this figure, the blocks that were classified by both methods as exploitable are demonstrated in blue. The block percentages classified as exploitable by the MSC-kriging and IC-kriging methods are %58 and %63, respectively. Most of the northwestern blocks that were determined by the IC-kriging as unexploitable were established as extractable by the MSC-kriging. At the eastern part of the quarry, the number of extractable blocks obtained from the IC-kriging is more than that of the MSC-kriging.

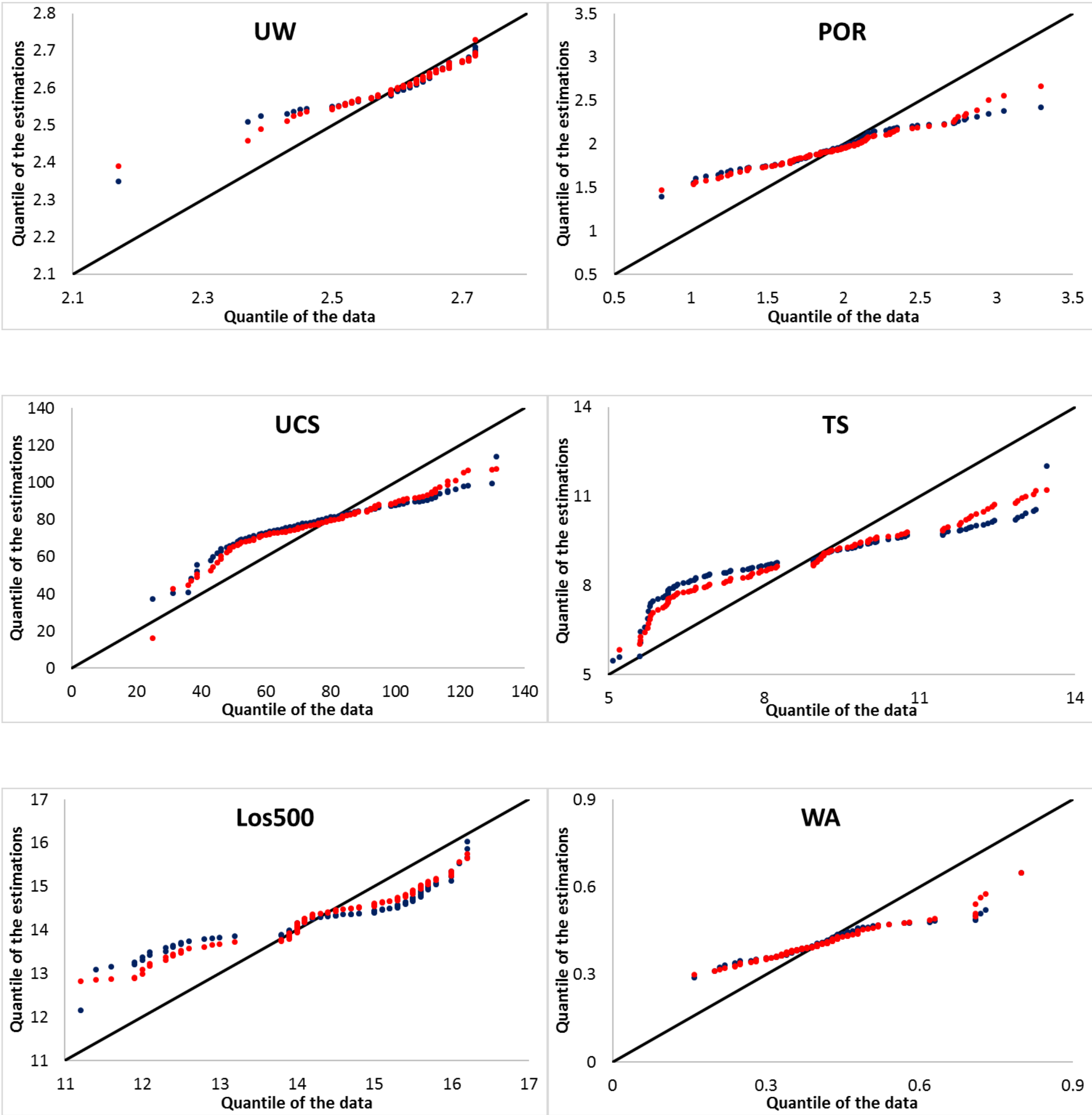


Figure 10. Quantiles of data against those of MS-kriging (red) and IC-kriging (blue) results.

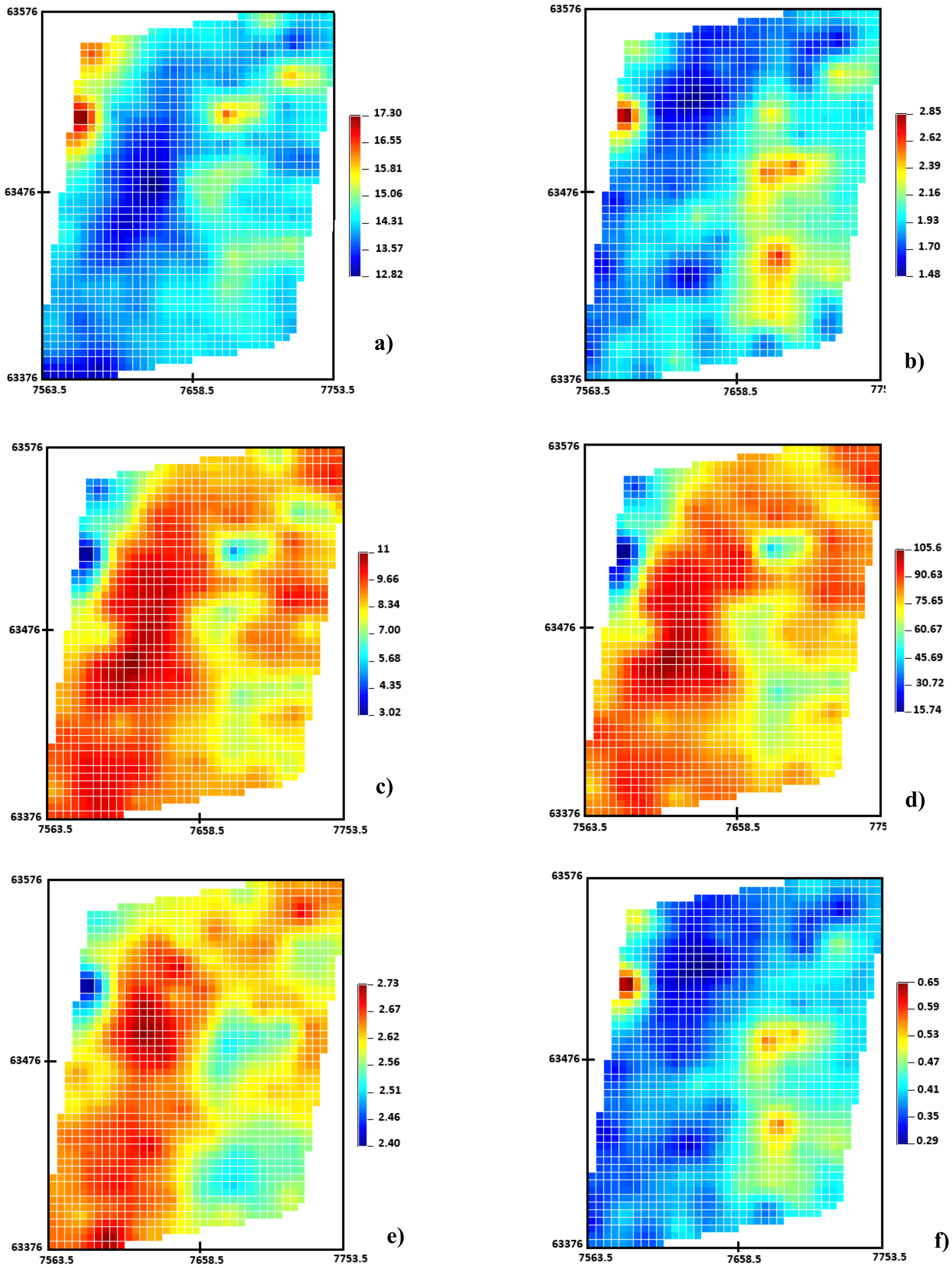
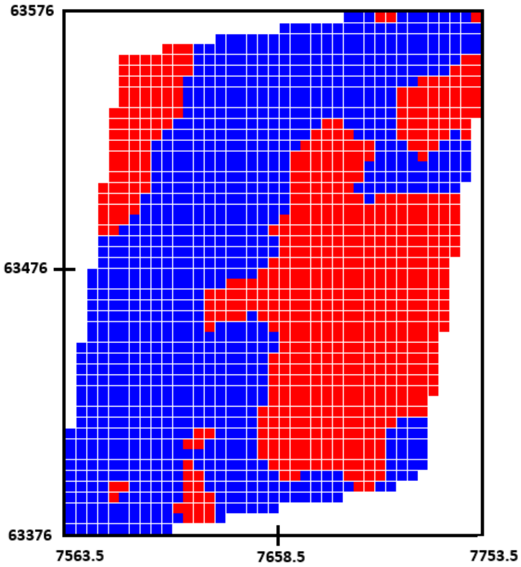
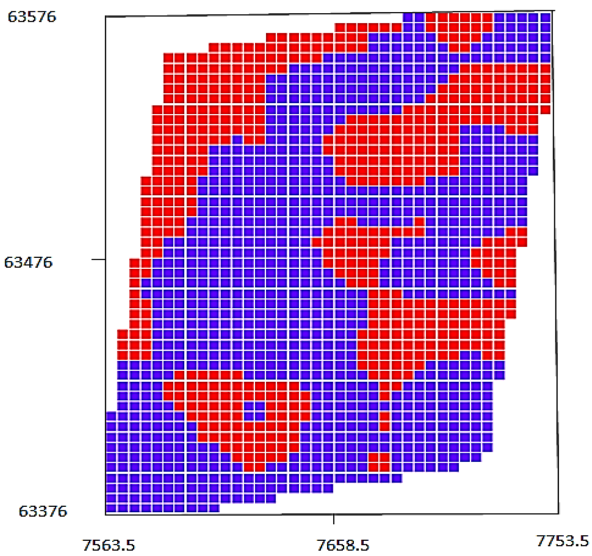


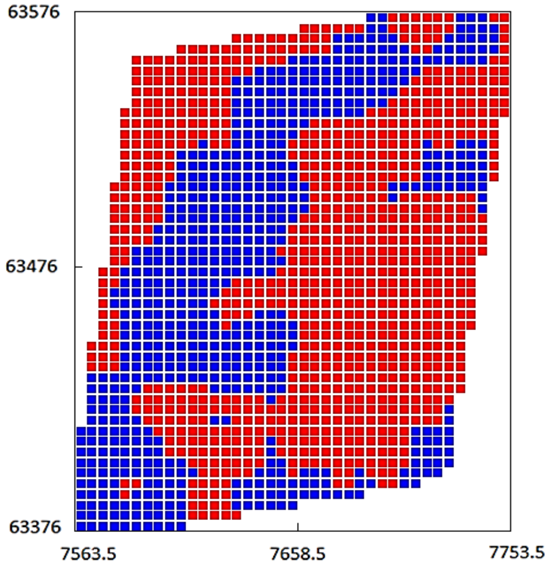
Figure 11. MSC-kriging estimation maps for a) Los500, b) P (%), c) TS (MPa), d) UCS (MPa), e) UW (g/cm^3), f) WA (%).



a)



b)



c)

Figure 12. Exploitable blocks (blue colored) obtained using a) MSCK, b) ICK, and c) both methods considered together.

5. Conclusions

Multivariate geostatistical estimation can be performed using the conventional cokriging methods or kriging of the orthogonal factors. The kriging estimation of the orthogonalized factors eliminates the tedious procedure of auto/cross-variogram analysis, and avoids the probability of facing an unsolvable system of equations.

In this work, at several lag distances, approximately orthogonal factors were produced through the Minimum Spatial Cross-correlation (MSC), and then the factors were estimated using the kriging method to be used in the selective extraction of andesite blocks. By running the cross-validation test, the method's efficiency was proved against the Independent Components kriging (IC-kriging) and cokriging.

The MSC factors have negligible cross-variograms that lie in a tighter interval than those of the IC factors so that the purpose of generating almost orthogonal factors was achieved. The MSC-kriging, IC-kriging, and cokriging cross-validation results reasonably reproduce variables' means. Among these methods, the MSC-kriging demonstrates the lowest bias due to possessing the lowest ME's. The MSC-kriging outperforms IC-kriging and cokriging for having the highest correlation coefficients of the estimated and observed values.

The proportion of the exploitable blocks to the total number of blocks were obtained to be 58% and 63% for the MSC-kriging and the IC-kriging, respectively. These blocks are mainly located around an imaginary N30°E oriented line, which intercepts the southwest corner of the studied area. The MSC-kriging is practical and user-friendly, and shows advantages over the cokriging and the IC-kriging; thus we recommend it for multivariate studies.

References

[1]. Taboada, J., Vaamonde, A., Saavedra, A. and Arguelles, A. (1998). Quality index for ornamental slate deposits. Engineering Geology. 50 (1-2): 203-210.

[2]. Taboada, J., Rivas, T., Saavedra, A., Ordóñez, C., Bastante, F. and Giráldez, E. (2008). Evaluation of the reserve of a granite deposit by fuzzy kriging. Engineering Geology. 99 (1-2): 23-30.

[3]. Tutmez, B. and Tercan, A.E. (2007). Spatial estimation of some mechanical properties of rocks by fuzzy modelling. Computers and Geotechnics. 34 (1): 10-18.

[4]. Ayalew, L., Reik, G. and Busch, W. (2002). Characterizing weathered rock masses—a geostatistical approach. International Journal of Rock Mechanics and Mining Sciences. 39(1): 105–114.

[5]. Exadaktylos, G. and Stavropoulou, M. (2008). A specific upscaling theory of rock mass parameters exhibiting spatial variability: Analytical relations and computational scheme. International Journal of Rock Mechanics and Mining Sciences. 45 (7): 1102-1125.

[6]. Saavedra, A., Ordonez, C., Taboada, J. and Armesto, J. (2010). Compositional kriging applied to the reserve estimation of a granite deposit. Dyna. 77 (161): 53-60.

[7]. Taboada, J., Saavedra, Á., Iglesias, C. and Giráldez, E. (2013). Estimating Quartz Reserves Using Compositional Kriging. Abstract and Applied Analysis. 2013: 1-6.

[8]. Walvoort, D.J.J. and de Gruijter, J.J. (2001). Compositional Kriging: A Spatial Interpolation Method for Compositional Data. Mathematical Geology. 33 (8): 951-966.

[9]. Huang, D. and Wang, G. (2015). Stochastic Simulation of Regionalized Ground Motions using Wavelet Packet and Cokriging Analysis. Earthquake Engineering and Structural Dynamics. 44: 775-794.

[10]. Isaaks, E.H. and Srivastava, R.M. (1989). An introduction to applied geostatistics. Oxford University Press. New York. 561 P.

[11]. Journel, A.G. and Huijbregts, C.H.J. (1978). Mining geostatistics. Academic Press. London. 600 P.

[12]. Taboada, J., Vaamonde, A. and Saavedra, A. (1999). Evaluation of the quality of a granite quarry. Engineering Geology. 53: 1-11.

[13]. Martínez-Martínez, J., Benavente, D., Ordóñez, S. and García-del-Cura, M.Á. (2008). Multivariate statistical techniques for evaluating the effects of brecciated rock fabric on ultrasonic wave propagation. International Journal of Rock Mechanics and Mining Sciences. 45 (4): 609-620

[14]. Tercan, A.E. (1993). Nonparametric methods for estimating conditional distributions and local confidence intervals. Unpublished Ph.D. dissertation. The Department of Mining and Mineral Engineering. The University of Leeds. 156 P.

[15]. Lajaunie, C.H. (1992). Comment on “indicator principal component kriging”. Mathematical Geology. 24 (5): 555-561.

[16]. Wackernagel, H. (1995). Multivariate geostatistics: An introduction with applications. Springer-Verlag. Berlin. 256 P.

[17]. Tercan, A.E. (1999). Importance of orthogonalization algorithm in modeling conditional distributions by orthogonal transformed indicator methods. Mathematical Geology. 31 (2):155-173.

[18]. Leuangthong, O. and Deutsch, C.V. (2003). Stepwise Conditional Transformation for Simulation of Multiple Variables. Mathematical Geology. 35 (2): 155-173.

[19]. Tercan, A.E. and Özçelik, Y. (2000). Geostatistical evaluation of dimension-stone quarries. Engineering Geology. 58 (1): 25-33.

[20]. Desbarats, A.J. (2001). Geostatistical Modeling of Regionalized Grain-Size Distributions Using Min/Max Autocorrelation Factors. geoENV III - Geostatistics for Environmental Applications. pp. 441-452.

[21]. Vargas-Guzmán, J.A. and Dimitrakopoulos, R. (2003). Computational properties of min/max autocorrelation factors. Computers & Geosciences. 29 (6): 715-723.

[22]. Rondon, O. (2011). Teaching Aid: Minimum/Maximum Autocorrelation Factors for Joint Simulation of Attributes. Mathematical Geosciences. 44 (4): 469-504.

[23]. Sohrabian, B. and Ozcelik, Y. (2012). Joint simulation of a building stone deposit using minimum/maximum autocorrelation factors. Construction and Building Materials. 37: 257-268.

[24]. Shakiba, S., Asghari, O., Keshavarz Faraj Khah, N., Sarallah Zabihi, S. and Tokhmechi, B. (2015). Fault and non-fault areas detection based on seismic data through min/max autocorrelation factors and fuzzy classification. Journal of Natural Gas Science and Engineering. 26: 51-60.

[25]. De Freitas Silva, M. and Dimitrakopoulos, R. (2016). Simulation of weathered profiles coupled with multivariate block-support simulation of the Puma

nickel laterite deposit, Brazil. Engineering Geology. 215: 108-121.

[26]. Ruessink, B.G., van Enckevort, I.M.J. and Kuriyama, Y. (2004). Non-linear principal component analysis of nearshore bathymetry. Marine Geology. 203 (1-2): 185-197.

[27]. Nielsen, A.A. (2011). Kernel Maximum Autocorrelation Factor and Minimum Noise Fraction Transformations. IEEE Transactions on Image Processing. 20 (3): 612-624.

[28]. Liu, S., Luo, M. and Zhang, G. (2013). Face recognition based on symmetrical kernel principal component analysis. Journal of Computer Applications. 32 (5): 1404-1406.

[29]. Musafer, G.N. and Thompson, M.H. (2017). Non-linear optimal multivariate spatial design using spatial vine copulas. Stochastic Environmental Research and Risk Assessment. 31 (2): 551-570.

[30]. Sohrabian, B. and Ozcelik, Y. (2012). Determination of exploitable blocks in an andesite quarry using independent component kriging. International Journal of Rock Mechanics and Mining Sciences. 55: 71-79.

[31]. Tercan, A.E. and Sohrabian, B. (2013). Multivariate geostatistical simulation of coal quality data by independent components. International Journal of Coal Geology. 112: 53-66.

[32]. Boluwade, A. and Madramootoo, C.A. (2015). Geostatistical independent simulation of spatially correlated soil variables. Computers & Geosciences. 85: 3-15.

[33]. Minniakhmetov, I. and Dimitrakopoulos, R. (2017). Joint High-Order Simulation of Spatially Correlated Variables Using High-Order Spatial Statistics. Mathematical Geosciences. 49 (1): 39-66.

[34]. Goovaerts, P. (1993). Spatial orthogonality of the principal components computed from coregionalized variables. Mathematical Geology. 25 (3): 281-302.

[35]. Xie, T. and Myers, D.E. (1995). Fitting matrix-valued variogram models by simultaneous diagonalization (Part I: Theory). Mathematical Geology. 27 (7): 867-875.

[36]. Cardoso, J.F. and Souloumiac, A. (1993). Blind beamforming for non-gaussian signals. IEE Proceedings F Radar and Signal Processing. 140 (6): 362-370.

[37]. Cardoso, J.F. and Souloumiac, A. (1996). Jacobi Angles for Simultaneous Diagonalization. SIAM Journal on Matrix Analysis and Applications. 17 (1): 161-164.

[38]. Quintana, Y. and Rodríguez, J.M. (2014). Measurable diagonalization of positive definite matrices. Journal of Approximation Theory. 185: 91-97.

[39]. Joho, M. and Rahbar, K. (2002). Joint diagonalization of correlation matrices by using Newton methods with application to blind signal separation. Sensor Array and Multichannel Signal Processing Workshop Proceedings.

[40]. Joho, M. (2008). Newton Method for Joint Approximate Diagonalization of Positive Definite Hermitian Matrices. SIAM Journal on Matrix Analysis and Applications. 30 (3): 1205-1218.

[41]. Mueller, U.A. and Ferreira, J. (2012). The U-WEDGE Transformation Method for Multivariate Geostatistical Simulation. Mathematical Geosciences. 44 (4): 427-448.

[42]. Tichavsky, P. and Yeredor, A. (2009). Fast Approximate Joint Diagonalization Incorporating Weight Matrices. IEEE Transactions on Signal Processing. 57 (3): 878-891.

[43]. Sohrabian, B. and Tercan, A.E. (2014). Introducing minimum spatial cross-correlation kriging as a new estimation method of heavy metal contents in soils. Geoderma. 226-227: 317-331.

[44]. Hyvarinen, A., Karhunen, J. and Oja, E. (2001). Independent component analysis. JohnWiley & Sons. 481 P.

[45]. Sohrabian, B. and Tercan, A.E. (2014). Multivariate geostatistical simulation by minimising spatial cross-correlation. Comptes Rendus Geoscience. 346 (3-4): 64-74.

[46]. Little, R.J.A. and Rubin, D.B. (2002). Statistical analysis with missing data. 2nd ed. John Wiley & Sons. Hoboken.

[47]. Ozcelik, Y. (1999). Investigation of the working conditions of diamond wire cutting machines in marble industry. Ph.D Thesis. Hacettepe University. Ankara (In Turkish).

[48]. ISRM. (1981). Rock Characterization, Testing and Monitoring. In: Brown, E.T. (Ed). ISRM (International Society for Rock Mechanics) suggested methods. Pergamon. Oxford.

[49]. TSE, (1993). Andesite-Used as Facing and Building Stone (TS 10835). TSE Publication.

[50]. Goovaerts, P. (1992). Factorial kriging analysis: a useful tool for exploring the structure of multivariate spatial soil information. Journal of Soil Science. 43 (4): 597-619.

[51]. Bourennane, H., Nicoullaud, B., Couturier, A. and King, D. (2004). Exploring the Spatial Relationships Between Some Soil Properties and Wheat Yields in Two Soil Types. Precision Agriculture. 5 (5): 521-536.

[52]. Olea, R.A. (1999). Geostatistics for Engineers and Earth Scientists. Springer.

[53]. Sohrabian, B., Ozcelik, Y. and Hasanpour, R. (2017). Estimating major elemental oxides of an andesite quarry using compositional kriging. International Journal of Mining, Reclamation and Environment. 31 (7): 475-487.

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چکیده:

خصوصیات کیفی آندزیت از قبیل وزن مخصوص، مقاومت فشاری تک‌محوره، مقاومت در برابر سایدگی در ۵۰۰ دور چرخش دستگاه لس‌آنجلس و ... برای تعیین بلوک‌های قابل استخراج و توالی بهره‌برداری آن‌ها مورد نیاز است؛ اما تعداد نمونه‌هایی که برداشت شده و مورد آنالیز قرار می‌گیرد، محدود است؛ بنابراین، در نقاط اندازه‌گیری نشده، خواص کیفی سنگ باید تخمین زده شود. کوکریجینگ، روش سنتی تخمین متغیرهای با ارتباط متقابل فضایی بوده است. هرچند، این روش ممکن است با ماتریس‌های لاینحل در الگوریتم‌اش مواجه شود. جایگزینی برای کوکریجینگ، تبدیل متغیرها به مؤلفه‌های متعامد و تخمین هر یک از این مؤلفه‌ها به وسیله کریجینگ است. آنالیز مؤلفه‌های مستقل از جمله روش‌هایی است که می‌تواند در تولید این مؤلفه‌ها مورد استفاده قرار گیرد؛ اما این روش فقط در فاصله گام برابر با صفر قابل اجرا بوده؛ لذا استفاده از روش‌هایی که در آن‌ها پارامتر فاصله لحاظ شده است، سودمند خواهد بود. در این پژوهش، روش حداقل همبستگی متقابل فضایی برای تبدیل شش ویژگی مکانیکی سنگ معدن آندزیت جوبوک واقع در آنکارا، ترکیه به مؤلفه‌های تقریباً متعامد در چند فاصله گام، مورد استفاده قرار گرفته است. در ۱۵۴۴ نقطه واقع بر یک شبکه منظم مربعی (با ابعاد ۵ متر)، مؤلفه‌های مورد اشاره با استفاده از روش کریجینگ مورد تخمین قرار گرفته و نتایج تخمینی به فضای داده‌های اصلی برگردانده شد. توسط اعتبار سنجی متقابل، کارآمدی کریجینگ با استفاده از مؤلفه‌های با حداقل همبستگی متقابل فضایی در مقابل روش‌های کوکریجینگ و کریجینگ مؤلفه‌های مستقل تست شد. با وجود نااریب بودن تخمینگرهای یاد شده، کریجینگ با استفاده از مؤلفه‌های با حداقل همبستگی متقابل فضایی، به دلیل دارا بودن کمترین مقدار میانگین خطا و بیشترین ضریب همبستگی میان مقادیر تخمینی و اندازه‌گیری شده، عملکرد بهتری نسبت به دو روش دیگر داشته است. نتایج تخمینی برای تعیین سود آورترین بلوک‌ها و بهترین امتداد برای استخراج، مورد استفاده قرار گرفته است.

کلمات کلیدی: همبستگی متقابل فضایی، کریجینگ، واریوگرام، سنگ ساختمانی.