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Investigation of linear and non-linear estimation methods in highly-skewed gold distribution

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Abstract
The purpose of this work is to compare the linear and non-linear kriging methods in the mineral resource estimation of the Qolqoleh gold deposit in Saqqez, NW Iran. Considering the fact that the gold distribution is positively skewed and has a significant difference with a normal curve, a geostatistical estimation is complicated in these cases. Linear kriging, as a resource estimation method, can be problematic and gives an unrealistic gold grade. In order to check and correct the errors in the linear methods, the non-linear kriging method has been deployed. One of the applicant's non-linear estimation methods is Indicator Kriging (IK). The IK method converts grade values into binary units of 0 and 1 using multiple thresholds that can be selected by the number-size (N-S) fractal model. The N-S model identifies important and critical thresholds based on the grade distribution. In IK, the Multiple Indicator Kriging (Multiple IK) and Median Indicator Kriging (Median IK) methods could be involved due to the number of indicator thresholds. IK is not sensitive to high values. Here, we make a comparison between Median IK and Multiple IK as well as those with ordinary kriging (OK), which is a linear kriging method. Overall, we conclude that all of these methods are suitable for resource estimation among these methods, although the IK method is better for estimation in different categories of gold grades.

Keywords: *Median Indicator Kriging, Multiple Indicator Kriging, Mineral Resource Estimation, Ordinary Kriging, Qolqoleh Gold Deposit.*

1. Introduction

Geostatistics is a branch of science that deals with the numerical prediction of natural variables in space or space-time such as ore grade, depth or thickness of a mineral layer. In geostatistical methods, the variables in an area have both spatial and random properties based on the idea of regionalized variables [1].

Kriging is a general approach that is applied to a wide range of estimation methods, in one point or one block; this depends on the lowest estimation error using the least squares approaches. Kriging is called the best unbiased linear estimator, and is an estimation method based on the weighted moving average [2]. The mean-based linear kriging methods are known or unknown, and can be classified as simple kriging (SK) and ordinary kriging (OK).

OK can be used to estimate the point and block grades, and is the most common geostatistical method for resource estimation. In this case study, the Qolqoleh gold deposit, the gold values are highly skewed, and the outliers and high grades have a huge impact on the amount of block grades that may differ from the actual grade. In order to provide satisfactory results, mineral resource analysts should adapt mathematical methods [3].

One of the main disadvantages of OK is the smoothing effect on the estimated results, which

usually indicates estimates lower than the actual grade value [4]. Also to calculate the estimation, the outlier should be capped and have a more control over the distribution [5]. The presence of outlier values introduces a considerable variability of the statistical parameters and calculation of the experimental variogram [6].
One of the alternative methods may be Indicator Kriging (IK). IK has been introduced by Journal [7] to estimate the local uncertainty through the inference process of a local cumulative distribution function (CDF) [8]. IK can do both tasks; in fact, a variable can be used without deleting high-quality data or making risk assessments [9]. Based on the number of thresholds, IK is categorized according to Median Indicator Kriging (Median IK) and Multiple Indicator Kriging (Multiple IK) [10]. For the calculation of Multiple IK, cut-offs usually get base CDF, which requires many cut-offs to cover all grades. Rahimi et al. [11] have introduced a new method to reduce the completion volume for multiple IK. They used the Number Size (N-S) model to select the thresholds that do not use all the unnecessary thresholds, and only the economic thresholds are selected. In the case of the N-S model, the fractal methods are based upon the geochemical distribution of grades and distribution of the corresponding samples [12, 13].
In the present work, we compared the accuracy of the three methods OK, Multiple IK, and Median IK with respect to the resource estimate in the Qolqoleh gold deposit. For calculation of Multiple IK, four thresholds were obtained using the N-S model.

2. Methods
2.1. Linear kriging
In linear interpolation, the variable of interest is approximated by a parametric function whose form is postulated in advance either explicitly (example. g., polynomials) or implicitly (example. g., a minimal curvature condition). The parameters are selected to optimize the best fit to the data points. Once the approximation function is determined, it is easy to define where it is required. Linear kriging prepares an interpolation function based on a data-derived variogram or covariance model [14]. The main forms of linear kriging include simple kriging (SK), ordinary kriging (OK), universal kriging (UK), and kriging with external drift (KED) [14].

2.1.1. Ordinary Kriging (OK)
In this method, the mean value is unknown and constant m(x) = m. OK operates under simple assumptions of stationarity, and requires no knowledge of the average [14]. Assuming that the region is stationary second order, the OK method implicitly evaluates the mean in a moving neighborhood, and thus minimizing the estimation variance [15].

2.2. Non-linear kriging
In fact, all the non-linear kriging algorithms are just linear kriging (OK or SK) applied to certain non-linear transformations of real data [16]. If the predicted grade distributions are nearly normal, a linear estimator is ideal [17]. However, if the variable distribution is skewed, the linear kriging methods are also disabled.
There are many non-linear kriging methods for estimation. These methods include Disjunctive Kriging (DK), IK (multiple IK and Median IK), Probability Kriging (PK), Multi-Gaussian Kriging (MGK), Uniform Conditioning (UC), and Residual Indicator Kriging (RIK) [18].

2.2.1. Indicator Kriging (IK)
This kriging method is an estimation method based on the linear kriging theory, which is non-Gaussian and is transformed into an indicator value that is a non-linear distribution [19]. For this method, which is based upon one or more specified cut-off grades, all data is converted to 1 (lower bound) and 0 (upper bound), as Equation (1).

$$I(z \leq p) = \begin{cases} 1 & if\ z \leq p \\ 0 & if\ z > p \end{cases} \tag{1}$$

and then all kriging steps (SK or OK) would be performed based on the binarized data (values 0 and 1). By interpolation of these values [0, 1], a probability function is obtained. The local error variance in IK is minimized [20]. Natural processes have a variable dependence on different quantiles or percentiles of the distribution. Instead of calculating a single variogram model representing a complete distribution, we should calculate several variogram models for different cut-offs described by indicators.
IK rates the CDF at a location, where it was not sampled for a given threshold z_k [21]. The values for the estimated indicators will be between 0 and 1.
In this article, the indicator data is estimated using simple kriging (SK).

2.2.2. Multiple IK and Median IK

In Multiple IK, the IK process is repeated for a series of K cut-off values in z_k = 1 ..., K, which discretizes the variability interval of the continuous attribute z. It should be noted that multiple IK is used in some resources as full indicator kriging (FIK) [11]. In Multiple IK, each indicator threshold requires an indicator variogram. The N-S model and CDF can be used to select cut-offs for converting the real data to the indicator data. The N-S model selects fewer thresholds, and does not select unnecessary thresholds. Median IK is an approximation of multiple IK, hypothesizing that the spatial continuity of multi-threshold indicators can be approximated by a single structural function that is for zc = ḿ, where ḿ is the median of the grade distribution [22]. Multiple IK has a greater complexity in various thresholds, in which the variogram and estimation parameters must be calculated. However, Median IK uses a single indicator threshold to obtain the variogram and estimate.

In this work, we examined these two methods on the gold deposit and evaluated the accuracy of the results of these methods.

2.3. Number–Size Fractal Model

The Number-Size (N-S) model is used to describe the distribution of a grade in order to evaluate the behavior of the data without a prior knowledge. This model represents a relationship between some specific features and the cumulative number of samples with such features that gold grade is desired in this research work [11]. This model can be represented by the following equations (Eqs. 2 and 3) [13, 23].

$$N(\geq c) \propto c^{-D} \tag{2}$$

also it can be re-written as:

$$log[N(\geq c)] = C - Dlog(\mathrm{c}) \tag{3}$$

in which c represents the concentration of an element, N (≥c) denotes the cumulative number of the samples with an elemental concentration equal to or greater than c, C is a constant, and D is the power scale (or fractal dimension) of the concentration distribution of that element. One of the most common applications of the N-S model consists of geochemical exploration for anomalous cut-off grades.

3. Case study

3.1. Qolqoleh deposit

The Qolqoleh gold deposit (Figure 1) is located SE of the Piranshahr-Sardasht-Saqqez (PSSZ) region, NW of the Sanandaj-Sirjan metamorphic zone (SSZ), 30 km SW of Saqqez (Kurdistan Province). The Qolqoleh gold deposit is located in an area of 10 km^2. This deposit was discovered in the 1990s through a geochemical exploration by geological surveys of Iran [24].

PSSZ is comprised of a NE-trending gold field belt hosted. SSZ of western Iran is a metamorphic belt (green shale-amphibolite) that was raised during the Late Cretaceous between the Afro-Arab continent and the Iranian microcontinent [25]. The geochemical, structural, and geological manifestations of the Saqqez region clearly demonstrate its strong potential for different types of gold mineralization, particularly in the eastern, NW, and SW parts of the region [26]. The gold deposits of Qolqoleh, Kervian, and Ghabaqloujeh, and the gold events of Hamzehqaranin and Kasnazan are in an NE-SW shear zone and a general dip of 30-70° NW called the Qolqoleh-Kasnazan shear zone [27]. The 20-km long Qolqoleh-Kasnazan shear zone is associated with a sinistral straight-sided strike fault with an inverse component that stimulated the metamorphic rocks attributed to percamberian in the vicinity of the Cretaceous metamorphic unit [27]. In all situations where gold mineralization is comprised within this shear zone, granitoid masses that have been injected along with the tectonic zone into the shear zone host a significant portion of the gold mineralization [11].

The Qolqoleh gold deposit, part of the SSZ, was affected by organic phases in the metamorphic greenschist facies and by deformation of several stages. Studies show that mineralization of the Qolqoleh deposit occurs both in the ductile and brittle states simultaneously and in the second deformation stage at the junction of two units of chlorite and granitoid. What distinguishes the two types of ductile and brittle deformation in the investigation of the core drilled in the Qolqoleh deposit shows the gold grade in the ores with a ductile deformation occurring in the hosts of chlorite Schist. Sulfide minerals are relatively higher than gold ore with a fragile deformation that extends the granitoid host [24, 27].

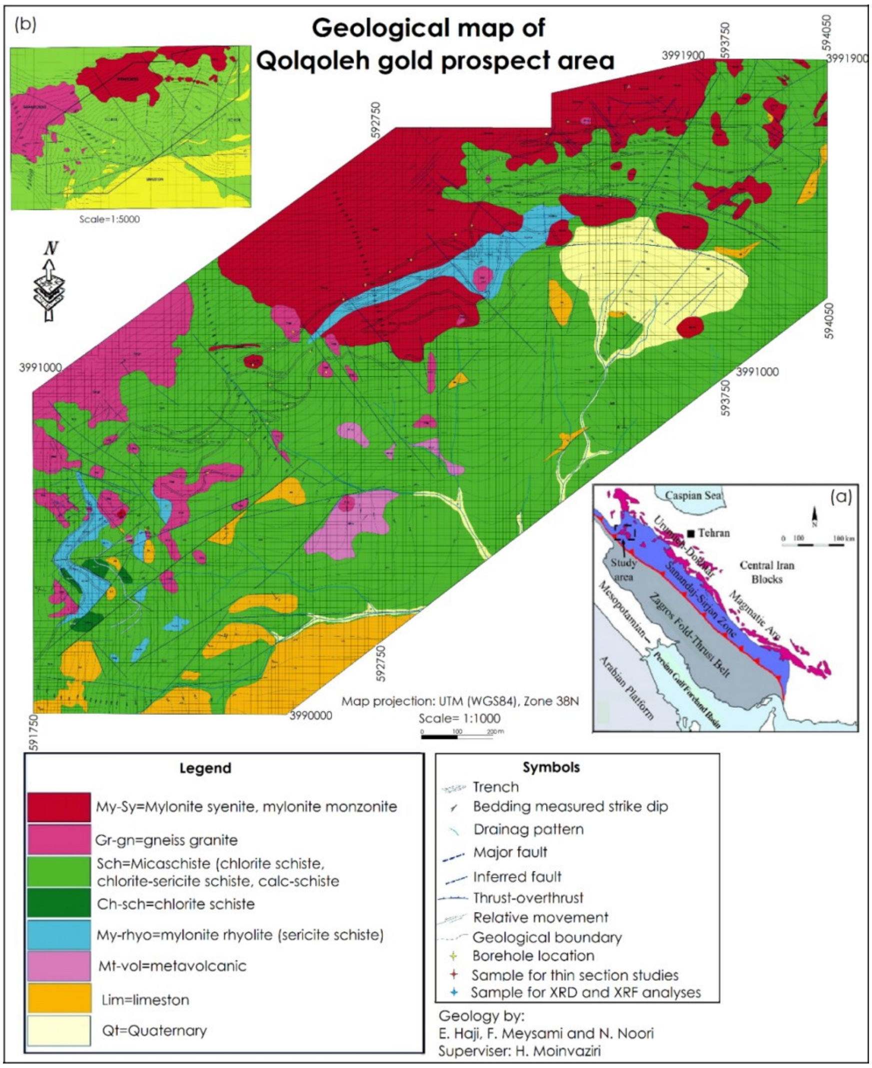

Figure 1. (a) Schematic tectonic framework of the Sanandaj–Sirjan metamorphic belt and location of the studied area (b) Geological map of the Qolqoleh gold prospect area.

4. Results and discussion

4.1. Descriptive statistics

The data used in this work was collected from 101 drill holes and 15 trenches from the Qolqoleh gold deposit (Figure 2). Before applying the estimation method, we first need to check the accuracy of the dataset. The first step is to manage the data and recognize the outliers. Some estimation methods are sensitive to outliers and have wrong results. The determination of outliers has a very important position. One method of identifying outliers is the use of graphical diagrams, e.g. cumulative frequency chart. In this graph, the data that is far from the original distribution curve is considered as an outlier.

After evaluating the gold grade in the studied area, the data above 6.345 (g/t) is considered as outliers. These outliers were corrected by converting them to the amount of closures (i.e. 6,345). The cumulative frequency chart is shown in Figure 3.

As it can be seen in the middle of Figure 2, the data is partially de-clustered. Therefore, the dataset was placed on a regular grid by de-clustering. The de-clustering techniques assign a weighting to each data value as a function of the proximity of the surrounding data w_i, i = 1, ..., n [4, 11]. In order to find the best cluster size, the data is de-clustered into multiple sizes and the best size for de-clustering, where the mean of the data is minimized. This issue is illustrated in Figure 4.

After de-clustering and removing the outliers, the final data is used for analysis. The histogram of the gold content in 5366 samples shows right-handed skewness (Figure 5). This skewness is very common in the distribution of gold deposits. It shows that the distribution of gold has many differences from the normal distribution. The statistical parameters of this distribution are listed in Table 1.

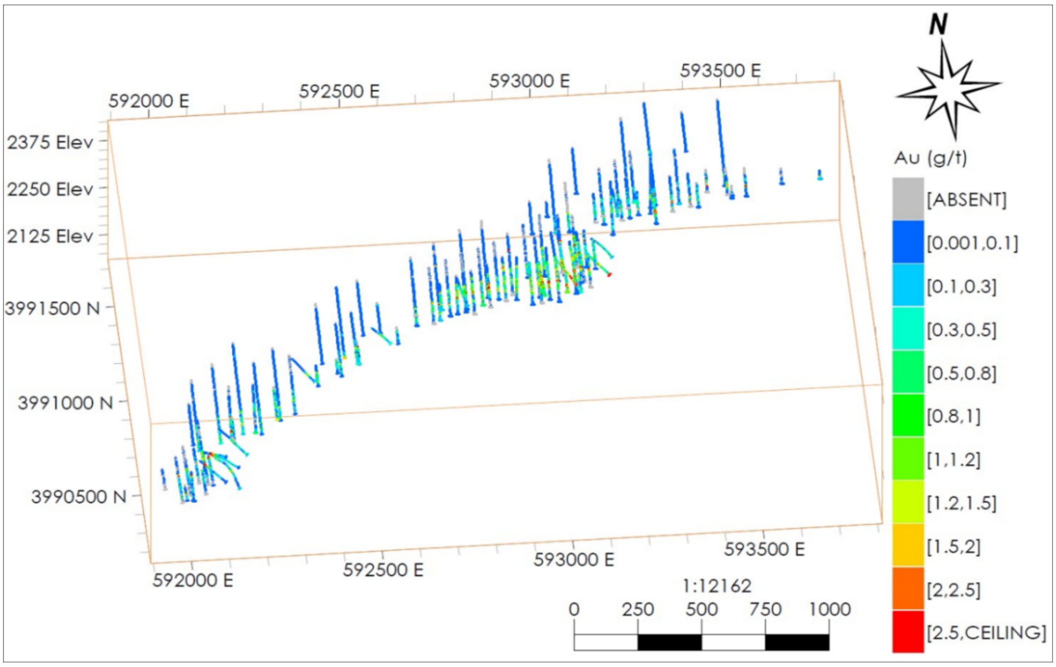

Figure 2. A 3D view of boreholes and trenches in the Qolqoleh gold deposit.

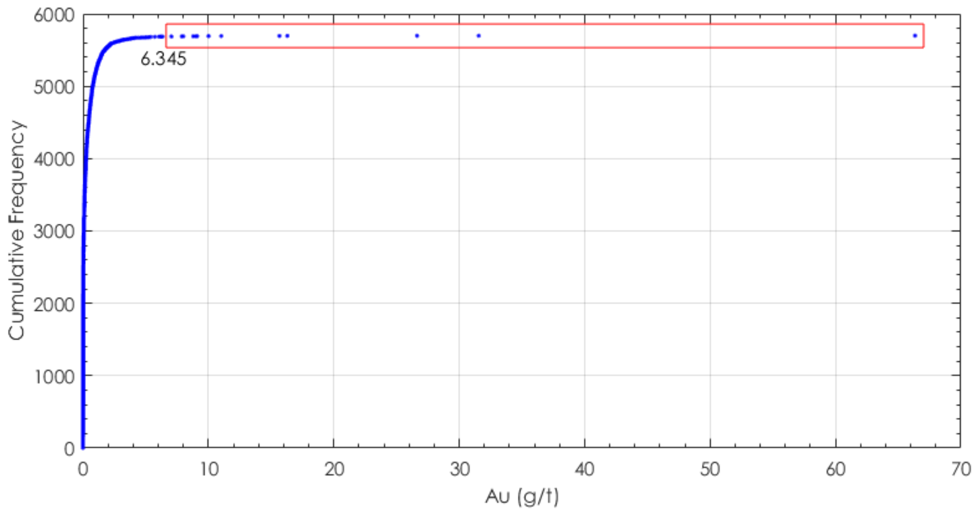

Figure 3. Cumulative frequency chart for determination outliers.

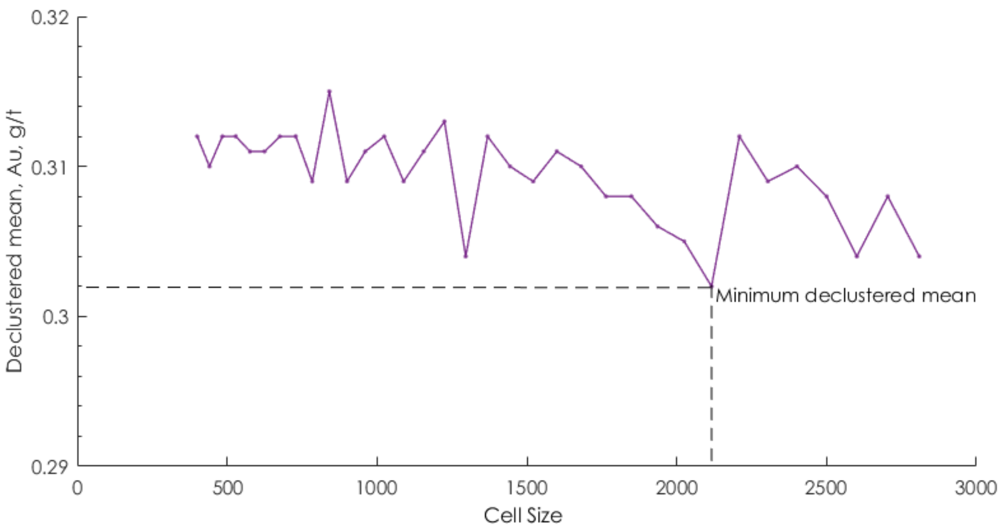

Figure 4. Plot of de-clustered mean versus cell size.

Table 1. Statistical value of Au.

Nb of data	Minimum (ppm)	Maximum (ppm)	Median (ppm)	Mean (ppm)	Variance $(ppm)^2$	Std. Deviation (ppm)	Coef. of variation	Skewness	Kurtosis
5366	0.001	6.345	0.045	0.302	0.389	0.624	2.065	4.551	29.349

4.2. Description of N–S Model

For estimation with IK, cut-offs were selected using the N-S model. As mentioned in this procedure, only economic and essential thresholds are selected, and the volume of inference is reduced compared to CDF. The resulting thresholds are all shown in the logarithmic curve of the N-S model (Figure 6). As one can see, four threshold values are selected via this method.

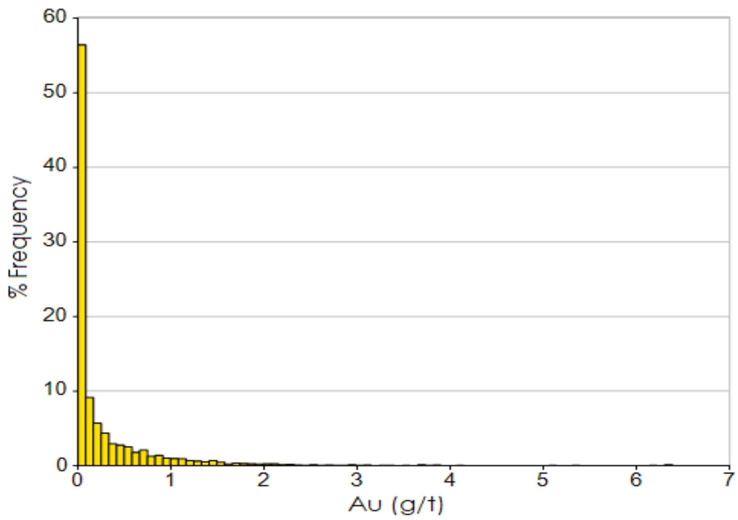

Figure 5. Histogram of gold data from Qolqoleh gold deposit.

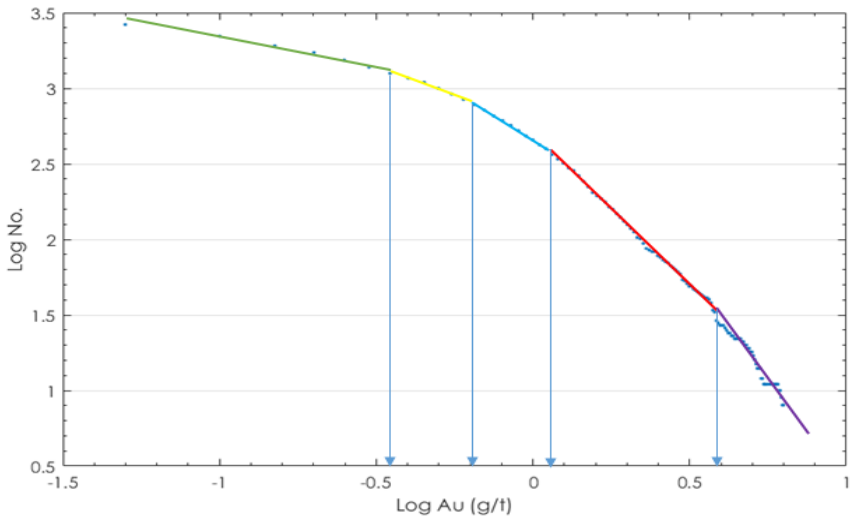

Figure 6. Log-log plots for N–S model of gold grades in Qolqoleh gold deposit.

4.3. Structural analysis

The structural analysis reflects the variability of regional variables. The first step in the geostatistical estimation is to plot an experimental variogram [19]. Variograms show an important feature of geological genesis as they provide an analytical tool for quantifying the anisotropy and domain of the underlying formation process [28].

The variogram model allows a linear estimation using a software, and its results are presented in a graph showing the spatial extent and co-relative direction of the variables being studied. Variograms of samples must first be estimated and modeled to determine the spatial variability of the random variables as a function of their separation distance [29]. In order to draw the indicator variogram, the first step is to transform the data into indicator values, and the variogram should be plotted using the values for the indicators. The variogram must be calculated in different directions to detect anisotropy of spatial variation [11].

The search ellipsoid parameters are calculated by anisotropy direction, which requires two or three major variogram models for this purpose. After this step, the range and direction of the ellipsoid are characterized. Figures 7, 8 and 9 show variogram modeling and modeling of the indicator variogram for each threshold and median.

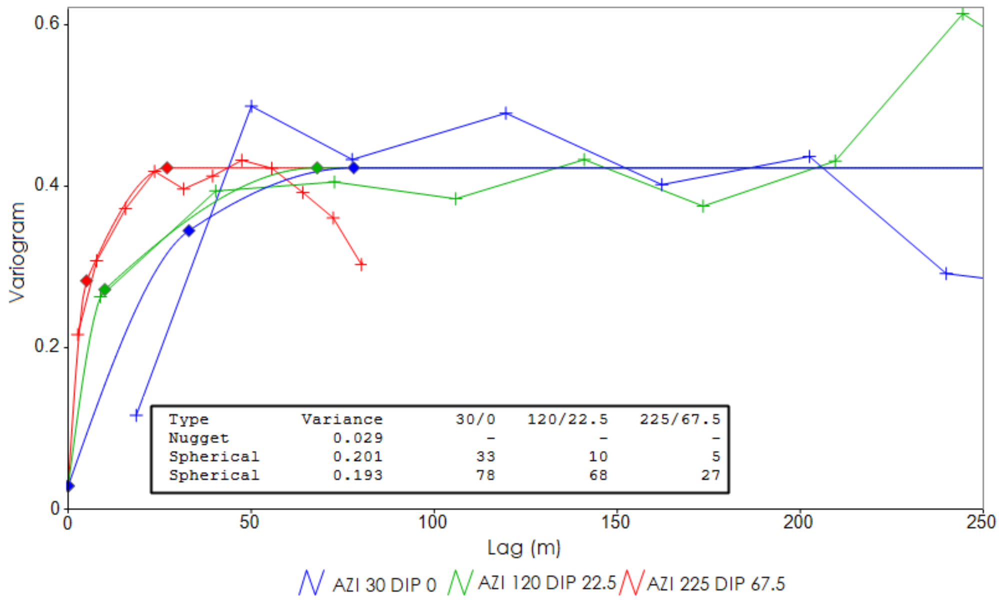

Figure 7. Experimental variogram and variogram modeling of Au variable in three main directions.

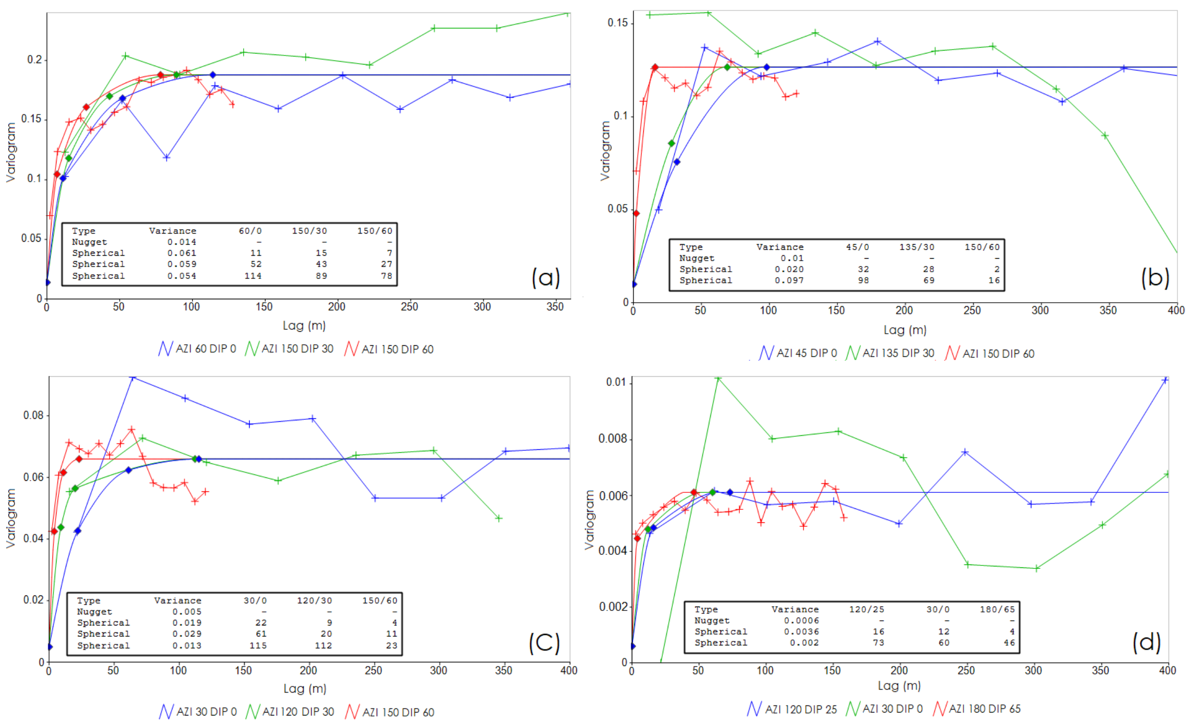

Figure 8. Experimental variogram and variogram modeling of indicator data in three main directions for (a) Threshold 1, (b) Threshold 2, (c) Threshold 3, and (d) Threshold 4.

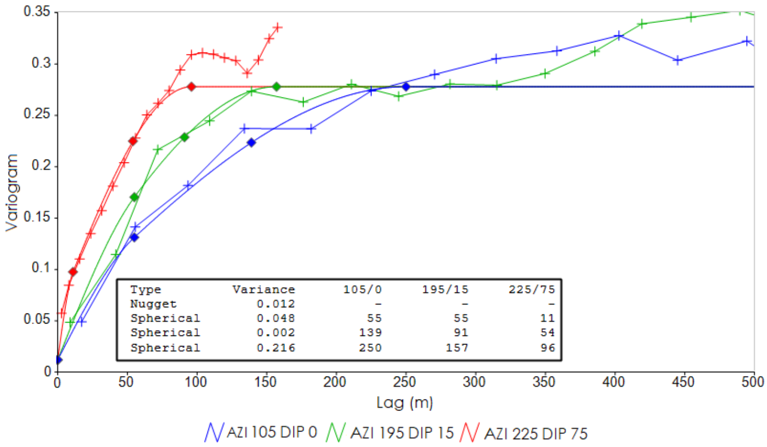

Figure 9. Experimental variogram and variogram modeling of indicator data in three main directions for median of data.

4.4. Resource estimation

To do OK, after deleting the outliers and determining the correct range of values, the OK estimator for the original data may be executed. However, to calculate the estimate OK, the variogram of the original data must first be drowned, and then the dimensions of the search ellipsoid are determined according to the parameters of the variogram.

The Multiple IK as OK requires the indicator variograms shown in transmissive sections, and at the end, a linear kriging estimate must be selected. In this work, SK was chosen to calculate the IK estimate in both modes of Multiple IK and Median IK. After this step, the multi-threshold indicator variogram models must be verified by a number of order relations (inequalities) provided by the characteristics of a general bivariate CDF [30]. For this case, the order relation was checked, and there was no order relation problem in these thresholds, and the results did not change. Since the results of IK are the probabilistic results in each block, the mean of each interval is used to convert the probabilistic results to degrees.

As mentioned earlier, Median IK is a simple Multiple IK mode that uses only the median indicator threshold and the Median IK indicator variogram for that indicator data. Therefore, an IK estimate was made for median and other cut-offs obtained from the N-S model using only one median indicator variogram. In the last step, Median IK needs as Multiple IK each interval to convert to the gold grades.

After performing the resource estimation with all the methods discussed, a 3D view of the estimated block is shown in Figures 10 and 11. Traces of drill holes (points) are shown on this plot. As it can be seen in these graphs, in general, we can say that the results of all methods are correct. In order to find the best method, each method must be validated.

To get the final results in comparison, the results are displayed in the Cutoff grade-Tonnage Curve and Cutoff grade-Average grade (Figure 12). As in Figure 12, in Multiple IK tonnage and average grade in high grades is higher than the other methods, and OK predicted the lowest tonnage that could be due to the smoothing effect in this procedure.

4.5. Validation techniques

There are several methods for validating the estimation results. Some of these methods are statistical and some are graphical. In this work, we use three validation methods. First, we use cross-validation to validate the estimation parameters and to compute these methods on real data. Then we use the swath plot to compare the mean of grades in cells and samples in different directions. Finally, the QQ plot was used to compare different quantiles in estimation cells with samples.

4.5.1. Cross-validation

This technique, also called leave-one-out, was used to validate the estimation results and replace the variogram models [4]. The idea is to re-estimate each sample $z(x_\alpha)$ ($\alpha = 1, ..., n$) independent from the sample at this point and to use the other (n-1) samples in the re-estimation [4]. Therefore, we have used cross-validation to validate all the methods used in this research work for the real data. The results of this technique are shown in the scatter plot of Figure 13. As it can be seen, all of these methods have a correlation close to each other, of which we could not conclude that one of them is better. According to the correlation coefficient obtained for each method, all methods are acceptable.

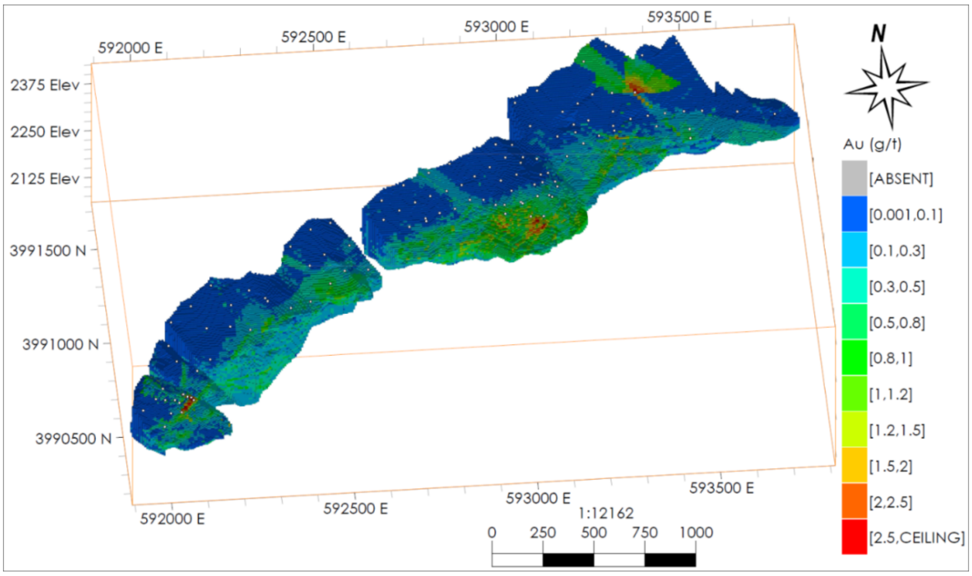

Figure 10. A 3D view of data estimated by OK. Locations of drill-holes are marked with points.

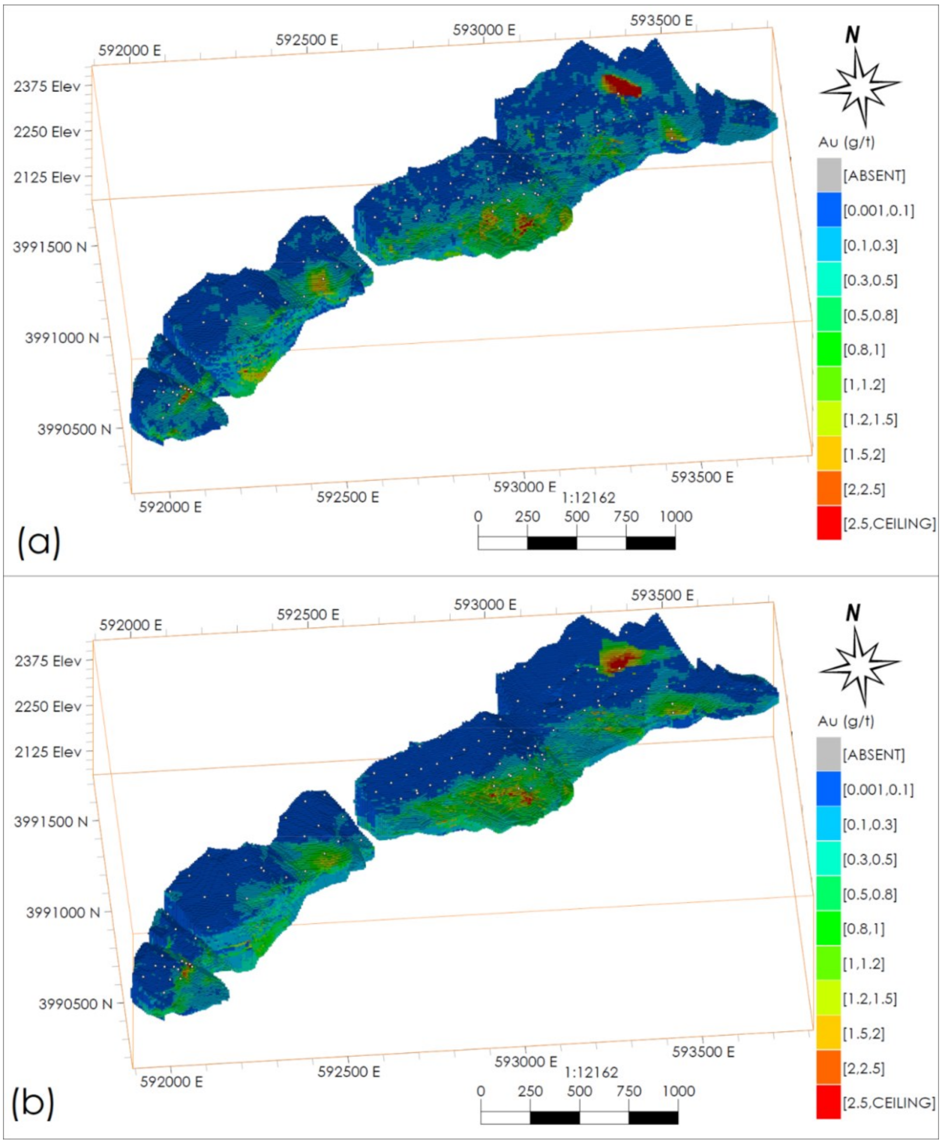

Figure 11. 3D view of data estimated by (a) Multiple IK and (b) Median IK. Locations of drill-holes are marked with points.

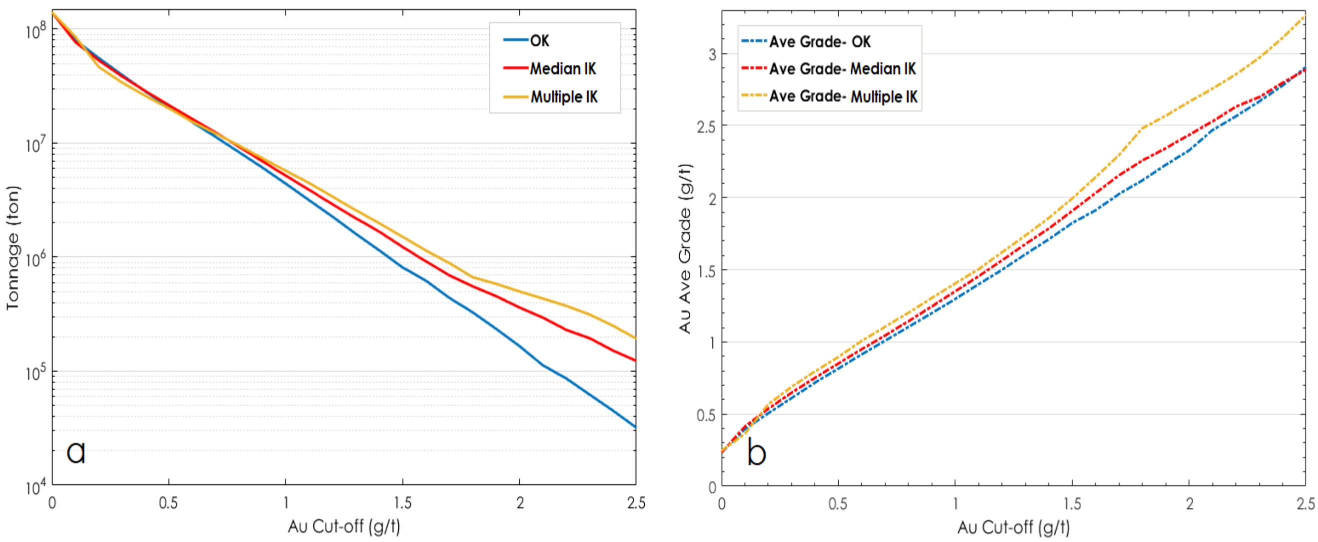

Figure 12. (a) Cutoff grade–Tonnage curve (b) Cutoff grade–Average grade Au curve for each methods of OK, Multiple IK, and Median IK.

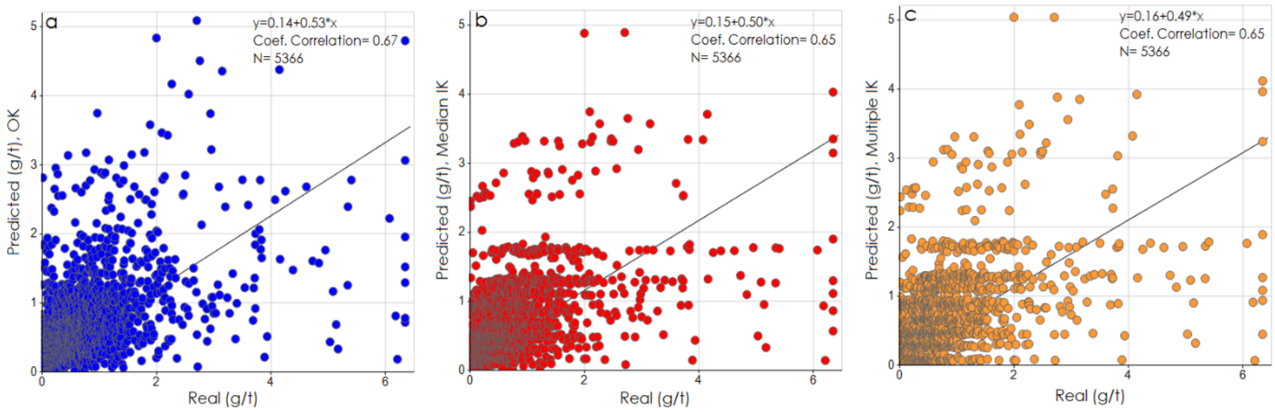

Figure 13. Scatter plot between validated Au with (a) OK, (b) Median IK, and (c) Multiple IK.

4.5.2. Swath plot

Swath plots are an important validation tool for making comparisons between the sampling points and estimated values to identify bias by under-estimation or over-estimation or a smoothing effect in the results. The effects of different estimation methods and parameters can also be compared. The swath plot is a 1D diagram in a particular direction of interest. This graph shows the mean grade for the blocks in the swath along with the mean swath sample values.

For this purpose, the swath plot is used to validate the estimated cells. The result of the swath plot is shown in Figure 14. As it can be seen in this figure, in general, the curves for the three methods are close together but in detail, the curve for Median IK in the directions of Z and Y is closer to the de-clustered data, and Multiple IK is closer to the real grade in the direction of X. With increase in the number of cells and blocks, curves of estimation cells and sample data are close to each other, and wherever the number of data is less, the curves have more differences. Also in high-grade, the results of Multiple IK are closer to the de-clustered data. The smoothing effect, which is one of the disadvantages of OK, is observed in the results of OK that in high-grades is lower than the other estimation results. Thus with these plots, we can conclude that the IK estimate for both modes has a better estimate than OK.

4.5.3. QQ-plot

The last method used for validation of the estimation cells in this research work is the QQ-plot. This method compares the distribution of sample data and cell data. In order to draw this diagram, a quantile of both data sets is calculated and recorded. For this purpose, QQ-plot drowned between the de-clustered data and the estimate cells, which is indicated in Figure 15. Checking this figure shows that the Multiple IK diagram has a less distance than the original distribution.

Meanwhile, it is separated later than the diagonal line (in the grade above 1.3 ppm), although other graphs are a greater distance than the original distribution and also their distribution is earlier separated from the y=x line (in the grade above 0.4 ppm). By comparing the results of all the estimated methods, we can conclude that Multiple IK looks better because the match with the high grade is much better.

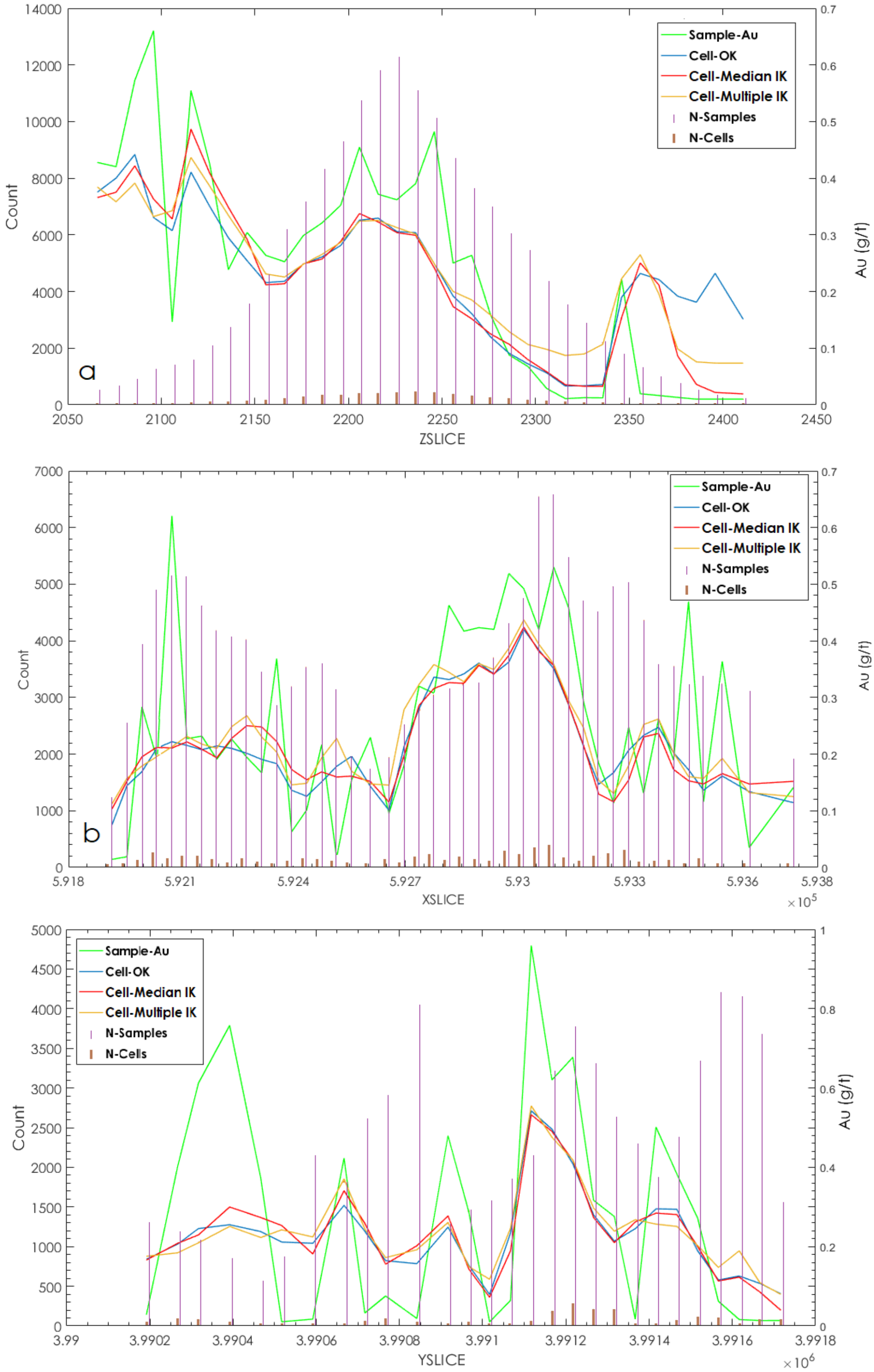

Figure 14. Swath plot in directions of (a) Z, (b) X, and (c) Y for validation of estimation cells.

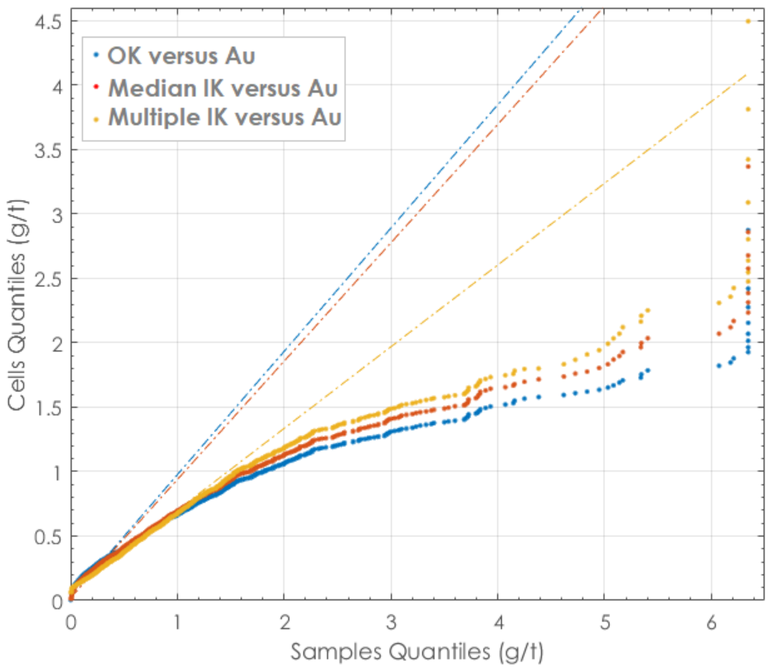

Figure 15. QQ-plot between samples and estimated cells.

5. Conclusions

In this work, estimates of OK, Multiple IK, and Median IK were tested with gold from the Qolqoleh gold deposit.

In the estimation of OK, although it has a high correlation with the real data, in this method, we had to remove the outliers and the accuracy of the original data. The OK method is suitable for the data following a normal distribution, while the distribution of gold has many differences from the normal curve. As it can be seen in the Cutoff grade-Tonnage curve and Cutoff grade-Average grade curve, the results of OK are lower than the other methods. These show the effects of smoothing, which could possibly be termed the most disadvantages of OK. Also in the swath plot and in the QQ-plot, the validation results of OK are weaker than the other two methods.

In the IK method, it is less necessary to remove outliers because this data is removed from the overall distribution as they are converted into indicator values.

The Median IK method fits better with the real data in the Swath plot than Multiple IK, although in the QQ-plot, Multiple IK is better. By examining the indicator variogram, we can see that the median indicator variogram is better than the rest of the indicator variograms, and has a high level of safety. Therefore, the results in the swath plot are better than Multiple IK. However, because Multiple IK better covers different variograms and a high grade, high quantile has a better match with the real data, and the QQ-plot for this method is better than the others. However, one of the biggest advantages of Median IK is the simplicity and the lower complexity of this method since the median is the only indicator variogram that can be done in different cut-offs.

The Multiple IK allows more control over the dataset compared to Median IK, although the drawback is its great complexity since we need to record different indicator variograms, for which some of the indicator variograms are difficult to fit since the number of dates on this threshold is low.

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References

[1]. Journel, A.G. and Huijbregts, C.J. (1978). Mining geostatistics. Academic press. 600 P.

[2]. Nas, B. (2009). Geostatistical Approach to Assessment of Spatial Distribution of Groundwater Quality. Polish Journal of Environmental Studies. 18 (6): 1073-1082.

[3]. Arroyo, D., Emery, X. and Peláez, M. (2012). Sequential simulation with iterative methods. In: Abrahamsen, P., Hauge, R., and Kolbjornsen, O. (EDs.), Quantitative Geology and Geostatistics, Vol. 17: Geostatistics Oslo 2012. Springer. pp. 3-14.

[4]. Rossi, M.E. and Deutsch, C.V. (2013). Mineral resource estimation, Springer, Dordrecht. 332 P.

[5]. Sinclair, A.J. and Blackwell, G.H. (2002). Applied mineral inventory estimation, Cambridge University Press, Cambridge. 346 P.

[6]. Machado, R.S., Armony, M. and Costa, J.F.C.L. (2012). Field Parametric Geostatistics- A Rigorous Theory to Solve Problems of Highly Skewed Distributions. In: Abrahamsen, P., Hauge, R., and Kolbjornsen, O. (EDs.), Quantitative Geology and Geostatistics, Vol. 17: Geostatistics Oslo 2012.Springer. pp. 383-395.

[7]. Journel, A.G. (1983). Nonparametric estimation of spatial distributions. Journal of the International Association for Mathematical Geology. 15 (3): 445-468.

[8]. Glacken, I. and Blackney, P. (1998). A practitioners implementation of indicator kriging. In Proceedings of a one day symposium: Beyond Ordinary Kriging.

[9]. Juang, K.W. and Lee, D.Y. (1998). Simple indicator kriging for estimating the probability of incorrectly delineating hazardous areas in a contaminated site. Environmental Science & Technology. 32 (17): 2487-2493.

[10]. Keogh, A. and Moulton, C. (1998). Median indicator kriging- a case study in iron ore. In Proceedings of a one day symposium: Beyond Ordinary Kriging.

[11]. Rahimi, H., Asghari, O. and Hajizadeh, F. (2018). Selection of Optimal Thresholds for Estimation and Simulation Based on Indicator Values of Highly Skewed Distributions of Ore Data. Natural Resources Research. pp. 1-17.

[12]. Agterberg, F. (1995). Multifractal modeling of the sizes and grades of giant and supergiant deposits. International Geology Review. 37 (1): 1-8.

[13]. Deng, J., Wang, Q., Yang, L., Wang, Y., Gong, Q. and Liu, H. (2010). Delineation and explanation of geochemical anomalies using fractal models in the Heqing area, Yunnan Province, China. Journal of Geochemical Exploration. 105 (3): 95-105.

[14]. Chiles, J.P. and Delfiner, P. (2009). Geostatistics: modeling spatial uncertainty, John Wiley & Sons, New Jersey. 628 P.

[15]. Pishbin, M., Fathianpour, N. and Mokhtari, A.R. (2016). Uniaxial Compressive Strength spatial estimation using different interpolation techniques. International Journal of Rock Mechanics and Mining Sciences. 89: 136-150.

[16]. Deutsch, C.V. and Journel, A.G. (1998). GSLIB: Geostatistical Software Library and User's Guide. 2nd ed. Oxford University Press, New York. 369 P.

[17]. Issaks, E.H. and Srivsatava, R.M. (1989). Applied geostatistics, Oxford University Press, New York. 592 P.

[18]. Vann, J. and Guibal. D. (1998). Beyond Ordinary Kriging–An overview of non-linear estimation. in Proceedings of a one day symposium: Beyond Ordinary Kriging.

[19]. Mendes, R.M. and Lorandi, R. (2006). Indicator kriging geostatistical methodology applied to geotechnics project planning. IAEG2006 Paper. 527: 1-12.

[20]. Marshall, J.K. and Glass, H.J. (2012). The influence of geostatistical techniques on geological uncertainty. In: Abrahamsen, P., Hauge, R., and Kolbjornsen, O. (EDs.), Quantitative Geology and Geostatistics, Vol. 17: Geostatistics Oslo 2012.Springer. pp. 467-477.

[21]. Pardo-Igúzquiza, E. and Dowd, P.A. (2005). Multiple indicator cokriging with application to optimal sampling for environmental monitoring. Computers & Geosciences. 31 (1): 1-13.

[22]. Badel, M., Angorani, S. and Panahi, M.S. (2011). The application of Median indicator kriging and neural network in modeling mixed population in an iron ore deposit. Computers & geosciences. 37 (4): 530-540.

[23]. Mandelbrot, B.B. (1983). The fractal geometry of nature. New York: Henry Holt and Company.

[24]. Rahimi, H. (2016). Modeling and mineral resource estimation of Qolqoleh gold deposit (southwest of Saqqez) using Ordinary Kriging, Indicator Kriging and Probability Kriging. MSc. thesis, Urmia University: Urmia, Iran. 92 P.

[25]. Mohajjel, M., Fergusson, C. and Sahandi, M. (2003). Cretaceous–Tertiary convergence and continental collision, Sanandaj–Sirjan zone, western Iran. Journal of Asian Earth Sciences. 23 (4): 397-412.

[26]. Almasi, A., Jafarirad, A., Afzal, P. and Rahimi, M. (2015). Prospecting of gold mineralization in Saqez area (NW Iran) using geochemical, geophysical and geological studies based on multifractal modelling and principal component analysis. Arabian Journal of Geosciences. 8 (8): 5935-5947.

[27]. Tajeddin, H. (2011). Gold ore controlling factors in metamorphic rocks of Saqez-Sardasht, NW of Sananda-Sirjan metamorphic zone. PhD thesis, Tarbiat Modarres University: Tehran, Iran. 436 P.

[28]. Marinoni, O. (2003). Improving geological models using a combined ordinary–indicator kriging approach. Engineering Geology. 69 (1): 37-45.

[29]. Antunes, I. and Albuquerque, M. (2013). Using indicator kriging for the evaluation of arsenic potential contamination in an abandoned mining area (Portugal). Science of the Total Environment. 442: 545-552.

[30]. Journel, A.G. and Posa, D. (1990). Characteristic behavior and order relations for indicator variograms. Mathematical Geology. 22 (8): 1011-1025.

بررسی روش‌های تخمینی خطی و غیرخطی در توزیع‌های با چولگی بالای طلا

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چکیده:

هدف از این پژوهش، مقایسه روش‌های کریجینگ خطی و غیرخطی در تخمین منبع معدنی کانسار طلای قلقله واقع در سقز، شمال غربی ایران، است. با توجه به این واقعیت که توزیع طلا دارای چولگی مثبت است و دارای یک تفاوت معنی‌داری با منحنی نرمال است، تخمین‌های زمین‌آماری در این موارد پیچیده است. کریجینگ خطی، به عنوان یک روش تخمین منابع، در این موارد می‌تواند مشکل‌ساز باشد و عیار غیرواقعی را برآورد کند. به منظور بررسی و اصلاح خطاهای روش‌های خطی، روش کریجینگ غیرخطی مورد استفاده قرار می‌گیرد. یکی از کاربردی‌ترین روش‌های غیرخطی، روش کریجینگ شاخص (IK) است. روش کریجینگ شاخص با استفاده از آستانه‌های متعدد مقادیر عیاری را به واحدهای شاخص ۰ و ۱ تبدیل می‌کند که برای انتخاب آستانه‌ها می‌توان از مدل فراکتال عیار- عدد استفاده کرد. مدل عیار- عدد آستانه‌های مهم و حساس را بر اساس توزیع عیاری تعیین می‌کند. روش کریجینگ شاخص بر اساس تعداد آستانه‌های شاخص به روش‌های کریجینگ شاخص چندگانه و کریجینگ شاخص میانه دسته‌بندی می‌شود. روش کریجینگ شاخص به مقادیر عیاری زیاد حساس نیست. در پژوهش، مقایسه‌ای بین روش‌های کریجینگ شاخص میانه و کریجینگ شاخص چندگانه و همچنین مقایسه این روش‌ها با روش کریجینگ معمولی (که یک روش کریجینگ خطی است) انجام شده است. به طور کلی، نتیجه‌گیری شد که همه این روش‌ها برای ارزیابی منابع معدنی در میان این روش‌ها مناسب هستند، اگرچه روش کریجینگ شاخص برای تخمین در بازه‌های مختلف عیاری طلا بهتر است.

کلمات کلیدی: کریجینگ شاخص میانه، کریجینگ شاخص چندگانه، تخمین منبع معدنی، کریجینگ معمولی، کانسار طلای قلقله.