Bayesian Data Fusion: a Reliable Approach for Descriptive Modeling of Ore Deposits

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- Data Assimilation
- Complexity
- Decision-making
- Economic Geology
- Uncertainty Reduction

Abstract
Recognition of ore deposit genesis is still a controversial challenge for economic geologists. Here, this task was addressed by the virtue of Bayesian data fusion (BDF), implementing available proofs: semi-schematic examples with two (Cu and Pb + Zn) and three (Cu, Pb + Zn, and Ag) evidences. The data, in the current paper being just concentrations of the indicated elements, was collected from the Angouran deposit in Iran at the prospecting and general exploration stages. BDF was used for discrimination between the three geneses of Massive Sulfide, Mississippi, and SEDEX types. A better genesis recognition with clear discrimination between the geneses was achieved by BDF, as compared to the earlier studies. The results obtained showed that uncertainties were reduced from 50% to less than 30%, and deposit recognition was greatly improved. Furthermore, we believe that using more properties can have a beneficial effect on the overall outcome. The comparison made between 2 and 3 properties showed that the amount of probable belonging values to any type of deposit was greater in 3 properties. It was also confirmed that using the completed information from the various stages of exploration progress can be amplified and be used for genesis recognition via BDF.

1. Introduction
Identification of the ore deposit genesis, one of the main duties of economic geologists, is an important step in exploration, surveying, sampling, and reserve modeling. For a proper identification, as proposed elsewhere [1-5], numerous information databanks, as listed below, needs to be put in place. The necessary information are the tectonic regime (magma tecton), mineral host rock and age, alteration or metasomatic zones of mineral host rock by hydrothermal or magmatic fluids, overall figure of the deposit (e.g. vein, layer, mass, porphyry), mineralization tissue (how deposit is placed in the host rock such as dispersed, massive or vein types), ore and gangue mineral (mineralization, e.g. iron, which might be as oxide, carbonate or sulfide as well as type of gangue), grade and ore deposit tonnage, and physico-chemical properties of fluid or magma (fluid inclusion and sustainable isotopes such as H, C, O, and S studies).

In the earlier methodologies, the use of a univariate data analysis to explore ore deposit genesis was common. As a result, the researcher could have ignored vast amounts of information and existing complexities, leading to probable misinterpretation and poor comprehension of what has happened during the course of genesis [6-13]. In contrast, the shift towards multivariate analysis approaches such as pattern recognition via genesis classification [14], mapping neural network [15], dynamic clustering [16], and hybrid clustering [17] has revolutionized our view towards recognition of ore deposit genesis. Nevertheless, even in the case of multivariate analysis, the
resulting information is heavily dependent on the implemented statistical approach. This so-called fuzzy genesis recognition can be misleading; raising questions in regards to uncertainties, and how to obtain more reliable information out of the considered evidences.

A relatively new concept in geoscience is the use of sensor data fusion, with earliest applications in military cases towards promoting machine-human relationship and reducing uncertainties in reliable decisions. Data fusion has found its merits in other sciences [18, 19] through gathering more data from varieties of sensors [20-23]. Such improvements have allowed optimization of computational efficiency, removal of data redundancy, reduction of uncertainties and cost, improving the resolution for signal-to-noise ratio, and achieving more reliable and comprehensive results.

Pattern fusion, which is the integration in the level of decision, is the highest level of data fusion [18, 23]. In this paper, BDF was used for identification of schematic ore deposit genesis. It was assumed that three geneses might be considered for a deposit: Massive Sulfide, Mississippi, and SEDEX. The characteristics of these deposits are summarized in Table 1. The role of data fusion is to amplify the most possible genesis for a certain deposit. The results obtained were compared with the common methods to give a clear view of the benefits for the applied method.

### 2. Bayesian Data Fusion (BDF)

Conditional probability is the basis of BDF. The Bayes law [19, 23] is:

$$ P(B | A) = \frac{P(A | B)P(B)}{P(A)} \quad (1) $$

in which $P(A | B)$ is a priori probability, $P(B)$ is the likelihood function, and $P(A)$ is a normalization factor. $P(A | B)$, the posteriori probability, is an indicator of the correctness of proposition of $B$. The result of the conditional probability $P(A | B)$ is in the range of $[0, 1]$. One means absolute belief to correctness of $A$ when $B$ is known. $P(A | B)$ is equal to zero when $A$ is absolutely incorrect and $B$ is known.

Suppose that $n$ properties of $S_i$ to $S_n$ are $n$-measured values from $X_1$ to $X_n$. There is a conditional probability for uncertainty as property of $S_i$, which is introduced by the $X_i$ value. The likelihood function would be the first parameter to be calculated in the Bayesian algorithm:

$$ L(X_i | Y) = \frac{P(X_i | Y)}{P(X_i | \neg Y)} \quad (2) $$

The Priori estimation can be calculated as:

$$ O(Y) = \frac{P(Y)}{P(\neg Y)} \quad (3) $$

where $P(Y)$ and $P(\neg Y)$ are the probability of occurrence and non-occurrence $Y$, respectively. $O(Y)$ represents the odds of the event. Posteriori estimation of proposition $Y$ equals to:

$$ O(Y | X_1, X_2, ..., X_n) = O(Y) \times \prod_{i=1}^{n} L(X_i | Y) \quad (4) $$

where the likelihood functions and priori estimation are calculated by Equations 2 and 3, respectively. Posteriori probability ($Y$) in the case of knowing $X_i$ to $X_n$ is equal to:

$$ P(Y | X_1, X_2, ..., X_n) = \frac{O(Y | X_1, X_2, ..., X_n)}{1 + O(Y | X_1, X_2, ..., X_n)} \quad (5) $$

Suppose that there are two properties; if the properties were measured in two individual times, then the equation would be modified as follows [27]:

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### Table 1. Brief characteristics of the MVT, SEDEX, and VMS (Besshi) type deposits.

<table>
<thead>
<tr>
<th>Type deposit</th>
<th>Host rock</th>
<th>Alteration</th>
<th>Form</th>
<th>Texture</th>
<th>Ore minerals</th>
<th>Gangue minerals</th>
<th>Main Metals</th>
<th>Second metals</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>MVT</td>
<td>Carbonate-Dolomite</td>
<td>Dolomitization</td>
<td>Stratiform</td>
<td>Disseminated, Veinlets</td>
<td>Galena, Sphalerite, Pyrite, Marcasite</td>
<td>Fluorite, Barite, Calcite, Quartz</td>
<td>Pb-Zn</td>
<td>Cu-Ba-F</td>
<td>[11, 24]</td>
</tr>
<tr>
<td>SEDEX</td>
<td>Sedimentary-Volcanic rocks</td>
<td>Dolomitization</td>
<td>Stratiform</td>
<td>Massive and fine Banded, Breccia, Disseminate</td>
<td>Galena, Sphalerite, ChalcoPyrite, Pyrite, Pyrrhotine</td>
<td>Calcite, Barite, Quartz, Dolomite</td>
<td>Zn-Pb</td>
<td>Mn-Fe, Mg-As-Sb-Ti</td>
<td>[11, 25]</td>
</tr>
<tr>
<td>VMS (Besshi)</td>
<td>Volcanic-Sedimentary rock</td>
<td>Metamorphism</td>
<td>Stratiform</td>
<td>Massive</td>
<td>Sphalerite, Galena, Pyrite, ChalcoPyrite, Pyrrhotine, Arsenopyrite</td>
<td>Quartz, Calcite, Barite</td>
<td>Zn-Pb</td>
<td>Ti-Au-Bi-Mg-Mn</td>
<td>[11, 26]</td>
</tr>
</tbody>
</table>
\[
P(x | Y \cdot Y') = \frac{P(x | Y \cdot Y) P(Y \cdot Y')}{P(x | Y') P(Y \cdot Y')} \times \text{Normalization Factor}
\]

Equation 6

In the case of having more properties or more, Equation 6 will be modified based on 7 [27]:

\[
P(x | Y \cdot Y') = \frac{P(x | Y \cdot Y) P(Y \cdot Y') P(x | Y') P(Y \cdot Y')}{P(x | Y') P(Y \cdot Y') \times \text{Normalization Factor}}
\]

Summation of \( P(x \cdot Y \cdot Y') \) has to be equal to one, achieved with the normalization factor. To describe the procedure, an example is solved in Section 4.

3. Problem Definition

The knowledge about ore deposit genesis would, of course, put miners in great advantage in terms of cost reduction. Thus implementing proper exploratory techniques will save us the benefits of doubts during decision-making. During the last half century or so, varieties in the exploration techniques have been developed to be used in mines and further optimized [28-31].

Since characteristics of deposits are unique (see Table 2, for example), an identical genesis pattern for each is expected, which makes the identification process associated with great amounts of uncertainties. To be clear, in Fig. 1, three triangles are extracted from the information in Table 2, which displays similarity between the numbered deposits and three well-known genuses of sulfide deposits, i.e. Mississippi Valley (MV), Massive Sulfide (MS), and SEDEX (S). In Fig. 1, concentrations of the characteristic elements in MVT, SEDEX, and VMS are displayed, where the numbers correspond to the deposit values in Table 2. Centers of different panels in Fig. 1 are considered as the absolute uncertainty point (33.3% membership to three genesis types). Accordingly, in places within Table 2, where the values for metal contents are missing, a spot has appeared in the middle of triangle (Fig. 1), i.e., considered as a dummy spot. Location of the circles and square in all panels in Fig. 1 are assigned based on the analysis of Cu, Zn + Pb, and Ag, respectively.

Often evidences are present that approve and simultaneously reject belonging of a deposit to a certain type, which shows uncertainties associated with genesis cognition. For instance, suppose that Zn in one deposit is around 5%. Based on Table 3, extracted from Table 2, rough ranges of some parameters in three well-known geneses of sulfide deposits are shown. The genesis might be SEDEX, MVT or VMS.

The aim of the current paper is to find an answer to this question that how it is possible to consider a unique, reliable, and reproducible genesis for a deposit based on the visible evidences. The results of the BDF approach will be presented. Three evidences are considered for the study: Cu, Pb + Zn, and Ag; their rough ranges are abstracted in Table 3.

<table>
<thead>
<tr>
<th>Number and name of district</th>
<th>Type of deposit</th>
<th>Host rock</th>
<th>Alteration</th>
<th>Form</th>
<th>Ore mineral</th>
<th>Gangue mineral</th>
<th>Main metals (ppm)</th>
<th>Minor metals (ppm)</th>
<th>References</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Tyndrum, Scotland</td>
<td>MVT</td>
<td>Quartzite, Carbonate</td>
<td>-</td>
<td>Strataband</td>
<td>Galena, Sphalerite, ChalcoPyrite</td>
<td>Quartz-Barite</td>
<td>Zn-Pb</td>
<td>-</td>
<td>[32]</td>
</tr>
<tr>
<td>2. Blazana-Gustet, Romania</td>
<td>MVT</td>
<td>Carbonate, Dolomite</td>
<td>Metamorphic</td>
<td>Strataband</td>
<td>Galena, Sphalerite, ChalcoPyrite, Pyrite, Galena, Sphalerite, ChalcoPyrite</td>
<td>Calcite, Barite, Quartz, Dolomite</td>
<td>Zn-Pb</td>
<td>Mn-Fe-Mg-Ag</td>
<td>[33]</td>
</tr>
<tr>
<td>3. Berian, Germany</td>
<td>SEDEX</td>
<td>Volcanic-sedimentary</td>
<td>Silicification, sericitation</td>
<td>Strataband</td>
<td>Galena, Sphalerite, Pyrite</td>
<td>Quartz, Calcite, Dolomite</td>
<td>Pb-Zn-Cu</td>
<td>Ag-Sb-Se-Te As=21-210 (Cu=76)</td>
<td>[34]</td>
</tr>
<tr>
<td>4. Fankou, China</td>
<td>MVT</td>
<td>Carbonate- Shale</td>
<td>Dolomitization</td>
<td>Strataband</td>
<td>Galena, Sphalerite, Pyrite</td>
<td>Quartz, Calcite, Dolomite</td>
<td>Pb-Zn</td>
<td>Ni-Cu</td>
<td>[35]</td>
</tr>
<tr>
<td>5. Navan, Ireland</td>
<td>SEDEX</td>
<td>Carbonate-Shale-volcanic rocks</td>
<td>Dolomitization</td>
<td>Strataband</td>
<td>Sphalerite, Ga</td>
<td>Lena, Pyrite, Marcasite</td>
<td>Calcite, Barite, Dolomite</td>
<td>Zn-Pb&gt;30%</td>
<td>Mn-Fe-Mg-Ag</td>
</tr>
<tr>
<td>6. Benue Trough, Nigeria</td>
<td>MVT</td>
<td>Shale, Siltstone, limestone</td>
<td>-</td>
<td>Strataband</td>
<td>Galena, Sphalerite, ChalcoPyrite, bornite, Pyrite</td>
<td>Quartz, Calcite, Dolomite, Barite, Fluorite</td>
<td>Pb-Zn</td>
<td>Fe-Mn Ag=26-140 (Cu=350)</td>
<td>[37]</td>
</tr>
<tr>
<td>8. Pyrenees, France</td>
<td>SEDEX</td>
<td>Carbonate-Shale-rhyolite</td>
<td>Dolomitization</td>
<td>Stratiform</td>
<td>Sphalerite, Galena, Pyrite, Marcasite</td>
<td>Calcite, Barite, Quartz</td>
<td>Zn-Pb</td>
<td>Ni-Cu-Ti</td>
<td>[38]</td>
</tr>
<tr>
<td>9. Malines, France</td>
<td>MVT</td>
<td>Carbonate-Shale</td>
<td>-</td>
<td>Strataband, Stratiform</td>
<td>Galena, Sphalerite, Pyrite</td>
<td>Quartz, Calcite, Dolomite</td>
<td>Zn-Pb</td>
<td>F-Ba</td>
<td>[39]</td>
</tr>
</tbody>
</table>

Table 2. Selected properties of MVT, SEDEX, and Besshi type deposits.
10 Zlate Hory, Czechoslovakia
  Besshi, Schist, Quartzite, marble Metamorphic Strataband, Massive Sphalerite, Galena, ChalcoPyrite, Pyrite, Pyrite, Sphalerite, Galena, ChalcoPyrite, Pyrite, Sphalerite, Galena, ChalcoPyrite, Pyrite, Arsenopyrite Quartz, Calcite, Dolomite Zn-Pb-Cu Au-Bi-Ag [28]

11 Tharsis mine, Spain
  VMS Carbonaceous black slate-volcanic group Metamorphic Stratiform Calcite, Dolomite, Quartz Ph-Zn< 2.6% (Cu=8000) Bi-Te [40]

12 Bløkavassli, Norway
  SEDEX Amphibolites, schist, gneiss, marble Metamorphic Stratiform-Lenses Quartz Zn-Pb<12% (Cu=4000) Ag-Sb [41]

13 Malmani, South Africa
  MVT Carbonate-Dolomite - Stratiform, Strataband Sphalerite, Galena, ChalcoPyrite, Pyrite, Pyrite Calcite, Dolomite Pb-Zn<4.3% Fe-Mn (Ag=70-300) [42]

14 Yindongzi, China
  SEDEX Meta Silstone, Shale, limestone Argillic-silice-albitization Strataband Calcareous, Barite, Quartz Ph-Zn<11% - Cu Carbonate, Barite, Pyrite Cu (Ag=48) [43]

15 Ponferrada, Spain
  MVT Carbonate-dolostone-Shale - Strataband Pyrite, Sphalerite, Galena Calcite, Quartz Zn - Pb<17.6% (Cu=1600) Ba (Ag=3) [44]

16 Pacara Basin, Peru
  MVT Carbonate-Dolomite - Strataband, Strataband Sphalerite, Galena, Marcarite, Pyrite Calcite, Dolomite Pb-Zn<10% F-Ba (Ag=31) [45]

17 Lengenbach, Switzerland
  SEDEX Dolostone - green schist- amphibolites Silicic Strataband, Stratiform Carbonate, Barite, Pyrite, Dolomite Zn - Pb<6% (Cu=1600) Cu (Ag=10-426) [46]

18 Santa Lucia, Cuba
  SEDEX Dolostone-Shale-limestone-schist Propylitized Stratiform, Lenticular Carbonate, Barite, Quartz Zn - Pb<17.6% (Cu=1600) Ba (Ag=3) [47]

19 Yenefrito, Spain
  SEDEX Silstones, marls and limestone propyritic sills Propylitized Stratiform, Lenticular Quartz, Calcite Zn - Pb - Cu [48]

20 Damaran Lufilian, Central Africa
  SEDEX MetaCarbonates, Dolomite Metamorphism - Strataband Sphalerite, Galena, ChalcoPyrite, Pyrite, Calcite, Dolomite, Barite, Zn-Pb<13% - Cu Ge, Cd, As, Pb<35% Sb, Ag, Au [49]

21 Kuh-e-Surmeh, Iran
  MVT Carbonate - Strataband Sphalerite, Galena, ChalcoPyrite, Pyrite, Pyrite, Sphalerite, Galena, ChalcoPyrite, Pyrite, Arsenopyrite Quartz, Calcite, Dolomite Zn-Pb<17.5% [50]

22 Angouran, Iran
  SEDEX MetaSulfide? Amorpholites, gneiss, marble Metamorphism Strataband, Stratiform Sphalerite, Galena, Pyrite, ChalcoPyrite, Pyrite, Sphalerite, Galena, ChalcoPyrite, Pyrite, Arsenopyrite Quartz, Dolomite, Anhydrite, Calcite Zn-Pb<29% - Cu, Ni, (Ag=250) [51]

23 George Fisher, Australia
  SEDEX Shale, Silstone, Carbonate, tuff Silica Dolomite Alteration Strataband, Stratiform Sphalerite, Galena, Pyrite, Pyrite, Calcite, Dolomite, Quartz, Fluorite Zn-Pb<16.5% - Cu (Ag=93-150) [52]

24 Namisivik, Canada
  MVT Dolomitic Mudstone - Strataband, Lenses Sphalerite, Galena, Pyrite, Pyrite, Pyrite, Calcite, Dolomite, Quartz, Fluorite Zn-Pb<10% - Cu (Ag=46), (Ag=35) [53]

25 Mount Isa, Australia
  SEDEX Dolomitic Shale, Silstones, and Mudstones Silica-Dolomite Strata-Bound Sphalerite, Pyrite, Calcite, Dolomite, Barite Zn-Pb<13% - Cu (Ag=1000) [54]

26 Upton, Canada
  MVT Limestone, Clastic Rocks - Strataband Sphalerite, Pyrite, Calcite, Dolomite, Barite Zn-Pb<2.1% - Ba (Ag=13.5) - Cd [55]

27 Red Gog, Northern Alaska
  SEDEX Chert, Carbonate, Mudstone and Shale - Stratiform - Lens Sphalerite, Galena, Pyrite, Pyrite, Carbonate, Dolomite, Barite, Calcite Zn-Pb<21% - Cu (Ag=230) [56]

28 McArthur, Australia
  SEDEX Dolomitic Silstone Silica-Dolomite Stratiform Zn-Pb<19% - Cu (Ag=60) - Ti, Fe [57]

29 Maestran basin, Spain
  MVT Limestones - Strataband Sphalerite, Galena, ChalcoPyrite, Pyrite, Sphalerite, Arsenopyrite, Pyrite, Calcite, Dolomite, Barite Zn-Pb<8% - Cu [58]

30 Baqueza, Cantabrian, Spain
  MVT Limestones Dolomitized Strataband Sphalerite, Galena, Pyrite, Calcite, Dolomite, Barite Zn-Pb<9.4% (Cu=50) [59]

31 Illinois-Kentucky, USA
  MVT Carbonate, Clastic Units - Strataband Sphalerite, Galena, Pyrite, Pyrite, Piritite, Dolomite, Fluorite Zn-Pb Cu-Ag [60]

32 Cordilieran, Canada
  MVT Limestones Dolomitized Strataband Sphalerite, Galena, Pyrite, Pyrite, Calcite, Dolomite, Barite Zn-Pb<7.1% - Cu [61]

33 McArthur River (HVC), Australia
  SEDEX Limestone-Shale - Stratiform-lenses Sphalerite and Galena, Pyrite Barite, Calcite, Dolomite Zn-Pb<10% - Cu (Ag=41) [62]

34 Lengshuikeng, China
  SEDEX Volcano sedimentary rocks - Strataband Sphalerite, Galena, Pyrite, Calcite, dolomite Zn-Pb<4.6% (Ag=204) [63]

35 Wasiu, China
  MVT Carbonate - Strataband Sphalerite, Pyrite, galena, Sphalerite, Pyrite, Calcite, dolomite Zn-Pb<11% - Cu [64]

36 Chahmir, Iran
  SEDEX Volcano sedimentary rocks Silicification, carbonitization Strataband Sphalerite, Pyrite, galena, Sphalerite, Pyrite, galena, Calcite, dolomite, quartz Zn-Pb<8% - Cu [65]

37 Howards Pass, Yukon
  SEDEX Volcano sedimentary rocks - Strataband Sphalerite, Pyrite, galena, Sphalerite, Pyrite, galena, Calcite, dolomite, quartz Zn-Pb<6% - Cu [66]
3.1 Case Study

The Angouran Zn-Pb-Ag deposit is located in the Western Zanjan Province, NW Iran, about 450 km NW Tehran (Fig 2; compiled from [65, 66]). The Angouran deposit is a world class and the largest zinc deposit in Iran. The ore deposit resources are about 14.6 MT with 22.6% zinc, 4.6% lead, and 110 ppm silver [67]. The Angouran deposit is located within the Sanandaj-Sirjan metamorphic belt, and the host rocks are marble, micaschist, amphibolites, and gneiss of Cambrian. Many have worked on the genesis of the Angouran deposit since 1968 with numerous developed models, to be named a few Proterozoic volcanogenic massive sulfide (VMS)-type mineralization [68], sedimentary-exhalative (SEDEX) process during the Mesozoic [67], and the Mississippi Valley type (MVT) deposit [51]. Therefore, it seems that the genesis for this deposit is rather conflicting between reports appearing in the early 2000s. Something we believe that is required to be rectified is through implementing a proper methodology that minimizes the uncertainty during identification of a deposit.
4. Schematic dataset

Properties (3), Cu, Pb+Zn, and Ag, are listed in Table 3 for analysis of the Angouran’s genesis. The data was analyzed and compared for the case of 2, Cu and Pb + Zn (Table 4), and 3 properties (Table 5), as indicated. Furthermore, prospecting and the general exploration stages were considered for analysis via BDF with the assumptions of 1) availability of the information for only 3 properties, Cu, Pb + Zn, and Ag 2) considering only 3 genesis types, Mississippi Valley, Massive Sulfide or SEDEX Sulfide, according to Fig. 1.

Results of analysis of the prospecting stage were fused together using Equations 2, 3, 4, and 5, and these results were reported in the last rows of Tables 4 and 5. A priori knowledge for the three common deposit types was considered the same and equal to:

\[ P(MV) = P(MS) = P(S) = \frac{1}{3} \]

The comparison of the Angouran deposit belonging probability with three genuses based on the previous datasets (first row in Table 4) and the results of data fusion (when Cu and Pb + Zn are available) (third row in Table 5) show that they have changed as follow:

- Mississippi Valley Type: from 50% (average of two properties) to 70.5%,
- Massive Sulfide: from 35% (average of two properties) to 26.5%,
- SEDEX Sulfide: from 15% (average of two properties) to 3.0%.

This shows that data fusion has amplified the probability of Mississippi Valley Type for Angouran, while it has been attenuated for the two other types.

Comparisons between Tables 4 and 5 are interesting. The probabilities of third property (Ag, in this case) were considered to be equal to average of properties 1 and 2 (Cu and Pb + Zn, in this case). Therefore, it is anticipated that the results of fusion of three properties (third row,
Table 5) should be similar to the results of fusion of two properties (third row, Table 4). However, the results were completely different (Data fusion when Cu, Pb + Zn, and Ag are available):

- Mississippi Valley Type: from 50% (average of two properties) to 81.7%,
- Massive Sulfide: from 35% (average of two properties) to 17.7%, and
- SEDEX Sulfide: from 15% (average of two properties) to 0.6%.

It should be emphasized that in both case studies (2 or 3 properties), based on the situations, the average belonging probability was considered to be the same (first row in Tables 4 and 5). The comparison shows that BDF amplifies discrimination between genoses when more properties are used. Of a great interest is the considerable reduction of uncertainties (smaller than 30%) in both cases with 2 and 3 properties. For the sake of clarification, the average belonging value improved from 50% to 70.5% for 2 properties (Table 4) and 81.7% for 3 properties (Table 5).

5. Result of data fusion

In this part, the results of the previous datasets and current datasets (prospecting and general exploration in Tables 4 and 5) are fused. The following equation is used for fusion of the results for two properties (in different exploration stages) in the Mississippi Valley type:

\[
\text{Fusion Value} = \frac{1}{2} \left( \text{Value}_1 + \text{Value}_2 \right)
\]

Figure 2. a) Simplified tectonic map of Iran (compiled from [65 – 66]). The star shows the location of the Angouran deposit. b) A panorama photo of Angouran lead and zinc mine.
where all the terms were calculated in Tables 4 and 5. It should be reiterated that the summation of probabilities have to be equal to one, which is the role of the normalization factor. The equation for the data fusion calculation result in the case of three properties in the Mississippi Valley type is abstracted as follows:

\[
P(MV | Y_1 Y_2 Y_3) = \frac{P(MV | Y_1 Y_2 Y_3) P(MV | Y_1) P(MV | Y_2) P(MV | Y_3)}{P(MV | Y_1) P(MV | Y_2) P(MV | Y_3)} \times \text{Normalization Factor}
\]

\[
P(MV | Y_1 Y_2 Y_3) = \frac{P(MV | Y_1 Y_2 Y_3) P(MV | Y_1) P(MV | Y_2) P(MV | Y_3)}{P(MV | Y_1) P(MV | Y_2) P(MV | Y_3)} \times \text{Normalization Factor}
\]

Table 4. Fuzzy values for a schematic study on Angouran’s deposit considering all 3 geneses (Figures 1) based on the results of analysis of two sensors: Cu and Pb + Zn.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>First Sensor (Cu)</th>
<th>Second Sensor (Pb+Zn)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous Dataset (Prospecting)</td>
<td>(P(MV</td>
<td>Y_1^2) = 0.4)</td>
</tr>
<tr>
<td></td>
<td>(P(S</td>
<td>Y_3^2) = 0.4)</td>
</tr>
<tr>
<td></td>
<td>(P(MS</td>
<td>Y_3^3) = 0.2)</td>
</tr>
<tr>
<td>Current Dataset (General Exploration)</td>
<td>(P(MV</td>
<td>Y_1^2) = 0.7)</td>
</tr>
<tr>
<td></td>
<td>(P(S</td>
<td>Y_3^2) = 0.17)</td>
</tr>
<tr>
<td></td>
<td>(P(MS</td>
<td>Y_3^3) = 0.13)</td>
</tr>
</tbody>
</table>

The results of applying BDF to all data are summarized in Tables 6 (in the case of two properties, Cu and Pb + Zn) and 7 (in the case of three properties, Cu, Pb + Zn, and Ag). Of a great interest is the fact that the application of data fusion improved the probability of the belonging from an average of 75% to nearly 100% (99.6%, 99.36% for 2 and 3 properties, respectively), which means less uncertainties in identification of the deposit (Tables 6 and 7).
Table 5. Fuzzy belonging values of schematic study on Angouran’s deposit considering all 3 geneses (Figures 1) based on the results of analysis of three sensors: Cu, Pb+Zn, and Ag.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>First Sensor (Cu)</th>
<th>Second Sensor (Pb+Zn)</th>
<th>Third Sensor (Ag)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Previous Dataset</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Prospecting)</td>
<td>( P(MV</td>
<td>Y_1^0) = 0.40 )</td>
<td>( P(MV</td>
</tr>
<tr>
<td></td>
<td>( P(S</td>
<td>Y_1^0) = 0.40 )</td>
<td>( P(S</td>
</tr>
<tr>
<td></td>
<td>( P(MS</td>
<td>Y_1^0) = 0.20 )</td>
<td>( P(MS</td>
</tr>
<tr>
<td>Current Dataset</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(General Exploration)</td>
<td>( P(MV</td>
<td>Y_1^2) = 0.70 )</td>
<td>( P(MV</td>
</tr>
<tr>
<td></td>
<td>( P(S</td>
<td>Y_1^2) = 0.17 )</td>
<td>( P(S</td>
</tr>
<tr>
<td></td>
<td>( P(MS</td>
<td>Y_1^2) = 0.13 )</td>
<td>( P(MS</td>
</tr>
</tbody>
</table>

\[
O(Y_i | Y_0) = \frac{1}{2} \cdot \frac{0.6 \cdot 0.4 \cdot 0.5}{0.5} = 0.333
\]

\[
P_{\text{before normalization}}(Y_i | Y_0) = \frac{0.333 + 0.072 + 0.0025}{1} = 0.177 = 17.7\%
\]

\[
O(Y_i | Y_0) = \frac{1}{2} \cdot \frac{0.4 \cdot 0.3 \cdot 0.35}{0.375} = 0.077
\]

\[
P_{\text{before normalization}}(S | Y_0) = \frac{0.077 + 0.0025}{1} = 0.079 = 7.9\%
\]

\[
O(Y_i | Y_0) = \frac{1}{2} \cdot \frac{0.2 \cdot 0.1 \cdot 0.15}{0.3} = 0.0025
\]

\[
P_{\text{before normalization}}(MS | Y_0) = \frac{0.0025 + 0.0025}{1} = 0.005 = 0.5\%
\]

MV, MS, and S stand for Mississippi Valley, Massive Sulfide, and SEDEX types, respectively.

Table 6. Summarized results of data fusion-based genesis recognition of the studied schematic deposit using the dataset for two sensors: Cu and Pb + Zn.

<table>
<thead>
<tr>
<th>Genesis Type</th>
<th>Data Fusion of Previous Dataset (Prospecting Stage)</th>
<th>Current Dataset (General Exploration Stage)</th>
<th>Final Data Fusion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>First Sensor (Cu)</td>
<td>Second Sensor (Pb + Zn)</td>
</tr>
<tr>
<td>Mississippi Valley</td>
<td>70.5%</td>
<td>70%</td>
<td>80%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_{\text{before normalization}}(MV</td>
<td>Y_1^1 Y_2^1 Y_3^1) = 0.705 \times 0.10 \times 0.70 = 0.49 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_{\text{before normalization}}(S</td>
<td>Y_1^1 Y_2^1 Y_3^1) = 0.40 \times 0.30 \times 0.10 = 0.20 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_{\text{before normalization}}(MS</td>
<td>Y_1^1 Y_2^1 Y_3^1) = 0.20 \times 0.10 \times 0.05 = 0.00975 )</td>
</tr>
<tr>
<td>SEDEX</td>
<td>26.5%</td>
<td>17%</td>
<td>15%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_{\text{before normalization}}(MV</td>
<td>Y_1^2 Y_2^2 Y_3^2) = 0.265 \times 0.17 \times 0.15 = 0.0563 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_{\text{before normalization}}(S</td>
<td>Y_1^2 Y_2^2 Y_3^2) = 0.40 \times 0.30 \times 0.10 = 0.0563 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_{\text{before normalization}}(MS</td>
<td>Y_1^2 Y_2^2 Y_3^2) = 0.20 \times 0.10 \times 0.05 = 0.00975 )</td>
</tr>
<tr>
<td>Massive Sulfide</td>
<td>3%</td>
<td>13%</td>
<td>5%</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_{\text{before normalization}}(MV</td>
<td>Y_1^3 Y_2^3 Y_3^3) = 0.03 \times 0.13 \times 0.05 = 0.00975 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_{\text{before normalization}}(S</td>
<td>Y_1^3 Y_2^3 Y_3^3) = 0.03 \times 0.13 \times 0.05 = 0.00975 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>( P_{\text{before normalization}}(MS</td>
<td>Y_1^3 Y_2^3 Y_3^3) = 0.03 \times 0.13 \times 0.05 = 0.00975 )</td>
</tr>
</tbody>
</table>
Results of the Angouran deposit belonging probability to three geneses are based on fusion of prospecting and general exploration dataset (first and second rows in Tables 6 and 7), in the cases that two or three properties generated interesting results. BDF concluded that with a probability more than 99%, the Angouran’s type is Mississippi Valley, with an uncertainty in decision-making close to zero. As a result, in the detailed exploration procedure, the patterns of the Mississippi Valley type must be utilized.

When two-stage datasets are available, numbers of properties are not so important, and there are no significant differences between the final results. In the case study, the Angouran’s belonging probability to the Mississippi Valley type were equal to 99.6 % and 99.36 % for two and three properties, respectively.

6. How proposed method could be applied to exploration programs

Identification of the ore deposit genesis, which is one of the main duties of economic geologists, is an important step in exploration, surveying, sampling, and reserve modeling. There are also usually evidences for known genesis (e.g. Tables 1 and 2); the similarity between those evidences and field observations helps to identify ore deposit genesis. The scientists have often different ideas about the genesis of a certain deposit (more example is in case 3.1). This uncertainty makes the exploration activities rather costly with disparity in classification outcome. The introduced procedure helps to integrate the evidences or even different hypotheses about the genesis of ore deposits in order to decrease the uncertainty associated with, which leads to utilize the suggested exploratory pattern for the identified genesis type.

<table>
<thead>
<tr>
<th>Genesis Type</th>
<th>Data Fusion of Previous Dataset (Prospecting Stage)</th>
<th>Current Dataset (General Exploration Stage)</th>
<th>Final Data Fusion</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>First Sensor (Cu)</td>
<td>Second Sensor (Pb + Zn)</td>
<td>Third Sensor (Ag)</td>
</tr>
<tr>
<td>Mississippi Valley</td>
<td>81.7%</td>
<td>70%</td>
<td>80%</td>
</tr>
<tr>
<td>SEDEX</td>
<td>17.7%</td>
<td>17%</td>
<td>15%</td>
</tr>
<tr>
<td>Massive Sulfide</td>
<td>0.6%</td>
<td>13%</td>
<td>5%</td>
</tr>
</tbody>
</table>

For example, as implied in Tables 6 and 7, data fusion has been concluded that with a probability more than 99%, Angouran’s type is Mississippi Valley. Therefore, it means that certainly, genesis is Mississippi Valley type and uncertainty in decision-making nears zero. As a result, in the detailed exploration procedure, the patterns of Mississippi Valley type must be utilized.

On the other hand, the introduced procedure might help to recognize the belts of lead and zinc with the geneses MVT, SEDEX, and Massive Sulfide. This will lead to useful prospecting patterns. Similar procedure could be developed for Porphyry-type Copper deposits, Manto-type Copper deposits, Iron ore deposits, etc.
7. Conclusion

Identification of the ore deposit genesis, which is important in optimization of exploration activities, is a challenging decision in economic geology. There are usually evidences for known genesis; similarity between those evidences and field observations help to identify ore deposit genesis. The scientists have often different ideas about genesis of a certain deposit. This uncertainty makes the exploration activities rather costly with disparity in classification outcome. 

In the current paper, Bayesian Data Fusion (BDF) was introduced and applied to achieve a unique genesis type for a deposit based on various stages of exploration datasets. A schematic problem was designed to show how BDF works and how it helps to discriminate between various possible genesis. Angouran’s challenge matched with the designed problem in order to make a well-defined issue. The results obtained show that data fusion amplifies the deposit belonging probability to a genesis and attenuation of other types. Therefore, it helps to decrease the uncertainty associated with knowledge of scientists’ judgments, and further helped enormously in identification of a deposit.

References


چکیده:
شناسایی زئین ذخایر معدنی، یکی از پرسش‌های اقتصادی بسیار حائز اهمیت است. در این مقاله، روش ترکیب اطلاعات بیزین برای رسیدن به اطمینان بیشتر در مورد شناسایی ژئن‌های معدنی و کاهش عدم قطعیت تشخیص زئین سونگون از اثر ویژگی‌های بررسی‌پذیر و نسبی آن مورد بررسی قرار گرفته است. بنابراین، در این مقاله، روش ترکیب اطلاعات بیزین برای کاهش عدم قطعیت در شناسایی و تشخیص زئین ذخایر معدنی به‌کارگیری گردیده است.

کلمات کلیدی: ترکیب داده، تصمیم‌گیری، زئین، اقتصادی، شناسایی، کاهش عدم قطعیت.