



## Application of Sequential Gaussian Conditional Simulation to Underground Mine Design Under Grade Uncertainty

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### Keywords

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*geostatistical simulation*

*heuristic algorithm*

*stope layout optimizer*  
*3D*

### Abstract

In mining projects, all uncertainties associated with a project must be considered to determine the feasibility study. Grade uncertainty is one of the major components of technical uncertainty that affects the variability of the project. Geostatistical simulation, as a reliable approach, is the most widely used method to quantify risk analysis to overcome the drawbacks of the estimation methods used for an entire ore body. In this work, all the algorithms developed by numerous researchers for optimization of the underground stope layout are reviewed. After that, a computer program called stope layout optimizer 3D is developed based on a previously proposed heuristic algorithm in order to incorporate the influence of grade variability in the final stope layout. Utilizing the sequential gaussian conditional simulation, 50 simulations and a kriging model are constructed for an underground copper vein deposit situated in the southwest of Iran, and the final stope layout is carried out separately. It can be observed that geostatistical simulation can effectively cope with the weakness of the kriging model. The final results obtained show that the frequency of economic value for all realizations varies between 6.7 M\$ and 30.7 M\$. This range of variation helps designers to make a better and lower risk decision under different conditions.

### 1. Introduction

The usage of the term “risk” as a synonym for uncertainty is not right because its definitions are not the same. Ross [1] has stated that the risk (or chance) can be described as the probability that a discrete event will or will not occur. The risk is denoted by a single probability estimation. In contrast, uncertainty denotes the inability to estimate a value exactly. Uncertainty can be denoted by a continuous distribution that defines a range of estimates and the likelihood of occurrence of an event. Therefore, the risk and uncertainty should be understood and managed during all stages of an investment such as a mining project.

In today’s mining operations, particularly in underground mining, the kriging method is the most dominant approach used to estimate ore resources. Hence, this technique is used to

produce a block model and the subsequent operations such as determination of underground stopes and production scheduling. On the other hand, this method leads the mining operations to several shortcomings, i.e. (i) a single NPV can be obtained, (ii) the uncertainty in numerous responses such as NPV, dilution, production rate, etc. is not accessible through this method, and (iii) the quantification of the influence of the smoothing effect of the kriging method is too problematic. In a mining project, the sources of uncertainty such as ore reserve, metal grade distribution, and metal price must be thoroughly examined to make a better decision; otherwise, the project may face serious financial calamities. A number of gold mines could not produce pre-defined gold from 1983 to 1987 because of inaccurately estimated grades [2]. A majority of

mining companies failed due to overestimation of ore grade in the South African mines [3]. In order to resolve the mentioned problems, conditional simulation with considering uncertainty during ore-body grade estimation can be useful and increase the accuracy of the project design [4, 5]. A number of researchers have tried to determine the influence of grade uncertainty in the open-pit mine limit and production planning. Smith and Dimitrakopoulos [6], and Kumral and Dowd [7] have studied the impact of geological uncertainty on open-pit short-term production scheduling using conditional simulation. Ramazan and Dimitrakopoulos [8] have used the integrated conditional simulation and Stochastic Integer Programming (SIP) to maximize Net Present Value (NPV) for a long-term production scheduling. Koushavand *et al.* [9] have presented a Mixed Integer Linear Programming (MILP) formulation, considering grade uncertainty to obtain maximum NPV during long-term mine planning in northern Alberta. Many researchers have studied the influence of grade uncertainty in open-pit mines [10-13]. Nevertheless, a few works have been carried out related to the impact of grade uncertainty on underground mining limit and production scheduling. Gerico and Dimitrakopoulos [14, 15] have developed a probabilistic mixed integer programming (MIP) formulation to determine the size and location of stopes and pillars in sub-level stope mines under pre-defined risks and grade uncertainties. Both the open-pit and underground mine plannings using only one of the estimation methods for the entire ore-body may cause a serious deviation from the target. In this paper, in the first step, a comprehensive review of the underground stope layout methods and algorithms are presented. Then a new heuristic algorithm and a computer program are introduced. After that, an underground copper deposit is selected in order to determine the stope boundaries under grade uncertainty using geostatistical analysis and conditional simulation.

## 2. Underground stope Layout determination

In 1977, the first algorithm was developed by Riddle [16] for optimization of underground mining limit to design the block-cave mine layout. Putting some blocks with negative values as pillars to provide separate stopes is the main difference between this algorithm and Dynamic Programming (DP). The algorithm was written in FORTRAN and implemented on hypothetical economic block models in a 2D space. Deraisme

*et al.* [17] have implemented the downstream geostatistical approach in a uranium underground mine. They used large panels considering non-linear geostatistics to analyze the grades of mining units. This method is able to consider slope angle, which is ignored in most of the proposed methods. Octree division algorithm was developed by Cheimanoff *et al.* [18]. The algorithm identifies mineable volumes using the data obtained by drillholes, geostatistical data, and mineral forms in a 3D space. This algorithm is contained in the GEOCAD [19] package but its implementation on a well-known block model has not been reported. The main drawback of this algorithm is that the minimum dimensions of blocks, which have a lower amount of minerals, are included in the final limit. This issue affects the overall profit of an operation due to the existence of several waste blocks [20]. The floating stope method has been developed by Alford [21]. During the process of this algorithm, a stope with pre-defined dimensions is floated within the block model in a 3D space, and finally, the underground mining limit can be generated based on an objective function, which can be defined as the highest tonnage, economic value or ore grade. This algorithm is available on the CAE Studio DATAMINE [22] software, and the optimality is not guaranteed due to the existing overlapping stopes in the area. Ovnicek and Young [23] have developed the Branch and Bound technique using the Mixed Integer Programming (MIP) technique in combination with the piecewise linear function as a method for stope design. In this algorithm, the boundaries of extraction stopes are determined by exploring the starting and ending points in a defined row. There are a lot of softwares available to solve the mathematical problems, especially the branch and bound techniques including the LINGO [24], GAMS [25], and CPLEX packages [26]. Mirzaeian and Ataee-pour [27] have used the GAMS software tool to optimize the underground stope geometry on numerical examples. In addition, the MPS [28] software usage for stope layout optimization in the Pea Ridge iron ore mine has been reported in 1995 [23]. Ataee-pour [29] has proposed the maximum value neighborhood (MVN) algorithm. The fundamentals and basic concepts of this algorithm are the same as the floating stope method. This method uses the concept of the neighborhood with the highest value of production. However, this algorithm that eliminates the drawbacks of the floating stope method and has been considered as one of the most popular methods in underground

mining industry has two main drawbacks: (i) Moving the starting location of evaluation alters the set of stope layouts generated from the same orebody, (ii) Blocks that are examined earlier in the process are given a preferential treatment. Ataee-pour [30] has developed a computer program named SLO<sup>1</sup> for stope limit optimization in a 3D space. In order to improve and facilitate the procedure of the floating stope method, Cawrse [31] has suggested the Multiple Passes Floating Stope Process (MPFSP). In this method, the input data including the maximum grades, cut-off grade, and maximum tonnage of wastes are defined by the user. Then after producing several stopes, the final statistical results are saved in an excel format (CSV). However, selection of the final stopes has been facilitated by this method, although it does not guarantee the optimum limit. Jalali and Ataee-pour [32], and Jalali *et al.* [33, 34] have proposed a 2D and mathematical logic algorithm called OLIPS<sup>2</sup>. In this algorithm, the dynamic programming (DP) method is used to find the underground optimum limit. One of the most important characteristics of this algorithm is that it is run on a special block economic value called integrated probable stope model. Based on this algorithm, a computer program called SBO<sup>3</sup> has been developed and validated by hypothetical models. In addition to the OLIPS algorithm, the GOUMA<sup>4</sup> algorithm has been presented by Jalali *et al.* [35]. This algorithm can be implemented for a variable value economic model (VVEM), and is able to solve large-scale problems in a 2D space. They introduced a computer program named GOUMA-CP, designed in the C++ language, and applied to optimize the underground mining area of a gold mine in Australia. Grieco and Dimitrakopoulos [36] have developed a probabilistic MIP model to determine the stope and pillar design based on the geological uncertainty and pre-defined risks. In this method, at first, the block model is divided into several layers. Next, each layer is separated into several panels and rings. Every ring is defined as a binary variable in mixed integer programming, whose objective function is to maximize the metal content in an entire pre-defined time period. In addition, the solution time of this method depends on the number of variables in the complex mixed integer programming model, which can limit its

application in a real industry operation [37]. Jalali and Hosseini [38] have introduced a greedy algorithm to determine the optimal stope layout. The algorithm's logic follows a searching method on a graph model corresponding to an economic block model, and is solved using Dijkstra [39], as a powerful solver. Topal and Sens [40] have proposed a methodology to find the best and profitable stope layout for an entire block model using the MATLAB software considering the economic values of the blocks. The main disadvantage of their algorithm was the elimination of several stopes with lower values, while removing the overlapping stopes. Bai *et al.* [41] have proposed an algorithm based on the graph theory to optimize the underground mining limits. This method is only applicable to the sub-level stoping mines. In this method, a raise is defined and each block can be expressed in the cylindrical system ( $r, \theta, z$ ). The final result can be obtained by establishing the best position of the raise and vertical extent. The main defect of their algorithm is that it is limited to the underground mines, which are extracted by the sub-level stoping method. Sandanayake *et al.* [42, 43] have proposed an algorithm based on heuristics considering various possible stopes to optimize the underground stope layout. The results of their implementation on a real copper deposit showed 10.7% more valuable solution than the MVN algorithm. Nikbin *et al.* [44] have proposed a new hybrid algorithm, which is a combination of the dynamic programming and the greedy algorithm. They implemented their algorithm on an actual case study. The results obtained by the new algorithm was compared with the previous algorithms including the floating stope, MVN, and greedy algorithm. Even though their proposed algorithm is not able to provide a truly optimum solution, it has been able to find a higher profit compared to the mentioned algorithms.

### 3. Proposed heuristic algorithm

In order to determine the underground stope layout, a heuristic algorithm developed by Sandanayake *et al.* [42] was utilized. In order to facilitate the implementation of this algorithm with some strategies, which will be explained in the following, a user-friendly interface (UI) computer program (Figure 1) was developed in the C# programming language [45] named Stope Layout Optimizer 3D (SLO3D) [46]. This computer program has three main steps:

<sup>1</sup> Stope Limit Optimizer

<sup>2</sup> Optimum Limit Integrated Probable Stope

<sup>3</sup> Stope Boundary Optimizer

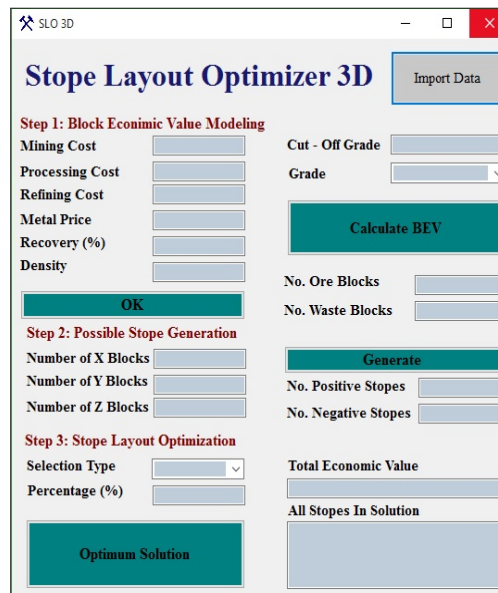
<sup>4</sup> Global Optimization for Underground Mining Area

1. Creating the Block Economic Value (BEV)
2. Generating the possible stopes
3. Determination of the final underground stope layout

In the following, a comprehensive explanation of SLO3D is presented.

**Table 1. Summary of the proposed algorithms in the underground stope layout optimization.**

Year	Author	Algorithm	Mining Method	Mathematical Formulation	Dimension
(1997)	Riddle [16]	Dynamic Programming (DP)	Block caving	Yes	2D
(1984)	Deraisme et al. [17]	Geostatistical Approach	Sub-level Stopping Cut & Fill	No	2D
(1989)	Cheimanoff et al. [18]	Octree Division	All	No	3D
(1995)	Alford [21]	Floating Stope	All	No	3D
(1999)	Ovanic & Young [23]	Branch and Bound	All	Yes	1D
(2000)	Ataee-pour [20]	Maximum Value Neighborhood	All	No	3D
(2001)	Cawrse [31]	Multiple Pass Floating Stope Process	All	No	3D
(2007)	jalali et al. [33]	OLIPS	All	yes	2D
(2007)	Grieco and Dimitrakopoulos [36]	Mixed Integer Programming	Sub-level Stopping	Yes	2D
(2008)	Manchuk and Deutsch [47]	Simulated Annealing	Sub-level Stopping Cut & Fill	No	3D
(2009)	Jalali and Hosseini [38]	Greedy	All	Yes	2D
(2010)	Topal and Sens [40]	Heuristic	All	No	3D
(2013)	Bia et al. [41]	Network Flow Method	Sub-level Stopping	Yes	3D
(2015)	Sandanayake et al. [42]	Heuristic	All	No	3D
(2016)	Jalali et al. [35]	GOUMA	All	Yes	2D
(2018)	Nikbin et al. [44]	DP and Greedy	All	Yes	3D
(1997)	Riddle [16]	Dynamic Programming (DP)	Block caving	Yes	2D



**Figure 1. Stope Layout Optimizer (SLO3D).**

### 3.1. Creating block economic value

A grade block model with regularised dimension as the input file must be created in the Microsoft Excel format before running the program. The input file structure consists of seven major columns similar to a spreadsheet shown in Figure 2 including the coordinate of a regularised block (XC, YC, and ZC), block dimension (XINC,

YINC, and ZINC), grade of a block (percent or gram per ton), value of a block (BEV (\$)), density (ton/m<sup>3</sup>), total weight, and metal weight of every block. After importing the input file by clicking on the specified button named 'Import Data', the user should import the economic factors such as the mining cost (C<sub>m</sub>), processing cost (C<sub>p</sub>), refining cost (R), metal price (P), grade (g),

weight of a block (T), and recovery (Y). Then with the defined cut-off grade, the blocks below the cut-off grade are termed as the waste blocks. In the “Grade” combo box, there are two options for definition of ore grade, in which the first one is related to metals where the grade is defined in percent (%) and the second one is for metals that are defined as grams/tonne. After clicking on the

“Calculate BEV” button, the grade block model is converted into an economic model based on Equation 1 and the number of ore and waste blocks with consideration given to the cut-off grade all reported.

$$\left\{ (P - R) \times G \times Y \times (C_m + C_p) \right\} \times T \quad (1)$$

Block	XC	YC	ZC	XINC (m)	YINC (m)	ZINC (m)	Grade (%)	density (ton/m3)	BEV (\$)	Block Weight	Metal Weight
1	3348413	223248	1525.5	5	5	5	0.0192559	2.54			
2	3348413	223248	1530.5	5	5	5	0.0409784	2.54			
3	3348413	223248	1535.5	5	5	5	0.021165	2.54			
4	3348413	223248	1540.5	5	5	5	0.0099024	2.54			
5	3348413	223248	1545.5	5	5	5	0.021478	2.54			

Figure 2. Input file structure before running program (Excel format).

### 3.2. Generating possible stopes

In the second step, the algorithm produces all possible stopes for an entire ore body. By clicking on the “Generate” button, the algorithm specifies the origin of the economic block model. Then assuming that the block increases in the size increment in the X, Y, and Z directions as the stope dimension parameters, the last stope block is determined. The constructed stope is floated through the economic model, and all possible stopes are generated based on the conditions shown in Equations 2 and 3 [42].

$$(i', j', k') \leq (i, j, k) \leq (i'', j'', k'') \quad (2)$$

$$(i' \leq i_{\max}, j' \leq j_{\max}, k' \leq k_{\max}) \quad (3)$$

where (i' j' k') and (i'' j'' k'') are the origin and last block of a stope, respectively. (imax jmax kmax), as shown in Equation 3, is the maximum stope block (i j k) within the economic model. Finally, the average grade and the economic value of each stope are calculated. The output file structure are calculated. The output file structure consists of seven major columns including (Figure 3): Stope ID, Stope Dimension in the X, Y, and Z directions (XINC, YINC, and ZINC), Stope Grade (percent), Stope Economic Value (SEV), Total Weight, Metal Weight and identifier of origin, and the last block of each possible stope. Finally, based on the SEV column, the number of positive and negative stopes are reported in SLO3D.

Stope ID	XINC(m)	YINC(m)	ZINC(m)	Grade (%)	SEV (\$)	TotalWeight (ton)	MetalWeight (gram)	X-Origin	Y-Origin	Z-Origin	X-Last	Y-Last	Z-Last
1	15	15	15	0.45056	-2320.9109	8572.5	38624240.76	1	1	1	3	3	3
2	15	15	15	0.425464	-23862.393	8572.5	36472888.57	1	1	2	3	3	4
3	15	15	15	0.363886	-61263.955	8572.5	31194117.89	1	1	3	3	3	5
4	15	15	15	0.521916	36243.4358	8572.5	44741242.88	1	1	4	3	3	6
5	15	15	15	0.451764	-7337.854	8572.5	38727436.04	1	1	5	3	3	7

Figure 3. Output file structure after generating all possible stopes.

### 3.3. Determination of final underground stope layout

In the first step, all possible stopes with positive economic values, generated in the previous step, are selected and imported as an input file for this step. In order to determine the optimum location of the underground stopes, all possible sets of the non-overlapping stopes are generated. Two major null families of sets ST and SE are created. ST is all possible sets of non-overlapping stopes that are generated during the algorithm and SE is a unique derived set of ST. During the algorithm process, each stope is compared with all stopes within any

set of non-overlapping stopes (SP). If the imported stope does not overlap with other stopes, all stopes are combined and a new set of non-overlapping stopes is created (SPnew). While the algorithm is iterated once, all sets are inserted in a new set called SO. This process is iterated until all positive stopes participate in the algorithm. Finally, the highest value of the non-overlapping stopes is selected as the optimum solution. The steps of this algorithm can be found in Figure 4.

In large-scale problems, the sets of non-overlapping stopes (SP) increase dramatically.

Thus the solution time increases as well. Three strategies are added to the presented algorithm in order to overcome this problem. The first strategy is the sorting of all sets ( $S_T$  members) according to their economic value to the lowest value, and selects a percentage of the sorted collection. The major drawback of the first strategy is removing some stope sets with a low economic value, while this strategy discards the possibility of a combination of removed sets and other stopes that may be a new set with a higher total value. Due to this disadvantage, two strategies with probabilistic backgrounds are proposed and added to this algorithm. These two strategies are:

- Selecting a percentage of  $S_T$  members randomly and frequently.
- Selecting a percentage of  $S_T$  members based on the number of stopes in each set.

In SLO3D, these strategies are covered in the “selection type” combo box. If the user does not choose any condition, the algorithm will analyze all possible sets of stopes, which can increase the solution time in very large problems. On the other hand, for instance, by choosing the first strategy in this section, the number of sets ( $S_T$  members) can be varied based on the economic value of each set. To select the number of stopes, the user can define the percentage using the “% of stopes”. Finally, the total economic value of the underground mining layout and identification of stopes, and the entire underground layout are reported.

#### 4. Simulation background

Numerous conventional methods are available for estimation of the regional variable such as ore grade by the least square based methods. Ordinary kriging (OK) and simple kriging (SK) are developed in this manner. The major drawback and inaccuracy of these methods is bias conditionality due to some underestimation and overestimation [48]. Unlike these approaches, the geostatistical simulation methods produce models on a detailed scale to simulate the spatial and statistical characteristics of an ore deposit [49]. Furthermore, the geostatistical simulation methods do not suffer from the smoothing effect and aim to model the *in situ* spatial variabilities. Therefore, these methods can be used for uncertainty analysis and risk quantification [50].

Among the simulation methods, Sequential Gaussian Simulation (SGS) is the most widely

used and efficient method for simulating a multivariable field in the mining industry [51-54]. The simplicity and flexibility of this algorithm are the most important reasons that make it popular [55, 56]. In the SGS approach, grade simulation is conducted over the Gaussian transformation of the measurements. Therefore, any simulated grade value is conditional on the regional data and every location is randomly selected from the normal conditional cumulative distribution defined by kriging mean and variance. All simulated grades are dependent upon the previously simulated grade values. Finally, all simulated values are back-transformed to the original data. The basic steps in the SGS algorithm are listed below [57]:

1. Calculate statistical and histograms of parameters.
2. Transform the data to a Gaussian distribution.
3. Compute and model variogram of transformed data.
4. Define the coordinates of blocks.
5. Choose a random path.
6. Estimating a value using the kriging method (known and simulated).
7. Draw a value at random from the Gaussian distribution, which is known as the simulated value.
8. Proceed to the next node and simulate sequentially.
9. Repeat steps till all nodes have been simulated.
10. Back-transform all simulated values.

#### 5. Case study

In order to incorporate the grade risk quantification in the determination of underground stope layout, a real copper deposit, located in the north-west of the Zahedan Province (Iran) was selected. The data on this area was obtained from 35 drill hole samples. The core logs contain the assay, geological description, azimuth, and dips of boreholes. The assay data of these boreholes shows the main metal with an average of 0.57% Cu and 5.75% Cu as the high grade. In this work, a part of this area (a vein in the N23°W direction) that has enough exploration data was considered as the main studied area. This area is a vein with a thickness of 20 m with a spanning 400 m in the longitude direction and reaching 100 m in the vertical direction. The datamine software was used to generate the ore body model.

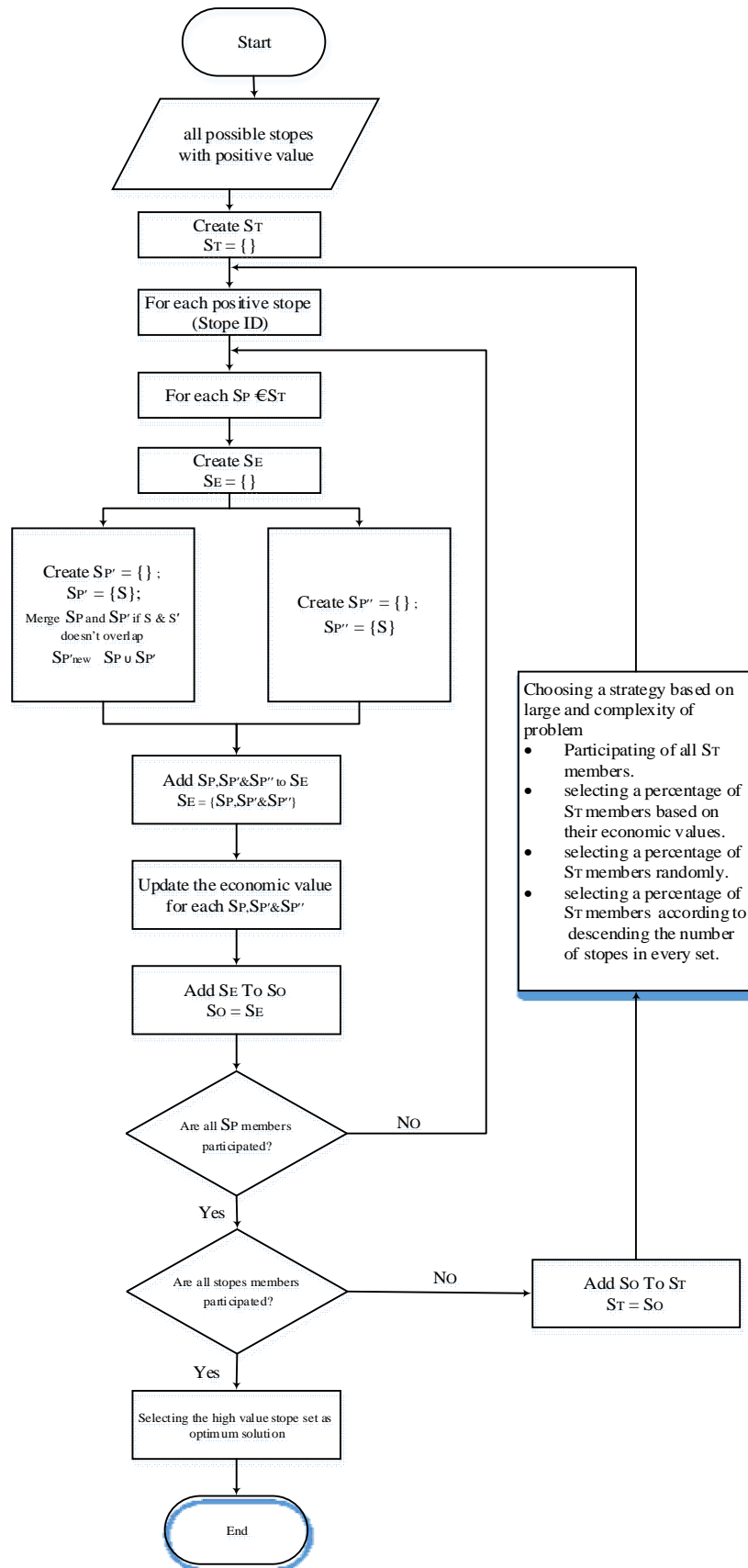


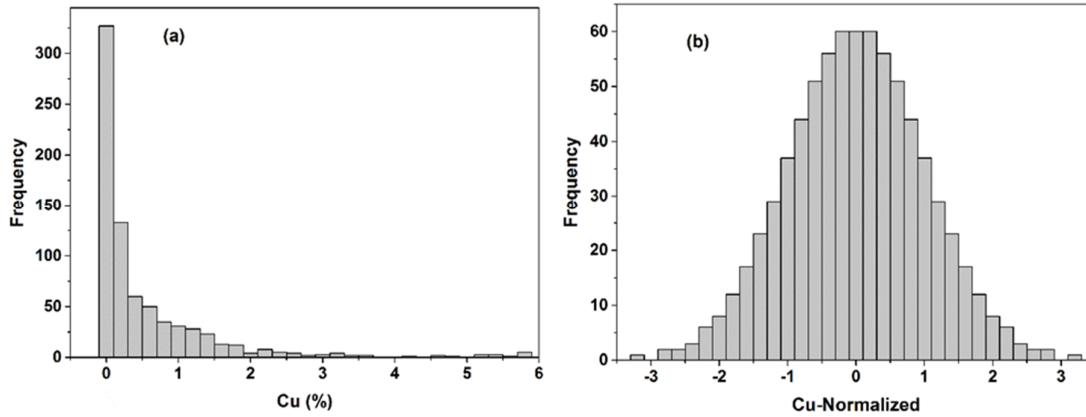
Figure 4. Revised stope layout optimization algorithm after Sandanayake et al. [42].

**6. Geostatistical modeling**

**6.1. Variogram modeling**

Variogram modeling and determination of anisotropic ellipsoid are the primary factors involved in the geostatistical analysis. In the first step, statistical analysis was carried out for grade

data. This analysis shows that the grade distribution in the studied area has a significant positive skewness as the histogram is skewed to the right (Figure 5a). Table 2 shows the statistical parameters of the exploration data after applying a compositing length of 5 m.



**Figure 5. Histogram of composite copper data; (a) original data, (b) normalized data.**

**Table 2. Summary statistics of the composite data.**

Number of samples	Average grade (Cu %)	Min. (%)	Max. (%)	Median	Variance	Std. Dev.	Kurtosis	Skewness
762	0.562	0.0005	5.75	0.161	0.911	0.954	11.842	3.153

Most statistical methods require assuming normality of data. Thus only the dataset with a normal distribution can be used to perform variogram modeling and simulation; otherwise, the geostatistical calculation will have systematic errors. In this work, using the S-GeMS [58] software, the row dataset is transformed to a normal distribution with the variance of one and the mean value of zero (Figure 5b). Table 3 shows the statistical parameters of the exploration data after normalization. Variography was carried out to provide the copper variability of the area. In order to identify the anisotropy axes, the

directional variograms were drawn. Due to the different range of variograms and equal sill, the anisotropy is geometric. Variograms for the directions of 30°, 90°, and 120° were performed and studied carefully. Figure 6 shows the directional variograms of the copper grade in different directions. The red points and the solid line represent the experimental and the theoretical variograms, respectively. According to the variography results, anisotropy is in the directions of N30°E and N60°W. Table 4 shows the parameters of variogram modeling.

**Table 3. Summary statistics of the normalized data.**

Number of samples	Min. (%)	Max. (%)	Mean	Std. Dev.
762	-3.213	3.213	0.00	1.00

**Table 4. Parameters of spherical model.**

Azimuth	Variogram		Model	Nugget effect	Sill	Range (m)
	Dip	Tolerance				
30	70	15	Spherical	0.2	0.8	69
120	70	15		0.4	0.5	54
Vertical	90	90		0.2	0.79	60



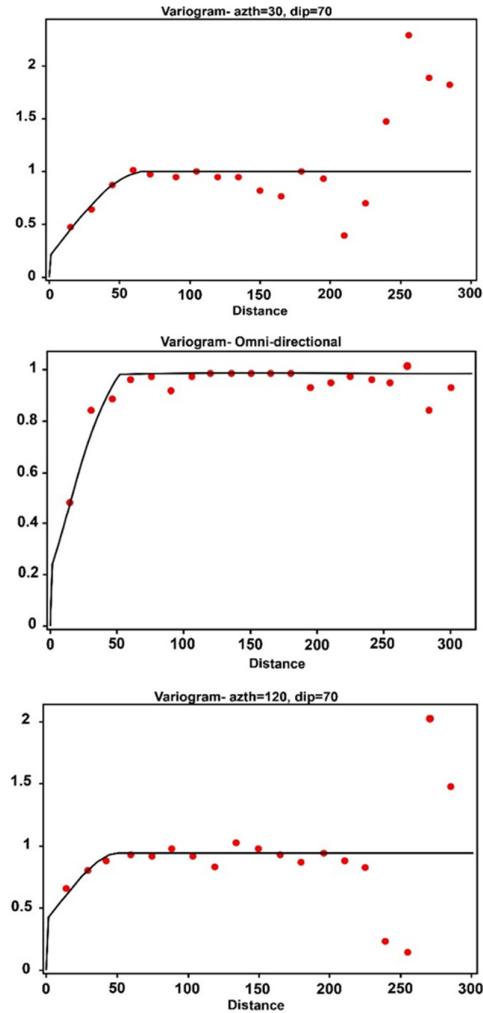


Figure 6. Directional variograms of copper grade.

**6.2. Copper value simulation**

In the second step, a 3D block model of the studied area was constructed. Since the sequential gaussian simulation is most widely used for obtaining grade distribution and considering that 50 realizations are enough to reach a reliable

result [36], this kind of simulation was used to produce 50 probable copper values for each unit of the entire grid block model. Figure 7 shows the horizontal plan of four randomly selected realizations.

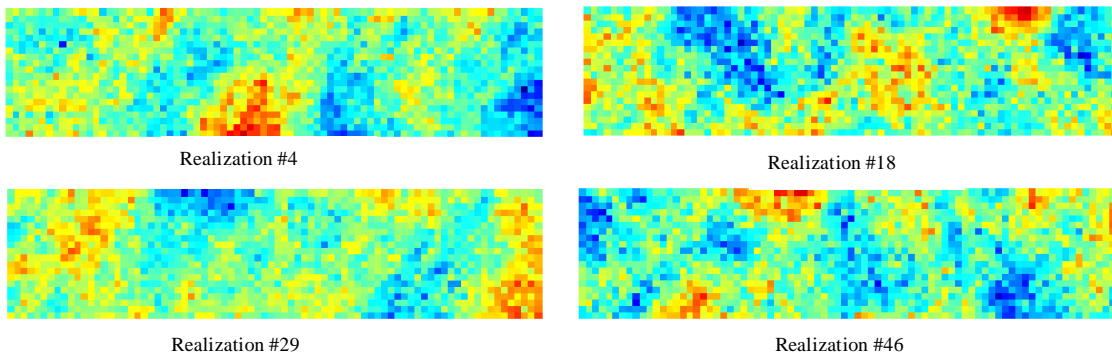


Figure 7. Horizontal plan of four randomly selected realizations.

### 6.3. Validation of simulation results

Validation of the simulated realizations must be undertaken by reproduction of histograms and variogram models, and it should be very similar to the original data features [59]. Figure 8 shows the matching of histogram reproduction of a randomly chosen from 50 realizations. Also to

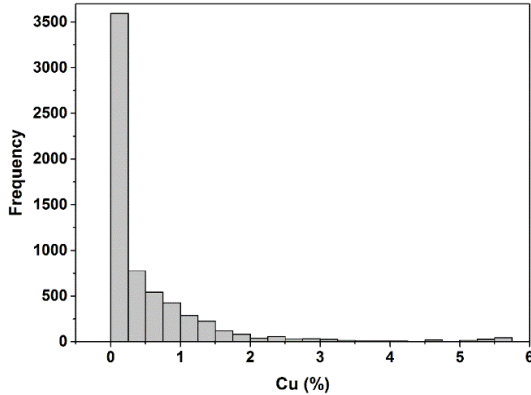


Figure 8. Histogram reproduction, randomly selected from 50 realizations.

### 6.4. Grade-Tonnage curves

One of the important factors involved for the investment of a mining project is a determination of risk in grade-tonnage curves that can be achieved through geostatistical simulation. By comparing these estimation and simulation curves, the effect of the uncertainty on the estimated model and average grade will be observed. Figure 10 shows the plotted grade-tonnage curves of all the 50 simulated models and kriging estimation method with different cut-off grades. The ore reserve tonnage is higher at lower

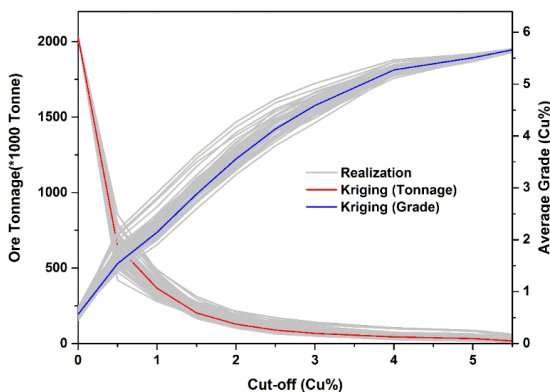


Figure 10. Grade-tonnage curves for 50 realizations.

## 7. Determining optimum stope layout

The mining method in this ore body is longitudinal stoping, a method similar to the sub-level stoping method. In longitudinal stoping, the

check the simulation performance, the variograms for 5 realizations were reproduced and compared with the variogram of the original dataset (Figure 9). The results obtained show that all realizations reproduce the histograms and variograms reasonably well.

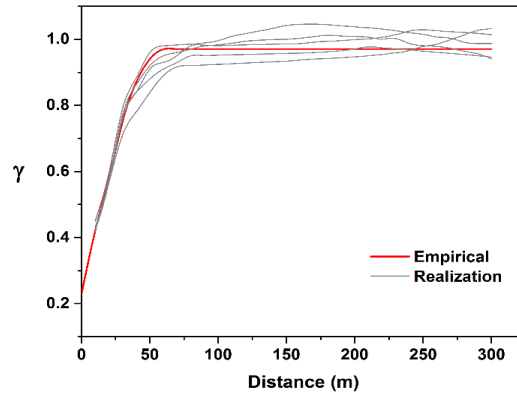


Figure 9. Empirical variograms for real data and five randomly selected realizations.

cut-off grades, and the tonnage obtained by the kriging method is in the middle of all simulated curves that can affect the economy of the mining project. Therefore, the related risk is more critical at lower grades. Since in this work the cut-off grade was 0.52%, the frequency of the ore reserve for all the 50 realizations was drawn (Figure 11). Based on this distribution, the ore reserve is between 453,000 and 799,000 tonnes with 95% confidential level. The ore reserve in the kriging method is 644,525 tonnes .

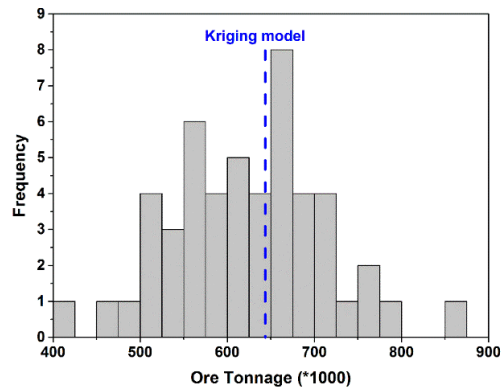


Figure 11. Ore reserve distribution for SGS-real data (cut-off = 0.52%).

direction of mining is in the same way as the sub-level stoping along the strike of the orebody (longitudinal direction). This method is designed for ore bodies with a thickness in the range of

circa 5-20 m [60]. Determination of stope dimension in these methods in most cases can be achieved by designing the stopes having high vertical and short horizontal dimensions or stopes having short vertical and long horizontal dimensions [61]. For this work, the stope dimensions are considered to be 50×20×25 m (10×4×5 m blocks in the X,Y,Z direction). Then considering the economic factors given in Table 5, all 50 realizations and kriging model were converted to block economic value incorporating

copper grades. Then 1136 possible stopes were generated for each model. After that, according to the proposed algorithm, the optimum layout, average grade, and metal content of all realizations, and the kriging model were obtained. For the kriging model, 24 stopes were obtained using the algorithm with a value of 15.6 M\$, while this value for other two realizations (simulation 5 and simulation 17) was 14.7 M\$ and 21.8 M\$, respectively.

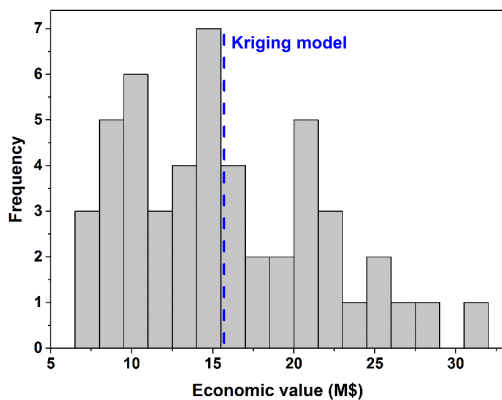
**Table 5. Economic factors.**

Parameter	Value
Mining cost (\$/tonne-ore)	20
Processing cost (\$/tonne-ore)	10
Smelting and refining costs (\$/tonne-metal)	90
Copper price (\$/tonne-metal)	6500
Recovery (%)	90
Cut-off (% Cu)	0.52

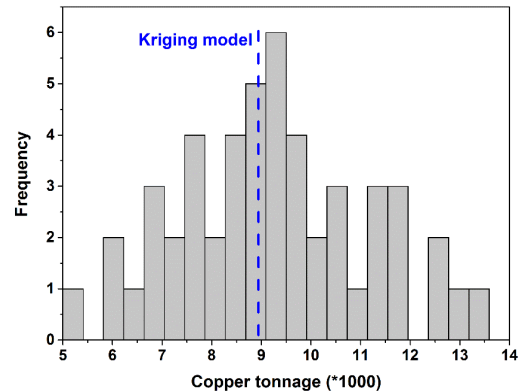
The economic value and metal content for each realization were calculated and studied carefully. Figure 12 shows the frequency of economic value for all realizations. It can be observed that the value varies between 6.7 M\$ and 30.7 M\$. The blue dash line shows the value of the kriging model. Figure 13 shows that according to the realizations, the minimum and maximum pure copper that can be extracted from the mine is between 5,227 and 13,583 tonnes, while this value for the kriging model is 8,971 tonnes. Moreover, in Table 6, details of the final underground limit for the realizations and kriging model are presented.

Gaussian Conditional Simulation approach generate a higher economic value as compared to the ordinary kriging. Hence, the results illustrate that ordinary kriging based underground mine planning can produce misleading outputs, which can lead to unrealistic expectations of net present value along with mine production scheduling, and so on. Since the traditional kriging methods rely on a single ore model assumed to be real deposit in the underground being mined, it is not able to evaluate the uncertainty of economic and operational consequences of the underground stope layout. On the other hand, the simulation methods give a great opportunity to describe the distribution of ore grade in each one of the realizations and quantification of uncertainty.

According to the results obtained in this section, it can be concluded that a large percentage of underground mining limits on the Sequential



**Figure 12. The economic value frequency for all realizations and kriging model.**



**Figure 13. The metal content frequency for all realizations and kriging model.**

**Table 5. Details of underground mining limit for SGS simulations.**

Parameter	SGS models			Kriging model
	Min.	Max.	Average	
Average grade (%)	0.53	0.88	0.67	0.71
Economic value (M\$)	6.74	30.71	15.86	15.62
Copper (tonne)	5257.6	13583.6	9250.4	8971.4
No. of stopes	12	30	22	24

## 8. Conclusions

Determination of the underground stope layout and production scheduling are the most important issues in regard to the underground mining. Numerous researchers have reported the influence of grade and economic uncertainty in determination of ultimate open-pit limit and production planning. In contrast, a few efforts have been made regarding the underground mine design under the grade uncertainty condition, which is one of the major components of technical uncertainty that affects the variability of the project. In this work, in the first step, according to a new heuristic algorithm, a user-friendly computer program called Stope Layout Optimizer 3D (SLO3D) was developed using the C# object-oriented programming in order to incorporate the influence of grade variability in the final stope layout in a 3D space. Subsequently, a set of stochastic realizations were produced to solve a practical engineering concern in a real copper vein, located in the south-west of Iran. The results of this research work showed that the ore reserve obtained by the kriging method was 644,525 tonnes, while this value varied between 453,000 and 799,000 tonnes with 95% confidential level for realizations. The economic value of underground stopes for all realizations yielded a value between 6.7 M\$ and 30.7 M\$, and for the kriging model, it was 15.6 M\$. The proposed methodology determined the underground stope layout over multiple geostatistical realizations rather than simply using geostatistical realizations in a post-processing framework to assess a layout based on a kriging model. According to the results obtained, it was observed that the performance of a mining project based on a single estimation method due to the inability to access uncertainty and risk quantification can influence the overall accuracy of the mine design and production planning, while simulation models obtained by geostatistical methods determined the range of

possible values instead of introducing a definite and unique existing value. In this way, it is possible to examine the risks and compare with the single estimation model. In this research work, the effect of grade uncertainty on the determination of underground optimum limit was studied.

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## تأثیر عدم قطعیت عیار در طراحی محدوده معدنکاری زیرزمینی با استفاده از شبیه سازی گوسی متوالی

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### چکیده:

در پروژه های معدنی، برای تعیین امکان سنجی باید تمام عدم قطعیت های مرتبط با یک پروژه در نظر گرفته شوند. عدم قطعیت عیار یکی از مؤلفه های اصلی عدم قطعیت فنی است که بر تغییرپذیری پروژه تأثیر می گذارد. شبیه سازی زمین آماری، به عنوان یک رویکرد معتبر، روشی است که به طور گسترده برای تعیین کمیت تحلیل ریسک برای غلبه بر اشکالات روش های تخمین مورد استفاده قرار می گیرد. در این مقاله، تمام الگوریتم های ایجاد شده توسط محققان برای بهینه سازی محدوده معدنکاری زیرزمینی بررسی شده است. پس از آن، یک برنامه کامپیوتری (SLO3D) براساس الگوریتم ابتکاری تهیه شده است. با استفاده از شبیه سازی شرطی گوسی متوالی، 50 شبیه سازی و یک مدل کریجینگ برای یک معدن مس زیرزمینی واقع در جنوب غربی ایران ساخته شده است. با استفاده از شبیه سازی کارگاه ها به صورت جداگانه انجام گرفت. مشاهده می شود که شبیه سازی زمین آماری می تواند به طور موثر با ضعف مدل کریجینگ مقابله کند. نتایج نهایی به دست آمده نشان می دهد که فراوانی ارزش اقتصادی برای کلیه دستاوردها بین 6/7 میلیون دلار و 30/7 میلیون دلار متغیر است. این دامنه تغییرات به طراحان کمک می کند تا در شرایط مختلف تصمیم بهتری در برابر ریسک کمتری بگیرند.

**کلمات کلیدی:** معدنکاری زیرزمینی، عدم قطعیت عیار، شبیه سازی زمین آماری، الگوریتم ابتکاری، SLO3D.

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