



## A Mathematical Model to Optimize Allocation Sequence in Dispatching Problem

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### Keywords

*Dispatch*  
*Dynamic allocation*  
*Simulation*  
*Genetic algorithm*

### Abstract

Truck-Shovel fleet, as the most common transportation system in open-pit mines, has a significant part of mining costs, for which an optimal management can lead to a substantial cost reduction. Among the available dispatch mathematical models, a multi-stage approach is well-suited for allocating trucks to the respected shovels in a dynamic dispatching program. However, with this kind of modeling, sequencing the allocated trucks is not possible, though it is important to find out the best solution to get the minimum cost. To comply with the shortcoming of the traditional model, in this work, a new hybrid model is developed and applied to the Sungon Copper Mine in Iran, in which, for each truck, an allocation matrix is considered as the input to the genetic algorithm implemented to determine the best solution. According to the results obtained, the optimal sequencing of the trucks can result in a significant (31%) cost reduction in a shift.

### 1. Introduction

Transportation is a vast part of the operating costs in surface mining, which should be decreased as much as possible for getting higher benefits [1]. The truck-shovel system is the most adopted way of handling mined materials in open-pit mines. In order to reduce the operating costs, a proper dispatching method has to be taught for the available fleet. In the dispatch problem, it is determined that each truck should move to which possible point. Also using an appropriate dispatching strategy, the total transportation costs including the truck traveling time, fuel consumption, idle time, and depreciation can be significantly reduced. Various solutions to the dispatching problem have been developed by different researchers. Rafael *et al.* have developed a mathematical model to maximize production with a minimum number of trucks [2]. Zhang and Xia have used an integer algorithm to determine the number of trips of each truck for minimizing the operational costs [3]. Chang *et al.* have considered revenue maximization as the objective function for the homogeneous fleet [4]. Kaveh

Ahangaran *et al.* have modeled the dispatch problem with the aim of minimizing the transportation cost for a heterogeneous fleet [5]. In evolution, Lutz has proposed a combined simulation and TS to deal with the transportation problem [6]. Fu and Healy have used the PA technique in the inventory models [7]. Faccenda and Tenga have presented an approach to incorporate the process of plant production operations into the design of a facility by combining simulation and genetic algorithm (GA) [8]. Morito *et al.* have applied a hybrid model simulation and simulated annealing (SA) to find an appropriate dispatching priority of operations [9]. Hill and Fu have applied simultaneous perturbation stochastic approximations (SPSAs) to a transportation problem [10]. By evaluation of the dynamic and fixed allocation, it proved that dynamic allocation is more efficient than fixed one [11-14].

In all of the above-mentioned models for dispatching a problem, the solving time is relatively high for which a multi-stage approach

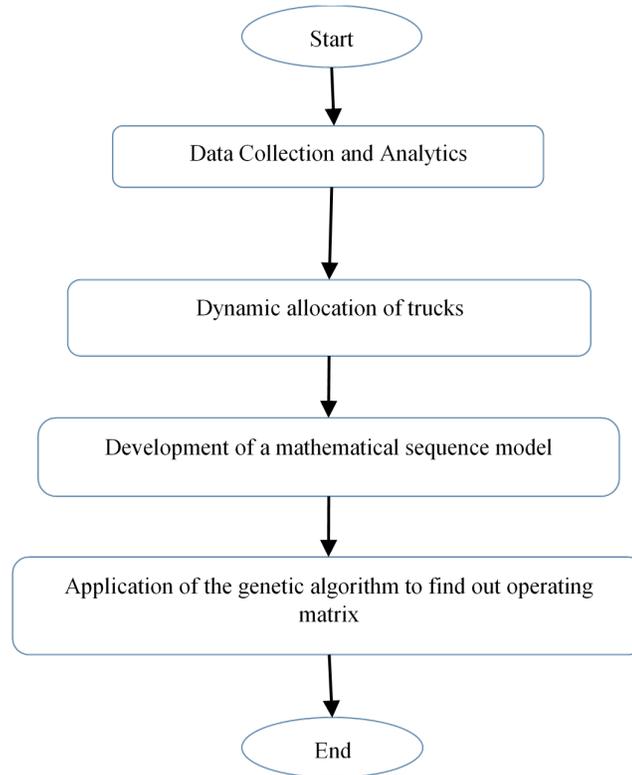
has been advised. Robert et al. have divided the dispatch problem into two stages of allocation and dispatching so that in the first stage, the maximum capacity of mine production and the optimal fleet are determined, and in the second stage, the simulation is applied to allocate trucks to shovels in a dynamic manner [15]. In the same way, Temeng and Otunye have solved the dispatch problem using a two-stage approach [15]. In a further research work, Gerrit and Subhash have employed a two-stage approach for solving a transportation problem. In the first stage, they determined the volume of rocks to be handled, and in the second stage, optimization was determined using integer programming [16].

In this work, a simulation-based mathematical model was developed for the heterogeneous fleet, in which an objective function of minimizing transportation cost was defined. A combination of the Zhang and Xia model with the proposed model could be used to perform the truck assignment in the form of a matrix. Since there are a huge number

of alternatives for truck sequences, a heuristic method such as GA has to be applied to find out the optimum sequence. The aforesaid multi-stage approach was determined to determine the best dispatching sequence in a real case study.

**2. Material and methods**

The algorithm for solving the dispatching problem with the aim of determining the operating matrix is shown in Figure. 1. As shown in this figure, in the first step, according to the Zhang and Xia model, the dynamic allocation of the trucks is determined. Using this approach, the truck number of trips between the loading and dumping points is determined, and finally, a displacement matrix is generated for each individual truck. For each truck, this matrix represents the number of trips for a specific route but the sequence of the trips is not known. However, it is clear that the dispatch sequence is effective in the production and costs. Therefore, a way should be thought of to find the best sequence to get the maximum benefit.



**Figure 1. A flowchart of the development model.**

For this purpose, in the second step, a new mathematical sequencing model was developed and considered as the cost function in GA. The developed model and its corresponding constraints are given in the Eqs. 1 to 8.

$$\min \sum_{s=1}^{N_s} \sum_{i=1}^{N_i} \sum_{t \in N'_i} \tilde{C}_t X^t_{si} + \tilde{C}'_t X'^t_{si} \tag{1}$$

$$\sum_{s=1}^S \sum_{i=1}^I Y^1_{k,0,j} = 1 \quad \forall k \tag{2}$$

$$\sum_{s=1}^S \sum_{i=1}^I Y_{k,s,i}^R = 1 \quad \forall k, R \quad (3)$$

$$\sum_{s=1}^S \sum_{i=1}^I Y_{k,s,i}^{R'} = 1 \quad \forall k, R' \quad (4)$$

$$Y_{k,s,i}^R \leq \sum_{i=1}^I Y_{k,s,i}^{R'} \quad \forall k, s, R' \quad (5)$$

$$\sum_{i=1}^I Y_{k,s,i}^{R'} \leq \sum_{i=1}^I Y_{k,s,i}^{R'-1} \quad \forall k, s, i \quad (6)$$

$$\sum_{r=1}^R Y_{k,s,i}^R = X_{k,s,i} \quad \forall k, s, i \quad (7)$$

$$\sum_{r=1}^R Y_{k,s,i}^{R'} = X'_{k,s,i} \quad \forall k, s, i \quad (8)$$

Wich in

$X_{k,s,i}$	Times of displacement of loaded truck k between shovel s and dumping point i
$X'_{k,s,i}$	Times of displacement of empty truck k between shovel s and dumping point i
$Y_{k,s,i}^r$	The binary variable, which equals to 1 if kth truck in r'th sequence of movement goes from the s'th shovel to the ith dumping points as a full type
$Y_{k,s,i}^{r'}$	The binary variable, which equals to 1 if kth truck in rth sequence of movement goes from ith dumping point to sth shovel points as an empty type
$\alpha_k$	Unit cost for kth truck in full mode per KM
$\beta_k$	Unit cost for kth truck in empty mode per KM
$C_k$	Capacity for kth truck by ton
$\bar{v}_k$	Average of speed for kth truck in full mode KM per hour
$\bar{v}'_k$	Average of speed for kth truck in empty mode KM per hour
$\bar{v}_s^{-l}$	Average of loading speed for sth shovel by ton per hour
$W_s$	Available waste and ore at sth shovels (ton)
$O_s$	Percentage of ore on near sth shovels (%)
$W'_i$	The demand for waste and ore at ith dumping points (ton)
$O'_i$	Percentage of ore required at i'th dumping point (%)
$\epsilon$	Accepted range for ore (%)
$d_{s,i}$	The distance between s'th shovels and ith dumping point
$W_s$	Average of loading speed for sth shovel by ton per hour

It is to be mentioned that to apply the developed model, initially, the assignment matrix should be determined from the Zhang and Xia model and then it should be considered as the input for this model. For converting the assignment matrix to the displacement matrix, GA can be applied. In this way, a simulation technique is used to determine the fitness of each solution generated by GA. The following steps have to be determined:

Step I: Randomly generate the initial population of chromosomes based on the result of the first stage for the dispatch plan. The jth chromosomes in the initial population for tth truck can be shown by  $P_{j,t}^0$  as:

$$P_{j,t}^0 = \begin{bmatrix} a & \dots & d \\ \dots & \dots & \dots \\ c & \dots & b \end{bmatrix}_{s,i} \quad (9)$$

Step II for each chromosome in the population, a simulation model is applied to determine the fitness function based on the objective function.

Step III If the stop condition is satisfied, stop and resume otherwise.

Step IV Create a new population by the operations of selection, crossover, and mutation.

Step V go to step II.

### 2.1. Procedure of GA

In order to make each chromosome from the first random generation, it is necessary to convert the assignment matrix of each truck to the operating matrix, which is done for each truck as below:

Step 1: Get an assignment matrix from the Zhang and Xia model.

Step 2: Define an empty displacement matrix.

Step 3: Finish the algorithm if there are no elements greater than zero in the assignment matrix.

Step 4: Randomly select an element greater than 0, which is considered as the next to the truck destination.

Step 5: Complete the primary displacement matrix by adding a new row, in which the first element is the row number of the selected intersection of the allocation matrix, and the second denotes the column number of the selected intersection of the allocation matrix, which is considered as a possible solution.

Step 6: The selected element is updated on the allocation matrix and is subtracted by one.

Step 7: Go to step three.

- Assume equal path lengths and zero discharge time and the following conditions:

- Two shovels with normal distribution of N (4.2) and N (5.3) for loading
- Two trucks with a normal distribution of N (10.1) and N (15.3) for a load capacity of one ton
- Two drainage points

With the Zhang's model solution, we will have:

First truck		Second truck	
1	$a$	1	$a$
2	$a$	1	$b$
2	$a$	1	$b$
1	$a$	1	$a$
2	$b$	2	$b$
1	$a$	2	$a$
1	$a$	2	$a$
1	$a$	1	$a$
2	$a$	1	$a$
2	$b$	2	$b$
1	$a$	2	$a$

After creating the initial population, a simulation model is used to determine the fitness of each solution. The simulation technique is implemented for traveling and loading time in both the empty and full trucks.

This generational process is repeated until a termination condition is reached. The common

terminating conditions are as follow: a solution is found that satisfies the minimum criteria, a fixed number of generations is reached, allocated budget (computation time/money) is reached, the highest ranking solution fitness is reached, and successive iterations no longer produce better results.

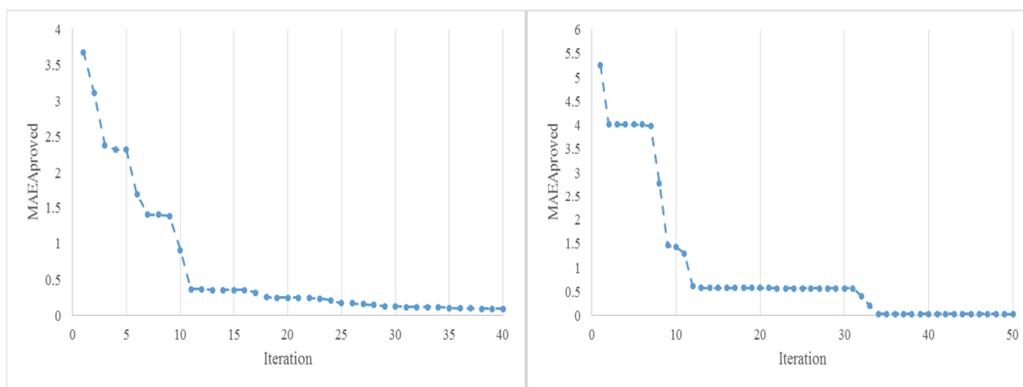


Figure 2. MAE approved with regard to appropriate iterations.

In GA, the crossover operation is used to create the second generation. In this operation, the parents are randomly selected using the roulette cycle; thereafter, the new parents (solutions) are compared with the matching matrix. The mutation operator is used to maintain the genetic variation

from one generation to the next. In order to determine the fitness of each one of the proposed solutions of GA, the simulation is repeated until the best answer is reached. The process of application of GA is shown in Figure 2.

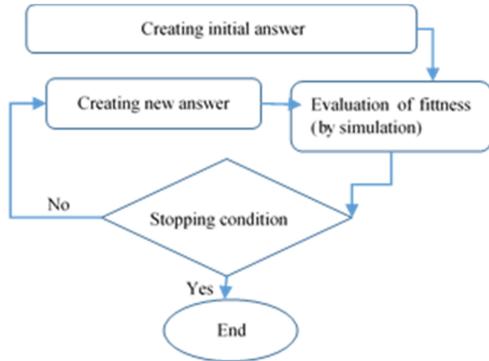


Figure 3. Process of application of GA.

**2.2. Case study**

The Songun Copper Mine is located in the East Azarbaijan Province, 130 km from Tabriz and 75 km from Ahar 30 km from Varzaghan. The mine operation takes place in three 8-hour shifts. A heterogeneous fleet with four different truck models including 12 Komatsu HD 785- 5, 16 Komatsu HD 325-6, one truck KOM HD 325-5, two TEREX, and a total of 5 Shovels. Also there are five operating shovels (i.e. two CAT 5090, one LIEBHERR-984, one LIEBHERR-992, and one Terex) in the mine. Note that in this work, there are 20 loading points for ore and 3 unloading points in

the executive year. Accordingly, for waste materials, there are 20 loading points and 12 unloading points. The proposed approach is implemented to obtain the best operating truck sequence matrix.

**3. Results**

The dispatching problem has three stochastic parameters including the movement time, loading time, and unloading time. In order to find out the probability distribution function for each one of the random parameters, the MINITAB software is used. Due to the multiplicity of routes and trucks, the distribution of the displacement time of each truck is not calculated; instead, the time of displacement of trucks in full and empty modes in all paths is clustered, and also instead of estimating the distribution of each truck's travel time in each path, the distribution time of the displacement in the clusters is computed.

Estimates of the statistical distribution of the movement time for each truck from each shovel to the unloading point and reverse are:

- The estimated statistical distribution of the load time for each shovel
- The estimated statistical distribution of the discharge time for each truck group

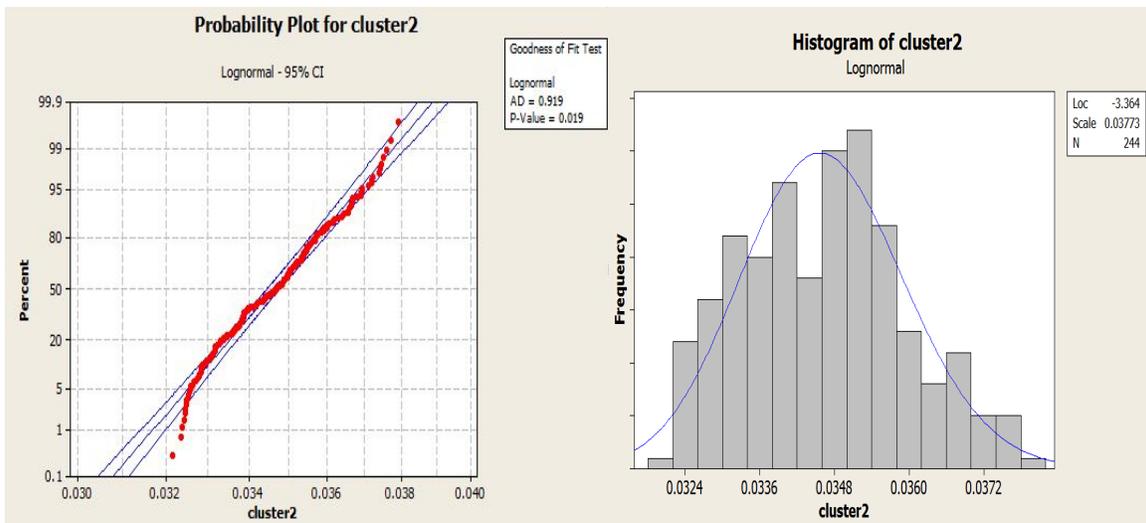


Figure 4. Probability and histogram graph for cluster 2.

Displacement distribution is estimated in four steps:

**Step 1.** At first, all records relating to the displacement time of each truck in each direction are collected in the full and empty modes.

**Step 2.** In each record, the time of displacement is divided by the route length so that the mean time of the unit length is obtained for each truck.

**Step 3.** Taking into account the average times of displacement of each truck in the full and empty parts for each route, the paths are clustered (using the k-means algorithm in the mini-tab software), as follows:

- First, the n cluster center is randomly determined.

- b) The distance between each record from the center of each cluster is determined.
- c) Each record is assigned to the closest center of the cluster.
- d) The cluster center is updated, and the average data for that cluster is replaced by the cluster.

In the clustering algorithm, steps c and d are repeated until no significant change is made in the center of the clusters.

Step 4. By considering the records for each cluster, the probability distribution function of the data is determined using the mini-tab software for each cluster.

After estimating the random parameters with GA and using the Zhang model results, the initial population is created, and then simulation based on the random parameters in accordance with the objective function of the model is determined.

The fitness of each solution is calculated by repeating the process of making a new generation until the condition of stopping GA is reached; the optimal sequence of truck dispatch is determined. The results obtained are given in the next section.

### 3.2. Application of proposed model

#### 3.1. Dynamic allocation of trucks

In the research literature, several articles have examined the optimization methods and have determined the optimal allocation of the truck and

shovel matrix, one of them is the Zhang's model, the result of which lead to the allocation matrix.

In this model, the assignment matrix is evaluated by optimization of the model by an objective function that minimizes the movement cost in the two types of loaded and empty conditions. The result of the model is an assignment matrix for each truck whose entries (elements) show the number of truck trips between two specified points as Eq. 10

$$x^1 = \begin{bmatrix} 5 & 4 & \dots & 3 \\ 0 & 0 & \dots & 6 \\ 4 & 0 & \dots & 0 \\ 0 & 0 & \dots & 10 \end{bmatrix} \quad (10)$$

Based on Eq. 10, the 1st truck must be moved 5 times between the first loading point and the first unloading point and 4 times between the first loading point and the second loading point.

#### 3.2. Determination of operating matrix using GA

By implementing the proposed model for extraction of minerals according to the planned level with the objective of minimizing the direct extraction costs, according to the results of the implementation of the model for 30 shifts, using the proposed model, the operational costs will be reduced by up to 31%.

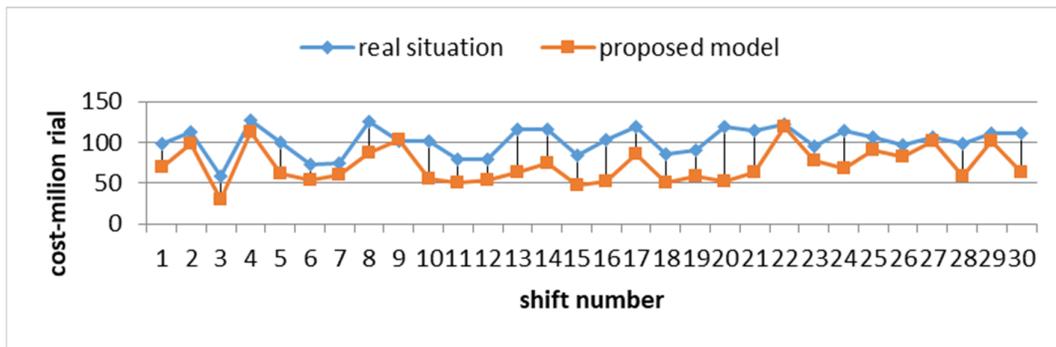


Figure 5. Comparing the cost of the real situation with the result of the proposed model.

## 4. Conclusions

Fleet management is one of the most important issues in my planning, which is dealt with in the dispatch problem. In this work, it was tried to determine the sequencing of truck displacement at the surface level through a multi-stage problem solution, use of dynamic allocation, and consideration of random parameters in the model. On the other hand, in solving the dispatch problem, there are generally various goals that are sometimes contradictory. In this case, the dispatch

problem was divided into two stages, the separation of the allocation process and the dispatch from one another such that in the first stage, the allocation process is followed, and in the second phase, using the simulation method, the optimal dispatch program for allocation of the first phase is done.

According to the results obtained from the implementation of the model on the true data of the Songun mine, the results obtained from the

implementation of the model are significantly better than the actual results.

Given the results obtained from the implementation of the model that take into account various extraction policies, and that in all cases significant achievements can be made, it can be argued that the targeted simulations along with dynamic deployment in mines can always be used as an alternative to enhance capabilities. The existing fleet can be considered by decision-makers in open-pit mines.

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## ارائه مدلی ریاضی جهت بهینه سازی توالی گسیل در مسئله دیسپچ

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### چکیده:

عملیات بارگیری و حمل و نقل به عنوان یک بخش مشترک در استخراج مواد معدنی اکثریت معادن روباز، نقش قابل توجهی در هزینه های تمام شده قیمت محصولات معدنی دارد. تعیین ترتیب و توالی حرکت تراکها که از جمله اساسی ترین و کاربردی ترین جنبه های مسئله گسیل در معادن است تاکنون مورد بررسی و تحقیق قرار نگرفته است. در این رساله مدلی ریاضی جهت بهینه سازی مسئله گسیل در چهار مرحله با در نظر گرفتن میزان تولید شاولها، تعداد تراکها، ماتریس تخصیص تراکها بین نقاط و تعیین توالی حرکت آنها ارائه می گردد. بدین معنی که میزان مواد جابه جا شده در هر مسیر و تعداد تراک مورد نیاز در هر شیفت تعیین، سپس میزان دفعات جابجایی هر تراک بین نقاط مشخص و در نهایت ترتیب توالی مراجعه تراکها به نقاط مختلف تعیین می گردد. به منظور کاهش زمان حل و هدفمند جستجو نمودن فضای جواب از الگوریتم فراابتکاری ژنتیک و به جهت اعمال ماهیت تصادفی پارامترهای زمانی از شبیه سازی استفاده شده است. به منظور اعتبارسنجی، مدل ارائه شده با داده های یک سال عملیات بارگیری معدن مس سونگون اجرا شده و نتایج بدست آمده با اطلاعات واقعی معدن مذکور طی همان سال و همچنین با نتایج بدست آمده از مدل ارائه شده توسط چانگ مقایسه و ارزیابی شده است. نتایج بدست آمده از اجرای مدل پیشنهادی و مقایسه با شرایط اجرایی نشان می دهد که به طور متوسط 21% هزینه های بارگیری و همچنین 18% تعداد کامیون های مورد نیاز کاهش داشته اند. همچنین نتایج بدست آمده در مقایسه با جواب های بدست آمده بر اساس مدل چانگ نشان می دهد که در بخش هزینه ها مدل پیشنهاد شده در این تحقیق منجر به کاهش 8% هزینه ها می شود که نشان می دهد عملکرد مدل مطلوب و قابل قبول است.

**کلمات کلیدی:** بارگیری، تراک، حمل و نقل، شاول، شبیه سازی و ژنتیک.