Numerical Modeling of Rock Slopes with a Potential of Block-Flexural Toppling Failure

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Abstract

One of the most important instabilities of rock slopes is toppling failure. Among the types of toppling failure, block-flexural failures are a more common instability, which occurs in nature. In this failure, some rock blocks break due to tensile stresses, and some overturn under their weights, and next to all of them topple together. In 2015, the physical and theoretical modeling of this failure has been studied by Amini et al. Due to the complexity of this failure mechanism, no appropriate numerical model has been proposed so far. In this research work, first, a literature review of the toppling failure is summarized. Then using the UDEC software, as a distinct element method (DEM), the experimental models are analyzed numerically, and the Voronoi joint model is applied to simulate the failure. The results of the numerical simulations are compared with the outcomes of the physical models and analytical solutions. This comparison illustrates that the numerical modeling has a good agreement with the corresponding experimental tests and theoretical approaches. Also, the results obtained show that although the mechanism of block-flexural toppling failure is complicated, the numerical code is well-capable of analyzing this failure.

1. Introduction

Toppling failure is a frequent instability in natural and human-made rock slopes. From a mechanical viewpoint, the primary toppling failure is categorized as flexural, blocky, and block-flexural [1]. If a rock mass is made up of a series of parallel discontinuities, dipping steeply against the facing slope, it will act like some rock columns that are placed on top of each other. In this case, rock columns are under tensile and compressive bending stresses due to their own weights. If the maximum tensile stress in every rock column surpasses its tensile strength, it fails and topples. Such an instability is classified as the flexural toppling failure (Figure 1-A). If one cross-joint series is added to the rock mass (Figure 1-B), the system cannot withstand the tensile bending stress, and consequently, the columns may overturn due to their weights. This type of failure is regarded as a typical blocky toppling failure. In real case histories, the above-mentioned perfect cases are rarely encountered, and toppling failure is mostly of block-flexural (Figure 1-C). This instability is a combination of the blocky and flexural toppling failure modes. Many research articles are available on the flexural and blocky toppling failures [2]. In this work, first, the theoretical method and physical modeling of Amini et al [4] are reviewed. Then the experimental tests are examined through a numerical simulation using the UDEC software, and the results obtained are discussed.
2. Literature Review

Müller [6] was the first who mentioned the overturning of rock blocks in 1968. He suggested that block toppling or rotation may have an effective factor in the failure of the north face of the Vaiont slide. In 1971, Ashby [7] analyzed the rotation of rock columns and presented some criteria based on the theoretical and experimental tests. Also Ashby recommended the appointment of “toppling” for such failures. In 1970, Erguvanli and Goodman [8] introduced a physical model to study the toppling failure through a base friction table apparatus. Goodman and Bray classified the toppling failure into the primary (flexural, blocky, and block-flexural) and secondary types [1]. For the primary type of toppling failures, the weight of the rock mass is the governing factor of the instability. Secondary toppling failure is stimulated by some external factors, and is entirely various. Some studies have been carried out for these types of failures [3, 5, 9-15]. In 2019, Sarfaraz et al. [16] numerically modeled the slide-head-toppling failure using the finite element method, and illustrated acceptable agreements with the pre-existing physical modeling and analytical approach results.

From 1976 till now, many physical tests, numerical modelings, analytical methods, design charts, and case studies of toppling failure have been published based on the Goodman and Bray classification [17-22]. Aydan and Kawamoto [23,24] modeled the toppling failure of the rock slopes, employing a friction table apparatus during 1987 and 1992. In 1993, Shimizu et al. [25] modeled some examples of the flexural toppling failure with finite element and discrete element methods. Adhikary et al. [26] modeled the flexural toppling failure using a geotechnical centrifuge apparatus in 1997. In 2007, Adhikary and Dyskin [27] conducted a new series of centrifugal model tests, where glass and concrete samples with the potential of flexural toppling were used as the materials. For the study of the kinetic conditions in toppling failure, Yeung and Wong [28] conducted a physical modeling and a 3D discontinuous deformation analysis. Based on the governing compatibility principles of cantilever beams, Amini et al. presented a straightforward solution for the stability analysis of flexural toppling failure [29, 30]. Also in 2012, Amini et al. [2] combined the method of Goodman and Bray with the method of Aydan and Kawamoto for the analysis of block-flexural toppling failure. In 2018, Zheng et al. [30] suggested a theoretical solution for rock slopes against sliding or flexural-toppling failure based on the limit equilibrium theory and two experimental model tests. Furthermore, they investigated the mechanisms of flexural toppling failure using the limit equilibrium theory and numerical modeling [32]. In 2019, Liu et al. [33] employed a 3D-DDA analysis method to analyze the toppling failure.

Figure 1. Schematic diagrams and real case studies of primary toppling failure: A) flexural, B) Blocky, C) Block-Flexural [3, 5].
3. A review of theoretical method for block-flexural toppling failure

Amini et al. [4] proposed an analytical approach to analyze and calculate the value of the safety factor of block-flexural toppling failure on the basis of the equilibrium and compatibility laws; this method is identified as the equivalent length (ψ) approach.

The parameter ψ can be computed as follows [4]:

\[ \psi = \frac{B \pm (B^2 - 4AC)^{0.5}}{2A} \]

where,
- \( \psi \): Equivalent length of rock slope (meter);
- \( \delta \): Angle of rock mass stratification with respect to the horizontal (degree);
- \( \varphi \): Angle between overall failure plane and the line of normal discontinuities (degree);
- \( \beta \): Angle of upper surfaces of rock slopes with respect to the horizontal (degree);
- \( \theta \): Angle of face slope with respect to the horizontal (degree);
- \( H \): Slope height (meter).

The parameter can be computed as follows [4]:

\[ F_s = \frac{t\sigma_v}{3\psi^2\gamma \cos \delta} \]  (2)

Also the safety factor of the block with the equivalent length \( \psi \) for block toppling failure can be calculated as follows [4]:

\[ F_s = \frac{t}{\psi \tan \delta} \]  (3)

In block-flexural toppling failures, the blocks with the potential of toppling failure exert part of their weight force on the cantilever rock column. Accordingly, it is recommended that a combination of the above relationships is used to analyze the rock slopes against block-flexural toppling failure [4]:

\[ F_s = k \frac{t}{\psi \tan \delta} + (1-k) \frac{t\sigma_v}{3\psi^2\gamma \cos \delta} \]  (4)

in which the parameter \( k \) is a dimensionless correction factor that differs between 0 and 1; this factor indicates the percentage of blocks with a pure blocky potential compared to all blocks in the rock slopes. If all of the blocks are cantilevers under flexure, the slope will be capable of a pure flexural toppling failure, so this coefficient is equal to 0; else, it will be less than 1. Furthermore, if all the blocks have the potential of blocky toppling, this coefficient will be equal to 1 [4].

4. Modeling of block-flexural toppling failure

4.1. A review of physical modeling

Base friction, tilting table, and centrifuge apparatus are conventional geotechnical methods used to study the behavior of soil and rock structures.

Amini et al. [4] conducted physical models using a tilting table device (Figure 3) that had a box placed over a pneumatic jack to set up the models. The columns used for the physical models of a single
column and layered rock slopes were created through the consolidation of a special mixture consisting of BaSO$_4$, ZnO, and Vaseline oil under a chosen pressure. The jack gradually increases the table angle, and the dip of the blocks and the slope angle vary proportionately. The other components in the tilting table include the air compressor, air-transfer hoses, compressed air fitting and fasteners, table’s angular velocity control equipment, and devices to read the table slope. After adjusting the model, the table is tilted until a failure happens. Hence, the angle at which the model initiates to fail or slide can be considered as the angle of instability [4].

Figure 3. A schematic representation of the tilting table machine used for physical modeling [4].

4.2. Numerical modeling
Numerical methods are commonly used as the tools for solving many problems of rock mechanics. The results of the physical models are simulated using the numerical software UDEC. This software is a 2D numerical program based on the distinct element method for discontinued media, for instance, rock slopes, toppling failure, and crack propagation. This software is based on the Lagrangian computational technique, which is suitable for simulating the movements and distortion of a block scheme. The discontinuities are treated as boundary conditions between blocks, and they are allowed large displacement alongside discontinuities and rotations of blocks [34]. For numerical modeling, the physical and mechanical properties of the materials should be available. The block properties are presented in Table 1. Also the properties of the joints between blocks are listed in this Table. The Mohr-Coulomb criterion was used in the numerical modeling. The Voronoi model (internal flaws) was applied to simulate the failure in which the parameters of this model were similar to the geo-mechanical parameters of the blocks.

<table>
<thead>
<tr>
<th>Model parameters</th>
<th>Blocks</th>
<th>Internal flaws</th>
<th>Joint element</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit weight (kN/m$^3$)</td>
<td>23.4</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Modulus of elasticity (MPa)</td>
<td>7</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Poison ratio</td>
<td>0.25</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Tensile strength (kPa)</td>
<td>31</td>
<td>31</td>
<td>0</td>
</tr>
<tr>
<td>Cohesion (kPa)</td>
<td>15</td>
<td>15</td>
<td>0</td>
</tr>
<tr>
<td>Friction angle (degree)</td>
<td>35</td>
<td>35</td>
<td>30</td>
</tr>
<tr>
<td>Normal stiffness (MPa/m)</td>
<td>-</td>
<td>50</td>
<td>20</td>
</tr>
<tr>
<td>Shear stiffness (MPa/m)</td>
<td>-</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

Amini et al. [4] modeled the block-flexural toppling in two sections. In the first step, ideally, it was supposed that the geometries of the slope block were divided into two portions as blocky and
flexural so that each other block was possibly blocky or flexural. However, these models are actually different from real rock slopes since slope blocks are frequently arranged randomly in nature. Thus in the second section, the random patterns are modeled to be more similar to the real layered rock slopes. In both sections, after the models were built, the table slowly tilted to cause failure to happen. Parameters such as the table angle and angle of overall plane failure were measured [4]. In the numerical modeling, the models were prepared at the angle at which the physical models initiate to fail or slide that could be considered as the instability angle. Next, the models were analyzed by the shear strength reduction method. In the following section, these two series of modeling are explained separately. Pictures of all the physical and numerical models (tests 1 to 8) are shown in the appendix at the moment of the failure. The tests 1 and 2 are for an ideal block-flexural toppling failure. Also the tests 3 to 8 are for a non-ideal block-flexural toppling failure.

### 4.2.1. Modeling ideal block-flexural toppling failure

In Figure 4, an example of these physical models is shown before testing and during failure. In these models, in fact, two consecutive blocks are similar to the two-block models; one block is fixed at its pivot, and is capable of carrying tensile stresses but the next block is free at its end and imposes its weight, after the table tilts, on the fixed block. Due to some constraints on the construction and movement of the blocks, two physical models were performed [4]. The modeling results are displayed in Table 2.

![Figure 4. Physical modeling of ideal block-flexural toppling failure (Test 1) [4].](image)

<table>
<thead>
<tr>
<th>Test No.</th>
<th>$\beta$ (°)</th>
<th>$\phi$ (°)</th>
<th>$\delta$ (°)</th>
<th>$\theta$ (°)</th>
<th>H (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-13</td>
<td>18</td>
<td>56</td>
<td>99</td>
<td>47.44</td>
</tr>
<tr>
<td>2</td>
<td>-5</td>
<td>26</td>
<td>48</td>
<td>91</td>
<td>37.45</td>
</tr>
</tbody>
</table>

Table 2. Geometrical parameters of ideal block-flexural toppling failure with physical modeling [4].

Figure 5 shows the numerical analysis results corresponding to the physical model of Test 1. The stress reduction factor of this model is equal to 0.975. In Figure 5, the symbols (o) and (*) illustrate the tensile and yield points, respectively. As it can be seen in this figure, the rock blocks have failed under the tensile stresses. Furthermore, distribution of the displacement vector is presented in this figure. According to this figure, the angle between the normal to discontinuities and overall toppling failure plane is 20° in the numerical modeling, which has an acceptable agreement with the corresponding physical model. Also the results of numerical models presented in Table 3 at the angle that the SRF value is equal to one. It is believed that in numerical methods, this value of critical stress reduction factor can be assumed to be equivalent to a safety factor [5].
4.2.2. Modeling of random set up of block-flexural toppling failure
In the second section, the blocks were placed entirely randomly in the model so that some of them were broken and the others were overturned freely. By choosing a random set of length blocks, the models fail against the block-flexural toppling failure. This behavior is almost similar to the actual rock slopes in which the rock columns are randomly bent or overturned [4]. The schematic view and photograph of the model from these experiments are shown in Figure 6. The results of the modeling can be seen in Table 4.

Table 3. Geometrical parameters of ideal block-flexural toppling failure with physical modeling.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>β (°)</th>
<th>φ (°)</th>
<th>δ (°)</th>
<th>θ (°)</th>
<th>H (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-14</td>
<td>20</td>
<td>57</td>
<td>100</td>
<td>47.27</td>
</tr>
<tr>
<td>2</td>
<td>-7</td>
<td>22</td>
<td>50</td>
<td>93</td>
<td>37.28</td>
</tr>
</tbody>
</table>

4.2.2. Modeling of random set up of block-flexural toppling failure with random set-up by physical modeling [4].

<table>
<thead>
<tr>
<th>Test No.</th>
<th>β (°)</th>
<th>φ (°)</th>
<th>δ (°)</th>
<th>θ (°)</th>
<th>H (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-26</td>
<td>15</td>
<td>69</td>
<td>112</td>
<td>44.46</td>
</tr>
<tr>
<td>4</td>
<td>-18</td>
<td>27</td>
<td>61</td>
<td>104</td>
<td>46.56</td>
</tr>
<tr>
<td>5</td>
<td>-21</td>
<td>30</td>
<td>64</td>
<td>107</td>
<td>45.88</td>
</tr>
<tr>
<td>6</td>
<td>-12</td>
<td>29</td>
<td>55</td>
<td>98</td>
<td>47.54</td>
</tr>
<tr>
<td>7</td>
<td>-26</td>
<td>34</td>
<td>69</td>
<td>112</td>
<td>44.46</td>
</tr>
<tr>
<td>8</td>
<td>-22</td>
<td>22</td>
<td>65</td>
<td>108</td>
<td>45.62</td>
</tr>
</tbody>
</table>
The results of numerical modeling such as plastic points and distribution of displacement vector of Test 6 (Figure 6) are shown in Figure 7. According to this figure, some blocks were broken under the tensile stress (flexural portion), and the others were separated along the secondary joint (blocky portion), causing a general toppling failure. The failure pattern of the numerical model is in a reasonable agreement with the corresponding physical model. Furthermore, the results of the tests 3 to 8 at the moment of failure are presented in Table 5.

Figure 6. Physical modeling of block-flexural toppling failure (Test 6) [4].

Figure 7. Numerical modeling of block-flexural toppling failure (Test 6) (unit of the displacement vector is m/s).
Table 5. Geometrical parameters of block-flexural toppling failure with random set-up by numerical modeling.

<table>
<thead>
<tr>
<th>Test No.</th>
<th>(\beta) (°)</th>
<th>(\varphi) (°)</th>
<th>(\delta) (°)</th>
<th>(\theta) (°)</th>
<th>H (cm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>-29</td>
<td>23</td>
<td>72</td>
<td>115</td>
<td>43.46</td>
</tr>
<tr>
<td>4</td>
<td>-22</td>
<td>19</td>
<td>65</td>
<td>108</td>
<td>45.62</td>
</tr>
<tr>
<td>5</td>
<td>-22</td>
<td>24</td>
<td>65</td>
<td>108</td>
<td>45.62</td>
</tr>
<tr>
<td>6</td>
<td>-10</td>
<td>30</td>
<td>53</td>
<td>96</td>
<td>47.75</td>
</tr>
<tr>
<td>7</td>
<td>-24</td>
<td>24</td>
<td>67</td>
<td>110</td>
<td>45.07</td>
</tr>
<tr>
<td>8</td>
<td>-24</td>
<td>25</td>
<td>67</td>
<td>110</td>
<td>45.07</td>
</tr>
</tbody>
</table>

4.3. Results and discussion

In this section, the results of the numerical modeling were compared with the corresponding physical models and analytical method. As it can be seen in Figures 4 to 7 and also Table 2 to 5, by comparing the geometrical parameters and how the blocks are toppling, the numerical modeling has a good agreement with the physical modeling. Additionally, the most appropriate quantity for comparison between these models is the value of the critical stress reduction factor. Since the value of the safety factor of the physical model is equal to 1 at the moment of failure, the critical stress reduction factor of the numerical model must also be equal to 1. In Table 6, the stress reduction factor of the numerical models is compared with the safety factor of the physical models. The differences between the numerical and physical results are less than 10%, which appear to be reasonable due to the complexity of the failure mechanism. It is also possible to compare the values for the safety factor obtained from the numerical and physical models with the values for the safety factor obtained from the theoretical solution proposed by Amini et al. (relationships 1 to 4). The results of this comparison are presented in Figure 8.

Table 6. Comparison of the numerical modeling results with the corresponding physical models.

<table>
<thead>
<tr>
<th>Models</th>
<th>Test 1</th>
<th>Test 2</th>
<th>Test 3</th>
<th>Test 4</th>
<th>Test 5</th>
<th>Test 6</th>
<th>Test 7</th>
<th>Test 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>(F_S) in physical modeling</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>SRF in numerical modeling</td>
<td>0.974</td>
<td>0.946</td>
<td>0.932</td>
<td>0.903</td>
<td>0.971</td>
<td>1.042</td>
<td>1.057</td>
<td>0.948</td>
</tr>
<tr>
<td>Difference (%)</td>
<td>2.6</td>
<td>5.4</td>
<td>6.8</td>
<td>9.7</td>
<td>2.9</td>
<td>4.2</td>
<td>5.7</td>
<td>5.2</td>
</tr>
</tbody>
</table>

Figure 8. Comparison of the safety factor in numerical modeling and analytical methods with the corresponding physical model.

5. Conclusions

In this work, the mechanism of the block-flexural toppling failure was examined through a series of numerical models, analyzed using the UDEC software as a distinct element code. The physical and theoretical modeling of this failure was investigated by Amini et al. in 2015. In this numerical modeling, for simulating a failure, the Voronoi joint model was applied. In the ideal
models, the number of blocks having a potential of block toppling failure is equal to the number of blocks having a potential of flexural toppling failure, and the failure plane is such that half the blocks are broken under tensile stresses. However, in non-ideal models, the number of blocks with the potential of blocky and flexural toppling failure is not necessarily the same, and the failure plane is formed so that fewer blocks are broken under tensile stresses due to blocks placed randomly. The results of numerical models were compared with the outcomes of the experimental tests and theoretical method. This comparison demonstrates that numerical modeling has a good agreement with the corresponding physical models, where the differences between the numerical and experimental results are less than 10%. Correspondingly, the results show that although the mechanism of block-flexural toppling failure is complex, the distinct element method is well-capable of analyzing the block-flexural toppling failure, and the UDEC software is an efficient tool for evaluating the stability analysis of this failure.

Conflicts of interest
The authors wish to confirm that there are no known conflicts of interest associated with this publication, and there has been no considerable financial support for this study that could have influenced its outcome.

Acknowledgement
The authors express their science thanks to Professor Ömer Aydan from Ryukyu University, Ryukyu, Japan, for his invaluable help and guidance throughout this research work.

References
[7]. Ashby, J. (1971). Sliding and toppling modes of failure in models and jointed rock slopes, Imperial College, University of London.


Appendix
Pictures of all physical and numerical models at the moment of failure.

Test 1
Test 2
Test 3
Test 4
Test 5
مدل سازی عددي شیروانی های سنگی با تناسب شکست وازگونی بلوکی - خمشی

حسن سرفراز 1، مهدي امیني 1

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ارسال 05/9/2019، پذیرش 14/11/2019

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چکیده:

یکی از مهم‌ترین ناپایداری‌ها در شیروانی‌های سنگی، شکست‌های وازگونی است. از میان انواع شکست‌های وازگونی، شکست وازگونی بلوکی- خمشی در طیب‌البین مشاهده می‌شود. در این شکست، بعضی از بلوک‌ها در اثر خمش شکسته شده و در برخی دیگر آزاد شده. همین‌که با افزایش خود‌خاک‌گیری و چرخند شدن در دلیل پیچیدگی مکانیسم‌های نویغ وازگونی، مدل‌سازی وجود شکست تاکنون، نه تنها در سطح این تحقیق، از سیم‌های توانایی شکست‌های وازگونی ارائه می‌شود، سپس با استفاده از نرم‌افزار UDEC در نظر داشته شده است. در این تحقیق، از نرم‌افزار UDEC و نرم‌افزار VORONOI برای مشابه‌کردن مدل‌سازی‌های عددي از دو روش انتخاب می‌شود. نتایج بدست‌آمده از مدل سازی عددي با نرم‌افزار UDEC و تحلیل مقایسه‌شده این نرم‌افزار با مدل‌سازی‌های اپی‌سی‌پی و یونی‌سی‌پی دارای همچنین نتایج به دست امده نشان می‌دهد که اگرچه مکانیسم شکست وازگونی بلوکی- خمشی به‌چنین است، مدل عددي یپشنهادی، آبرژ مسئولی برای تحلیل و پیش‌بینی و ارائه این نوع شکست است.

کلمات کلیدی: شیروانی سنگی، شکست وازگونی بلوکی- خمشی، مدل‌سازی عددي، روش‌های مجاری.