Investigation of Bishop’s and Janbu’s Models Capabilities on Slope Stability Problems with Special Consideration to Open-Pit Mining Operations

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1. Introduction

Generally, one of the most important purposes of open-pit mine planners is to maximize the net present value [1, 2]. Therefore, in the first step, the effective factors of break-even stripping ratio, cut-off grade, block economic value, and open-pit slope are considered as the important parameters in the design and production planning steps [1, 2]. Figure 1 shows the design stages of an open-pit mine, intending to determine the final pit. According to Figure 1, the economic uncertainties play an important role in the process of determining the final pit. However, two important points indicate that the economic conditions of a mine are completely dependent on the slope angle [1,3,4]: a) considering the slope angle more than the optimal amount, the probability of local failure in benches, and consequently, a large failure in the pit wall is greatly increased, which, if it occurs, it can lead to damages such as loss of large amounts of minerals and imposition of high waste costs, separation and recovery of ore, increasing costs associated with the construction or repair of ramps, and human casualties and various damage to mining machinery and even mine closures; b) considering the slope angle less than the optimal amount leads to an increase in the stripping ratio, a decrease in the pit depth, and consequently, a decrease in the net present value of mine. Finally, in the homogenous and isotropic media, the results of the Bishop, Janbu, and numerical modelings are close together.
A. Describe appropriately the properties of rock samples in a wide range of stress conditions in different depths.
B. Predict the effect of one or more discontinuities on the behavior of intact rock samples.
C. Show proportionally the characteristics of the rock mass behavior.

Despite the previous studies, there is no failure criterion that can satisfy the above-mentioned conditions yet. In reality, each failure criterion is potent in some of the above, and in the other part, they are weak. Among the proposed failure criterion, the linear Mohr-Coulomb failure criterion and the non-linear generalized Hoek and Brown failure criterion have been most widely used due to their proper adaptation to the rock mass properties [16]. The Mohr-Coulomb failure criterion and the generalized Hoek and Brown failure criterion are biaxial criteria based on the maximum and minimum principal stresses. In reality, it is assumed that the failure process of rock is controlled by the maximum and minimum principal stresses, and the effect of the intermediate principal stress is neglected [16]. Moreover, recently, some researchers, using the results of the exact laboratory tests such as the true triaxial strength test, have investigated the effect of the intermediate principal stress on the failure criteria [17-20]. Accordingly, applying the triaxial failure criteria, the effect of intermediate principal stress on the stability conditions of rock mechanics projects has been studied, and it has been concluded that the stability conditions can be a function of the intermediate principal stress [21-26]. Investigation of the recent studies shows that none of them has paid attention to the effect of the overall slope angle value on the ultimate pit depth, and consequently, determination of the net present value [3-14, 21-26]. Therefore, this work intend to investigate the effect of open-pit slope angle on determining the final pit and some parameters related to economic calculations. The following steps were undertaken:

1) Considering an example, the performance of the modified Bishop and Janbu methods for slope stability analysis will be investigated.
2) Considering the modified Bishop and Janbu methods and numerical modeling, determination of the open pit underground limit will be investigated.

**2. A brief explanation of modified Bishop and modified Janbu methods**

In the limit equilibrium method, firstly, the shear strength of the slide surface and the force required to maintain the balance is evaluated and then compared to calculate the safety factor. In the circular failure analysis, the sliding mass is usually divided into vertical slices and the stability analysis is performed as a piecewise slice. If the balance condition exists for each component, the balance condition for the sliding mass is established. Therefore, the number of equations required for the analysis depends on the two factors of equilibrium conditions and number of slices. Two important limit equilibrium methods are the modified Bishop method and the modified Janbu method [6].

In the modified Bishop method, it is assumed that the slip surface is completely circular, and in the interface of slices intended for analysis, there is a horizontal force. Considering the Mohr-Coulomb failure criterion, the safety factor is calculated as follows [6]:

$$FS = \frac{\sum \left[ c + \left( \gamma_z h - \gamma_s h_s \right) \tan \varphi \right] \Delta x / \cos \psi_o}{1 + \tan \psi_o \tan \varphi}$$

$$\sum \left[ \gamma_z h \Delta x \sin \psi_o + \frac{1}{2} \gamma_s Z^2 \frac{\alpha}{R} \right]$$

(1)
where FS is a safety factor, \( \gamma_r \) is the unit weight of soil or rock, \( \gamma_w \) is the water unit weight, \( h \) is the slice height, \( \delta w \) is the water height within slice, \( \psi_b \) is the base angle of slice, \( c \) is the cohesion of slip surface, \( \phi \) is the friction angle of slip surface, \( \Delta x \) is the slice width, \( Z \) is the water depth within tension crack, \( R \) is the slip radius, and \( \alpha \) is the distance between the failure arch center and two thirds of the depth of tension crack.

In the modified Janbu method, the slip surface can be non-circular, and the balance of forces is satisfied only in the vertical direction, and it is assumed that in the interface of slices intended for the analysis, there is a horizontal force. Considering the Mohr-Coulomb failure criterion, the safety factor is calculated as follows [6]:

\[
FS = \frac{f_0}{\sum \left[ \frac{e + (\gamma_r h - \gamma_w \delta w) \tan \phi}{1 + \tan \psi_b \tan \phi} \frac{h \Delta x \tan \psi_b}{FS} \right]}
\]

\[
f_0 = 1 + \frac{K(d/L - 1.4(d/L)^2)}{if : c = 0 \Rightarrow K = 0.31 \\
if : c > 0, \phi > 0 \Rightarrow K = 0.50}
\]

3. Investigating performance of modified Bishop and modified Janbu methods

In order to investigate the performance of the modified Bishop and Janbu methods, according to Figure 2, an example is presented. In this example, a slope consisting three benches of 15m is created within a weak sandstone medium. The bench width is 2m, the slope angle of the bench is 75 degrees, and the natural gradient of the ground surface is 45 degrees. The friction angle and cohesion are 43 degrees and 0.145 MPa, respectively, and the unit weight of rock mass is 0.025 MN/m\(^3\) (dataset1). According to Figure 2, the sliding mass is divided into 8 slices; their characteristics are mentioned in Table 1. According to these characteristics, the safety factors, based on the modified Bishop and Janbu methods, are equal to 1.39 and 1.26, respectively. In addition, this slope was analyzed by the finite element method using the RS2 software (ver. 2019) (Figures 3 to 5), and the safety factor was obtained to be 1.25.

Table 1. The data required for the stability analysis [6].

<table>
<thead>
<tr>
<th>Slice number</th>
<th>( \psi_b ) (deg)</th>
<th>( \gamma_r h \Delta x ) (MN)</th>
<th>( \gamma_w h \Delta x ) (MPa)</th>
<th>( \Delta x ) (m)</th>
<th>( \phi ) (deg)</th>
<th>( c ) (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
<td>1.312</td>
<td>0.017</td>
<td>6.1</td>
<td>43</td>
<td>0.145</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>1.597</td>
<td>0.047</td>
<td>6.1</td>
<td>43</td>
<td>0.145</td>
</tr>
<tr>
<td>3</td>
<td>34</td>
<td>2.603</td>
<td>0.071</td>
<td>6.1</td>
<td>43</td>
<td>0.145</td>
</tr>
<tr>
<td>4</td>
<td>39</td>
<td>2.635</td>
<td>0.087</td>
<td>6.1</td>
<td>43</td>
<td>0.145</td>
</tr>
<tr>
<td>5</td>
<td>44</td>
<td>3.501</td>
<td>0.095</td>
<td>6.1</td>
<td>43</td>
<td>0.145</td>
</tr>
<tr>
<td>6</td>
<td>50</td>
<td>3.914</td>
<td>0.087</td>
<td>6.1</td>
<td>43</td>
<td>0.145</td>
</tr>
<tr>
<td>7</td>
<td>57</td>
<td>3.592</td>
<td>0.047</td>
<td>6.1</td>
<td>43</td>
<td>0.145</td>
</tr>
<tr>
<td>8</td>
<td>65</td>
<td>2.677</td>
<td>0</td>
<td>6.1</td>
<td>43</td>
<td>0.145</td>
</tr>
</tbody>
</table>

Also for more investigation, two more datasets of resistance properties of slip surface were considered (dataset2: \( c=0.152 \) and \( \phi=45.5^\circ \) and dataset3: \( c=0.16 \) and \( \phi=48^\circ \)). According to Table 2, the results obtained for the three understudied methods are close together but the safety factor obtained from the modified Janbu method is much closer to that obtained from the FEM model, the most important factor being the ability of the modified Janbu method to predict the non-circular slip surface.
Figure 3. Rock slope modeling by the finite element method using the RS2 software.

Figure 4. Maximum shear strain changes in FS=1.25.

Figure 5. Total displacement changes in FS=1.25.

Table 2. Comparing the results of slope stability analysis by the limit equilibrium methods and the numerical method.

<table>
<thead>
<tr>
<th>Dataset No.</th>
<th>φ (deg.)</th>
<th>c (MPa)</th>
<th>Modified Bishop</th>
<th>Modified Jambu</th>
<th>FEM (RS2 software)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>43</td>
<td>0.145</td>
<td>1.39</td>
<td>1.26</td>
<td>1.25</td>
</tr>
<tr>
<td>2</td>
<td>45.5</td>
<td>0.152</td>
<td>1.49</td>
<td>1.36</td>
<td>1.35</td>
</tr>
<tr>
<td>3</td>
<td>48</td>
<td>0.16</td>
<td>1.59</td>
<td>1.46</td>
<td>1.43</td>
</tr>
</tbody>
</table>

4. Investigating overall slope angle in economic conditions of an open-pit mine

In this section, we aimed to investigate the effect of the overall slope angle (final slope angle) on the mine economic conditions. For this purpose, as shown in Figure 6, a vertical section of an iron ore mine, with the characteristics listed in Table 3, was considered. The depth of the final pit was considered to be 180m, which was to be exploited by twelve benches of 15m. The bench width in the final pit was 8m and the floor width of the pit was 70m (equal to the apparent width of iron ore).

Table 3. Geomechanical properties of orebody and rock masses.

<table>
<thead>
<tr>
<th>Type</th>
<th>Resistance properties considering</th>
<th>Unit weight (MN/m³)</th>
<th>Young’s modulus (GPa)</th>
<th>Poisson ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Cohesion (MPa)</td>
<td>Friction angle (deg.)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hanging wall</td>
<td>0.165</td>
<td>42</td>
<td>0.025</td>
<td>1.1</td>
</tr>
<tr>
<td>Footwall</td>
<td>0.155</td>
<td>35</td>
<td>0.025</td>
<td>1</td>
</tr>
<tr>
<td>Ore</td>
<td>1.1</td>
<td>46</td>
<td>0.025</td>
<td>2.5</td>
</tr>
</tbody>
</table>
Based on the given information, the optimum slope angles of the right and left walls of the final pit were analyzed by the modified Bishop and modified Janbu methods. Moreover, according to Figures 7 and 8, the slope stability of the pit was investigated by RS2. According to Table 4, considering FS=1.3 as an acceptable criterion for an optimum design [27], the stable and optimum slope angles of the right wall and left wall of pit was obtained. As shown in Table 4, the results are close together so that the optimum slope angles of the right wall and left wall were evaluated to be about 44° and 41°, respectively. To calculate the waste and ore volumes, a thickness of 1m was considered for this section, and therefore, the volumes of waste and ore within the pit were 70873m³ and 12600m³, respectively, and consequently, the overall stripping ratio (OSR) was 5.62.

From the view point of design slope and economic conditions, an accurate calculation of the geomechanical properties of the rock masses surrounding the pit is very important. To investigate the effect of the geomechanical properties on the stability of the pit slope and economic conditions of the open pit, a sensitivity analysis was carried out.
It was assumed that there were, respectively, about 7% and 11% errors in calculating \( c \) and \( \phi \), and therefore, the actual values for \( c \) and \( \phi \) were in accordance with Table 5. Again, the right and left pit walls, considering the obtained slope angles for them and new values of resistance properties (44° and 41°, respectively), were analyzed, and it became clear that the safety factors of both walls increased by about 15% to 1.49, and therefore, the slope angles of 44° and 41° were no longer optimal. Considering FS=1.3 as an acceptable criterion for optimum design, the stability conditions of pit walls were analyzed by the limit equilibrium and numerical methods. It was detected that the optimum slope angles for the right wall and left wall of the pit increased by 32% and 34% to 58° and 55°, respectively. In Figures 9 and 10, the numerical modelling by RS2 is shown. Under these conditions, and with the constant pit depth, the waste volume was obtained to be 42,977 m³ and the ore volume, due to a constant pit depth, did not change, and therefore, OSR of the new pit was 3.41, which decreased by 39.3% compared to the previous pit. If the stripping cost was considered to be about $0.8 per tone, the cost savings in stripping operation of the new pit would be (only for this section, with a thickness equal to 1) about $22317, which would surely be a large number for all the mine.

### Table 5. Actual geomechanical properties of orebody and rock masses.

<table>
<thead>
<tr>
<th>Type</th>
<th>Cohesion (MPa)</th>
<th>Friction angle (deg.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hanging wall</td>
<td>0.180</td>
<td>46</td>
</tr>
<tr>
<td>Foot wall</td>
<td>0.166</td>
<td>39</td>
</tr>
<tr>
<td>Ore</td>
<td>1.18</td>
<td>51</td>
</tr>
</tbody>
</table>

According to the earlier pit characteristics, OSR is equal to 5.62. On the other hand, in a new pit, OSR is equal to 3.41, and therefore, the pit can be deeper. This idea is typically shown in Figure 11. It is assumed that two pits are feasible for mining operations; in pit No.2, the final slope angle is greater than pit No.1, thus it can be deeper. Therefore, considering OSR=5.62 as one of the acceptable economic criteria, the goal is to increase the depth of a new pit so that the safety factor of stability of pit walls is equal to 1.3.

As mentioned earlier, the optimum slope angles of the right and left walls of the pit with a depth of 180m were obtained to be 58° and 51°, respectively. It should be noted that with increase in the pit depth, the slope of the pit will not be stable, and therefore, several simultaneous analyses are required to calculate 1) the maximum pit depth and 2) the optimum slope angle. To determine the maximum depth, the stripping ratio of open pit and underground mining (SR_{op&u}) is used. According to this ratio, the depth of the mine, which is the cost of open-pit mining for each ore ton is equal to the cost of underground mining for each ore ton, and is considered as the mining limit of open pit and underground (the costs after extraction are the same for both mining methods).
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Figure 11. A typical model to show the effect of the slope angle of pit walls on the pit depth.

Therefore, the stripping ratio of the open-pit-underground limit is obtained as follows:

\[ SR_{u&o} = \frac{C_u - C_o}{C_w} \]  

(4)

where \( C_u \) is the mining cost per ore ton in underground mining, \( C_o \) is the cost of extraction per ore ton in open-pit mining, and \( C_w \) is the stripping cost per ton. To determine the open-pit-underground mining (the maximum pit depth), the overall stripping ratio is considered \( SR_{u&o} \). The volumes of waste and ore can be defined as a function of pit depth, as follow:

\[ V_o = \int_0^{H_{u&o}} f(h)DH \]  

(5)

\[ V_w = \int_0^{H_{u&o}} g(h)DH \]  

(6)

Where \( H_{u&o} \) is the transition depth to underground mining (maximum pit depth), and \( f(h) \) and \( g(h) \) are the volume functions of waste and ore, respectively. In Figure 12, \( V_o \) and \( V_w \) are obtained by Eqs. (7) and (8).

\[ V_o = (W_o \sin \beta)(W_h H_{u&o}) = W_o H_{u&o} \]  

(7)

\[ V_w = \left[ \left( \frac{H_{u&o}}{\tan \theta_1} + \frac{H_{u&o}}{\tan \theta_2} + W_o \right) + W_o \right] \frac{H_{u&o}}{2} - W_o H_{u&o} = \frac{H_{u&o}^2}{\tan \theta_1} + \frac{H_{u&o}^2}{\tan \theta_2} \]  

(8)

Figure 12. Pit geometry for determining the transition depth to underground mining.
In Eqs. (7) and (8), \( \theta_1 \) and \( \theta_2 \) are the slope angles of the pit walls, \( W_r \) is the apparent thickness of the ore layer, and \( \beta \) is the dip of ore. Moreover, in Figure 12, \( W_f \) is the floor width of the pit. Considering the recovery coefficient in open-pit mining (\( R_o \)) and recovery coefficient in underground mining (\( R_u \)), Eq. (4) is re-written as follows:

\[
SR_{\text{total}} = \frac{R_u \cdot C_u - R_o \cdot C_o}{C}
\]  
(9)

Now, the transition depth is obtained by equating Eq. (9) with OSR and then replacing Eqs. (7) and (8) in it.

\[
\text{OSR} = \frac{W_o}{V_o} = \frac{R_u \cdot C_u - R_o \cdot C_o}{C} \cdot \frac{H_{\text{trap}}}{W_o} + \frac{H_{\text{trap}}}{W_o} = \frac{W_o (R_u \cdot C_u - R_o \cdot C_o)}{C} \cdot \frac{1}{\cot \theta_1 + \cot \theta_2}
\]  
(10)

The data required to determine the transition depth is given in Table 6. As mentioned, to determine the maximum pit depth (transition depth), it must be analyzed for the slope stability conditions and the economic conditions simultaneously so that the safety factor of the slope stability of pit walls is 1.3 and the overall stripping ratio is 5.62. Based on the results obtained from the limit equilibrium and numerical methods, the right and left walls of a pit with a depth of 256m are stable at the slope angles of 54° and 51°, respectively. In this state, OSR=5.62, and therefore, from the stripping ratio point view, the mining operations can be economic. For the investigated section (Fig 6), the comparison of a 256m pit with a 180m pit showed that the ore extraction increased by about 5324t, which if this increase is extended for the whole pit, it will be a significant volume that will increase the mine life and profitability.

<table>
<thead>
<tr>
<th>( R_o )</th>
<th>( C_o ) ($/t)</th>
<th>( R_u )</th>
<th>( C_u ) ($/t)</th>
<th>( W_r ) ($/t)</th>
<th>( W_o ) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.84</td>
<td>7.5</td>
<td>0.8</td>
<td>2.25</td>
<td>0.8</td>
<td>70</td>
</tr>
</tbody>
</table>

5. Conclusions

In this paper, using the Bishop’s and Janbu’s methods and numerical modeling, it was shown that the open-pit mining operations and the economic conditions are a function of the final slope angle of pit. The most important results obtained from the stability analyses by modified Bishop and modified Janbu methods and numerical modeling (FEM) by the RS2 software are:

1) In homogenous and isotropic media, the results of the modified Bishop and modified Janbu methods and numerical modeling are close together. Therefore, in these media, the use of equilibrium methods for analyzing the slope stability is recommended.

2) The low accurate in determining the geomechanical properties affects the optimum of final pit slope and the economic conditions of the mine, consequently. In the investigated example, by increasing the accurate calculations of the geomechanical properties, the overall stripping ratio was reduced by 39.3%, from 5.62 to 3.41.

3) The transition depth to underground mining is a function of the accurate geomechanical properties of the rock masses surrounding pit and the final slope angle of pit. In the investigated example, by neglecting about 7% and 11% errors in calculating \( c \) and \( \phi \), the maximum pit depth (transition depth) was increased by 42%, from 180m to 256m.

References


بررسی قابلیت‌های روش‌های بیش‌پیت و جانبو در مسائل تحلیل پایداری شیب با در نظر گرفتن شرایط عملیات معدن کاری رویاز

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چکیده:

یکی از مهم‌ترین پارامترهای تأثیرگذار در شرایط اقتصادی معادن رویاز، روش‌های بیش‌پیت و جانبو است. به طوری که با افزایش شیب بین شناخته شده و کاهشیده می‌شود. با این حال، اثربخشی روش‌های نسبی با روش‌های متنوع و محاسباتی شیب بینی می‌شود. در این مقاله، با در نظر گرفتن شرایط اقتصادی رویاز کاری، می‌توان به اثربخشی روش‌های بیش‌پیت و جانبو در مطالعه شیب بینی دست برید. نتایج نشان می‌دهد که روش‌های بیش‌پیت و جانبو با نتایج مدل‌های سایر روش‌های محاسباتی مشابه می‌باشند.

کلمات کلیدی: پایداری شیب، پیت، روش‌های بیش‌پیت و جانبو، مدل سایر، روش‌های بیش‌پیت