A New Method for Forecasting Uniaxial Compressive Strength of Weak Rocks

H. Fattahi*

1- Department of Earth Sciences Engineering, Arak University of Technology, Arak, Iran

Received 24 January 2020; received in revised form 24 February 2020; accepted 29 February 2020

Keywords:
Uniaxial compressive strength
Weak rocks
Relevance vector regression
Cuckoo search algorithm
Harmony search algorithm.

Abstract
The uniaxial compressive strength of weak rocks (UCSWR) is among the essential parameters involved for the design of underground excavations, surface and underground mines, foundations in/on rock masses, and oil wells as an input factor of some analytical and empirical methods such as RMR and RMI. The direct standard approaches are difficult, expensive, and time-consuming, especially with highly fractured, highly porous, weak, and homogeneous rocks. Numerous endeavors have been made to develop indirect approaches of predicting UCSWR. In this research work, a new intelligence method, namely relevance vector regression (RVR), improved by the cuckoo search (CS) and harmony search (HS) algorithms is introduced to forecast UCSWR. The HS and CS algorithms are combined with RVR to determine the optimal values for the RVR controlling factors. The optimized models (RVR-HS and RVR-CS) are employed to the available data given in the open-source literature. In these models, the bulk density, Brazilian tensile strength test, point load index test, and ultrasonic test are used as the inputs, while UCSWR is the output parameter. The performances of the suggested predictive models are tested according to two performance indices, i.e. mean square error and determination coefficient. The results obtained show that RVR optimized by the HS model can be successfully utilized for estimation of UCSWR with R² = 0.9903 and MSE = 0.0031203.

1. Introduction
A proper determination of the uniaxial compressive strength of weak rocks (UCSWRs) is of significant importance in the design of rock mechanics structures, for instance, tunnels, slopes, and dams. Nevertheless, there are some impeding parameters in the direct determination of UCSWR in the laboratory. For example, preparing the required rock core samples is often difficult, particularly for the rocks that exhibit a significant foliation and those that are fractured [1,2]. Therefore, a direct determination of UCSWR can be time-consuming and costly [3]. Many researchers have attempted to find the alternative and indirect methods in order to estimate UCS using different methods. In this paper, the well-known research works are addressed. Ghose and Chakraborti [4] have suggested an empirical relation between the Schmidt rebound number and UCS for Indian coal. Meulenkamp, Grima [5] have estimated UCS by a back-propagation neural network. In their research work, the density, grain size, porosity, Equotip hardness reading, and rock type were considered as the inputs for the UCS estimation. Singh et al. [6] have suggested a number of relations between some index factor (area weighting, grain size, orientation of weakness (foliation) planes, aspect ratio, mineral composition, and form factor) and strength parameters (UCS, tensile strength, and axial point load strength) of schistose rock. Gokceoglu, Zorlu [7] have estimated the Young’s modulus and UCS of problematic rocks by the regression methods and a fuzzy model. Sonmez et al. [8] have utilized a fuzzy inference system for estimation of UCS.

*Corresponding author: h.fattahi@arakut.ac.ir (H. Fattahi).
based on the petrographic data. Fener et al. [9] have proposed an empirical relation between the UCS values and the Schmidt hardness for two sedimentary, six igneous, and three metamorphic rocks. Kilic, Teymen [10] have found strong relationships between the UCS values and the Schmidt hardness for 19 different rock samples. Dehghan et al. [11] have utilized feed forward neural network to estimate UCS. In their research work, porosity, Schmidt hammer rebound number, p-wave velocity, and point load index were considered as the inputs to estimate UCS. Cevik et al. [12], for sedimentary rock samples, have evaluated the application of artificial neural network (ANN) in forecasting UCS. Yagiz et al. [13] have developed the non-linear regression and ANN techniques to estimate UCS for 54 carbonate rocks. Minaeian and Ahangari [14] have suggested an empirical relationship between the UCS values and the Schmidt hardness for some samples of weak conglomeratic rock. Mishra, Basu [15] have used the fuzzy inference system model for the prediction of UCS in three different rocks. Yesiloglu-Gultekin et al. [16], for the granite samples in Turkey, have proposed the superiority of the adaptive neuro-fuzzy inference system (ANFIS) model compared to the ANN model in forecasting UCS. Aboutaleb et al. [17] have evaluated the relationship between UCS with dynamic poisson ratio and the dynamic Young’s modulus using a simple and multivariate regression analysis, an ANN, and support vector regression (SVR).

Although ANN is an alternative for forecasting UCS, it is a trouble to determine the architecture for ANN, and the stochastic events are present during the building of the model. Also ANNs do have some shortages: they have a slow learning rate. In contrast, SVR is deterministic and global. However, it still has the difficulty to determine the factors involved (e.g. penalty weight C and insensitivity ε) and selected a suitable kernel function. The relevance vector regression (RVR) approach is a good competitor of SVR. In the RVR case, there is no restriction on the basis functions, unlike the SVR framework, where the basis functions must satisfy the Mercer’s kernel theorem [18,19]. Also the kernel width σ is the only factor to be tuned in the RVR method. Therefore, the sparse RVR method could generalize better with a very less computation time than SVR. In this research work, the improved RVR is suggested for the indirect estimation of UCSWR. The optimization algorithms applied for optimizing RVR are the harmony search (HS) and cuckoo search (CS) algorithms. The HS and CS algorithms are utilized to choose the suitable kernel parameters of their RVR method. The goodness of each hybrid method was considered using the data available in the literature.

2. Materials and methods

In this part, first, the literature review relevant to the RVR model is described, and then there are descriptions about the HS and CS algorithms.

2.1. Relevance vector regression (RVR)

RVR, presented by Tipping [18], is actually a special case of a Gaussian process. Unlike SVR, the uncertainty in the output estimation value can be characterized. Also RVR has a better sparseness than SVR, which can reduce online prediction complexity. In addition, RVR does not require to estimate the error/margin trade-off parameter C, which can reduce the computational time, and the kernel function does not need to satisfy the Mercer condition. For those advantages of the RVR approach compared with SVR, RVR has received a great attention, and is successfully employed in the regression problems of estimation [20-22].

In the RVR approach, supposing the system is multiple-input-single-output, given a dataset of N input vectors with N corresponding scalar-valued target \( \{x_n, t_n\} \), the output \( t = (t_1, \ldots, t_N)^T \) can be expressed as the sum of an approximation vector \( y(x) \). The targets are from the model with additive noise:

\[
t_n = y(x_n, w) + e
\]

where \( w \) is the weight vector and \( e \) is the random noise. The function \( y(x) \) is defined as follows:

\[
y(x, w) = \sum_{i=1}^{N} w_i K(x, x_i) + w_0
\]

\[
= \sum_{i=1}^{N} w_i \Phi(x)
\]

(2)

\( \Phi(x) \) is given as \( \Phi(x) = [1, K(x, x_1), K(x, x_2), \ldots, K(x, x_N)] \).

The targets can be given as \( p(t|x_n) = N(t|y(x_n), \sigma^2) \). The likelihood of the complete dataset can be written as:

\[
p(t|w, \sigma^2) = \frac{1}{\pi \sigma^2} \exp \left\{ - \frac{1}{2 \sigma^2} \| t - \Phi(x)w \| \right\}
\]

(3)

where \( w = (w_0, w_1, \ldots, w_N) \), \( t = (t_1, t_2, \ldots, t_N) \), and \( \Phi \) is the \( N \times (N + 1) \) design matrix. Here, the RVR approach adopts a Bayesian perspective and
limits $w$ and $\sigma^2$ by defining a prior probability

distribution over the weights:

$$p(w|a) = \prod_{i=1}^{N} N(w_i|0, \alpha_i^{-1})$$

$$= \frac{1}{2\pi^{N+1/2}} \prod_{i=1}^{N} \alpha_i^{1/2} \exp \left( - \frac{\alpha_i w_i^2}{2} \right) \quad (4)$$

$$p(a) = \prod_{i=1}^{N} \text{gamma}(\alpha_i|a,b)$$

$$p(\beta) = \text{gamma}(\beta|a,b) \quad (6)$$

where $b = \sigma^2$, $a$ is an N+1 hyper-parameter, and

$$\text{gamma}(a|a,b) = 1 \Gamma(a)^{-1} b^a a^{-1} e^{-ba} \Gamma(a) = \int_0^\infty t^{a-1} e^{-zt} dt \quad (7)$$

Also the posterior over weights can be considered through the Bayesian rule:

$$p(w|t, a, \sigma^2) = \frac{p(t|w, \sigma^2)p(w|a)}{p(t|a, \sigma^2)}$$

$$= \frac{1}{2\pi^{N+1/2}} |\Sigma|^{-1/2} \exp \left( - \frac{1}{2} (w-\mu)^T \Sigma^{-1} (w-\mu) \right) \quad (8)$$

where the posterior covariance and mean are defined as follow:

$$\Sigma = (\sigma^{-2}2\Phi^{-2}\Phi + A)^{-1} \quad (9)$$

$$\mu = \sigma^{-2} \Sigma \Phi t$$

$$\text{where } A = \text{diag}(\alpha_1, \alpha_2, \ldots, \alpha_N). \text{ The likelihood distribution over the training targets is given by Tipping [18]:}$$

$$p(t|a, \sigma^2) = \int p(t|w, \sigma^2)p(w|a)dw$$

$$= (2\pi)^{-N/2} |C|^{-1/2} \exp \left\{ \frac{1}{2} t^T C^{-1} t \right\} \quad (11)$$

where the covariance is given by

$$C = \sigma^{-2} I + \Phi A^{-1} \Phi^T. \text{ A detailed explanation of the RVR approach can be found in [18,23].}$$

### 2.2. HS algorithm

HS [24] is a metaheuristic algorithm that simulates the improvisation process of musicians. The HS algorithm does not require the initial values for the decision variables, and uses a stochastic random search that is based on the harmony memory considering the rate and the pitch adjusting rate. The method is very easy to implement, and there are few parameters to adjust. HS is described below:

**Step 1. Initialization of the optimization problem and algorithm parameters**

The optimization problem can be defined as:

Minimize $f(x)$ subject to $x_u \leq x_i \leq x_u$ (i $= 1,2,\ldots,N$), where $x_u$ and $x_l$ are the upper and lower bounds for decision variables, respectively. The HS algorithm parameters are also specified in this step. They are the harmony memory size (HMS) or the number of solution vectors in harmony memory, pitch adjusting rate (PAR), harmony memory considering rate (HMCR), distance band width (bw), and the number of improvisations (K) or stopping criterion. K is the same as the total number of function evaluations.

**Step 2. Harmony memory initialization**

HS is initialized in the harmony memory (HM). The harmony memory is a memory location, where all the solution vectors (sets of decision variables) are stored. The initial harmony memory is randomly generated in the region $[x_u, x_l]$ (i $= 1,2,\ldots,N$). This is done based on the following equation:

$$x_i' = x_u + \text{rand}() \times (x_u - x_i) \quad j = 1,2,\ldots,\text{HMS} \quad (12)$$

where rand() is a random number from a uniform distribution of [0,1].

**Step 3. Improvise a new harmony from HM.** $x' = (x_1', x_2', \ldots, x_N')$ is improvised based on the following three mechanisms [25-27]: random selection, memory consideration, and pitch adjustment. In the random selection, the value of each decision variable in the new harmony vector is randomly chosen within the value range with a probability of (1 - HMCR). HMCR, which varies between 0 and 1, is the rate of choosing one value from the historical values stored in HM, and (1 - HMCR) is the rate of randomly selecting one value from the possible range of values [28].

$$x_i' \in \{x_i^1, x_i^2, \ldots, x_i^{\text{HMS}}\} \text{ with probability } \text{HMCR}$$

$$x_i' \in x_i \text{ with probability } (1 - \text{HMCR}) \quad (13)$$

The value for each decision variable obtained by the memory consideration is examined to determine whether it should be pitch adjusted. If the pitch adjustment decision for $x_i$ is made with a probability of PAR, $x_i$ is replaced with $x_i' \pm u(-1, +1) \times \text{bw}$, where bw is an arbitrary distance band width for the continuous design variable, and $u(-1, +1)$ is a uniform distribution between -1 and 1. The value of (1 - PAR) sets the rate of performing nothing. Thus pitch adjustment is applied to each variable as follows:

**Step 4. UpdateHM with the new harmony**

The initial harmony memory is randomly generated in the region $[x_u, x_l]$ (i $= 1,2,\ldots,N$). This is done based on the following equation:

$$x_i' = x_u + \text{rand}() \times (x_u - x_i) \quad j = 1,2,\ldots,\text{HMS} \quad (12)$$

where rand() is a random number from a uniform distribution of [0,1].
\[ x'_i = x'_i \pm \mu(-1, +1) \times bw \text{ with probability } \text{HMCR} \] (14)

\[ x'_i = x'_i \text{ with probability } \text{HMCR} \times (1 - \text{PAR}) \]

**Step 4.** Training the SVR model and fitness evaluation

For this purpose, the whole dataset is separated into two independent and non-overlapping datasets of testing set and training set arbitrarily; the former is employed for the training and optimal parameter selection procedures and the latter assesses the model prediction robustness and ability.

**Step 5.** Harmony memory update

After a new harmony vector \( x^{\text{new}} \) is generated, the harmony memory will be updated. If the fitness of the improvised harmony vector \( x^{\text{new}} = (x'_1, x'_2, \ldots, x'_{n_H}) \) is better than that of the worst harmony, the worst harmony in HM will be replaced with \( x^{\text{new}} \) and become a new member of the HM.

**Step 6.** Termination

Repeat steps 3-5 until the stopping criterion (maximum number of improvisations \( K \)) is met.

In this paper, the kernel factor of Gaussian RBF kernel \( K_{\text{RBF}}(x_i, x_j) = \exp \left( -\frac{||x_i - x_j||^2}{\sigma^2} \right) \) is selected by the HS algorithm.

### 2.3. CS algorithm

The CS algorithm [29] is inspired by some species of a bird family called cuckoo because of their special lifestyle and aggressive reproduction approach [30-32]. In order to describe the CS algorithm, the following three idealized rules are used [32]: (a) each cuckoo lays one egg at a time and dumps it in a randomly selected nest; (b) the best nests with high quality of eggs are carried over to the next generations; and (c) the accessible host nest number is constant, and the egg, which is laid using a cuckoo, is discovered by the host bird with a probability in the range of \([0, 1]\). A detailed description of the CS algorithm can be found in [29]. Also Figure 1 presents a flow chart of the CS algorithm. In this work, the kernel parameter of Gaussian RBF kernel \( K_{\text{RBF}}(x_i, x_j) = \exp \left( -\frac{||x_i - x_j||^2}{\sigma^2} \right) \) is selected by the CS algorithm.

![Figure 1. A flow chart of the CS algorithm [29].](image)

### 2.4. RVR Optimized by HS and CS Algorithms

In RVR, the HS and CS algorithms are applied as an optimizer for the hyper-parameters of RVR. Usually, RVR is hybridized by the HS and CS algorithms, where here, the prediction results achieved by RVR act as a fitness function evaluation. The optimized value of RVR hyper-parameters can be obtained after a maximum iteration number has been reached. In this work, the objective function is served by root mean squared error (RMSE), where the lower the RMSE, the better is the estimation accuracy. The procedure of optimizing the RVR variables with the HS and CS algorithms is presented in Figure 2.
3. Forecasting UCSWR using RVR-HS and RVR-CS models

In order to forecast UCSWR, all the relevant parameters should be determined, due to the fact that RVR-HS and RVR-CS work based on the given data and do not have a previous knowledge about the subject of prediction. The following sections describe the input and output parameters and prediction of UCSWR using the RVR-HS and RVR-CS models.

3.1. Database information

The main scope of this work is to implement the above methodology in the problem of UCSWR. The dataset applied in this work for determining the relationship among the set of input and output variables are gathered from the open source literature [33]. A dataset that includes 40 case studies was employed in the current study, while 32 data points (80%) were utilized for constructing the models and the remainder (8 data points) was utilized for the model performance evaluation. The partial datasets in Table 1 contain data for 5 data points: the bulk density (BD), point load index test (Is(50)), Brazilian tensile strength test (BTS), ultrasonic test (V_p), and UCSWR.

<table>
<thead>
<tr>
<th>Rock type</th>
<th>BD (Kg/m^3)</th>
<th>BTS (MPa)</th>
<th>Is(50) (MPa)</th>
<th>V_p (m/s)</th>
<th>UCSWR (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shale</td>
<td>3516</td>
<td>3.8</td>
<td>3.9</td>
<td>2897</td>
<td>55.9</td>
</tr>
<tr>
<td>Shale</td>
<td>3,435</td>
<td>3.7</td>
<td>3.7</td>
<td>2,857</td>
<td>47.3</td>
</tr>
<tr>
<td>Iron pan</td>
<td>2,455</td>
<td>1.7</td>
<td>0.4</td>
<td>1,820</td>
<td>8.4</td>
</tr>
<tr>
<td>Iron pan</td>
<td>2,522</td>
<td>1.6</td>
<td>0.5</td>
<td>1,852</td>
<td>14.4</td>
</tr>
<tr>
<td>Old alluvium</td>
<td>2,236</td>
<td>2.7</td>
<td>0.2</td>
<td>1,909</td>
<td>14.5</td>
</tr>
</tbody>
</table>

3.2. Performance Criterion

To measure the accuracy, the difference between the output of the model and the real output is considered as the error and represented in two ways including mean squared error (MSE) and squared correlation coefficient (R^2) [34-40]. Let t_k be the actual value, \( \hat{t}_k \) be the predicted value of the k^th observation, and n be the number of observations; then MSE and R^2 could be defined, respectively, as follow:

\[
MSE = \frac{1}{n} \sum_{k=1}^{n} (t_k - \hat{t}_k)^2 \quad (15)
\]

\[
R^2 = 1 - \frac{\sum_{k=1}^{n} (t_k - \hat{t}_k)^2}{\sum_{k=1}^{n} t_k^2 - \frac{\sum_{k=1}^{n} t_k^2}{n}} \quad (16)
\]

3.3. Algorithm Configuration

In the proposed RVR-HS and RVR-CS, many parameters are required to be set carefully. In the CS algorithm, the maximum iteration number = 50, number of nests = 8, population number (number of cuckoos) = 25, discovery rate of alien eggs/solutions = 0.75, and beta = 3.2. Also in the HS algorithm, the maximum iteration number = 50,
population number = 25, harmony memory consideration rate = 0.4, pitch adjustment rate = 0.1, number of new harmonies = 15, and fret width damp ratio = 0.995. In order to obtain a good performance of the RVR model, the parameter is set differently in each operation process. At last, the one much better than the mean value is chosen in this work.

4. Results and Discussion
In this work, the RVR-HS and RVR-CS models were utilized to build a prediction model for forecasting UCSWR from the available data using MATLAB environment. All data (40 data points) were randomly divided into two subsets: 80% of the total data was allotted to train data of model construction and 20% of the total data was allocated for test data used to assess the reliability of the developed model. In these models, BD, BTS, Is(50), and Vp were utilized as the input parameters, while UCSWR was the output parameter.

In the data-driven system modeling methods, some pre-processing steps are commonly implemented prior to any calculations to eliminate any outliers, missing values or bad data. This step ensures that the raw data retrieved from the database is perfectly suitable for modeling. In order to soften the training procedure and improve the accuracy of prediction, all data samples are normalized to adapt to the interval [0, 1] according to a linear mapping function. After modeling, a correlation between the estimated values of UCSWR by the RVR-HS and RVR-CS models and measured values for training and testing phases is shown in Figures 3 and 4. As shown in these figures, the results of the RVR-HS model in comparison with the actual data show a good precision of the RVR-HS model.

Figure 3. Correlation between the measured and estimated UCSWR using the RVR-HS model for a) training datasets b) testing datasets.
Figure 4. Correlation between the measured and estimated UCSWR using the RVR-CS model for a) training datasets b) testing datasets.

Also the performance analysis of the RVR-HS and RVR-CS models for predicting UCSWR is shown in Table 2. As presented in this table, the RVR-HS model with $R^2 = 0.9903$ and MSE = 0.0031203 is found to be the best predictive model.

Table 2. Performance analysis of the RVR-HS and RVR-CS models for forecasting UCSWR.

<table>
<thead>
<tr>
<th>Description</th>
<th>MSE (Train)</th>
<th>$R^2$ (Train)</th>
<th>MSE (Test)</th>
<th>$R^2$ (Test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>RVR-HS model</td>
<td>Training</td>
<td>0.0022038</td>
<td>0.9889</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Testing</td>
<td>0.0031203</td>
<td>0.9903</td>
<td></td>
</tr>
<tr>
<td>RVR-CS model</td>
<td>Training</td>
<td>0.0022207</td>
<td>0.9884</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Testing</td>
<td>0.0039557</td>
<td>0.9684</td>
<td></td>
</tr>
</tbody>
</table>

In addition, according to Figure 5 and Table 3, the MSE and $R^2$ of RVR-HS model (for training/testing = 80/20) is less than those for the other models in almost all the cases, indicating that it can be a better choice for a prediction process. It is worth mentioning that the presented model was developed based upon the limited sets of data, and cannot be generalized for all rocks. However, it is open for more development if more data is available.

Table 3. Comparing the performance of RVR-HS model in forecasting UCSWR with different fractions of training and testing data.

<table>
<thead>
<tr>
<th>Training/testing (%)</th>
<th>Model</th>
<th>MSE (Train)</th>
<th>MSE (Test)</th>
<th>$R^2$ (Train)</th>
<th>$R^2$ (Test)</th>
</tr>
</thead>
<tbody>
<tr>
<td>90/10</td>
<td>RVR-HS</td>
<td>0.0023631</td>
<td>0.0044612</td>
<td>0.98032</td>
<td>0.99023</td>
</tr>
<tr>
<td>80/20</td>
<td>RVR-HS</td>
<td><strong>0.0022038</strong></td>
<td><strong>0.0031203</strong></td>
<td><strong>0.98194</strong></td>
<td><strong>0.99037</strong></td>
</tr>
<tr>
<td>70/30</td>
<td>RVR-HS</td>
<td>0.002483</td>
<td>0.005784</td>
<td>0.97731</td>
<td>0.98681</td>
</tr>
<tr>
<td>60/40</td>
<td>RVR-HS</td>
<td>0.0057377</td>
<td>0.022629</td>
<td>0.96933</td>
<td>0.96977</td>
</tr>
</tbody>
</table>
5. Conclusions
UCSWR is a very important parameter for rock classification and design of structures either upon or inside rocks. In addition, this parameter is essential for judgment about its suitability for various construction purposes. However, determination of UCSWR is time-consuming and expensive, and involves destructive tests. Therefore, an indirect test is often used to predict UCSWR. In this paper, a new approach, namely RVR optimized by the HS and CS algorithms, was proposed for predicting UCSWR. In our methodology, the HS and CS algorithms were applied as the optimization tool for determining the optimal value of user-defined parameters existing in formulation of RVR. The optimization implementation increases the performance of the RVR model. The following conclusions were obtained:

- RVR-HS with $R^2 = 0.9903$ and MSE = 0.0031203 is a reliable system modeling technique for forecasting UCSWR with a highly acceptable degree of accuracy and robustness.

- Application of evolutionary algorithms significantly increases the speed and accuracy of finding the optimal values of kernel parameters.

- It is worth mentioning that the presented model was developed based upon the limited sets of data, and cannot be generalized for all rocks. However, it is open for more development if more data is available.

References


یک روش جدید برای تخمین مقاومت فشاری تک محوره سنگ‌های ضعیف

هادی فتحی

1- گروه مهندسی زمین‌شناسی، دانشکده مهندسی علوم زمین، دانشگاه صنعتی اراک، ایران
2- ارسال ۲۴/۱/۲۰۲۰، پذیرش ۹/۳/۲۰۲۰

* نویسنده مسئول مقاله h.fattahi@arakut.ac.ir

چکیده:
مقاومت فشاری تک محوره سنگ‌های ضعیف از جمله پارامترهای مهم در طراحی فضاهای زیرزمینی، معدن‌نوردی و زیربتنی است. مقاومت‌های روش‌های استاندارد مانند RMI و RMR معمولاً به دلیل پرهزینه و زمان‌بگیری استفاده در این پرونده در محیط‌های زیرزمینی عمل نمی‌کنند. از این رو با بررسی مدل‌های جدیدی از این روش‌ها برای تخمین مقاومت فشاری خاصی که به عنوان پارامتر وزن، دما، نرخ فرآیند واریانس نیروی نیروی میانگین که به آن ارتباط دارد مختصات می‌گیرد. در این مقاله پژوهشی بررسی کاربرد روش‌های استاندارد ساختاری برای تخمین مقاومت فشاری تک محوره سنگ‌های ضعیف در محیط‌های زیرزمینی و معدن‌نوردی ارائه گردیده است. روش‌های استاندارد ساختاری برای مدل‌های ساختاری، مثال‌هایی از نحوه عملکرد آنها در محیط‌های مختلف توضیح گرند. این مقاله به بررسی آنالیز مدل‌های استاندارد ساختاری و مقاومت فشاری تک محوره سنگ‌های ضعیف ارائه می‌شود.

کلمات کلیدی: مقاومت فشاری تک محوره سنگ‌های ضعیف، رگرسیون بردار ارتباط، تکنیک کم‌ریزه کردن، تکنیک جستجوی هارمونیک، کوکومبی، فشردگی.