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A Comprehensive Study of Several Meta-Heuristic Algorithms for Open-Pit Mine Production Scheduling Problem Considering Grade Uncertainty

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Abstract

It is significant to discover a global optimization in the problems dealing with large dimensional scales to increase the quality of decision-making in the mining operation. It has been broadly confirmed that the long-term production scheduling (LTPS) problem performs a main role in mining projects to develop the performance regarding the obtainability of constraints, while maximizing the whole profits of the project in a specific period. There is a requirement for improving the scheduling methodologies to get a good solution since the production scheduling problems are non-deterministic polynomial-time hard. The current paper introduces the hybrid models so as to solve the LTPS problem under the condition of grade uncertainty with the contribution of Lagrangian relaxation (LR), particle swarm optimization (PSO), firefly algorithm (FA), and bat algorithm (BA). In fact, the LTPS problem is solved under the condition of grade uncertainty. It is proposed to use the LR technique on the LTPS problem and develop its performance, speeding up the convergence. Furthermore, PSO, FA, and BA are projected to bring up-to-date the Lagrangian multipliers. The consequences of the case study specifies that the LR method is more influential than the traditional linearization method to clarify the large-scale problem and make an acceptable solution. The results obtained point out that a better presentation is gained by LR–FA in comparison with LR-PSO, LR-BA, LR-Genetic Algorithm (GA), and traditional methods in terms of the summation net present value. Moreover, the CPU time by the LR-FA method is approximately 16.2% upper than the other methods.

1. Introduction

One of the main steps in mine planning is a long-term production scheduling optimization process. Its main purpose is to make the most of the net present value of the entire profits from the production process, while filling all the operational constraints include ore production, grade blending, mining capacity, mining slope, etc. during each scheduling period with a pre-arranged high degree of probability. Optimization and mathematical formulation have been used so as to solve the long-term production scheduling problems since the 1960s. The deterministic and uncertainty-based approaches are the two main mathematical optimization ones practiced to solve these kinds of problems. Nevertheless, the supposition of input certainty is not always accurate. The truth is that certain limits affect the verification of some e data including ore grades, future product demand, future product price, and production costs. Thus decisions on production plan have to be made before knowing the exact values of those data. Any deterministic method is incapable of coping with uncertainty in a quantitative manner. This will affect the producing infeasible plans regarding the production necessities. figThe major methods applied to solve the long-term production scheduling (LTPS) problem are heuristics, dynamic programming, and integer programming. A common heuristic method is based

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on the Lerchs-Grossmann algorithm [1], which is a precise approach to determine the ultimate pit. For the sake of computing time, the floating-cone methods have been used as heuristic and precise alternatives to the Lerchs-Grossman algorithm. Various heuristics have also been expanded for this problem. Pana [2] has presented a heuristic as a moving cone or dynamic cone. This heuristic has been enhanced by Korobov [3, 4], and then amended by Dowd and Onur [5, 6]. A dynamic programming approach to the LTPS problem has been demonstrated by Onur and Dowd [7]. They have mentioned that a solely dynamic programming approach is improbable to be able to solve the large-scale problem cases. Laurich and Kennedy [8] have also examined the application of a constructive heuristic called incremental pit expansion. These algorithms have not yielded an optimal solution yet [9] as the researchers have come to great accomplishments in this regard. As presented in Table 1, the direction of production planning and the provision of optimal algorithms have been greatly deliberated within the last few decades.

Table 1. Review of the presented models since 1969.

Year	Authors	Model	D[1]	U[2]	HMM[3]	Ref.
1969	Johnson	Linear Programming	*			[10]
1974	Williams	Dynamic Programming, Integer Programming, Network Flow, Parametric Programming	*			[11]
1983	Gershon	Linear Programming , Mixed Integer Programming	*			[12]
1986	Dagdelen and Johnson	Lagrangian Relaxation Method	*			[13]
1992	Ravenscroft	Conditional Simulation		*		[14]
1994	Dowd	Geostatistical simulation		*		[15]
1995	Elevli	Operation Research, Artificial Intelligence	*			[16]
1995	Denby and Schofield	Genetic Algorithm		*	*	[17]
1998	Tolwinski	Dynamic Programming	*			[18]
1999	Akaike and Dagdelen	4D Network Relaxation	*			[19]
2000	Whittle	Milawa	*			[20]
2002	Johnson *et al.*	Mixed Integer Programming	*		*	[21]
2002	Dimitrakopoulos *et al.*	Generalized Sequential Gaussian Simulation, Direct Block Simulation		*		[22]
2004	Godoy and Dimitrakopoulos	Simulated Annealing Algorithm		*	*	[23]
2004	Dimitrakopoulos and Ramazan	Linear Programming	*			[24]
2004	Ramazan and Dimitrakopoulos	Mixed Integer Programming	*			[25]
2004	Ramazan and Dimitrakopoulos	Mixed Integer Programming		*		[26]
2006	Gholamnejad *et al.*	Chance Constrained Programming		*		[27]
2007	Gholamnejad and Osanloo	Chance Constrained Integer Programming		*		[28]
2007	Ramazan and Dimitrakopoulos	Stochastic Integer Programming	*			[29]
2009	Boland *et al.*	Mixed Integer Programming	*			[30]
2010	Bley *et al.*	Integer Programming	*			[31]
2010	Kumral	Robust stochastic optimization		*		[32]
2012	Lamghari and Dimitrakopoulos	Tabu Search		*	*	[33]
2012	Gholamnejad and Moosavi	Binary Integer Programming		*		[34]
2013	Latorre Nanjari and Golosinski	Dynamic Programming, Mining Heuristic				[35]
2013	Sattarvand and Niemann-Delius	Ant Colony Optimization	*		*	[36]

Table 1. (Continued).

Year	Authors	Model	D[1]	U[2]	HMM[3]	Ref.
2013	Goodfellow and Dimitrakopoulos	Simulated Annealing Algorithm		*	*	[37]
2013	Dimitrakopoulos and Jewbali	Stochastic integer programming	*			[38]
2014	Leite and Dimitrakopoulos	Stochastic Integer Programming		*		[39]
2014	Moosavi *et al*.	Lagrangian Relaxation Method, Genetic Algorithm	*		*	[40]
2014	Moosavi *et al*.	Augmented Lagrangian Relaxation Method, Genetic Algorithm	*		*	[41]
2014	Koushavand *et al*.	Mixed Integer Linear Programming		*		[42]
2014	Asad et al.	Stochastic Network Flow, Lagrangian Relaxation Method		*	*	[43]
2014	Lamghari *et al*.	Variable Neighbourhood Descent Algorithm		*	*	[44]
2015	Shishvan and Sattarvand	Ant Colony Optimization	*		*	[45]
2016	Mokhtarian and Sattarvand	Imperialist Competitive Algorithm	*		*	[46]
2016	Mokhtarian and Sattarvand	Commodity Price Distribution function, Median Latin Hypercube sampling method, Integer Programming	*			[47]
2016	Goodfellow and Dimitrakopoulos	Simulated Annealing Algorithm, Particle Swarm Optimization, Differential Evolution		*	*	[48]
2016	Lamghari and Dimitrakopoulos	Rockafellar and Wets Progressive Hedging Algorithm		*		[49]
2016	Lamghari and Dimitrakopoulos	Tabu Search Heuristic Incorporating a Diversification Strategy, Variable Neighborhood Descent Heuristic, Very Large Neighborhood Search Heuristic, Network Flow Techniques, Diversified Local Search		*	*	[50]
2017	Bakhtavar *et al*.	Stochastic Chance-Constrained Programming		*		[51]
2018	Khan	Particle Swarm Optimization, Bat Algorithm		*	*	[52]
2018	Rahimi *et al*.	Logical mathematical algorithm	*		*	[53]
2018	Tahernejad *et al*.	Information Gap Decision Theory		*		[54]
2018	Jelvez *et al*.	Expected Time Incremental Heuristic Algorithm	*		*	[55]
2018	Khan and Asad	Mixed Integer Linear Programming	*			[56]
2018	Alipour *et al*.	Robust counterpart linear optimization, Genetic Algorithm		*	*	[57]
2019	Chatterjee and Dimitrakopoulos	Lagrangian Relaxation Method, Sub-Gradient Method, Branch and Cut Algorithm		*	*	[58]
2019	Dimitrakopoulos and Senécal	Multi-Neighborhood Tabu Search		*	*	[59]

[1] Deterministic, [2] Uncertainty, [3] Heuristic, and Meta-heuristic Methods.

The present paper dissects the application of meta-heuristic algorithms to solve the problems of long-term scheduling of open-pit mining with consideration of uncertainty. The authors presented hybrid models by Lagrangian relaxation (LR), particle swarm optimization algorithm (PSO), firefly algorithm (FA), and bat algorithm (BA) to elucidate the LTPS problem under the condition of grade uncertainty. To develop its performance, speeding up the convergence, the present study suggests practicing the LR methods on the LTPS problem. Additionally, PSO, FA, and BA are applied to bring

up-to-date the Lagrangian multipliers. A case study with its limitations was evaluated to inspect the efficacy of the proposed methods. While many limits reveal a useful system, it is distinguished that the whole generation price can be lessened. In comparison with the first method, the results obtained demonstrate that the complete version has taken significant progress. The consequences are very close to the best results achieved in the literature.
The subsequent parts of this paper are scheduled as below. Section 2 indicates the objective functions and their associated limitations. Sections 3 and 4 indicate a summary of the methodology and hybrid models, and the proposed models will be developed. Section 5 includes an evaluation of the outcomes. Above and beyond, the data collection and preparation are defined in this section. Validation of the established models is done. Finally, Section 6 shows the deduction.

2. LTPS Problem Formulation

The long-term production scheduling model is put into practice to estimate the production purposes and ore material current within several years. Totally, it takes a basic image of the production and voices it as a linear problem.

2.1. Objective Function

The simplest method is to illustrate a full space optimization model, each period of the planning horizon, to consider the decision-making fortitudes. Remarkably, the obtainability of restraints is bonded into the model. Henceforth, the LTPS problem is indicated in the subsequent parts.

$$Maximize \quad Z = \sum_{n=1}^{N} \sum_{t}^{T} \frac{NV_n^t}{(1+r)^t} \times X_n^t \qquad (1)$$

In the constructed model, the following indications were accepted. n is the block identification number; n = 1, 2,…, N; N is the total number of blocks to be scheduled; t is the scheduling period index, t =1, 2,…, T; T is the total number of scheduling periods; NV_n^t is the net value to be generated by mining block n in period t; r is the discount rate in each period; and X_n^t is the binary variable.

2.2. Constraints in model

2.2.1. Grade Blending Constraints

One of the most important hitches in production scheduling is the ore grade that has to be set aside steady while leading to the processing plant. For this reason, the grade of ore that is being steered to the mill should be well-defined between two limits.

2.2.1.1. Upper Bound Constraints.

It is significant that the average grade of the material directed to the mill should be a lesser quantity or equal to the certain grade value, G_{max} for each period, t:

$$\sum_{n=1}^{N} (g_n - G_{max}) \times X_n^t \leq 0 \qquad (2)$$

where g_n is the average grade of block n and O_n is the ore tonnage in block n.

2.2.1.2. Lower Bound Constraints.

Extraordinarily, the average grade of the material conducted to the mill has to be more or alike the definite value G_{min} for each period, t:

$$\sum_{n=1}^{N} (g_n - G_{min}) \times X_n^t \geq 0 \qquad (3)$$

2.2.2. Reserve Constraints

Reserve restrictions made for each block specify that all measured blocks in the model should be mined on one occasion.

$$\sum_{t=1}^{T} X_n^t = 1 \quad , \forall n = 1,2,3,\dots,N \qquad (4)$$

2.2.3. Processing Capacity Constraint

Total tons of the treated ore should not surpass the processing capacity (PC_{max}) in every period, t:

$$\sum_{n=1}^{N} (O_n \times X_n^t) \leq PC_{max} \qquad (5)$$

2.2.4. Mining Capacity Constraint

The entire quantity of material (waste and ore) to be mined cannot be more than the whole accessible mining capacity (MC_{max}) for each period (t):

$$\sum_{n=1}^{N} (O_n + W_n) \times X_n^t \leq MC_{max} \qquad (6)$$

where W_n is the tonnage of waste material within block n.

2.2.5. Wall slope Constraints

These restraints ratify that it is indispensable to mine all blocks limited directly through the mining of block k, a target block before the extraction of block k is begun. There are two methods for these constraints:
Using one constraint for each block per each period:

$$X_k^t - \sum_{r=1}^{t} X_y^r \leq 0 \quad , \forall k = 1,2, \dots, N \quad , \forall t = 1,2, \dots, T \tag{7}$$

where k is the index of a block considered as extraction at period t and Y is the total number of blocks followed by block k.
Using Y-constraints for each block at each period:

$$YX_k^t - \sum_{y=1}^{l} \sum_{r=1}^{t} X_y^r \leq 0 \quad , \forall y = 1,2, \dots, l \quad , \forall k = 1,2, \dots, N \quad , \forall t = 1,2, \dots, T \tag{8}$$

2.3. LTPS model considering grade uncertainty

Fundamentally, mining space is denoted as a possible space based on the uncertainty that leads to this space. In mining engineering procedures, the uncertainty makes a decision built on uncertain results. Dimitrakopoulos (1998) [60] held the arrangement of uncertainties in mining projects because of the importance of this subject.

- The uncertainty in the ore deposit model is connected to the uncertainty in tonnage and grade.
- Technical uncertainty like extraction structures: slope constraints, drilling capacity, etc.
- Economic uncertainties, capital costs, comprising product prices, operating costs.

Amongst the uncertainties, grade uncertainty leads to a large portion of probabilities caused by grade uncertainty.

An integer programming-based model is provided in this section to inspect the grade uncertainty. In this method, a possibility is apportioned to each block (PI_n), which indicates the probability made from n for each block in the block model. Now, it is time to organize our objective function in such a way that earlier production periods are given to mine the blocks with higher certainty. When additional information usually becomes obtainable, the uncertain blocks are gone for later periods. Subsequently, one more objective function is presented to the objective function of the traditional model in the subsequent form of:

$$Maximize \quad Z_2 = \sum_{n=1}^{N} \sum_{t}^{T} \frac{NV_n^t}{(1+r)^t} \times PI_n \times X_n^t \tag{9}$$

This objective function brings about the constraints (2) to (8).

3. LR Function for LTPS Problem

The Lagrangian relaxation (LR) method is recognized as one of the mathematical means of a mixed-integer programming problem. In the presentation [61-65] of this technique in LTPS, Lagrangian multipliers relax the system limitations and introduce them to the objective function. Next, the relaxed problem intensified into a more controllable sub-problem for separate units and solved through dynamic programming. Next, the relaxed problem is intensified into a more controllable sub-problem for separate units and solved through dynamic programming. The convergence standard is satisfied in case the convergence standard is achieved.

Basically, LR is based on the viewpoint to relax the system restrictions as a result of Lagrangian multipliers. For the next step, the relaxed problem is split into some smaller sub-problems [66]. The constant Lagrangian function can be made by dint of assigning non-negative Lagrangian multipliers λ^t, μ^t, and η^t in terms of processing type at period t to the constraints (3), (5), and (6), respectively.

$$\begin{aligned} Max \quad L(X,\lambda,\mu,\eta) &= Z_2(X) \\ &- \sum_{t=1}^{T} \lambda^t \left(\sum_{n=1}^{N} (g_n - G_{min}) \times O_n \times X_n^t \right) \\ &+ \sum_{t=1}^{T} \mu^t \left(PC_{max} - \sum_{n=1}^{N} (O_n \times X_n^t) \right) \\ &+ \sum_{t=1}^{T} \eta^t \left(MC_{max} - \sum_{n=1}^{N} (O_n + W_n) \times X_n^t \right) \end{aligned} \tag{10}$$

LTPS is elucidated via the Lagrangian relaxation method by relaxing or momentarily ignoring the preventing constraints and solving the problem as if they have never been. While maximizing due to the control variable X_n^t in LTPS, this is done over the dual optimization process, which strives to affect the constrained optimum by lessening the Lagrangian function L due to the Lagrangian multipliers λ^t, μ^t, and η^t.

$$j^* = \min_{\lambda,\mu,\eta} j(\lambda, \mu, \eta)$$

Where

$$j(\lambda, \mu, \eta) = \max_{X} L(X, \lambda, \mu, \eta)$$

Subjected to the constraint (4), assume that λ, μ, and η are fixed, we maximize the Lagrangian function L as follows. From Eq. (10), we have:

$$\begin{aligned} Max \quad L(X,\lambda,\mu,\eta) = & \sum_{n=1}^{N}\sum_{t}^{T}\frac{NV_n^t}{(1+r)^t} \times PI_n \times X_n^t \\ & - \sum_{t=1}^{T}\lambda^t\left(\sum_{n=1}^{N}(g_n - G_{min}) \times O_n \times X_n^t\right) \\ & + \sum_{t=1}^{T}\mu^t\left(PC_{max} - \sum_{n=1}^{N}(O_n \times X_n^t)\right) \\ & + \sum_{t=1}^{T}\eta^t\left(MC_{max} - \sum_{n=1}^{N}(O_n + W_n) \times X_n^t\right) \end{aligned} \tag{11}$$

According to the Lagrangian multipliers, the amendment of Lagrangian multipliers should be rationally done to make the best use of the Lagrangian function. To regulate the Lagrangian multipliers, most references practice a combination of the sub-gradient method and several heuristics achieve a fast solution. In the current work, PSO, FA, and BA are applied to amend the Lagrangian multipliers and improve the performance of the Lagrangian relaxation technique.

4. Construction of initial solution

The meta-heuristic algorithms are measured as possible methods for making out the estimated problem. In this research work, PSO, FA, and BA were used to improve the Lagrangian multipliers.

4.1. PSO Algorithm

In 1995, Eberhart and Kennedy [67, 68] presented the first PSO methodology as an optimization method due to the possible directions. The researchers have observed the social behavior of the bird or fish groups during a food search to guide the population to a promising area for space search. Certain normal processes are practiced for the manners of the creatures of the ruling body. Birds are just searching for their food by changing their physical movements by escaping missions. Hence, every one of the group members supposedly uses the previous experiences and other detections from the members in order to find food. This kind of corporation is considered as a positive improvement within a competitive search for food. PSO is grounded on the idea of sharing information among the group members. In PSO, a particle is denoted to each answer to a problem that is the situation of a bird in the search space. All particles include a degree of the ability in which the quality of action optimizes it. Furthermore, each particle embraces a factor called velocity that identifies it in the search range [69-71].

PSO starts with a group of inadvertent replies. Next, it searches for the location and velocity of each particle so as to determine the best answer in the problem space. The two most remarkable values indicate that each particle is identified at each step of the population movement. As a result, the first step is recognized as the finest answer in terms of suitability ever obtained for each particle. This is actually the personal best and is termed pbest. The global best, identified as gbest, is another best value ever attained by means of PSO. In order to search for new solutions, swarms of particles are randomly initialized over the search space, and they move through a D-dimensional space. Authorized x_k^i and v_k^i, respectively, are the position and velocity of the *i*-th particle in the search space at the *k*-th iteration, and then the velocity and position of this particle at the (*k*+1)th iteration are updated using the following equations [72]:

$$\begin{aligned} v_{k+1}^i = w.v_k^i + c_1.r_1.(p_k^i - x_k^i) \\ + c_2.r_2.(p_k^g - x_k^i) \end{aligned} \tag{12}$$

$$x_{k+1}^i = x_k^i + v_{k+1}^i \tag{13}$$

where r_1 and r_2 demonstrate accidental numbers between 0 and 1, c_1 and c_2 are constants, p_k^i demonstrate the best position of the *i*-th particle, and p_k^g correlates with the global best position in the swarm up to the *k*th iteration. The PSO algorithm pseudo-code can be summarized as follows:

Objective function: $f(x)$, $x = (x_1, x_2, \ldots, x_D)$;

Initialize particle position and velocity for each particle and set $k = 1$.

Initialize the particle's best known position to its initial position i.e. $p_k^i = x_k^i$.

do

Update the best known position (p_k^i) of each particle and swarm's best known position (p_k^g).

Calculate particle velocity according to the velocity equation (12).

Update particle position according to the position equation (13).

While maximum iterations or minimum error criteria is not attained

Figure 1. Pseudo-code of the PSO algorithm.

4.2. Firefly Algorithm

Yang [73-75] has developed the firefly algorithm based on the perfect behavior of the flashing specifications of fireflies. For more clarification, we can overemphasize these flashing features as the following three rules:

- All fireflies are unisex to attract other fireflies notwithstanding of their sex;
- Attractiveness is comparative to their illumination, and hence, for any two flashing fireflies, the less bright one will move toward the brighter one. The attractiveness is proportional to the brightness, and they both decline as their distance surges. In case no one is brighter than a specific firefly, it moves arbitrarily;
- The brightness or light strength of a firefly is influenced or specified through the landscape of the objective function to be improved.

For the maximization problem, the brightness can merely be relative to the objective function. Extra forms of brightness can be expressed in a comparable way to the fitness function in genetic algorithms or the bacterial foraging algorithm (BFA) [76].

The variation in light intensity and formulation of attractiveness are the two significant issues in FA. For easiness, we can always suppose that the attractiveness of a firefly is specified by its brightness or light intensity, which, in turn, is related to the prearranged objective function. Nevertheless, the attractiveness is comparative; it should be realized in the eyes of the beholder or projected by the other fireflies. Though the attractiveness is comparative, it should be realized in the eyes of the beholder or protected by other fireflies as light intensity is reduced with the distance from its source, and light is also attracted to the media. Thus we should permit the attractiveness to diverge with the degree of absorption.

In the simplest form, the light intensity *I(r)* varies with the distance *r* monotonically and exponentially. That is:

$$I = I_0 e^{-\gamma r} \tag{14}$$

where I_0 is the original light intensity and is the light absorption coefficient. As a firefly's attractiveness is proportional to the light intensity seen by adjacent fireflies, we can now define the attractiveness β of a firefly by:

$$\beta = \beta_0 e^{-\gamma r^2} \tag{15}$$

where β_0 is the attractiveness at $r = 0$. It is worth pointing out that the exponent r can be replaced by other functions such as r_m when m > 0. Schematically, the Firefly Algorithm (FA) can be summarized as the pseudo-code. The FA algorithm pseudo-code can be summarized as follows:

```
Objective function f(x), x = (x1, ..., xd)^T
Initialize a population of fireflies xi (i = 1, 2, ..., n)
Define light absorption coefficient γ
While (t <MaxGeneration)
For i = 1 : n all n fireflies
For j = 1 : i all n fireflies
Light intensity Ii at xi is determined by f(xi)
If (Ij > Ii)
Move firefly i towards j in all d dimensions
end if
Attractiveness varies with distance r via exp[−γr]
Evaluate new solutions and update light intensity
end for j
end for i
Rank the fireflies and find the current best
end while
Postprocess results and visualization
```

Figure 2. Pseudo-code of the firefly algorithm.

The distance between any two fireflies i and j at x_i and x_j can be the Cartesian distance $r_{ij} = ||x_i - x_j||$ or the l_2-norm. For other applications such as scheduling, the distance can be time delay or any suitable forms, not necessarily the Cartesian distance.

The movement of a firefly *i* attracted to another more attractive (brighter) firefly *j* is determined by:

$$x_i = x_i + \beta_0 e^{-\gamma r_{ij}^2}\left(x_j - x_i\right) + \alpha \epsilon_i \tag{16}$$

where the second term is due to the attraction, while the third term is randomization with the vector of random variables *i* being drawn from a Gaussian distribution.

4.3. Bat Algorithm

One of the strongest optimization procedures is collective intelligence based on the group behaviour. In 2010, Yang [77] introduced an algorithm affected by the collective behaviour of bats in a natural environment based on the use of sound reflection by bats. Bats are able to navigate the precise trail and site of their bait via sending sound waves and receiving their reflections. The bird is able to draw a sound image of the obstacles connecting its sites and recognize them well when the sound waves turn back to the bat wave transmitter. This system makes it possible for the bats to identify the moving objects such as insects and trees. The micro-bats send short-duration loud sound beats with constant happening in the area of 25 kHz to 150 kHz and listen for the echo rebounding from the immediate objects to catch the food or escape the obstacles. Bats naturally issue 10 to 20 such sound pulses per second and are able to increase the pulse release rate to about 200 pulses per second as they approach their victim. Yang has presented the succeeding directions in order to convey these special properties of bats into an optimization algorithm:

- All bats practice their echolocation abilities so as to detect their distance from a definite object and distinguish between food/prey and background obstacle in some way. The fact is that all bats use their echolocation skills.
- Since bats are able to change the wavelength λ and loudness A_0 of their released sound pulses to discover the food, they are capable of flying inadvertently with velocity v_i at position x_i with a frequency f_{min}. Likewise, bats can modify the rate and wavelength or frequency of their emitted pulse according to their distance from the prey.
- The loudness varies from a large positive value A_0 to the least persistent value A_{min}.

Each bat's current position is regarded as a possible solution to the optimization problem [78-80].

According to the rules, the position x_i^t and the velocity v_i^t for each *i*-th virtual bat in the *t* repetition and also the frequency f_i are calculated as follow:

$$f_i = f_{min} + (f_{max} - f_{min})\beta \tag{17}$$

$$v_i^t = v_i^{t-1} + \left(x_i^t - x_*\right) \tag{18}$$

$$x_i^t = x_i^{t-1} + \upsilon_i^t \tag{19}$$

where x_i^t and x_i^{t-1} are the current and previous positions of particle i, v_i^t and v_i^{t-1} are the current and previous velocities of particle i, $\beta \in [0\,,1]$ is a random vector with uniform distribution, and x_* is the best current position that is selected in each replication after comparison with the position of the virtual bats. Usually, consider the frequency f with $f_{min} = 0$ and $f_{max} = 100$. In each replication, in the local search, one of the answers is selected as the best answer, and the new position of each bat is updated locally with the random step as follows:

$$x_{new} = x_{old} + \epsilon\overline{A^t} \tag{20}$$

where $\epsilon \in [-1\,,1]$ is a random number and $\overline{A^t}$ is the average loudness of the bats in the *t* repetition. Also the loudness of the A_i loudness and the pulse rate r sent each time, it is updated as follows:

$$A_i^{t+1} = \alpha A_i^t \tag{21}$$

$$r_i^{t+1} = r_i^0[1 - exp(-\gamma t)] \tag{22}$$

where α and γ are constant values and for $0 < \alpha < 1$ and $r > 0$. When $t \to \infty$, we have: $r_i^{t+1} \to r_i^0$ and $A_i^{t+1} \to 0$. The Bat algorithm pseudo-codes is as follows:

```
Initialize bat population with random initial positions xi and random velocities vi in the search space
Initialize each bat's frequency fi, pulse rate ri, and the loudness Ai
Calculate the fitness value of each particle at its initial position xi and determine the initial global
best position x*
While (t < maximum number of generations)
Generate a new solution by updating the frequency, velocity, and position using Eqs. (14), (15),
and (16), respectively
If (rand > ri
Select a solution among the best solutions
Generate a local solution around the selected best solution
end if
If (r and < Ai and f(xi) is better than f(x*) )
Accept the new solution
Increase ri, reduce Ai
end if
Rank the bats and find the current best x*
If the stopping criteria are met: end while
```

Figure 3. Pseudo-code of the Bat algorithm.

4.4. Framework of proposed models

In the present research work, two steps are required for hybrid methods. The first one states the Lagrangian function, which brings up-to-date the Lagrange multipliers. The second step is the precise global extension of the stated LR function, in which PSO, FA, BA, and GA are utilized to find out a new stochastic method close to the ideal maximum. Figure 4 shows a flowchart of the suggested approach.

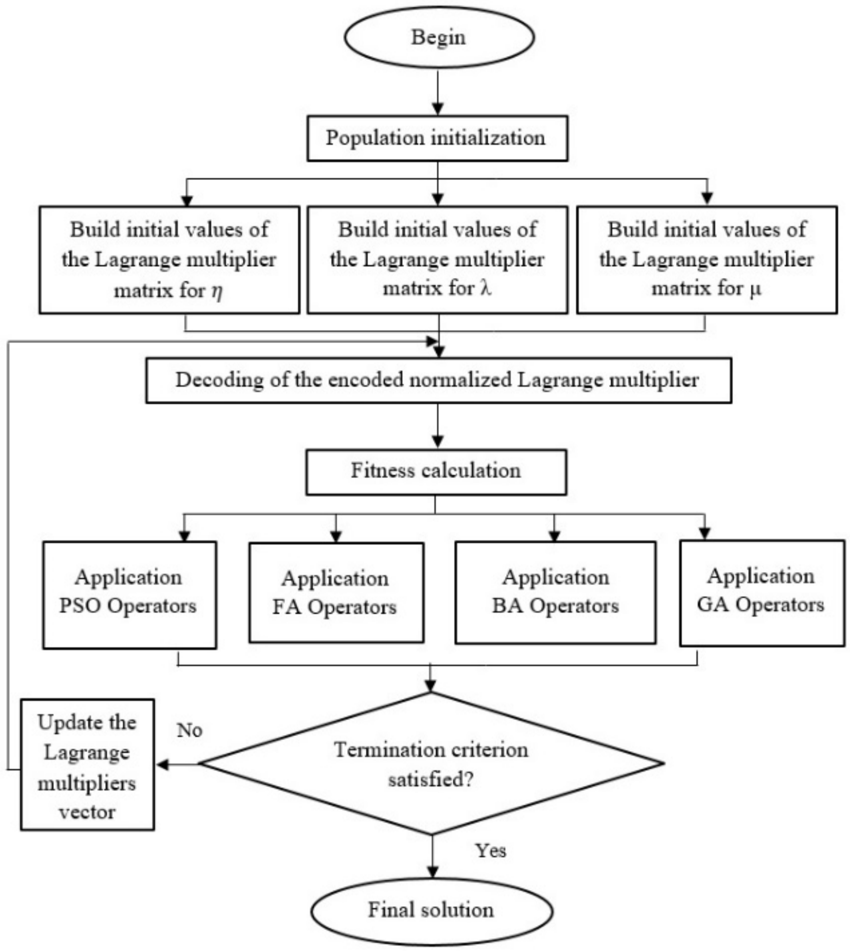

Figure 4. Flowchart of the suggested models.

5. Numerical results and discussion

All the developed formulations were confirmed by the numerical experiments on the artificial dataset including 150 blocks. In conclusion, the enactment by LR-FA is actually better than the other methods from the view of the duality gap (Table 2). An iron ore push-back data of the central Iranian Iron orebody was chosen as the case study to compare the proposed mathematical model for LTPS.

The presented model was implemented in the Chadormalu mine. Also its deposit was recognized as the major iron ore one in the central part of Iran. Chadormalu is located at the epicenter of Persia (Iran) Desert at the north of gray Chah-Mohammad Mountains. Figure 5 divulges that the Chadormalu deposit embraces some 400 million tons of resource and 320 million tons of reserves that are divided between the northern and southern ore bodies by average Fe- and P-contents of 55.2% and 0.9%, respectively.

Four push-backs were scheduled for the Chadormalu mine. The mathematical model presented in this paper was practiced in the second push-back. The 3D view of the second push-back is illustrated in Figure 6. This push-back includes 6854 blocks, 2754 of which are ore blocks and 4100 are waste blocks. The tonnage of ore and waste presented in the aforementioned push-back is 103.8 and 110.2 million tons, respectively. The push-back in this research work is located on part of low grade with an average grade of 52.6%. Table 3 shows the technical parameters of the Chadormalu mine.

Table 2. Numerical results for the synthetic dataset containing 150 blocks.

Iteration	Method	Duality gap
1	LR–FA	0.076
	LR–BA	0.082
	LR-PSO	0.092
	LR-GA	0.098
	LR-SG	1.16
2	LR-FA	0.065
	LR–BA	0.076
	LR-PSO	0.081
	LR-GA	0.088
	LR-SG	0.924
3	LR-FA	0.028
	LR–BA	0.032
	LR-PSO	0.044
	LR-GA	0.057
	LR-SG	0.491

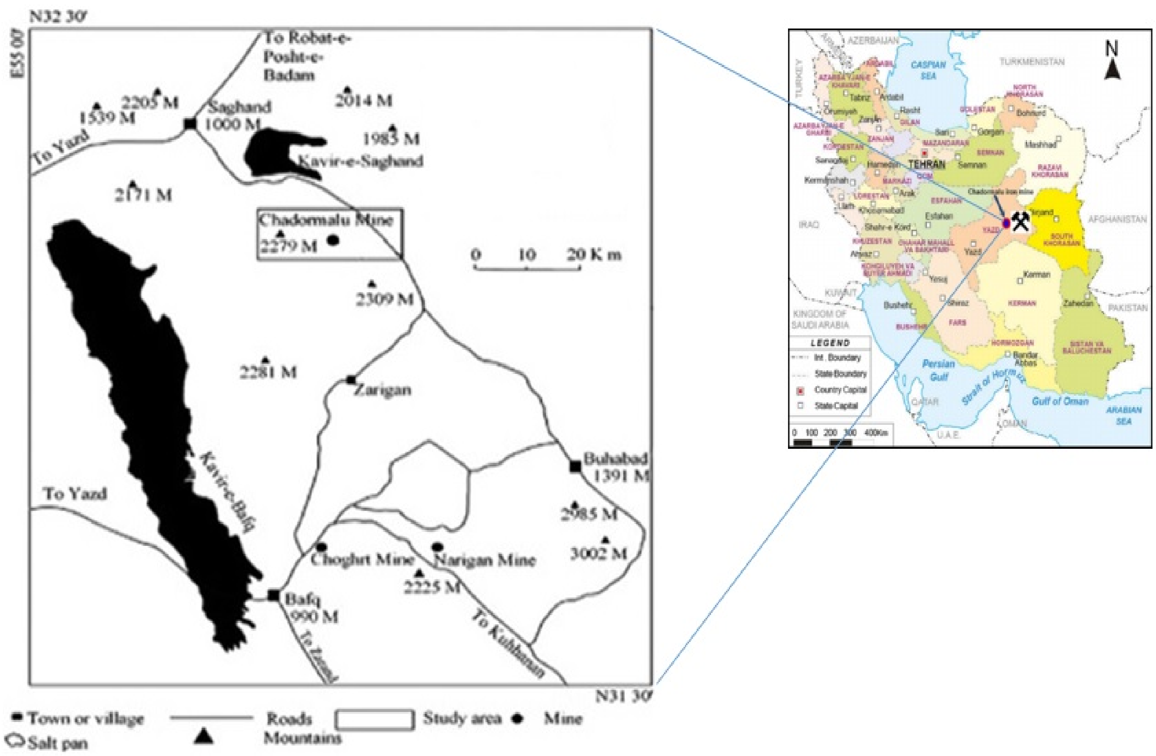

Figure 5. Geographical position of the Chadormalu deposit.

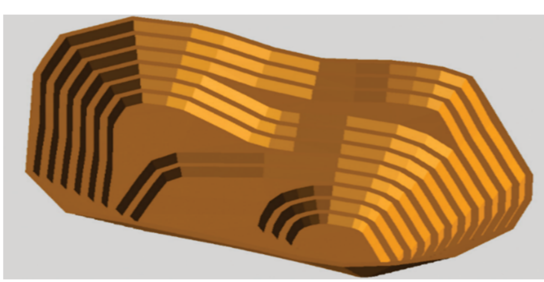

Figure 6. A 3D view of the second push-back in the Chadormalu mine [34].

Table 3. Technical parameters.

Mill cut-off grade (%)	47
Mining capacity (Mtone/year)	25
Processing capacity (Mtone/year)	8.1
Mining recovery (%)	90
Processing recovery (%)	74
Mine life (year)	12

Table 4 shows the numerical results of the proposed model for an iron ore push-back data set including more than twelve planning periods. As disclosed in Table 4, the summation annual net value using the LR–FA method is 41.669 M$ and the summation annual net value through the LR-BA, LR-PSO, LR-GA, and LR–SG method are 40.906 M$, 40.523 M$, 39.903 M$, and 39.523 M$.

Additionally, a comparison of the average grade of the ore for Chadormalu mine is displayed in Table 5. The average grade of ore in twelve years using the LR–FA method is 55.12%, and for the LR-BA, LR-PSO, LR-GA, and LR–SG methods are 54.89%, 54.64%, 54.46%, and 54.31%. Table 6 shows the annual ore and waste tonnage of the Chadormalu mine.

Table 4. Comparison of NV for Chadormalu mine among LR–FA, LR–BA, LR–PSO, LR-GA, and LR-SG.

Years	Net Value (NV), M$				
	LR-FA	**LR–BA**	**LR–PSO**	**LR–GA**	**LR–SG**
1	3.876	3.824	3.814	3.646	3.574
2	3.866	3.792	3.765	3.623	3.492
3	3.704	3.611	3.592	3.485	3.463
4	3.647	3.582	3.407	3.361	3.295
5	3.507	3.321	3.317	3.284	3.277
6	3.467	3.307	3.311	3.273	3.265
7	3.326	3.292	3.281	3.268	3.242
8	3.309	3.287	3.273	3.253	3.239
9	3.276	3.243	3.212	3.184	3.182
10	3.241	3.237	3.194	3.185	3.176
11	3.235	3.226	3.183	3.172	3.161
12	3.215	3.184	3.174	3.169	3.157

Table 5. Comparison of average grade of ore for Chadormalu mine among LR–FA, LR–BA, LR–PSO, LR-GA, and LR-SG.

Years	Average grade of ore (Fe), %				
	LR-FA	**LR–BA**	**LR–PSO**	**LR–GA**	**LR–SG**
1	58.47	58.33	58.28	58.21	58.19
2	58.26	58.18	58.15	58.07	58.02
3	58.11	57.84	57.82	57.55	57.52
4	57.62	56.84	56.27	56.04	55.87
5	57.19	56.65	56.21	55.79	55.68
6	56.55	56.29	55.18	55.21	55.16
7	55.63	55.34	55.32	55.04	54.78
8	54.81	54.75	54.69	54.55	54.27
9	53.56	53.44	53.38	53.29	53.04
10	52.23	52.18	52.08	52.05	51.84
11	50.76	50.66	50.41	49.83	49.75
12	48.27	48.14	47.92	47.88	47.63

In this section, we developed, implemented, and tested the proposed model in MATLAB R2019a environment. The test was performed on an Intel Quad-Core, 3.5 GHz PC with 32 GB of RAM. The computational time of each method is shown in Table 7. It was assessed that the CPU time was nearly 16.2% higher than that for the other methods by means of the LR-FA hybrid method suggested in the present study. The computational times are considerably less with the presented methods compared to the traditional methods, especially when the problem size increases. The results obtained demonstrate that the presented methods significantly diminish the computational time, while retaining a small gap duality. The suggested method appears to be a feasible option for solving the mine production scheduling problem, where the number of integer variables is huge. Also the small gap duality demonstrates the effectuality of the suggested approach. In other words, the results obtained show that the LR method is effective in minimizing the model, and thus reducing the computational time.

Table 6. Annual ore and waste tonnage.

Years	Annual ore and waste tonnage (Mton)									
	LR-FA		LR–BA		LR–PSO		LR–GA		LR–SG	
	Ore	Waste	Ore	Waste	Ore	Waste	Ore	Waste	Ore	Waste
1	9.12	1.74	9.03	2.11	9.25	2.38	8.95	2.19	8.91	2.37
2	8.85	1.92	8.82	2.24	8.79	2.25	9.34	2.27	8.86	2.34
3	8.78	2.21	8.74	2.37	8.72	2.32	8.56	2.35	8.87	2.23
4	8.91	6.21	8.87	6.71	8.85	6.91	8.83	6.84	8.82	7.16
5	8.82	6.28	8.85	6.86	8.87	6.87	8.79	6.83	8.81	7.02
6	8.68	6.32	8.76	6.73	8.79	6.84	8.76	6.86	8.78	6.88
7	8.76	8.59	8.72	8.64	8.32	8.91	8.59	8.89	8.39	9.05
8	8.71	8.68	8.69	8.92	8.77	8.83	8.67	8.83	8.41	8.87
9	8.47	8.75	8.66	9.07	8.69	8.71	8.45	8.71	8.35	8.77
10	8.84	9.94	8.38	10.07	8.01	10.26	8.47	10.18	8.42	10.14
11	8.73	10.12	8.48	10.16	8.52	10.09	8.51	10.04	8.51	9.97
12	8.75	8.57	8.29	9.19	8.81	9.93	8.31	9.92	8.74	9.62

Table 7. General information about the solution found by MATLAB for the proposed models.

Methods	Number of blocks (N)	Number of periods (T)	Computational time in minutes
LR-FA	6854	12	23.15
LR-BA	6854	12	26.9
LR-PSO	6854	12	31.72
LR-GA	6854	12	38.32
LR-SG	6854	12	43.86

6. Conclusions

The aim of this research work was to present a mathematical model for a long-term production planning and achieve the highest revenue in the grade uncertainty situation. In order to make the long-term production planning, grade uncertainty was applied as an input feature to the mathematical model of production planning. A long-term production development optimization model grade uncertainty was employed as the binary integer programming. Principally, the possibility index for each ore block was determined to point out the grade uncertainty. For the next step, getting the best out of the net present value with physical and operational constraints was practiced to model the objective function. The optimization intends to select parts of high-grade possibility reserves in the early years and parts of low-grade likelihood reserves in the subsequent years of mine life for extraction. The development of the proposed model displays the influence of grade uncertainty on the block extraction sequence. The present paper suggests the hybrid methods of Lagrangian relaxation with meta-heuristic methods in order to solve the long-term production problem in open-pit mines as it is hard to solve the production planning models in open-pit mines in large-scale and such problems. This research work also presents a new approach due to the optimization of Lagrange coefficients and comparing its performance with the traditional conventional method.

The results of the case study proved that the Lagrangian relaxation method could carry out a suitable solution to the main problem. The hybrid strategies could produce a more effective solution to the extent of the near-optimal solution in comparison with the traditional approximation method. Moreover, it was specified that the stable convergence property and prevention of early convergence could be identified as the chief advantages of the method suggested in this research work. In fact, the sub-gradient method could be considered as a compatible version of the gradient method. Although this method represents good convergence characteristics, it has been tested only for small-scale problems. The authors use the sub-gradient method that is commonly utilized to determine the multiplier values based on the past calculation results. Clearly, the sub-gradient method may converge very slowly on large problems due to the zigzag phenomenon and small steps. In other words, the sub-gradient directions often cause the multipliers to zigzag across sharp ridges. However, for the LR–Meta-heuristics method, since the direction is obtained by a weighted combination of the gradients of adjacent facets, zigzagging is significantly reduced. Within a definite period, the net present value by the LR-FA hybrid method was 1.87%, 2.83%, 4.43%, and 5.43% higher than the LR-BA, LR-PSO, LR-BA, and LR-SG process.

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بررسی جامع از الگوریتم‌های فراابتکاری متعدد برای مسئله برنامه‌ریزی تولید بلند مدت معادن روباز با در نظر گرفتن عدم قطعیت عیار

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چکیده:

یافتن یک بهینه سازی جهانی در مسائل بزرگ مقیاس برای افزایش کیفیت تصمیم‌گیری در عملیات معدنکاری، حائز اهمیت است. مسئله برنامه‌ریزی تولید بلند مدت نقش کلیدی را در پروژه‌های معدنکاری جهت افزایش عملکرد مرتبط با دستیابی به محدودیت‌ها ایفا می‌کند و در عین حال سود کل پروژه را در یک دوره معین به حداکثر می‌رساند. برای بدست آوردن راه‌حل مناسب، به سبب اینکه مسائل برنامه‌ریزی تولید، بزرگ مقیاس و پیچیده هستند، روش‌های برنامه‌ریزی نیازمند بهبود هستند. در این مقاله، مدل ترکیبی جهت حل مسئله برنامه‌ریزی تولید بلند مدت تحت شرایط عدم قطعیت عیار با استفاده از روش آزادسازی لاگرانژی، بهینه سازی ازدحام ذرات، الگوریتم کرم شب‌تاب و الگوریتم خفاش ارائه می‌گردد. استفاده از روش آزاد سازی لاگرانژی جهت توسعه عملکرد و تسریع همگرایی مسئله برنامه‌ریزی تولید پیشنهاد شده است. علاوه بر این، الگوریتم‌های فراابتکاری جهت به‌روزرسانی ضرایب لاگرانژ مورد استفاده قرار گرفته‌اند. نتایج مطالعه موردی نشان می‌دهد که روش ترکیبی پیشنهادی کارآیی بهتری نسبت به روش سنتی، برای حل مسائل بزرگ مقیاس دارد و یک راه‌حل قابل قبول ارائه می‌دهد. نتایج بدست آمده از روش ترکیبی آزادسازی لاگرانژی و الگوریتم کرم شب‌تاب از لحاظ مجموع ارزش خالص فعلی نسبت به سایر روش‌ها نتایج نزدیک به بهینه را تولید می‌کند. همچنین، زمان محاسباتی مدل پیشنهادی تقریباً ٪ 16/2 سریع‌تر از دیگر روش‌ها است.

کلمات کلیدی: معادن روباز، برنامه‌ریزی تولید بلند مدت، عدم قطعیت عیار، آزادسازی لاگرانژی، روش‌های فراابتکاری.