Development of An Integrated Mathematical Model to Optimize Waste Rock dumping Satisfying Environmental Aspects

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Keywords

Abstract

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Encapsulating
Block Sequencing
Optimization
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Waste rock dumping is very important in the production planning of open-pit mines. This subject is more crucial when there is a potential of acid-forming (PAF) by waste rocks. In such a type of mines, to protect the environment, the PAF materials should be encapsulated by non-harmful rocks. Therefore, block sequencing of the mined materials should be in such a way that both the environmental and economic considerations are considered. If non-acid forming (NAF) rocks are not mined in a proper time, then a stockpile is required for the NAF materials, which later on would be re-handled for encapsulation of PAF rocks. In the available models, the focus is on either block sequencing or waste dumping strategy. In this work, an attempt has been made to develop an integrated mathematical model for simultaneous optimization of block sequencing and waste rock dumping. The developed model not only maximizes the net present value (NPV) but also decreases the destructive environmental effects of inappropriate waste dumping. The proposed model, which is solved by a CPLEX engine, is applied to two different iron deposits. Also the performance of the proposed model is cross-checked by applying the available (traditional) models in a two-step manner. According to the results obtained, it can be considered that utilizing the developed model, because of extensive re-handling cost reduction, the NPV improvement is significant, especially when the overall stripping ratio is higher (deposit case A).

1. Introduction

In the process of long-term open-pit mine planning, block sequencing is very important. Sequencing is usually performed with the aim of maximizing the net present value (NPV) [1]. However, from the environmental viewpoint, waste rock dumping is very important in the sulfide mines. The potential acid-forming materials in such a type of the mines can seriously damage the environment due to the formation of acid mine drainage (AMD) [2-3]. The formation of AMD is a result of the oxidation of sulfide minerals [3]. Many studies have been conducted to neutralize AMD [3-7]. However, this method can impose a high cost to the project in hand [4]. Therefore, in an appropriate mine planning, the block sequencing determination with the NPV maximization tactic may lead to an improper economical evaluation of a project without considering the origin of the waste rocks. In this regard, in open-pit mining, encapsulation of the potential of acid-forming (PAF) materials with non-acid forming (NAF) materials is a reasonable way to protect the environment and to have an economical plan.

As mentioned earlier, the main objective of mine planning is to maximize NPV by determining a proper block sequencing. In this regard, Caccetta and Hill have developed a mathematical model for the block sequencing problem [8]. In order to obtain an optimum solution for this problem, various approaches have been used. Ramazan has implemented the clustering method and used a fundamental tree algorithm to reduce the number of binary variables [9]. However, this approach is not applicable to large-scale mines. Boland et al. have proposed a disaggregation approach with the ability to increase the accuracy of processing variables [10]. Jelves et al. have proposed an aggregation heuristic method to solve the block
sequencing problem [11]. They compared the performance of their approach with the datasets from the library MineLib. Ramazan and Dimitrakopoulos have presented a mathematical model to optimize production planning in the presence of supply uncertainty [12]. Finally, many applications of the heuristic methods such as lagrangian relaxation [13]; sliding time window [14]; hybrid LP-variable neighborhood descent [15]; LR-topological sorting [16]; genetic algorithm [17]; artificial neural network [18]; simulated annealing [19-22]; particle swarm optimization [23,24]; ant colony optimization [25,26]; tabu search [22, 21, 27]; and imperialist competitive algorithm [28, 29] can be addressed for solving large-scale problems. In most research works, the attention is solely given to find the most appropriate block sequencing, and there are few types of research works that are related to the dumping strategy. Regarding the waste dumping cost, the works done by Dincer [30] and Williams et al. [31] can be mentioned. Ben-Awuah et al. have also presented a mathematical model to solve the sequencing problem and waste management in oil sand mines [32]. The main focus of these models is on determining block sequencing and waste dumping considering cost minimization rather than the environmental aspects. However, a proper plan block sequencing, especially for the waste rocks, should be in such a way that both the re-handling cost and environmental side-effects are minimized. In order to diminish the undesirable effects of the PAF materials, Li et al. have proposed a new mathematical model to determine the most appropriate schedule for waste dumping [33]. In this model, the waste dumping is planned in such a way that the PAF materials are enclosed with the NAF ones. However, in their approach, waste dumping is performed according to the block sequencing from the conventional methods in which the composition of waste blocks (such as acid generation potential and heavy metal) is not taken into account. In this way, the NAF waste rocks may be extracted at an inappropriate time and will inevitably be sent to the stockpile. Therefore, unnecessary additional costs related to extensive re-handling due to improper sequencing may be imposed to the project. Reduction of total costs, adverse environmental effects, and re-handling will lead to huge savings for the mining project. Hence, for a comprehensive plan, waste dump planning must be integrated with the block sequencing problem. Recently, Fu et al. have proposed a model to simultaneously solve the block sequencing and waste dumping problems but their results have not been validated with those of the other available models. In this work, an attempt was made to develop an integrated mathematical model to simultaneously optimize ore and waste block sequencing taking into account the waste block environmental hazard potential. To do so, and keeping in mind the nature of the problem, the mixed integer programming (MIP) approach was considered to be appropriate for modeling. The integrated approach will generate good plans by maximizing NPV and creating waste dump with less adverse environmental effects. In order to solve the problem, the standard IBM ILGO-CPLEX solver was applied.

2. Modeling of block sequencing and waste rock dumping

In the mining process, for exposing ore, waste rocks of different types (i.e. environmentally harmful or non-harmful) have also to be mined. To protect the environment and to have an economic operation, an appropriate block sequencing has to be determined. Destination of the different extracted blocks is illustrated in Figure 1. Ore blocks are sent to the processing plant, whereas waste blocks are transferred to the main rock dump (MRD) according to their compositions. It has been mentioned that the NAF waste blocks can be transported to any location in MRD (n cells) without any limitations, whereas the PAF waste blocks have to be sent to certain places that can be enclosed by the NAF materials later on. In this way, waste blocks are arranged in such a way that the PAF rocks are located only in the center of the main rock dump (blue cells in Figure 1). It should be noted that sometimes the PAF waste rocks are situated at the bottom of the deposit, and hence, there should be a temporary stockpile for the NAF waste rocks that would be re-handled to encapsulate the uncovered PAF within MRD. As a matter of fact, material re-handling from stockpile will impose additional costs to the mining project. In order to overcome this, the extraction sequence of the blocks including the PAF waste rocks should be in such a way that produces the maximum possible NPV and minimizes the re-handling cost. In order to achieve these objectives, a mathematical model was developed to optimize block sequencing and waste rock dumping considering NPV maximization and minimization of the waste dumping net present cost (NPC) including the re-handling cost.
3. Development of mathematical model

An MIP mathematical model was developed to simultaneously optimize block sequencing and waste rock dumping. It was noted that both PAF and NAF could be placed in the PAF cells. Also NAF is placed in the stockpile for encapsulation of PAF, if necessary. The notations are as follow:

### 3.1. Parameters

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>T</td>
<td>Number of time periods with the index of t.</td>
</tr>
<tr>
<td>J, I</td>
<td>Number of ore and waste blocks, respectively, with the indices of j and i.</td>
</tr>
<tr>
<td>N, K, S</td>
<td>Number of cells within the MRD and PAF cells in the center of MRD and stockpiles with the indices of n, k, and s.</td>
</tr>
<tr>
<td>D</td>
<td>Number of destinations including the N and S sets with the index of d.</td>
</tr>
<tr>
<td>V_j</td>
<td>Discounted profit resulting from the mining an ore block j ($)</td>
</tr>
<tr>
<td>C_{i,d}</td>
<td>Discounted cost of the mining and hauling one cubic meter of waste block i to destination d ($/m^3$).</td>
</tr>
<tr>
<td>C_{s,n}</td>
<td>Discounted cost of the re-handling one cubic meter of waste material from stockpile s to dump cell n ($/m^3$).</td>
</tr>
<tr>
<td>M_d</td>
<td>Capacity of destination d.</td>
</tr>
<tr>
<td>M_{max}</td>
<td>Maximum mining capacity (m^3).</td>
</tr>
<tr>
<td>P_{max}, P_{min}</td>
<td>Maximum and minimum capacity of processing (m^3).</td>
</tr>
</tbody>
</table>

### 3.2. Decision Variables

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_j^t = \begin{cases} 1 &amp; \text{if ore block j is extracted in period t,} \ 0 &amp; \text{otherwise.} \end{cases}</td>
<td></td>
</tr>
<tr>
<td>Y_i^t = \begin{cases} 1 &amp; \text{if waste block i is extracted by the end of time period t,} \ 0 &amp; \text{otherwise.} \end{cases}</td>
<td></td>
</tr>
<tr>
<td>Z_n^t = \begin{cases} 1 &amp; \text{if cell n is fully filled by the end of time period t,} \ 0 &amp; \text{otherwise.} \end{cases}</td>
<td></td>
</tr>
</tbody>
</table>

\[ V_{i,d}^t \quad \text{Rock volume mined from a waste block i and sent to destination d in period t}. \]
\[ V_{s,n}^t \quad \text{Amount of the NAF waste rock re-handled from stockpile s to cell n within MRD in period t}, \]

as follows:
3.3. Objective function
In the problem of integrated block sequencing optimization and waste rock dumping, the objective function is to maximize NPV:

\[
\text{max } \sum_{t=1}^{T} \sum_{j=1}^{J} X_j \times f_j = \sum_{d=1}^{D} \sum_{t=1}^{T} \sum_{j=1}^{J} c_{id} \times V_{id} - \sum_{n=1}^{N} \sum_{t=1}^{T} \sum_{s=1}^{S} c_{s_n} \times V_{sn} 
\]

(1)

3.4. Constraints
The constraint (2) enforces that each ore block is mined once.

\[
\sum_{t=1}^{T} X_j \leq 1 \forall j = 1, 2, \ldots, J
\]

(2)

According to the constraints (3) to (5), in order to extract a given ore block, all the overlying blocks must be mined first. In addition, the constraints (6) to (8) control the precedence restrictions associated with the fractional extraction of waste blocks.

\[
X_j \leq \sum_{r=1}^{r} X_r \forall t = 1, 2, \ldots, T; \forall \{j, e \in E | j \neq e; e \in E_j\}
\]

(3)

\[
X_j \leq M(Y_j) \forall t = 1, 2, \ldots, T; \forall \{f \in I, j \in E | j \neq f; f \in F_j\}
\]

(4)

\[
X_j \geq M(-1 + Y_j) \forall t = 1, 2, \ldots, T; \forall \{f \in I, j \in E | j \neq f; f \in F_j\}
\]

(5)

\[
\sum_{d=1}^{D} V_{id} \leq \sum_{r=1}^{r} X_r \times U_e \forall t = 1, 2, \ldots, T; \forall \{i \in I, e \in E | i \neq e; e \in W_i\}
\]

(6)

\[
\sum_{d=1}^{D} V_{id} \leq M(Y_j) \forall t = 1, 2, \ldots, T; \forall \{i \in I, f \in E | i \neq f; f \in Q_i\}
\]

(7)

\[
\sum_{d=1}^{D} V_{id} \geq M(-1 + Y_j) \forall t = 1, 2, \ldots, T; \forall \{i \in I, f \in E | i \neq f; f \in Q_i\}
\]

(8)

The constraints (9) to (12) represent that each waste block can be extracted at the end of period \( t \) and sent to the appropriate dump cell(s) according to its acid production potential. These constraints cause that NAF blocks are sent to any cell in the MRD or stockpile, while the PAF blocks are placed only in the center of MRD.

\[
\sum_{d=1}^{D} \sum_{t=1}^{T} V_{id} - U_t \leq M(-1 + Y_j) \forall t = 1, 2, \ldots, T; \forall \{i \in I | i \leq A_o\}
\]

(9)

The constraints (13) and (14) control the MRD cell dumping sequencing. According to these constraints, each cell within MRD will only be available for receiving waste rock when the nine lower cells have already been filled (except the first level).

\[
\sum_{i=1}^{I} y_i \times V_{in} + \sum_{s=1}^{S} y_i \times V_{sn} \geq M(-1 + Z_s) \forall t = 1, 2, \ldots, T; \forall \{n \in N; \forall b \in B_n\}
\]

(10)

\[
\sum_{i=1}^{I} y_i \times V_{in} + \sum_{s=1}^{S} y_i \times V_{sn} \leq M(Z_s) \forall t = 1, 2, \ldots, T
\]

(11)

The constraints (15) and (16) state that the cumulative volume of material transported to a main waste dump cell must not be more than its maximum capacity.

\[
\sum_{i=1}^{I} y_i \times V_{in} + \sum_{s=1}^{S} y_i \times V_{sn} - \phi_s \geq M(-1 + Z_s) \forall t = 1, 2, \ldots, T; \forall \{n \in N; \forall b \in B_n\}
\]

(12)

\[
\sum_{i=1}^{I} y_i \times V_{in} + \sum_{s=1}^{S} y_i \times V_{sn} \leq M(Z_s) \forall t = 1, 2, \ldots, T
\]

(13)

The constraint (17) ensures that the total volume of material mined (waste and ore) in each period must be less than the maximum mining capacity.

\[
\sum_{j=1}^{J} X_j + \sum_{d=1}^{D} \sum_{t=1}^{T} V_{id} \leq M_{\text{max}} \forall t = 1, 2, \ldots, T
\]

(14)

The constraints (18) and (19) state that the total volume of ore processed in a beneficiation plant must be within the range of the plant capacity.

\[
\sum_{j=1}^{J} b_j \times X_j \leq P_{\text{max}} \forall t = 1, 2, \ldots, T
\]

(15)

\[
\sum_{j=1}^{J} b_j \times X_j \geq P_{\text{min}} \forall t = 1, 2, \ldots, T
\]

(16)

The constraint (20) indicates that the inventory of
stockpile must be less than the stockpile capacity in each time period. In addition, the constraint (21) forces that in a time period, the flow-out material from the stockpile to the main rock dump should be less than the stockpile inventory at the end of the previous time period. The constraint (22) states that the flow-out volume of the stockpile in the first period is equal to zero.

\[
\sum_{i=1}^{j} \sum_{t=1}^{T} y_{it} \times V_{it} - \sum_{n=1}^{N} \sum_{t=1}^{T} y_{tn} \times V_{tn} \leq \varphi_s \\
\forall s = 1, 2, \ldots, S; \{i \in I| A_t \leq A_o \} \\
\forall t = 1, 2, \ldots, T
\]

(20)

\[
\sum_{j=1}^{J} \sum_{i=1}^{I} y_{ij} \times V_{ij} \leq \sum_{n=1}^{N} \sum_{t=1}^{T} y_{it} \times V_{tn} - \sum_{n=1}^{N} y_{tn} \times V_{tn} \geq 0 \\
\forall s = 1, 2, \ldots, S; \{i \in I| A_t \leq A_o \} \\
\forall t = 2, \ldots, T
\]

(21)

\[
\sum_{n=1}^{N} V_{tn} = 0 \forall s = 1, 2, \ldots, S \\
\]

(22)

The constraints (23) and (24) specify that the summation of the proportions of a waste block extracted in various time periods should be equal to the volume of the block. Also the constraint (25) satisfies the capacity of a waste dump cell in MRD.

\[
\sum_{t=1}^{T} \sum_{d=1}^{D} V_{id} = U_i \forall \{i \in I| A_t \leq A_o \} \\
\sum_{t=1}^{T} \sum_{k=1}^{K} V_{ik} = U_i \forall \{i \in I| A_t \geq A_o \} \\
\sum_{t=1}^{T} \sum_{j=1}^{J} y_{ij} \times V_{tn} + \sum_{t=1}^{T} \sum_{n=1}^{N} y_{tn} \times V_{tn} = \varphi_s \\
\forall n = 1, 2, \ldots, N
\]

(23)

(24)

(25)

4. Implementation of developed mathematical model

In this section, a real iron mine (case study A) was considered to examine the efficiency of the proposed model. The economic parameters in the modeling process are summarized in Table 1. A 3D block model of 515 blocks with dimensions of 20 m × 20 m × 12 m was considered for this study. Also to investigate the effect of the overall stripping ratio (OSR), a hypothetical deposit was generated from case A (i.e. case B) with a lower OSR. It is obvious that in creating the hypothetical case B with a lower OSR, almost all the other deposit specifications including grade distribution, number of the NAF waste blocks, and number of the PAF waste blocks would be different from case A. Specifications of the two cases A (real) and B (hypothetical) are given in Table 2.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Unit</th>
<th>Quantity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of time periods</td>
<td>Years</td>
<td>3</td>
</tr>
<tr>
<td>Swell factor</td>
<td>%</td>
<td>1.25</td>
</tr>
<tr>
<td>Cut-off grade</td>
<td>%</td>
<td>25</td>
</tr>
<tr>
<td>Cut-off acidity</td>
<td>%</td>
<td>3</td>
</tr>
<tr>
<td>Iron price</td>
<td>$/m^3</td>
<td>40</td>
</tr>
<tr>
<td>Mining and haulage cost</td>
<td>$/m^3</td>
<td>3</td>
</tr>
<tr>
<td>Processing cost</td>
<td>$/m^3</td>
<td>6</td>
</tr>
<tr>
<td>Stockpile re-handling cost</td>
<td>$/m^3</td>
<td>1</td>
</tr>
<tr>
<td>Recovery of mining</td>
<td>%</td>
<td>95</td>
</tr>
<tr>
<td>Recovery of processing</td>
<td>%</td>
<td>80</td>
</tr>
<tr>
<td>Discount rate</td>
<td>%</td>
<td>10</td>
</tr>
</tbody>
</table>

In order to solve the model, a CPLEX engine was employed. Examples of block sequencing and dumping sequence obtained from the proposed model are shown in Figures 2 and 3, respectively. Also to compare the results obtained, the same problem was solved using the Ramazan & Dimitrakopoulos [35] and Li et al. [33] models in a stepwise process. It means that in the first step, the Ramazan and Dimitrakopoulos model was utilized for determination of the block sequencing, and in the second step, using the block sequencing obtained, the Li et al. model was employed for determination of the dumping sequence. Examples of the obtained block sequencing and waste dump sequencing are depicted in Figures 4 and 5, respectively. As it can be seen in Figure 3, there is no block that has to be re-handled in the dumping sequence of the proposed model, while in Figure 5 that is the outcome of the Li et al. model, it is seen that there is a large number of blocks required to be re-handled from temporary stockpile to the main permanent waste dump. Also the difference between block sequencing of the proposed model (Figure 2) with that of the Ramazan and Dimitrakopoulos model (Figure 4) is completely natural because in the proposed model a key factor of waste rock composition has been incorporated in the model development.
The economic outcomes of the application of the Ramazan & Dimitrakopoulos and Li et al. models and the proposed model are presented in Table 3. As it can be seen in this table, employing the proposed model, the re-handling cost is completely eliminated, and therefore, a higher NPV is obtained for both the A and B cases. However, the NPV improvement in the case of A is significantly higher than that for case B, which is due to the presence of a higher number of the PAF waste blocks (# 114) as compared to the PAF waste blocks (# 54) in case B. Therefore, it can be concluded that for the deposits with a high OSR, the application of the proposed model can convert an uneconomical deposit to an economical one. It can be noted that NPV of case B is much higher than that for case A, which shows a better quality of deposit B as compared to deposit A. This idea is also supported by comparing the amount of OSR in both cases. Another interesting point in Table 3 is that the NPC improvement in case A is higher than that for case B, which is due to the presence of a higher number of the PAF waste blocks as compared to the PAF waste blocks in case B.
5. Conclusions

In this work, a new integrated MIP model was developed to simultaneously optimize block sequencing and waste rock dumping taking into account the environmental protective measures. In the previous works, the focus has been mainly on
block sequencing, and only a few studies have been related to the waste dumping strategy. However, apart from one recently published paper (Fu et al.), there is no similar document like the present study. The main objective of the proposed model is to obtain competent plans by maximizing NPV and creating waste dump with minimum adverse environmental effects. The performance of the proposed model was examined by running it for two different iron deposits in the CPLEX environment, and then the outcomes obtained were compared with the Ramazan & Dimitrakopoulos and Li et al. approaches for the same case studies. The comparative results demonstrate the superiority of the proposed model. Application of the proposed model caused a full elimination of the re-handling cost, thus a higher NPV was obtained for both cases A and B. Enhancement of NPV is more considerable when the amount of the PAF waste blocks is higher. Employment of the proposed model is highly recommended for economically marginal deposits, where a slight reduction in the costs can change the situation of a deposit from negligibly uneconomical to economical.

References


توسعه یک مدل ریاضی یکپارچه برای بهینه سازی انباشت باطله با در نظر گرفتن جنبه‌های زیست محیطی

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چکیده:

انباشت باطله یکی از مهم‌ترین مراحل برنامه‌ریزی تولید در معدن روباز است. این موضوع در معدن بارای سنگ باطله با پتاسیل تولید بسیار اهمیت و وزهای برخوردار است. در این معدن، بارای کاهش اثرات زیست محیطی نامطلوب، مواد باطله سایر معدن‌های شونده باکریان، ترتیب استخراج بلوک‌ها باعث گونه‌ای یاده‌که هر دو دست ملاحظات اقتصادی و زیست محیطی در نظر گرفته شود. جایگاه مواد باطله غیر اسیدی در زمان نامحسوس استخراج شونده ایجاد انتقال‌گذاری مواد برای استخراج بلوک‌ها و انباشت باطله به طور قابل‌توجه بوده‌است. در این مقاله، یک مدل ریاضی برای بهینه سازی هزینه‌های توزیع باطله تولید اعضاها به همراه محیط‌زیست نشان داده شد. در این مقاله، مدل ریاضی توزیع باطله به همراه محیط‌زیست نشان داده شد. در این مقاله، مدل ریاضی توزیع باطله به همراه محیط‌زیست نشان داده شد. در این مقاله، مدل ریاضی توزیع باطله به همراه محیط‌زیست نشان داده شد. در این مقاله، مدل ریاضی توزیع باطله به همراه محیط‌زیست نشان داده شد.