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A New Mathematical Model for Production Scheduling in Sub-level Caving Mining Method

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Abstract

Production scheduling in underground mines is still a manual process, and achieving a truly optimal result through manual scheduling is impossible due to the complexity of the scheduling problems. Among the underground mining methods, sub-level caving is a common mining method with a high production rate for hard rock mining. There are limited studies about long-term production scheduling in the sub-level caving method. In this work, for sub-level caving production scheduling optimization, a new mathematical model with the objective of net present value (NPV) maximization is developed. The general technical and operational constraints of the sub-level caving method such as opening and developments, production capacity, sub-level mining geometry, and ore access are considered in this model. Prior to the application of the scheduling model, the block model is processed to remove the unnecessary blocks. For this purpose, the floating stope algorithm is applied in order to determine the ultimate mine boundary and reduce the number of blocks that consequently reduces the running time of the model. The model is applied to a bauxite mine block model and the maximum NPV is determined, and then the mine development network is designed based on the optimal schedule.

1. Introduction

Mineable reserve optimization is one of the most important issues in the mining industry, and in this regard, the ultimate mine limit and production scheduling optimization are the key aspects for both the surface and underground mines. The open-pit mining method is of great importance among surface mining methods, and there are many studies and commercialized approaches to optimize the operations in this method. Today, production scheduling optimization in open-pit mines is an integral part of designing and planning; nevertheless, optimization of the ultimate mine limit and production planning in other surface mining methods has not improved so much, and studies on optimization and planning in these methods are very limited and primitive. This is due to the fact that the grade and geometric variations in the reserves extracted by these methods are very low, and for this kind of reserves, the use of sophisticated optimization methods is not necessary. Among the underground mining methods, block caving, sub-level caving, cut and fill, and stope and pillar methods are usable for reserves with grade and geometric variations and complexity. Therefore, for the mentioned methods, optimization of the ultimate mine limit and production scheduling can be necessary. Ultimate mine limit decides which parts of the reserve are included in the stope and which parts are not, and production scheduling defines the tonnages and grades to be mined throughout the mine-life.

Among the underground mining methods, caving methods are somehow comparable with the surface mining methods in terms of production rate and operating costs. Sub-level caving is one of the methods with a high production rate applied for hard rock mining, and there are limited studies about its long-term production schedule

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optimization. The long-term production scheduling is a strategic plan for the mining operations that contain fewer details than a medium or short-term plan. However, a long-term plan includes clear definitions related to mining reserves, production sequence, and production rate. Moreover, several objectives can be considered for long-term production scheduling. Net present value (NPV) maximization, cost minimization, and mine life maximization are the common strategic objectives [1]. Among them, NPV maximization is preferred by the mining companies.

Manual planning methods, heuristic algorithms, and exact algorithms can be used for production scheduling in underground mines. Exact algorithms guarantee to find an optimal solution but manual planning methods or heuristic algorithms will not lead to an optimal solution. Though the exact algorithms are able to achieve the optimal schedule, usually the size of a mathematical formulation for the production scheduling problems is such that problem-solving in a reasonable time, it is not achievable. Therefore, some size reduction procedures should be applied prior to the application of mathematical models.

In this paper, a mathematical model is presented with the objective of NPV maximization for sub-level caving long-term production scheduling, and it is applied on a real bauxite mine of Iran. The production scheduling problem is formulated as an integer linear programming (IP) in the MATLAB framework. The general technical and operational constraints of the sub-level caving method such as production capacity, sub-level geometry, and developments are considered in this model. Prior to the application of the model, the floating stope algorithm is applied to determine the ultimate mine boundary. Then the blocks that are located outside the mine boundary are removed from the block model. This will reduce the number of decision variables and running time considerably.

2. Literature Review

Underground mine design and planning procedure start with stope boundary optimization. Several approaches have been developed for stope optimization [2, 3]. Dynamic programming [4] and branch and bound technique [5] have been used to optimize a stope in 2D problems. However, these methods fail to produce realistic stopes for complex 3D deposits that cannot be simplified in 2D. Some 3D techniques have also been reported including mathematical morphology tools [6, 7], floating stope [8], maximum value neighborhood method [9], and octree division [10]. Manchuk and Deutsch (2008) have provided a simulated annealing-based algorithm [11], and Bai *et al.* (2013) have developed a stope optimizer based on the graph theory [12]. Recently some models have been developed for underground mine reserve optimization under grade uncertainty [13].

When the mine stope layout is determined, production scheduling is conducted on the blocks located within the optimized boundary. In underground mining, various models have been developed for the optimization of production planning. In general, none of these models have been commercialized. Most of them are suitable for short-term planning with the aim of minimizing production deviations from the existing manual and non-optimized long-term programs [14-24]. Some of these models have been formulated for a specific mining operation, and they must be modified before application to other cases [25, 26]. Some models do not have real optimal solutions, which are some expert-oriented search-based methods [27-31]. In the process of developing these models for underground mines, various objective functions have been presented such as profit maximization [32] and project time minimization [33, 34]. Recently some models have been developed with the objective of NPV maximization for long-term production scheduling [1], and some other models developed to optimize both the stope layout designing and production scheduling in the sub-level stoping operation [35, 36].

Mathematical modeling is widely used in mining optimization problems. In the case of underground mining scheduling problems, much earlier, in 1973, Williams *et al.* presented a simple mathematical model with the aim of minimizing the deviation from the planned production for the sub-level stoping method [14]. Recently, several mathematical models have been proposed for underground mine production planning including long-term production scheduling with the objective of NPV maximization for the block caving method using the mixed-integer linear programming (MILP) formulation [1], mathematical optimization (MILP) for production scheduling a complex underground mine with the objective of maximize metal production over the life of the mine [37], and short-term production scheduling model (IP model) in a cut and fill gold mine with a ventilation constraint [38].

There are very limited studies for sub-level caving production scheduling. In 1998, Winkler presented a production planning model aimed at minimizing the diversion from targeted production [14]. This model has been used for a sub-level caving mine, and favorable results have been achieved. However, this model is not suitable for use in other mines, and there is no report of using this model in another mine. In 2004, Kuchta *et al*. provided a short-term mathematical model (MILP) with the goal of minimizing deviations from monthly planned production quantities in the Kiruna Mine of Sweden [22]. This mine produces about 24 million tons of iron ore yearly using the sub-level caving method. Newman and Kuchta (2007) have developed a heuristic-based model to reduce the problem-solving time [24]. They applied their model to modify Kuchta *et al*. (2004) for practical solutions. In 2011, Martinez and Newman provided a new model for the long-term production planning of the Kiruna mine based on the original model provided by Kuchta (2004), and the optimization was performed to the problem-solving time in 2007 [39]. In this model, like the initial model, the objective function is minimizing deviations from monthly planned production. There are some related models that have been developed for other underground mining methods [40-42].

As NPV maximization is a common objective for mine production scheduling, a model for long-term production planning in sub-level caving with the objective of NPV maximization is required. In this work, an integer linear programming (IP) model with the objective of NPV maximization is presented for sub-level caving long-term production scheduling.

3. Problem definition

Sub-level caving is simple in many respects. It can be used in orebodies with very different properties, and it is easy to mechanize. However, from other points of view such as recovery and dilution, the method is unfavorable. Sub-level caving is used to mine large steeply dipping tabular or massive orebodies. In this method, the ore is extracted via the sub-levels that are developed in the orebody at a regular vertical spacing. Each sub-level has a systematic layout of parallel drifts, along with or across the orebody. In this method, mining starts at the top of the orebody and develops downwards. Ore is mined from the sub-levels spaced at regular intervals throughout the deposit. A series of ring patterns is drilled and blasted from each sub-level, and the broken ore is mucked out after each blast. The mining cycle is illustrated in Figure 1.

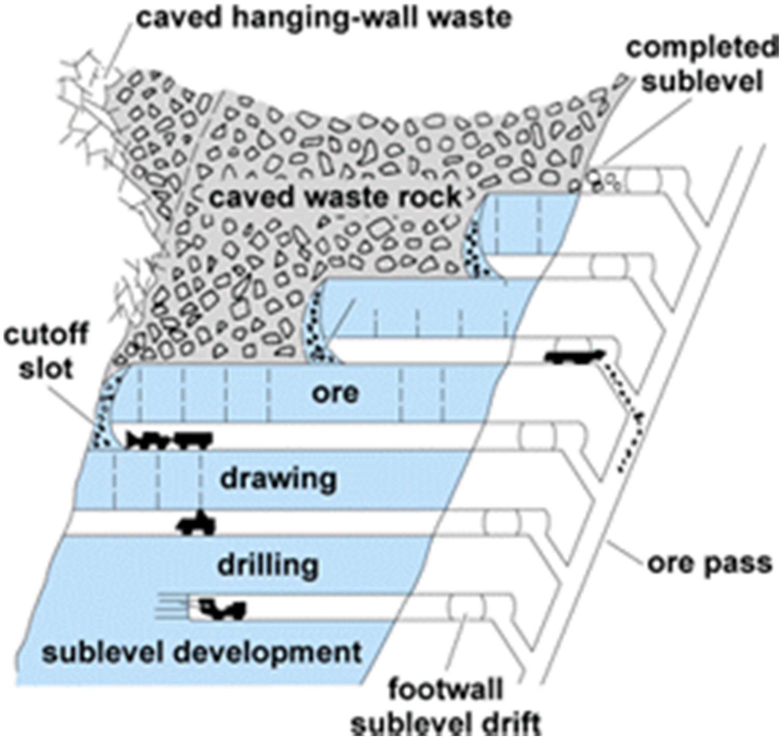


Figure 1. Sub-level caving mining cycle [13].

In the manual design methods, according to the rock mechanics, the shape, size, and dimensions of the ore and the material transportation method, the location of the main openings is determined. The location of the main openings dictates the direction of ore preparation and extraction. In this method, the extraction sequencing in the sub-levels and the number of active sub-levels depend on the capacity of the existing equipment and the production commitment. That is why, in the previous studies, it was observed that the objective function was to minimize deviations from manually planned production.

In computer-based mine planning models, the block model of the orebody is the main input. In the case of the reserves with grade and geometric variations and complexity, the grade and value of blocks are different in a block model, and the mining sequence has a significant effect on the mining project NPV. For example, a hypothetical economic block model is shown in Figure 2. The block economic value is written on each block. The presented block model (Figure 2) can be mined from the left to the right or vice versa. It is an important issue that the development direction and production sequencing can make different NPVs for mining operation in this hypothetical economic block model. It is the same in real mines as well, no doubt that development direction and production sequencing can make different NPVs for a mining operation in a real mine but so far, no particular attention has been paid to this issue.

Thus, different NPVs are achievable in sub-level cave mining for various mining directions, various numbers of active sub-levels, and various mining sequences. This simple example shows that production scheduling and mining sequence optimization are necessary for the sub-level caving method. In this work, in order to optimize the mining sequence, a mathematical model with integer linear programming (IP) formulation is presented that is suited for sub-level caving operations.

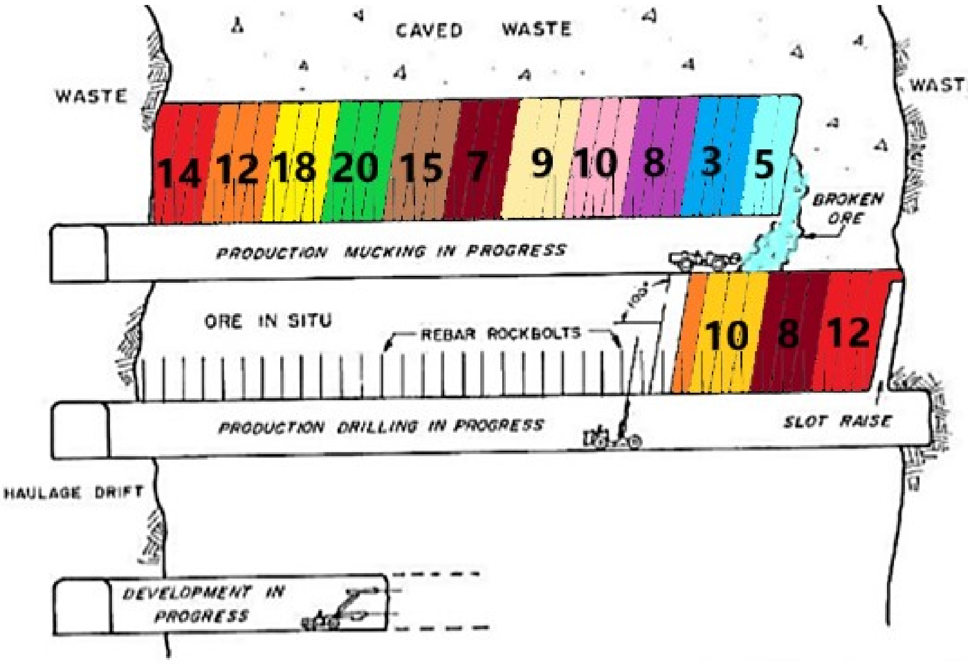


Figure 2. A hypothetical block model.

4. Mathematical Formulation

A long-term production scheduling model for the sub-level caving method is formulated within an integer linear programming (IP) framework. The decision variables dictate the block of orebody to be extracted or not during a specified time period. The objective is to maximize NPV of the mining process with respect to some operational constraints.

4.1. Sets and Parameters

The sets and parameters used in this work are presented as below:

- *I:* number of blocks in horizontal direction,
- *J:* number of blocks in vertical direction,
- *T:* number of scheduling periods (in years),
- B_{ij}: block located in horizontal location *i* and vertical location *j*
- P_{ij}: amount of rock (ore and waste) tonnage in block B_{ij}
- BEV_{ij}: economic value of B_{ij},
- *d:* discount rate (%),
- ppy_{min} *and* ppy_{max} *:* minimum and maximum annual production rates,

- D_i: opening and development network alternative i,
- N: number of opening and development network alternatives,
- D_{op}: optimum opening and development network,
- A: minimum number of blocks that a sub-level must be in advance from its underlying sub-level,
- S: maximum number of active sub-levels in each period.

4.2. Decision Variable

$$x_{i,j,t} = \begin{cases} 1 & \text{If } B_{ij} \text{ to be extracted in period } t \\ 0 & \text{Otherwise} \end{cases}$$

4.3. Objective Function

The objective function of the model is to maximize NPV of the mining process, and it is given in Eq. 1.

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{t=1}^{T}\frac{BEV_{ij}}{(1+d)^{t}} \times x_{i,j,t} \tag{1}$$

The objective function is composed of the block economic value (BEV), discount rate, and a decision variable that shows which block is extracted in each period. In order to maximize NPV, the most profitable blocks will be chosen.

4.4. Constrains

When discussing production planning in underground mines, there seem to be a lot of constraints such as development constraints, ventilation constraints, geomechanical constraints, machinery constraints, ore accessibility constraints, underground space supporting constraints, and roof caveability constraints. In fact, for different scheduling horizons, there are different constraints. For example, blending or machinery dispatching is a short-term scheduling constraint, ventilation is a medium, and production capacity is a long-term scheduling constraint. Therefore, a list of constraints in underground mining is developed, and by omitting the short-term and medium-term constraints, those constraints that are applicable in sub-level cave long term planning are selected. In this regard, production capacity, sub-level mining geometry, openings and development network, vertical and horizontal ore access, and number of active sub-levels and reserve constraints are the main long-term production scheduling constraints for the sub-level caving method that are considered in this research work.

4.4.1. Sub-level Caving Geometry

Sub-level caving geometry depends on the geomechanical conditions and mining machinery. It imposes the dimensions of sub-levels. In this research work, the block dimensions of the block model are selected in relation to the sub-level caving geometry. In this way, the height of the blocks is equal to the sub-level spacing (h_B), the width of the blocks is equal to the width of the slice that is blasted at each stage (burden (b)), and the thickness of the blocks is equal to the drift spacing (S_D). In thin layer deposits, the block thickness is equal to the thickness of the orebody (the parameters are shown in Figure 3).

The block model is one of the inputs to the production planning model. Therefore, the initial block model is re-blocked with respect to the requirements of sub-level caving geometries. If the block dimensions are determined according to the sub-level caving geometry, it means that the sub-level caving geometry is considered in the production planning model.

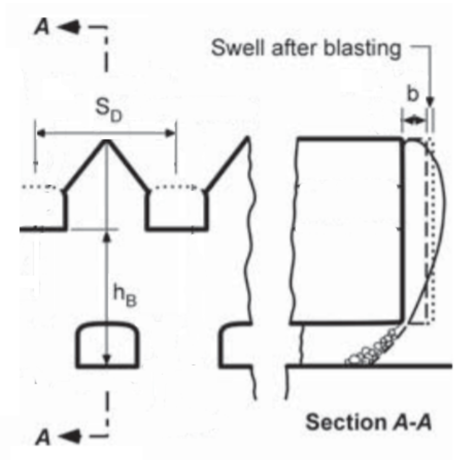


Figure 3. The sub-level caving geometry.

4.4.2. Mine Opening and Development Network

Openings and developments in underground mines are one of the main constraints on the mining design process, and usually, no mining design is done without considering them. As already mentioned, at the moment, in underground mines, initially, the development network is designed, and then the production planning is carried out. In this work, the conventional underground mining design is challenged and the importance of considering production planning before designing the development network is discussed. Therefore, in this paper, a new method is presented for designing underground mine developments with the consideration of the optimal production plan, which will be discussed further. In this method, the main alternatives are considered for mine openings (D_i). Then the optimal production planning by the presented model with the objective of maximizing NPV is determined (NPV_{max}). Afterward, according to Eq. 2, the closest alternative to the optimal production plan is selected as the mine optimal opening (D_{op}). Finally, the underground development network is designed to optimum the main opening and production schedule. Since the amount of developments is not related to the extraction sequencing and for the mine with a fixed sub-level geometry, the amount of developments is somehow constant, and as a result, the early determined optimal production schedule does not change significantly considering the underground development network.

$$D_{op} = \{D_i \mid NPV_{D_i} \cong NPV_{max} \quad \forall \ \ i = 1, \dots, N\} \qquad (2)$$

4.4.3. Production Capacity

The production capacity constraint, formulated as Eq. 3, forces a mining rate between the desired and maximum mining capacities available. In other words, this constraint controls the production capacity, and it ensures that the ore tonnage that must be mined each year is equal to the pre-determined capacity.

$$ppy_{min} \le \sum_{i=1}^{I} \sum_{j=1}^{J} x_{i,j,t} P_{ij} \le ppy_{max}, \quad \forall \ t \in T \qquad (3)$$

4.4.4. Vertical Access Constraint

As the sub-level caving method is a downward mining method, to extract a block in a sub-level, the block located on its top must be extracted before. This constraint is formulated in Eq. 4. This constraint is defined as the minimum number of blocks that a sub-level must be delayed from its overlaying sub-level. In other words, the vertical access constraint is the minimum number of blocks that a sub-level must be forwarded from its underlying sub-level. Operation safety, roof caveability, production capacity, number of active sub-levels, and some other parameters are effective in determination of the vertical access constraint. For example, consider Figure 4. In order to extract a block in the lowest sub-level, a number of blocks in the upper sub-level should be mined earlier. The mine designer sets the minimum number of blocks that a sub-level must be forwarded from its underlying sub-level (indicated by A in Eq. 4) with regard to the operation safety, roof caveability, and production capacity. If we assume that A is equal to 5, then extraction of block '2-1' should be halted until block '1-5' is extracted.

$$x_{i,j,t} \le x_{i+A,j-1,t'} \qquad \forall \ i \in \{1, \dots, I\}, j \in \{1, \dots, J\}; \ t \in T; t' \le t \qquad (4)$$

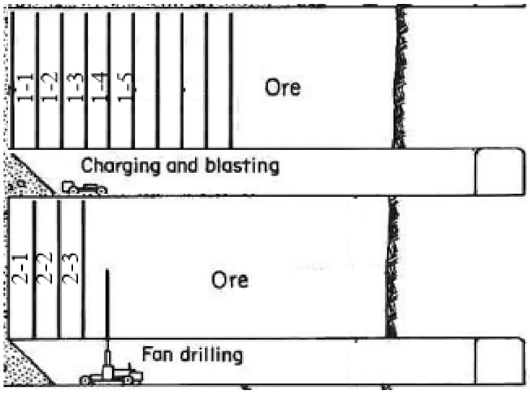


Figure 4. The sub-level caving vertical access constraint.

4.4.5. Horizontal Access Constraint

In the sub-level caving method, in addition to vertical access constraint, a horizontal access constraint should also be defined. This constraint is defined for the blocks located in each sub-level. It controls the extraction of blocks with respect to the mining direction within a sub-level. This constraint is formulated in Eq. 5.

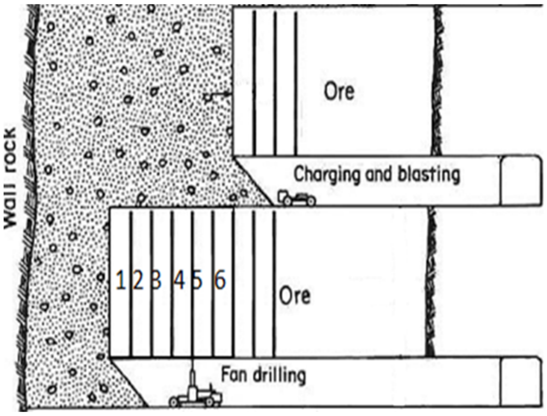


Figure 5. The sub-level caving horizontal access constraint.

$$x_{i,j,t} \le x_{i-1,j,t'} \qquad \forall \ i \in \{1, \dots, I\}, j \in \{1, \dots, J\}; \ t \in T; t' \le t \qquad (5)$$

The constraints 4 and 5 are the most important technical and operational constraints in the sub-level caving method.

4.4.6. Active sub-levels

There is a limitation on the number of active sub-levels due to equipment capacity, development capacity, and available capital expenditure. This constraint is formulated in Eq. 6.

$$x_{1,j+S,t'} \leq x_{I,j,t} \qquad \forall\, j \in \{1,\dots,J-S\};\; t,t \in T;\, t' \leq t \tag{6}$$

where S is the maximum number of sub-levels that can be developed due to equipment and development capacity for production in each period.

4.4.7. Reserve

The reserve constraint that is formulated in Eq. 7 ensures that each block is mined just once.

$$\sum_{t=1}^{T} x_{i,j,t} = 1 \qquad \forall\, i \in \{1,\dots,I\}, j \in \{1,\dots,J\} \tag{7}$$

4.5. Variable reduction

Determining maximum NPV and solving the mathematical models in small case examples is possible and tractable but in real deposits with large scale block models, solving the problem and determining the maximum NPV require powerful computers and so much time and effort. In such case, application of the methods that reduce the number of decision variables is required to speed up the optimization process. The simplest method available to reduce the number of decision variables is to reduce the number of blocks. In this regard, determination of the stope layout will omit non-profitable blocks from the block model. Hence, the number of blocks to be scheduled is reduced. As stated earlier, there are different methods for stope layout determination such as floating stope optimizer. Floating stope optimizer is the only underground stope optimizer that is formulated in a commercial software and it can be used for underground deposits. The inputs to this optimizer are the minimum stope dimension, cut-off grade, stope head grade, and objective function. This algorithm selects the blocks that have the potential to be mined by defining a mining envelope. In this regard, the blocks that are out of the outer envelope are removed from the model. This will considerably reduce the number of blocks that improves the running time in the next step.

5. Running model

The presented IP model (Eq. 1-9) is generated in the MATLAB programming platform. At first, the model is applied to the hypothetical block model presented in Figure 4. The other assumptions are as follow:

- number of blocks in the horizontal direction is 10,
- number of blocks in the vertical direction is 4,
- number of scheduling periods is 5 years,
- economic value of each block is given in Figure 6,
- discount rate is 10%,
- minimum and maximum annual production rates are equal to 8 blocks per year,
- minimum number of blocks that a sub-level must be in advance from its underlying sub-level is 2 blocks,
- maximum number of active sub-levels in each period is not considered in this example.

The model optimizes the mining sequence such that NPV is maximized. The maximum achievable NPV for the given hypothetical block model is $74.58, and the optimum mining sequencing is shown in Figure 7.

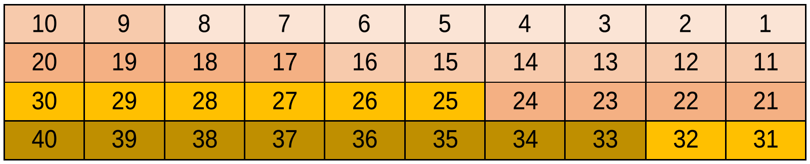
1	2	2	1	3	2	1	2	1	1
1	2	2	4	3	2	1	2	1	2
2	3	4	5	3	2	2	1	1	1
2	4	6	4	3	2	3	2	2	1

Figure 6. A hypothetical block model.

1	2	3	4	5	6	7	8	17	25
9	10	11	12	13	18	19	26	27	28
14	15	16	20	21	29	30	33	34	35
22	23	24	31	32	36	37	38	39	40

Figure 7. Production sequencing for achieving maximum NPV.

The manual production planning of the hypothetical block model is shown in Figures 8 and 9. The mining direction in Figure 8 is right to left and NPV of the mining process is obtained to be $71.5, and in Figure 9, the mining direction is left to right and NPV is $72.8. By comparing the above NPV values, there is a 4.3% difference in the manual and optimum program, and it seems that this difference in real block models can be greater in the long run.



10	9	8	7	6	5	4	3	2	1
20	19	18	17	16	15	14	13	12	11
30	29	28	27	26	25	24	23	22	21
40	39	38	37	36	35	34	33	32	31

Figure 8. Manual production sequencing in right to left direction.

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40

Figure 9. Manual production sequencing in left to right direction.

Prior to the application of the presented model to a real dataset, it is tested on small case examples in order to check and verify the codes, and the results obtained show a successful validation of the model. When the model is verified and validated, it is applied to a real block model. As a case study, the presented model is applied to the Golbini bauxite mine. This mine is located in the north of Iran, and it is in the design and production planning stage. According to the exploratory boreholes, the geologic block model contains 6300 blocks with a dimension of 5 m × 20 m × thickness of the layer, and the ore reserve is about 3 million tons of bauxite. Due to a large number of blocks (i.e. decision variables), the application of the model is somehow limited. In order to overcome this problem, a minable stope envelope is determined prior to the application of the model. Using the floating stope optimizer, the stope boundaries were determined and the unnecessary blocks were removed from the block model with respect to the maximum envelope of the optimized stope layout. This will reduce the number of blocks and improves the running time of the model in the next step. The total mineable reserve that can be extracted is almost 2.5 Mt. 4076 blocks are selected by the maximum envelope. The resulting block model is named "optimized economic block model", and it is shown in Figure 10. It is assumed that it is possible to extract the orebody by that sub-level caving method. The parameters considered for production planning are as follow:

- number of blocks in the horizontal direction is 240,
- number of blocks in the vertical direction is 27,
- planning horizon is 8 years,
- economic value of each block is calculated with respect to the metal content,
- discount rate is 10%,
- minimum and maximum annual production rates are equal to 500 and 520 blocks per year, respectively,
- minimum number of blocks that a sub-level must be in advance from its underlying sub-level is 3 blocks,
- maximum number of active sub-levels in each period is 10 sub-levels,
- mining direction is from East to West (due to geotechnical conditions).

It should be mentioned that there is a limited geotechnical and experimental data for determination of A and S for the case study, and the authors used the Gosfil mine engineers' experiences to calculate the values of A and S.

As stated earlier, the objective function is to maximize NPV of mining operations, while ensuring that all constraints are considered during the mine life. The model is applied with respect to the above-mentioned parameters. The resulting mining sequence that leads to the highest NPV is shown in Figure 11. According to the results obtained, the maximum achievable NPV is 74 M$. The amount of NPV in the manual planning mode was M$ 69; there was a 7% difference with the optimal program, and it seems that this difference in massive block models can be greater in the long run.

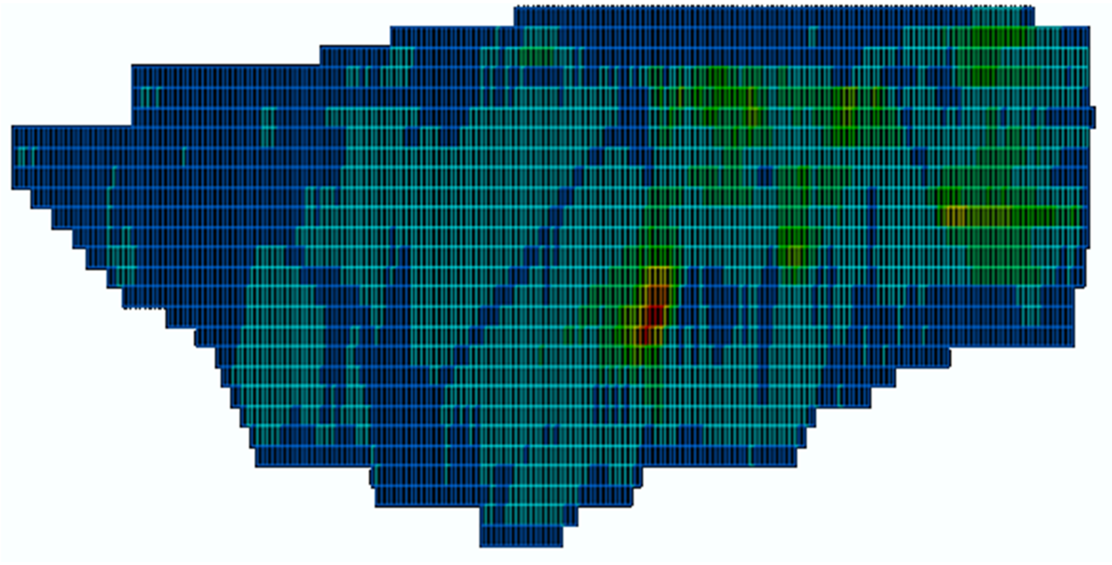


Figure 10. The optimized economic block model.

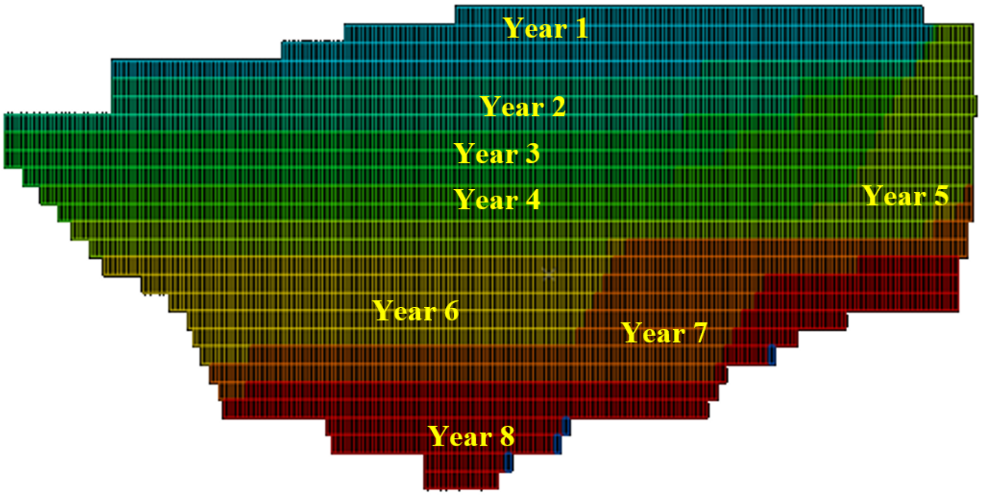


Figure 11. Production scheduling for an eight-year operation.

6. Discussion

When the goal is to maximize NPV, then efforts will be made to extract more valuable blocks earlier in time by taking into account the mining constraints. Consider Figure 12 that illustrates a section of a schematic mine by the sub-level caving method; the economic value of each block is given in the figure as well. In order to maximize NPV, first, the economic value of the blocks is identified, and then mining of the blocks with more economic value is prioritized by the production manager. The mining sequence is determined in such a way that the more valuable blocks are mined as soon as possible. The mining schedule is determined manually in this example. The 14 initial steps of the mining sequence are given in Figure 12. The same manner is applied for mining the rest of the blocks. The approach explained in the previous paragraph should be practiced in real case examples. In the example given in Figure 12, there is a limited number of blocks. However, in more complex models with a large number of blocks, production scheduling is impossible without using mathematical methods. This is the motivation for the generation of a mathematical model for production planning in the underground mines that are operated by the sub-level caving method.

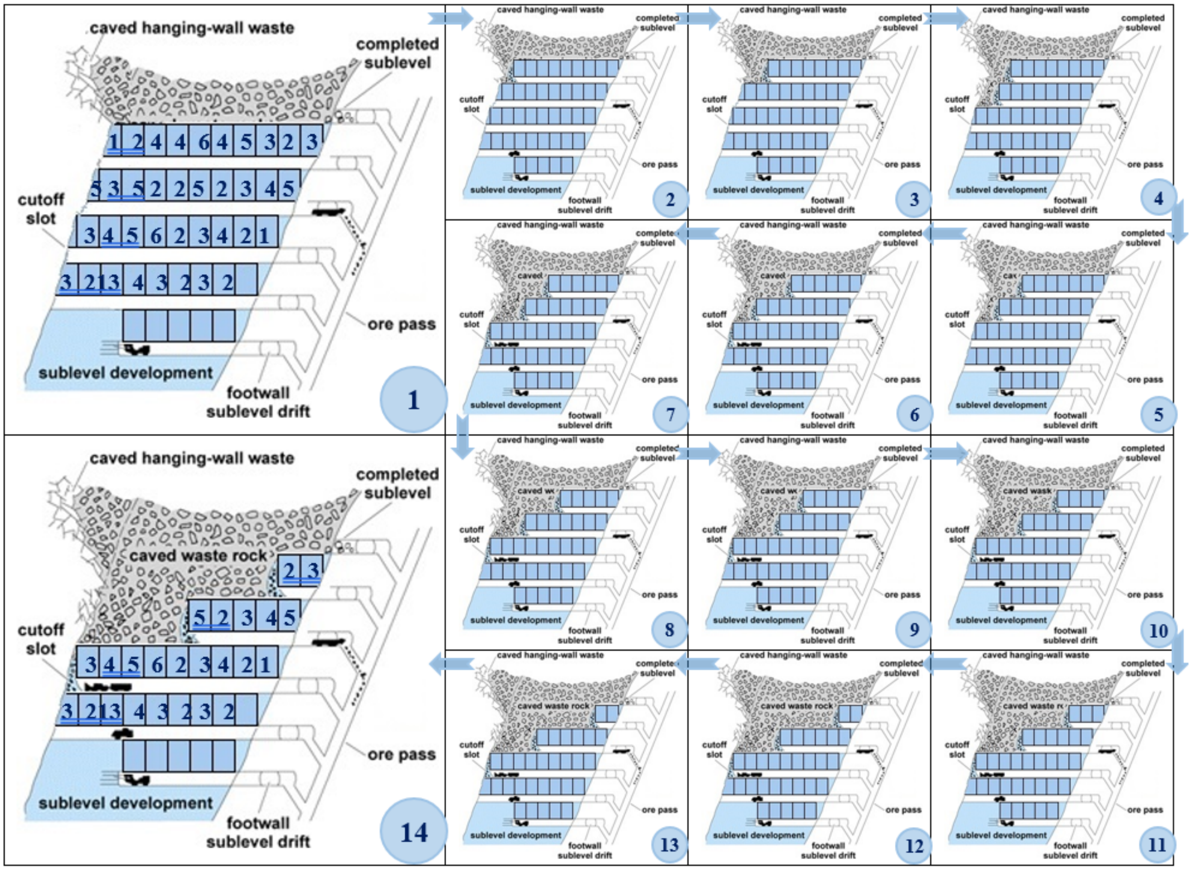


Figure 12. Production sequencing in a schematic sub-level caveing method.

Another important point to be mentioned here is the mine development. As shown in Figure 12, considering the block values, extraction of the first two levels causes a higher NPV, and thus the model prioritizes these blocks to be extracted earlier in time and then proceeds to the next sub-levels. However, due to the development capacity, the active sub-levels can be equal to 4 (i.e. S = 4) and the development network is completed for four levels; however, the extraction is planned and carried out in the first two levels. It means that the accomplishment of the development network can be more precise if it is scheduled with respect to the production sequence. In this regard, it is recommended that the development network be scheduled after the production sequencing. Thus the procedure that leads to an optimal production schedule has four stages, as follow:

Stage 1: Generate an economic block model according to the geological block model,

Stage 2: Optimize the stope boundaries using any stope optimizer and remove the unnecessary blocks from the block model,

Stage 3: Apply the model and determine the maximum NPV and the optimal mining sequence,

Stage 4: Determine the development schedule according to the optimal mining sequence.

Consider the case study in Section 5, where the mining sequence is determined for a layered deposit. According to the above-mentioned stages, when the mining sequence is determined, the mine development works are designed and scheduled with respect to the mining sequence. It means that the mine developments are accomplished to the extent that is required annually.

Figure 13 illustrates a schematic section of the mine development network. With respect to the optimal mining sequence, the development works that are required annually are calculated. Figure 14 shows the annual development operations, except for the main openings. According to the results obtained, the maximum development works are required in years 1 and 2.

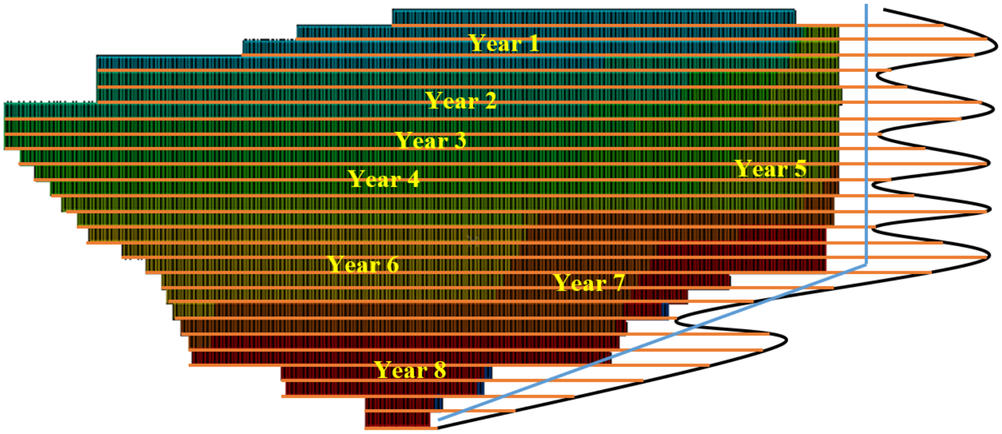


Figure 13. Development network and production scheduling.

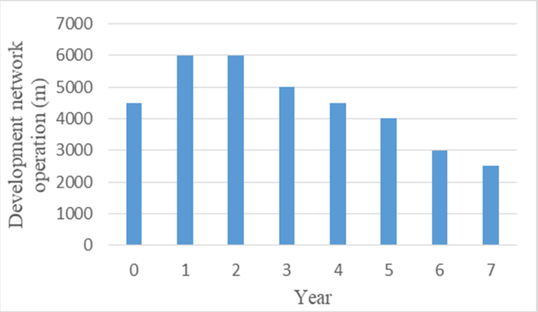


Figure 14. Development network annual operation.

It is clear that a declining grade strategy will lead to the highest NPV. Figure 15 shows the average grade of the material mined each year. As shown in the figure, the average grade has an upward trend in the first two years and then a downward trend over the rest of the mine life. This is due to the limitations imposed by the mine opening and development. However, it could be said that, on average, the average grade has a declining trend over the mine life. Then one could say that the model by prioritizing the mining of high-grade blocks optimizes the grade strategy as well.

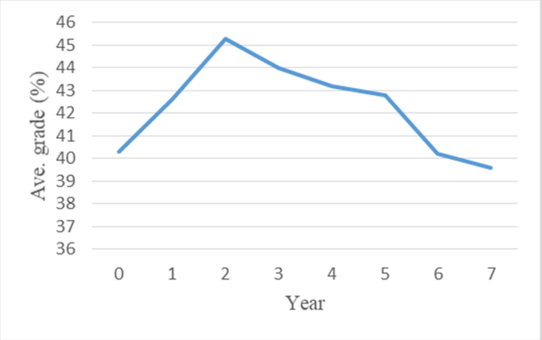


Figure 15. Average grade of production for each year (%).

7. Conclusions

Determining an optimum production plan is very important in an underground mine design. The production scheduling optimization techniques are not widely used in underground mining. This is due to the high investment requirements in mining projects, and even a small deviation from the optimal production plan may cause a large deviation in mining economics. Thus there is a need to improve the current production scheduling approaches that are applied to underground mines and specially sub-level caving. This is normally scheduled manually. The manual methods are time-consuming. Developing a mathematical method enables the mine planner to establish and compare different mining scenarios to produce an optimal mining schedule. In this work, a mathematical model was formulated for the optimization of the sub-level caving production schedule within an integer programming framework. The formulation maximizes the NPV with respect to several technical constraints. In this research work, the presented model was applied to a hypothetical example to show the improvement in NPV compared to the conventional hand methods. In real cases, due to a large number of blocks and decision variables, the application of the model is limited. In order to overcome this problem, a minable stope envelope was determined prior to the application of the model, and the blocks that were not selected by the outer envelope of the floating stope algorithm were removed from the model. This considerably reduced the number of blocks and decision variables and improved the running time. The presented procedure was applied to a 2D representation of a real bauxite deposit. The results obtained showed that an optimal and practical mining sequence was achievable.

According to the results, the optimal mining sequence had a declining grade strategy that fitted the objective of NPV maximization. Moreover, the accomplishment of development works should be scheduled with respect to the optimal mining sequence.

Future studies

This paper tries to develop a mathematical model to solve the production planning problem in 2D sub-level caving mine without considering the precise design of the developments. Future studies are devoted to address 3D planning problems and the effect of mining developments on production planning.

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مدل جدید ریاضی برای برنامه‌ریزی تولید در روش استخراج تخریب در طبقات فرعی

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چکیده:

برنامه‌ریزی تولید در معادن زیرزمینی همچنان به صورت دستی انجام می‌شود و با توجه به ماهیت پیچیده مسائل برنامه‌ریزی تولید، دستیابی به نتیجه بهینه واقعی از طریق برنامه‌ریزی دستی غیرممکن است. در میان روش‌های استخراج زیرزمینی، روش تخریب در طبقات فرعی یک روش استخراج رایج با نرخ تولید بالا برای استخراج سنگ‌های سخت است. تا کنون مطالعات محدودی با موضوع برنامه‌ریزی تولید برای روش تخریب در طبقات فرعی انجام شده است و در این تحقیق، یک مدل ریاضی جدید با هدف حداکثر کردن ارزش خالص فعلی (NPV) مناسب برای روش تخریب در طبقات فرعی توسعه داده شده است. در این مدل ریاضی، محدودیت‌های فنی و عملیاتی مانند بازکننده‌ها و آماده‌سازی‌ها، ظرفیت تولید، هندسه روش استخراج و دسترسی به ماده معدنی در نظر گرفته شده اند. در این مدل ریاضی و برای کاهش تعداد بلوک‌های مورد پردازش به منظور کاهش زمان اجرای مدل، ابتدا با استفاده از الگوریتم کارگاه شناور بلوک‌های غیر ضروری حذف و محدوده‌نهایی معدن تعیین می‌شود. در این تحقیق، مدل ارائه شده بر روی مدل بلوکی یک معدن بوکسیت اجرا، بیشینه ارزش خالص فعلی محاسبه و برنامه‌ریزی تولید آن تعیین شد و سپس بر اساس برنامه‌ریزی تولید بهینه به دست آمده برنامه‌ریزی آماده‌سازی‌های معدن انجام گرفت.

کلمات کلیدی: مدل‌سازی ریاضی، برنامه‌ریزی خطی، معدنکاری زیرزمینی، تخریب در طبقات فرعی، برنامه‌ریزی تولید بهینه‌سازی، ارزش خالص فعلی.
