

## A developed approach based on grinding time to determine ore comminution properties

N. Saeidi, M. Noaparast\*, D. Azizi, S. Aslani, A. Ramadi

*Mining Engineering Department, University College of Engineering, University of Tehran, Iran*

Received 01 November 2012; received in revised form 14 October 2013; accepted 31 December 2013

\*Corresponding author: [noparast@ut.ac.ir](mailto:noparast@ut.ac.ir) (M. Noaparast).

### Abstract

In this paper, iron ore sample from the Chadormalu was investigated to determine some comminution properties. Chadormalu deposit is one of the largest iron ore mine in Iran, which is located in Yazd province. The representative ore sample contained 57%Fe, 0.9%P and 0.17%S. The sample was crushed; afterward, it was ground in various grinding times according to the Bond Ball mill approach to specify the work index values. Based on different grinding times and the obtained results, a new work index equation was then simulated through which grinding time was considered as the main variable. The relationships between work index, the work input and  $P_{80}$  were then concluded. In addition, the results of tests were then used to estimate the selection function parameter. A new equation was applied to determine energy efficiency which could be implemented for energy consumption calculation. Two equations for  $E_B$  and  $E_B/E_{limit}$  were then obtained, where  $E_B$  is the efficiency of comminution, and the  $E_{Limit}$  is the maximum limiting energy efficiency for particle fracture under compressive loading. These equations could estimate the parameters of the iron ore would be precisely estimated. Indeed, by means of work index value; some crushing and grinding characteristics of the taken sample were assessed by which comminution circuit would be designed much better.

**Keywords:** *iron ore, chadormalu, work index, comminution properties, selection function, energy efficiency.*

### 1. Introduction

#### 1.1. Work index

It has been 50 years since Bond published his theory of comminution [1], and well over 100 years since von Rittinger [2] and Kick [3] published theirs. As pointed out by Hukki [4] all of these equations are special forms of the same differential equation as proposed by Walker et. al [5]. This equation was presented as follows:

$$dE = -C (dx / x^n) \quad (1)$$

where  $E$ =net energy required per unit weight (specific energy);  $x$ =index describing the size distribution, e.g.  $P_{80}$ ;  $n$ =exponent indicating the order of the process;  $C$ =constant related to material properties and the units chosen to balance the equation.

In the design of grinding circuits in a mineral processing plant, the Bond method is widely used for a particular material in dimensioning mills, determining power/energy required, and in the evaluation of performance. Its use as an industrial standard is very common, providing satisfactory results in all industrial applications. Despite having many advantages, this method has some drawbacks such as being tedious and time consuming, and also requiring a special standard mill [6, 7].

Due to these difficulties, a number of easier and faster methods have been developed to determine the Bond work index [8, 9, 10, 11, 12, 13, 14 and 15]. The general characteristic of all these methods is the need for either a Bond mill or a standard laboratory mill. Indeed, the bond equation has been changed due to these

investigations. The new equations were suggested by some investigators to decrease the work index estimation error.

In many investigations, bond work index have been used to estimate various kinds of ore properties. Deniz [7] has found new relationships for some dynamic properties such as elasticity modulus ( $E_d$ ), shear modulus ( $G_d$ ) and bulk modulus ( $K_d$ ) by means of bond equation. Ahmadi [16] has applied bond work index to estimate commercial operation. The bond work index was used to estimate the energy efficiency by Desmond [17]. Musci and et. al [18] found grindability characteristics of lateritic and karst bauxites, by means of bond work index. Velázquez and et. al [19] studied grindability of lateritic nickel ores based on bond work index.

### 1.2. Energy efficiency

It is generally recognized that comminution is an energy intensive process. Estimations of energy consumption by DOE[20] during mining operations show that about 39% is used for the whole of beneficiation and processing operations, of which 75% is accounted for by comminution, indicating that comminution consumes approximately 29.3% of the total mining energy in the USA. These numbers are likely to be generally applicable to mining worldwide and perhaps useful in assessing the fraction of the total national energy consumption attributable to comminution in different mining countries [17]. Tromans and Meech [21, 22] obtained theoretical estimates for C (in Equation 1), for over 60. They estimated the efficiency of crushing,  $E_B$ , for several minerals by comparing the calculated energy required to produce new fracture surface area with the actual energy input, based on average standard work index values  $W_i$  (kWh/st). Comminution would be analyzed by this concept that can be estimated by Equation 2 as follows.

$$E_B = (6 F_R \gamma / \rho D_{\text{aef}} W_i) \times 100 \quad (2)$$

where  $\rho$  is the mineral density ( $\text{kg/m}^3$ ), ( $W_i$ ) is the Bond work index in SI units (J/kg), and  $D_{\text{aef}}$  is the average effective diameter of the milled product (micron).

### 1.3. Selection function

Numerous methods are used to describe comminution process kinetics and particle breakage. One of the frequently used principles is the so-called conventional approach, in which "selection function" and "breakage distribution

function" are used. Defining these functions, the particle size distribution of the product formed by comminution can be predicted. Selection and the breakage distribution function make it possible to express the population balance of batch grinding [23, 24]:

$$\frac{dm_i(t)}{dt} = \sum_{j=1}^{n-1} b_{i,j} \cdot S_j \cdot m_j - S_i \cdot m_i \quad (3)$$

$$n \geq i \geq j + 1$$

where  $S_j$  is the selection function representing the probability of particle in the  $j$  size interval to break under the lower size limit of any size interval,  $j$  ( $x_j$ ).

If the grinding is treated as a rate process, breakage of the given size fraction usually follows the first-order law [25]. Thus, the breakage rate of material (the selection function) from the initial size interval ( $i=1$ ) would be expressed as:

$$\frac{dm_i(t)}{dt} = -S_1 \cdot m_1(t) \quad (4)$$

This study aims to develop a new equation which can be created based on grinding time and bond work index values. To do that, an iron sample was used in different experiments. In next step, the defined equation is applied to estimate the selection function and energy efficiency. In addition, relationships between bond work index values and some variables are investigated as well. In fact, it will be concluded that bond work index values can be significantly used to find out some crushing and comminution characteristics.

## 2. Materials and methods

### 2.1. Material characteristics

For this study, the required sample was provided from the Chadormalu, one of the largest iron mines in Iran, located in Yazd province. The representative sample was taken from phosphorous magnetite-hematite zone. The chemical analysis showed that this sample contained about 57%Fe, 0.9%P and 0.17%S. To crush the sample, two laboratory jaw and roll crushers were applied. After crushing, 2kg of material was sampled to determine its size distribution and  $d_{80}$  in sieve analysis test, the result of which is presented in Figure 1. It was found that the  $d_{80}$  of sample was 1256 microns. This crushed sample was used in to estimate bond work index values.

The sample was used in mineralogical studies which showed that it contained 8-10% hematite, 65-70% magnetite, 5-6% goethite, 2% pyrite, and about 15% gangue minerals. In addition, the XRD analysis results confirmed that the sample contained 10-15% hematite, 65-70% magnetite, 6-7% goethite, 2-3% calcite, 5-6% quartz, 2-3% apatite and 4-5% siderite.

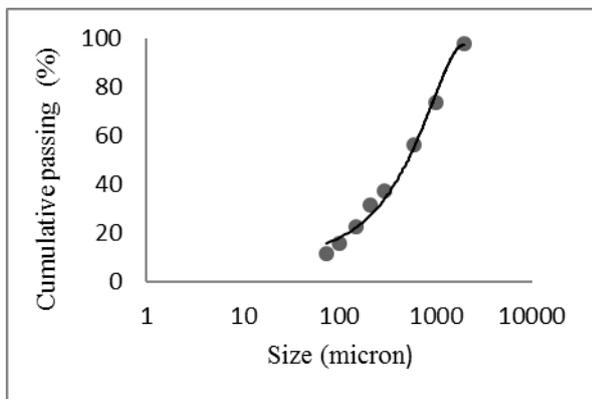


Figure 1. The size distribution of crushed sample

2.2. Apparatus and method

Bond work index is estimated by using a standard bond ball mill. In this study, one special ball mill was used to perform bond tests by which bond work index can be precisely obtained. This ball mill was designed and set up by Nematollahi [15], and is commonly called "calibrated bond ball mill". In fact, this mill is a sort of scaled down of bond ball mill, with a coefficient of two-third, and its dimensions are 200x200 mm (8x8 inches) [15]. Moreover, 5kg of ore sample can be ground in dry condition by this ball mill and 5.9kg of steel balls should be definitely used to satisfy comminution procedure. The number and diameter of used balls in experiments are given in Table 1[15].

Table 1. Characteristics of balls used in calibrated bond ball mill for work index test.

Diameter of balls (mm)	38.1	31.75	25.4	19.05	15.87
Number of balls	13	20	3	21	28

Besides, the results of work index tests are accordingly implemented in Equation 5 to calculate the ore work index values. Equation 5 is similar to Bond work index equation, but it has been defined for the foregoing calibrated bond ball mill [15].

$$W_i = \frac{11.76}{P_i^{0.23}} \times \frac{1}{G_i^{0.75}} \times \frac{1}{\frac{10}{\sqrt{P}} - \frac{10}{\sqrt{F}}} \tag{5}$$

Where,  $W_i$ =Bond laboratory ball work index (kWh/st);  $P_i$ =closing screen size in micron;  $G_i$ =net grams of control screen undersize per mill revolution;  $P$ =80% passing size of the product in micron;  $F$ =80% passing size of the feed in micron.

3. Results and discussion

3.1. Experiments

In this study, crushing properties of a particular iron ore sample from chadormalu mine was studied to describe its various grinding and crushing characteristics. In this case, a new work index equation based on grinding time was truly simulated to calculate work index, and relationships between some comminution parameters were also found. For this purpose, representative ore sample was ground in 4 different experiments in which their grinding times were changed. These tests were carried out in 20, 60, 120 and 180 seconds as grinding times and their size distributions are shown in Figure 2. Furthermore, results of comminution procedure such as work index, work input and  $P_{80}$  for each experiment are presented in Table 2.

Afterward, new equation was defined to calculate work index based on the above results and size distribution (Table 2 and Figure 2). To do that, relationships between  $G_i$  and  $P_{80}$  with grinding time were studied. In this case,  $G_i$  and  $P_{80}$  versus grinding time were plotted by which the best correlation was found. As matter of fact, it was essential to create a relation between these parameters, which could be replaced with  $G_i$  and  $P_{80}$ . These relations indicated that  $G_i$  and  $P_{80}$  are depended to the grinding time, by which work index can significantly be obtained. These relations are shown in Figures 3.

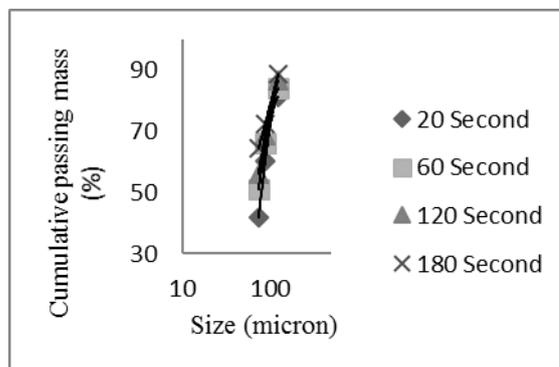
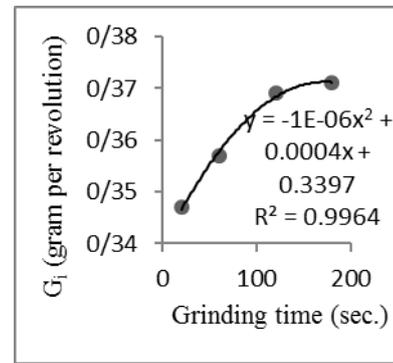
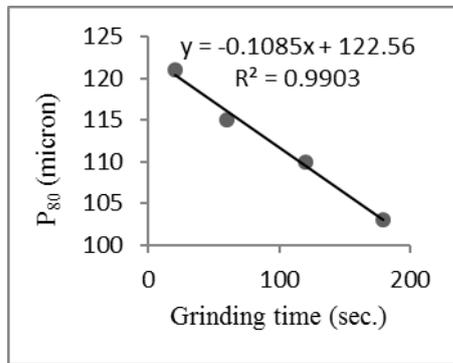


Figure 2. Size distribution of sample versus grinding time

**Table 2. Estimated  $W_i$ ,  $P_{80}$  and  $W$  for each experiment**

Time (second)	$W_i$ (kWh/st)	$P_{80}$ (micron)	$W$ (kWh/st)
20	13.1	121	8.21
60	12.36	115	8.04
120	11.68	110	7.84
180	11.11	103	7.81



**Figure 3. The trends of  $P_{80}$  with grinding time (a), and  $G_i$  with grinding time (b).**

The coefficient of  $R^2$  indicates that the defined equations are good enough to represent the system under the given experimental domain [26]. Joglekar and May [27] suggested that for a good fit of a model,  $R^2$  should be at least 0.80. In this work  $R^2$  was found to be 0.9903 and 0.9964 for  $P_{80}$  and  $G_i$  respectively. The value of  $R^2$  shows that there is an acceptable relationship between investigated values. These models are given as Equations 6 and 7 as follows:

$$P_{80} = -0.1085t + 122.56 \tag{6}$$

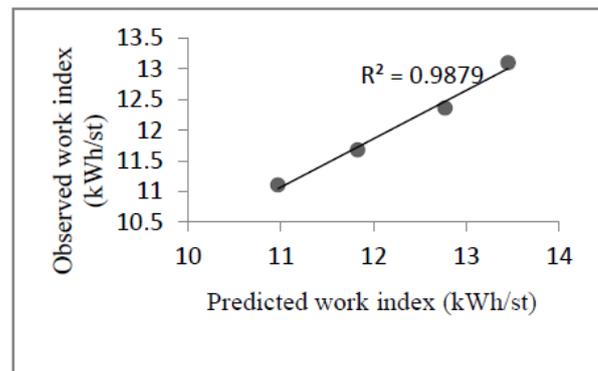
$$G_i = -1E-0.6t^2 + 0.0004t + 0.3397 \tag{7}$$

In next step, these equations (6 and 7) were replaced with  $G_i$  and  $P_i$  in the modified work index equation (Equation 4). Therefore, Equation (8) was accordingly defined as follows:

$$W_i = \frac{5.6}{(-1E - 0.6t^2 + 0.0004t + 0.3397)^{0.75}} \times \frac{1}{\frac{10}{\sqrt{-0.1085t + 122.56}} - \sqrt{F}} \tag{8}$$

Validation of this new equation (Equation 8) must be checked through which this equation can be approved for any further application. For this purpose, amounts of observed work index versus predicted work index values have been plotted in Figure 4. Predicted index was estimated using Equation 8 for the 20, 60, 120 and 180 seconds as

grinding times, and observed work index was achieved from experiments which were presented in Table 2. Figure 8 shows the high value of  $R^2$  (0.9997) which is an indication and confirmation for the significant correlation and accordingly validation of Equation 8.



**Figure 4. The trend between predicted and observed obtained  $W_i$**

For more confidence, one test was performed in 100 seconds as grinding time, and its relevant work index was then calculated. In addition, for this grinding time (100 seconds), the work index was estimated using Equation 8. The results are presented in Table 3. The low value of error  $[(12.18 - 12.13) / 12.18] * 100 = 0.41\%$  indicates that Equation 8 has enough accuracy to estimate the work index effectively using Equation 8.

### 3.2. Relationship between variables

In this work, the interactions of various variables on each other were also studied, and their relevant relationships with grinding parameters were precisely described. In the first step, the work input versus grinding time and  $P_{80}$  were

investigated. In fact, if their plots are defined, they can help to figure out the optimum grinding time and  $P_{80}$ . In addition, energy consumption may be significantly controlled by which costs of grinding can be significantly decreased. These plots are presented as follows in Figure 5.

Table 3. Derived results for validation of new work index equation

	Grinding time (second)	$P_{80}$ (micron)	$G_i$ (g)	$w_i$ (kWh/st)	Error (%)
Experimental results	100	113	0.358	12.18	
Calculated results	100	112	0.369	12.13	0.41

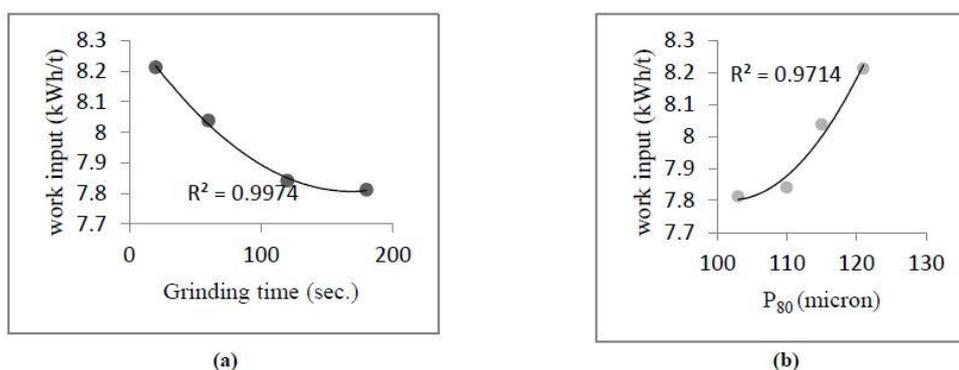


Figure 5. The trends of W and grinding time (a), W and  $P_{80}$  (b).

According to Figure 5, the work input increased with the increase of size of  $P_{80}$ , and the decrease of grinding time. Based on these results, it was concluded that increasing  $P_{80}$  had an increasing effect on the work input and energy consumption, on the other hand when grinding time increased, the work input reduced. This can be used to find out the best economical and effective grinding time for this procedure. Moreover, the trends of work index values versus grinding times and  $P_{80}$  were also studied to understand their relationships, and results indicated that the above mentioned trends were observed. These trends are presented in Figure 6.

Figure 6 shows that when grinding time increased, work index decreased. If this relationship is noticed, it will be concluded that amount of ground ore increases when grinding time increases. Therefore, the work index values should be definitely decreased due to use of Equations 8 and 4. Moreover, Figure 6b shows that  $P_{80}$  increases when work index increases. This happened because of the decrease in grinding. In fact, it defines when grinding is decreased, the amount of fine particles is accordingly decreased. It means that the amount of coarse particles is increased. These relationships could perhaps help to understand the grinding process and impacts of

grinding time on the change of relevant/effective variables on work index.

### 3.3. Selection function

The selection function can be implemented to design ball and SAG mills, and consumption energy calculation as well, and of course both of them are essential to perform the comminution process. Thus, because of this importance, it is definitely remarkable to estimated/found the selection function by which comminution process can be perfectly described. In this case, by plotting  $\ln(m(t)/m(0))$  versus grinding time, the selection function is estimated. Where  $m(t)$  is the amount of remained ground ore remained in the first/coarsest sieve and  $m(0)$  is initial amount of ore used in each test. This correlation is presented in Figure 7.

If the line is fitted to the experimental points very well, the slope of fitted line can be considered as the selection function. Because of this definition, the amount of the selection function of used ore for the size/sieve of 125 microns was equal to 0.0043 (1/minute). However, this can then be applied to calculate some other variables which are dependent on the selection function, such as breakage function.

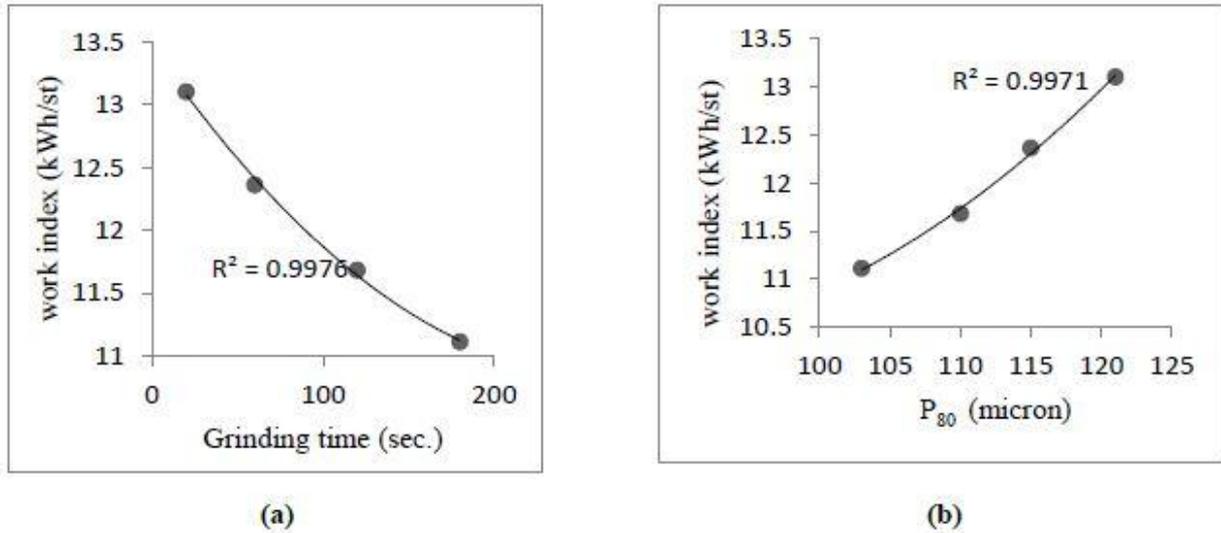


Figure 6. The trends of  $W_i$  and grinding time (a),  $W_i$  and  $P_{80}$  (b).

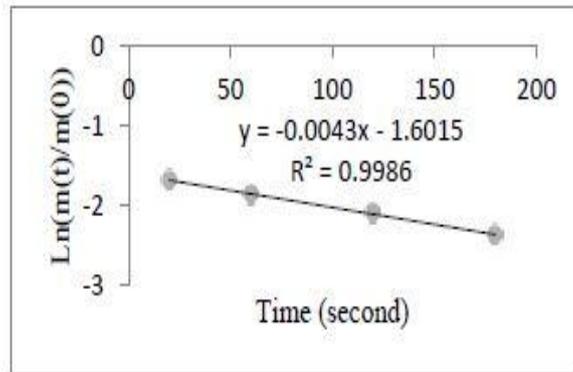


Figure 7. The selection function estimation plot

### 3.4. Limiting energy efficiency and relative efficiency

In next step of this research work, new equations were described for  $E_B$  and  $E_B/E_{limit}$ , based on  $W_i$  by which the crushing efficiency could accurately be calculated.  $E_B$  is the efficiency of comminution, and  $E_{Limit}$  is the maximum limiting energy efficiency for particle fracture under compressive loading. For this purpose,  $W_i$  was estimated based on the equation in various grinding times, and  $E_B$  and  $E_B/E_{limit}$  were accordingly calculated for every  $W_i$ . For more illustration,  $W_i$  of each grinding time was estimated by Equation 8, and all of them were used to gain energy efficiency (for this ore). Besides, it should be explained that the other required variables of Equation 2 were selected from the data of Tromans's paper [17]. It should be noted that some particular values for variables were considered to calculate  $E_B$  and  $E_B/E_{limit}$  and

in fact they made this investigation possible [17]. In addition, the strain energy analysis could present the limiting energy for particles fracture under comminution (compression). The limiting energy efficiency varies between 5% and 10%, depending on the value of the Poissons ratio, as shown in Figure 8. Required variables of Equation 2 and estimated  $W_i$ ,  $E_B$  and  $E_B/E_{limit}$  are presented in Tables 4 and 5 respectively.

Afterward,  $W_i$  versus  $E_B$  and  $E_B/E_{limit}$  was plotted and the best line was accordingly fitted for trends. These plots are shown in Figure 9, which indicates that there is one significant relationship between  $W_i$  versus  $E_B$  and  $E_B/E_{limit}$ , and the value of  $R^2$  is high enough to trust for the good fitness. Consequently, the fitted lines can be surely applied for new model equations. These models are shown as Equations 9 and 10 respectively.

$$E_B = 0.0304W_i^2 - 1.1193 W_i + 13.711 \quad (9)$$

$$E_B/E_{limit} = 0.377 W_i^2 - 13.922 W_i + 170.54 \quad (10)$$

The Equations 9 and 10 could be employed to find the amount of energy consumption and energy efficiency, which play significant roles in comminution process. These important new equations can be obtained by a few work index values, while they can perfectly help to calculate consumption energy. In fact, these new developed equations could be used to design crushing and grinding circuit by fewer numbers of tests. This would be more economic than previous method to calculate required energy.

Table 4. Values of used parameters for estimating  $E_B$

Variables	$\gamma(J/m^2)$	$D_{aef}(\text{micron})$	$\rho(Kg/m^3)$	$F_R$
Value	6.449	40	5197	3

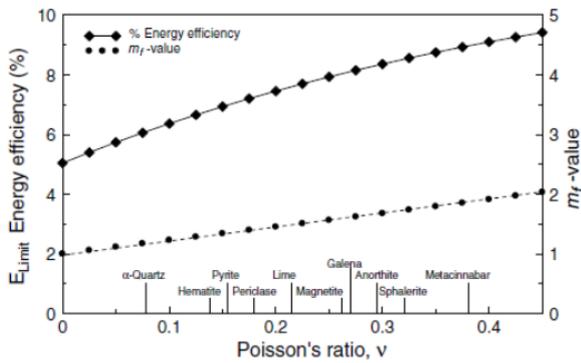
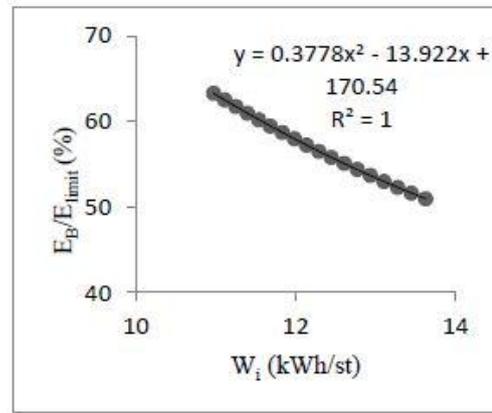


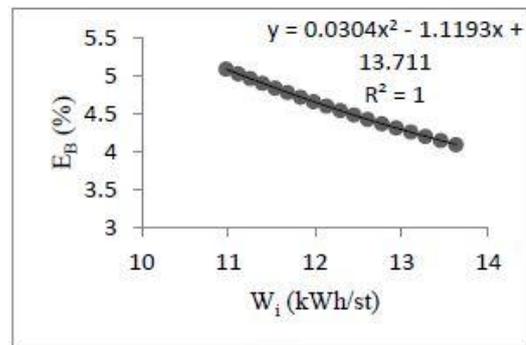
Figure 8. Effect of Poisson's ratio on the maximum limiting energy efficiency  $E_{Limit}$  for particle fracture under compressive loading [17].

Table 5. Crushing efficiencies  $E_B$ , based on the Equation 10 with  $FR = 3$ ,  $D_{aef} = 40$  microns, and ideal maximum limiting efficiencies  $E_{Limit}$  from Figure 8.

Grinding time (second)	$w_i$ (kWh/st)	$E_B$ (%)	$E_B/E_{limit}$ (%)
10	13.63	4.09	50.95
20	13.45	4.15	51.62
30	13.27	4.20	52.30
40	13.10	4.26	52.98
50	12.93	4.31	53.67
60	12.77	4.37	54.37
70	12.61	4.42	55.07
80	12.45	4.48	55.78
90	12.29	4.54	56.50
100	12.13	4.60	57.22
110	11.98	4.65	57.95
120	11.83	4.71	58.69
130	11.68	4.77	59.43
140	11.53	4.83	60.18
150	11.39	4.90	60.94
160	11.25	4.96	61.71
170	11.11	5.02	62.49
180	10.97	5.08	63.28



(a)



(b)

Figure 9. The trends of  $E_B/E_{limit}$  and  $W_i$  (a),  $E_B$  and  $W_i$  (b).

#### 4. Conclusions

In this research work, the iron ore sample provided from chadormalu iron mine was used in different work index and selection function experiments. According to the results, new developed equation of ore work index estimation was defined in which grinding time has a main role. Accordingly, applying grinding time in this equation, the work index can perfectly be calculated. Also, relations between variables of comminution such as work index, the work input and  $P_{80}$  were investigated. It was shown that there are significant relationship between these foregoing parameters which they could be influenced by increasing and decreasing of each other. Moreover, the selection function for size/sieve 125 microns was 0.0043 (1/minute), using chadormalu ore sample. This can be used to calculate some other relevant variables such as breakage function. Finally, energy efficiency was evaluated by this procedure, and new equations for  $E_B$  and  $E_B/E_{limit}$  were described by which energy consumption would be assessed very well.

#### References

- [1]. Bond, F.C. (1952). The third theory of comminution. Transaction of the AIME, 193, 484–494.
- [2]. Rittinger, P.R. (1867). Lehrbuch der Aufbereitungskunde, Ernst and Korn, Berlin, Germany.
- [3]. Kick, F. (1885). Das Gesetz der proportionalen Widerstande und seine anwendung felix. Verlag von Arthur Felix, Leipzig, Germany.
- [4]. Hukki, R.T. (1962). Proposal for a solomnic settlement between the theories of von Rittinger, Kick and Bond. Transactions of the AIME, 223, 403–408.
- [5]. Walker, W.H., Lewis, W.K., McAdams, W.H. and Gilliland, E.K. (1937). Principles of Chemical Engineering. Mc Graw-Hill, New York, USA.
- [6]. Bond, F.C. (1961). Crushing and Grinding Calculations. Brit.Chem. Eng. Part I, 6, 378–385, Part II, 6, 543–548.
- [7]. Deniz, V. H. and Ozdag. (2002). A new approach to Bond grindability and work index: dynamic elastic parameters, Minerals engineering, 16, 211-217.
- [8]. Berry, T.F. and Bruce, R.W. (1966). A simple method of determining the grindability of ores. Can. Min. J., 87, 63–65.
- [9]. Smith, R.W. and Lee, K.H. (1968). A comparison of data from Bond type simulated closed-circuit and batch type grindability tests. Trans. Soc. Min. Eng., AIME, 241, 91–99.
- [10]. Horst, W.E. and Bassarear, J.H. (1977). Use of simplified ore grindability technique to evaluate plant performance. Trans. Soc. Min. Eng., AIME, 260, 348–351.
- [11]. Karra, V.K. (1981). Simulation of Bond grindability tests. CIM Bull., 74, 195–199.
- [12]. Yap, R.F., Sepulude, J.L. and Jauregui, R. (1982). Determination of the Bond work index using an ordinary laboratory batch ball mill. Designing and Installation of comminution circuits. Soc. Min. Eng., AIME, New York, 176–203.
- [13]. Armstrong, D.G. (1986). An alternative grindability test: an improvement of the Bond procedure. Int. J. Min. Process., 16, 197–208.
- [14]. Magdalinovic, N. (1989). A procedure for rapid determination of the Bond work index. Int. J. Min. Process., 27, 125–132.
- [15]. Nematollahi, H. (1994). New size laboratory ball mill for Bond work index determination. Min. Eng., 352–353.
- [16]. Ahmadi, R. and Shasavari, SH. (2009). Procedure for determination of ball Bond work index in the commercial operations, Minerals engineering, 22, 104–106.
- [17]. Tromands, D. (2009). Mineral comminution: Energy efficiency considerations, Minerals engineering, 21, 613–620.
- [18]. Musci, G., Csoke, B. and K. Solymar. (2011). Grindability characteristics of lateritic and karst bauxites, Minerals Processing, 100, 96–103.
- [19]. Velázquez, A., Menéndez, J. and Brown, R. (2011). Grindability characteristics of lateritic and karst bauxites, Minerals Processing, 100, 96–103.
- [20]. DOE, Mining Industry of the Future Fiscal Year 2004 Annual Report, Industrial Technologies Program, US Department of Energy, Energy Efficiency and Renewable Energy, February, 2005.
- [21]. Tromans, D. and Meech, J.A. (2002). Fracture toughness and surface energies of minerals: estimates for oxides, sulphides, silicates and halides. Minerals Engineering, 15, 1027–1041.
- [22]. Tromans, D. and Meech, J.A. (2004). Fracture toughness and surface energies of covalent minerals: theoretical estimates. Minerals Engineering, 17, 1–15.
- [23]. Austin, L. G. and Bagga, P. (1981). An analysis of fine dry grinding in ball mills Powder Technology, 28, 83.
- [24]. Kotake, N., Suzuki, K., Asahi and Kanda, S. Y. (2002). Experimental study on the grinding rate constant of solid materials in a ball mill, Powder Technology, 122(2), 101–108.
- [25]. Fuerstenau, D.W., De, A. and Kapur, P.C. (2004). Linear and nonlinear particle breakage process in comminution systems, Int J Miner Process, 74, 317–327.
- [26]. Mehrabani, J. V., Noaparast, M., Mousavi, S. M. and Dehghan, R. (2010). A. Ghorbani, Process optimization and modeling of sphalerite flotation from a low-grade Zn-Pb using response surface methodology [J]. Separation & Purification Technology, 27, 2010, 242–249.
- [27]. Joglekar, A. M. and May, A. T. (1987). Product excellence through design of experiments [J]. Cereal Foods World 32, 12, 857–868.