Shahrood University of Technology
Iranian Society of Mining Engineering (IRSME)
Journal of Mining and Environment (JME)
journal homepage: www.jme.shahroodut.ac.ir
THOMSON REUTERS
Vol. 11, No. 3, 2020, 899-908.
DOI: 10.22044/jme.2020.9775.1897

A New Method for Predicting Indirect Tensile Strength of Sandstone Rock Samples

H. Fattahi*

Department of Earth Sciences Engineering, Arak University of Technology, Arak, Iran

Received 10 June 2020; received in revised form 21 June 2020; accepted 22 June 2020

Keywords

Tensile strength

Physical properties

Adaptive network-based fuzzy inference system

Subtractive clustering method

Fuzzy c-means clustering method

Abstract

The tensile strength (σt) of a rock plays an important role in the reliable construction of several civil structures such as dam foundations and types of tunnels and excavations. Determination of σt in the laboratory can be expensive, difficult, and time-consuming for certain projects. Due to the difficulties associated with the experimental procedure, it is usually preferred that the σt is evaluated in an indirect way. For these reasons, in this work, the adaptive network-based fuzzy inference system (ANFIS) is used to build a prediction model for the indirect prediction of σt of sandstone rock samples from their physical properties. Two ANFIS models are implemented, i.e. ANFIS-subtractive clustering method (SCM) and ANFIS-fuzzy c-means clustering method (FCM). The ANFIS models are applied to the data available in the open source literature. In these models, the porosity, specific gravity, dry unit weight, and saturated unit weight are utilized as the input parameters, while the measured σt is the output parameter. The performance of the proposed predictive models is examined according to two performance indices, i.e. mean square error (MSE) and coefficient of determination (R2). The results obtained from this work indicate that ANFIS-SCM is a reliable method to predict σt with a high degree of accuracy.

1. Introduction

The tensile strength (σ_t) of rocks is an important parameter involved in the design of a variety of engineering structures. There are basically two approaches used for determining σ_t, one of which is to collect and test the rock specimens in the laboratory (direct methods), and the other one is to use the empirical equations and/or statistical methods (indirect methods) [1]. The direct standard method (assesment of σ_t in the laboratory) is time-consuming and expensive, especially with highly fractured and inhomogeneous rocks [2]. The difficulties associated with performing a direct uniaxial tensile test on a rock specimen have led to a number of indirect methods for assessing σ_t. Several pertinent studies have previously been undertaken in order to develop the empirical correlations to predict the σ_t values in terms of the physical/mechanical properties of rocks. The estimator variables used for predicting σ_t are the mineralogical composition and the intrinsic rock properties such as the electrical resistivity [3], grain size, aspect ratio, form factor [4], strength ratio, unconfined compressive strength (UCS), tensile crack initiation stress [5], total porosity [6], angle between the planes of rock anisotropy and the loading direction, diameter of the central hole, contact condition of loading [7] point load strength [6, 8], Shore hardness, sound velocity, Schmidt hardness, porosity, and point load index [9].

Although previous efforts are valuable, in many cases, the aforesaid empirical approaches are not capable of distinguishing the sophisticated structures involved in the dataset. These reasons have been the main causes of interest to better find out the interaction between the physical properties for the indirect prediction of σ_t of rocks. For this

Corresponding author: h.fattahi@arakut.ac.ir (H. Fattahi).

purpose, recently, the adaptive network-based fuzzy inference system (ANFIS) [10-12] has been found to be a computational intelligence method that integrates the fuzzy inference system (FIS) concept into the artificial neural network (ANN), and has been widely used in the field of civil and mining engineering [13-15].

In a conventional FIS, the number of rules is decided by an expert who is familiar with the target system to be modeled. In an ANFIS simulation, however, no expert is available, and the number of membership functions (MFs) assigned to each input variable is chosen empirically, i.e. by plotting the datasets and examining them visually or simply by trial-and-error. For the datasets with more than three inputs and two outputs, the visualization techniques are not very effective, and most of the time, trial-and-error must be relied on. Generally, it is very difficult to describe the rules manually in order to reach the precision required with the minimized number of MFs when the number of rules is larger than 3. The better performance of ANFIS than the other intelligent methods is due to the FL and ANN combination. The path that an input would cover is like that of the input fuzzy inference system convey coordinates of sample to the input MFs, and then it passes through MF and changes; after that, its results go to the rules that according to available rules the category would be determined. One of the most important steps in the hybrid neuro–fuzzy modeling is the fuzzy membership value definition

In this research work, the ANFIS-subtractive clustering method (ANFIS-SCM) and the ANFIS-fuzzy c–means clustering method (ANFIS-FCM) are suggested for the indirect estimation of σ_t. In these models, porosity, specific gravity, dry unit weight, and saturated unit weight are utilized as the input parameters, while σ_t is the output parameter. The goodness of each hybrid model was evaluated using the data available in the literature. Finally, a statistical error analysis was performed on the modeling results in order to investigate the effectiveness of the proposed method.

2. Material and Methods

2.1. Adaptive network-based fuzzy inference system (ANFIS)

The ANFIS approach [16] is a combination of the neural learning and the Sugeno fuzzy to capture the input–output relationship. The structure of an ANFIS approach for two-input is presented in Figure 1.

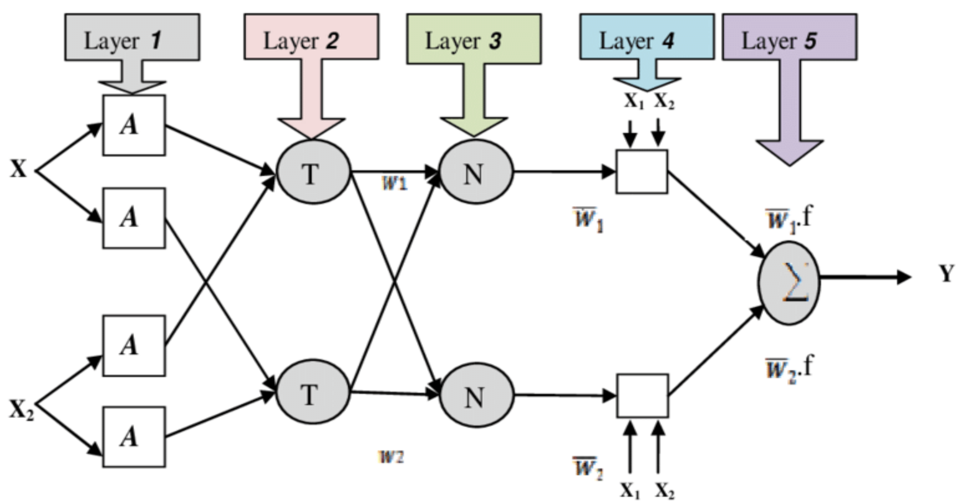


Figure 1. Structure of an ANFIS approach for two-input (after [16]).

Layer 1 is responsible for the fuzzification [17]:

$$Q_i^1 = \mu_{Ai}(x) = \frac{1}{1+\left[\left(\frac{x-v_i}{\sigma_i}\right)^2\right]^{b_i}}, \qquad (1)$$

where $\{\sigma_i, V_i, b_i\}$ is a series of parameters influencing the membership function (MF), *Ai* is the linguistic label, and *x* is the input.

Layer 2 is [16]:

$$Q_i^2 = W_i = \mu_{Ai}(x).\mu_{Bi}(y) \qquad i=1,2 \qquad (2)$$

Layer 3 is as follows [16]:

$$Q_i^3 = \overline{W}_i = \frac{w_i}{\sum_{j=1}^{2} w_j}, \qquad i=1,2 \qquad (3)$$

where w_i is the "firing strength" of the i^{th} rule, which is computed in Layer 2.

Layer 4 is as follows [17]:

$$Q_i^4 = \bar{W}_i f_i = \bar{W}_i (p_i x + q_i y + r_i), \quad (4)$$

where $\bar{W}_i$ is the output of Layer 3.

Layer 5 is the output layer:

$$Q_i^5 = Overall\ Output = \sum \bar{W}_i f_i = \frac{\sum w_i f_i}{\sum w_i} \quad (5)$$

Using different identification methods, and for a given dataset, different ANFIS models can be built. In this work, in order to identify the antecedent MFs, SCM, and FCM, two methods were used.

2.2 Subtractive Clustering Method

The mountain clustering method is simple and effective. However, its computation grows exponentially with the dimension of the problem. An alternative approach is the subtractive clustering method, introduced by Chiu [18], in which the data points are considered as the candidates for the center of clusters. The algorithm continues as follow:

Step 1: Consider a collection of n data points $\{X_1, X_2, X_3, ..., X_n\}$ in an M–dimensional space. Since each data point is a candidate for the cluster center, a density measure at data point X_i is defined as shown in Eguation (6):

$$D_i = \sum_{j=1}^{n} \exp\left(-\frac{\|x_i - x_j\|^2}{\left(\frac{r_a}{2}\right)^2} \right) \quad (6)$$

where r_a is a positive constant. Therefore, a data point will have a high density value if it has many neighboring data points. The radius r_a defines a neighborhood; the data points outside this radius contribute only slightly to the density measure.

Step 2: After the density measure of each data point is calculated, the data point with the highest density measure is selected as the first cluster center. Let X_{c1}, be the point selected and D_{c1} be its density measure. Next, the density measure for each data point x_i is revised as Eguation (7):

$$D_i = D_i - D_{ci} \exp\left(-\frac{\|x_i - x_{ci}\|^2}{\left(\frac{r_b}{2}\right)^2} \right) \quad (7)$$

where r_b is a positive constant.

Step 3: After the density calculation for each data point is revised, the next cluster center X_{c2} is selected and all the density calculations for the data points are revised again. This process is repeated until a sufficient number of cluster centers are generated.

2.3. Fuzzy C–Means Clustering Method (FCM)

FCM is a data clustering algorithm in which each data point belongs to a cluster to a degree specified by a membership grade; Bezdek introduced this algorithm in 1973 [19]. FCM partitions a collection of *n* vector $X_i,\ i = 1, 2, ..., n$, into *c* fuzzy groups and finds a cluster center in each group such that a cost function of dissimilarity measure is minimized. The steps of the FCM algorithm are, therefore, first described in brief.

Step 1: Chose the cluster centers $c_i,\ i = 1, 2, ..., c$, randomly from the n points $\{X_1, X_2, X_3, ..., X_n\}$.

Step 2: Compute the membership matrix *U* using Eguation (8):

$$\mu_{ij} = \frac{1}{\sum_{k=1}^{c} \left(\frac{d_{ij}}{d_{kj}}\right)^{2/m-1}} \quad (8)$$

where $d_{ij} = \|c_i - x_j\|$, is the Euclidean distance between the i^{th} cluster center and the j^{th} data point, and *m* is the fuzziness index.

Step 3: Compute the cost function according to Eguation (9). Stop the process if it is below a certain threshold.

$$J(U, c_1, ..., c_2) = \sum_{i=1}^{c} J_i = \sum_{i=1}^{c} \sum_{j=1}^{n} \mu_{ij}^m d_{ij}^2 \quad (9)$$

Step 4: Compute the new *c* fuzzy cluster centers $c_i,\ i = 1, 2, ..., c$, using Eguation (10).

$$c_i = \frac{\sum_{j=1}^{n} \mu_{ij}^m x_j}{\sum_{j=1}^{n} \mu_{ij}^m} \quad (10)$$

Go to step 2.

3. Experimental Database

The main scope of this work was to implement the above methodology in the problem of σ_t prediction. The dataset applied in this work for determining the relationship among the set of input and output variables was gathered from the open source literature [20]. A database composed of the measured σ_t values and physical properties was established using the data collected from a formation around the

Khouzestan Province (Iran). The 29 specimens of fresh sandstone blocks were cored in the laboratory. Each dataset contained the parameter porosity (%), specific gravity (G_s), dry unit weight (KN/m^3), saturated unit weight (KN/m^3), and measured σ_t (MPa). The σ_t values for the rock samples were determined using the Brazilian tensile strength tests. A detailed description of the database could be found in the refered resource [20]. Table 1 shows the statistical description of the datasets used in this work.

Table 1. Statistical description of the dataset utilized for construction of ANFIS models.

Parameter	Min.	Max.	Average
Porosity (%)	4.19	25.27	11.50
Specific gravity (G_s)	22.76	26.68	24.71
Dry unit weight (kN/m^3)	16.97	24.62	21.90
Saturated unit weight (kN/m^3)	19.42	25.11	23.04
Tensile strength (MPa)	0.19	13.23	5.90

4. Pre–Processing of Data and Performance Criterion

In order to start the training, the input and output data should be normalized to increase the efficiency of the networks in recognition of the relationships between the inputs and output data. Normalization is also really helpful in increasing the accuracy of the prediction and scaling the data to minimize the biasing of the networks. Data normalization can also reduce the time consumed for training. It is especially useful for modeling those applications where the input data is in different scales [21, 22]. There are many normalization techniques conventionally used to scale up the data including Z–Score normalization, Min–Max normalization, sigmoid normalization, statistical column normalization, etc. However, for the purpose of this work, the Min–Max normalization method was used. This was due to the capability of the Min–Max normalization in maintaining the variation in each feature after normalization. Beside, this normalization method can preserve all the relationships in the data [22]. The Min–Max normalization equation can be expressed as follows:

$$x_M = 2\left(\frac{x - x_{\min}}{x_{\max} - x_{\min}}\right) - 1 \qquad (11)$$

where x is the original value of the dataset, x_M is the mapped value, and x_{max} (x_{min}) denotes the maximum (minimum) raw input values, respectively .

In addition to normalization, the mean square error (MSE) and coefficient of determination (R^2) are two conventional criteria considered to assess the efficiency of the networks. MSE can be calculated using the following equation:

$$MSE = \frac{1}{n}\sum_{k=1}^{n}(t_k - \hat{t}_k)^2 \qquad (12)$$

where t_k is the actual value, $\hat{t}_k$ is the predicted value of the k^{th} observation, and n is the number of samples used for training or testing the network. MSE is routinely used as a criterion to show the discrepancy between the measured and estimated values of the network [23-26]. The coefficient of determination, R^2, can also be calculated as follows:

$$R^2 = 1 - \frac{\sum_{k=1}^{n}(t_k - \hat{t}_k)^2}{\sum_{k=1}^{n} t_k^{\,2} - (\sum_{k=1}^{n} \hat{t}_k^{\,2} / n)^2} \qquad (13)$$

R^2 is widely used as a representation of the initial uncertainty of the model. The best network model, which is unlikely to build, would have MSE = 0 and R^2 = 1.

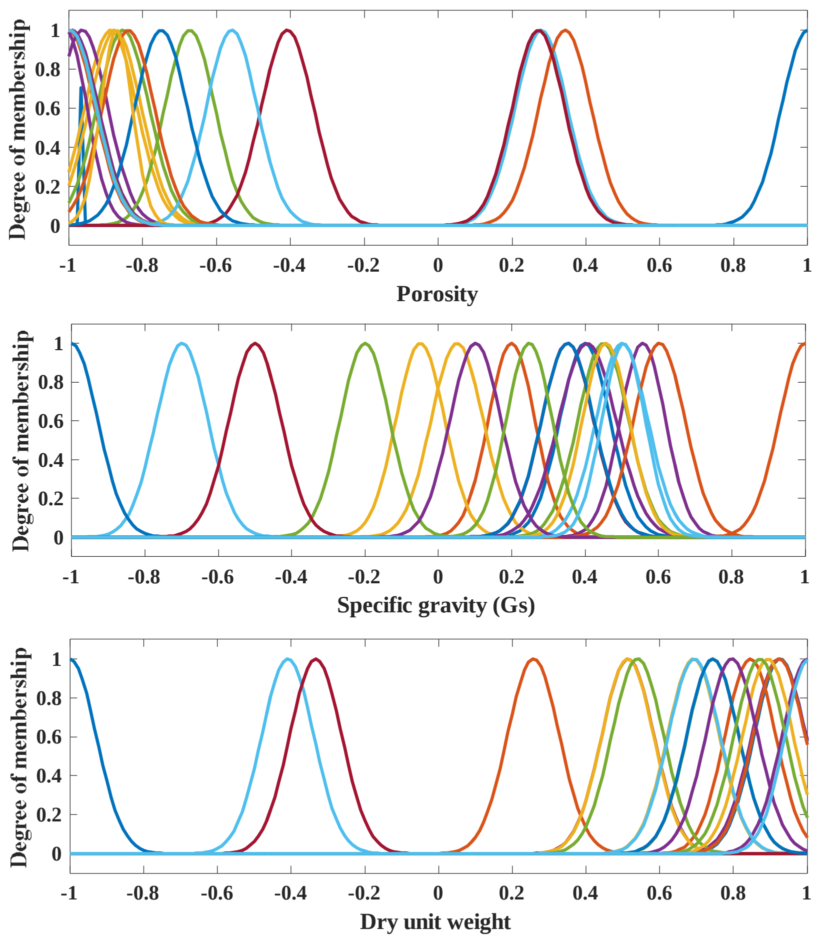
5. Results

The training and testing procedures of the two ANFIS models (ANFIS-SCM and ANFIS-FCM) were conducted from scratch for the five mentioned datasets. The MSE and R^2 values obtained for the training datasets indicate the capability of learning the structure of data samples, whereas the results of the testing dataset reveal the generalization potential and the robustness of the system modeling methods. The characterizations of the ANFIS models are revealed in Table 2.

Table 2. Characterizations of the ANFIS models.

ANFIS parameter	ANFIS–SCM	ANFIS–FCM
MF type	Gaussian	Gaussian
Output MF	Linear	Linear
Number of nodes	207	157
Number of linear parameters	100	75
Number of non-linear parameters	160	120
Total number of parameters	260	195
Number of training data pairs	20	20
Number of testing data pairs	9	9
Number of fuzzy rules	20	15

The number of rules obtained for the ANFIS-SCM and ANFIS-FCM models are 20 and 15, respectively. MFs of the input parameters for different models are shown in Figures. 2 and 3.



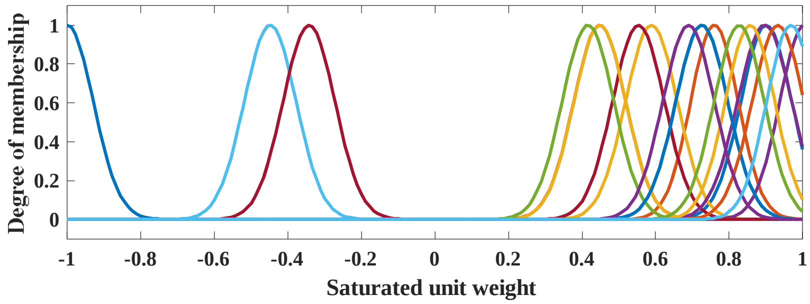
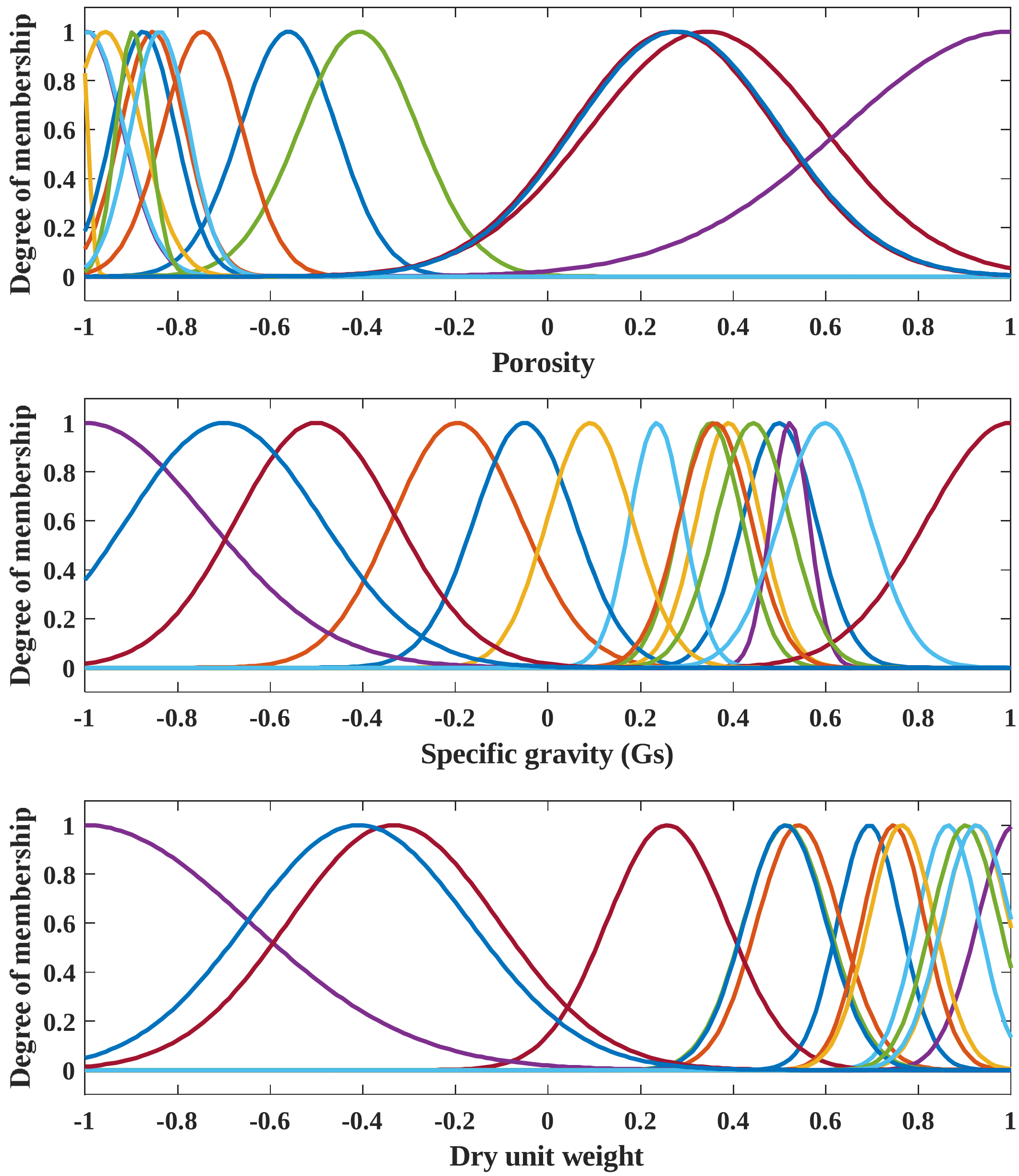


Figure 2. MFs obtained by the ANFIS–SCM model.



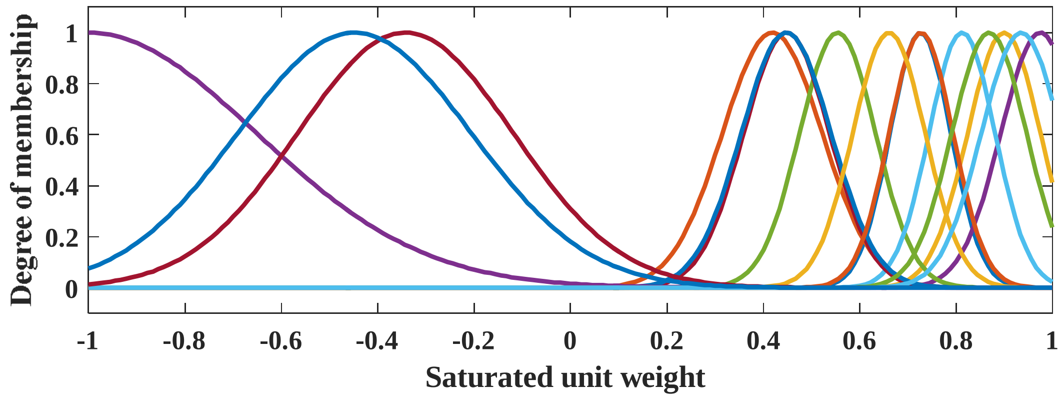
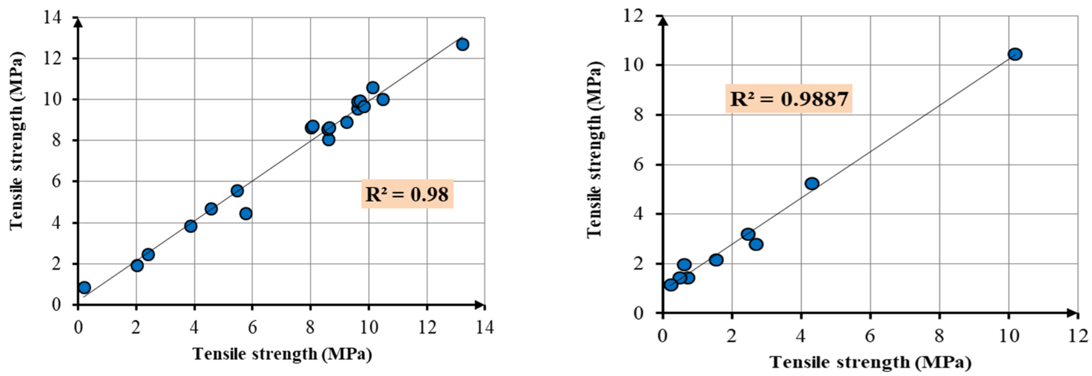


Figure 3. MFs obtained by the ANFIS–FCM model.

A comparison between the results of three models for the training and testing datasets is shown in Table 3. As it can be observed in this table, the ANFIS–SCM model with MSE = 0.016 and R^2 = 0.9887 for the testing datasets performs better than the ANFIS–FCM model for the indirect estimation of σ_t. Furthermore, correlations between the measured and predicted values of σ_t for the testing and training phases are shown in Figures. 4 and 5.

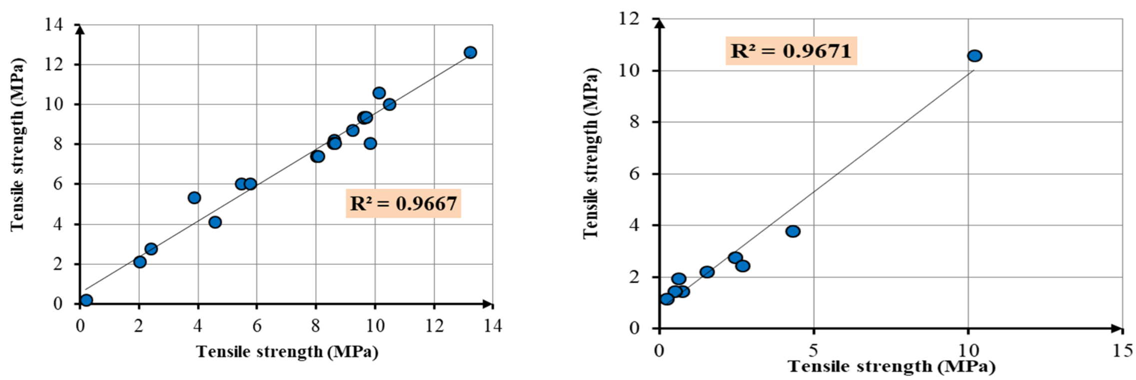
Table 3. A comparison between the results of the ANFIS models.

ANFIS model	Training		Testing	
	MSE	R^2	MSE	R^2
ANFIS–SCM	0.005	0.9800	0.016	0.9887
ANFIS–FCM	0.006	0.9667	0.017	0.9671



(a) **(b)**

Figure 4. Correlation between the measured and predicted values of σ_t by ANFIS–SCM model a) training data, b) testing data.



(a) **(b)**

Figure 5. Correlation between the measured and predicted values of σ_t by ANFIS–FCM model a) training data, b) testing data.

A comparison between the predicted values of σ_t by the ANFIS models and the measured values for the datasets at the testing phases is shown in Figure 6. As shown in this figure, the results of the ANFIS–SCM model in comparison with the actual data show a good precision of the ANFIS–SCM model.

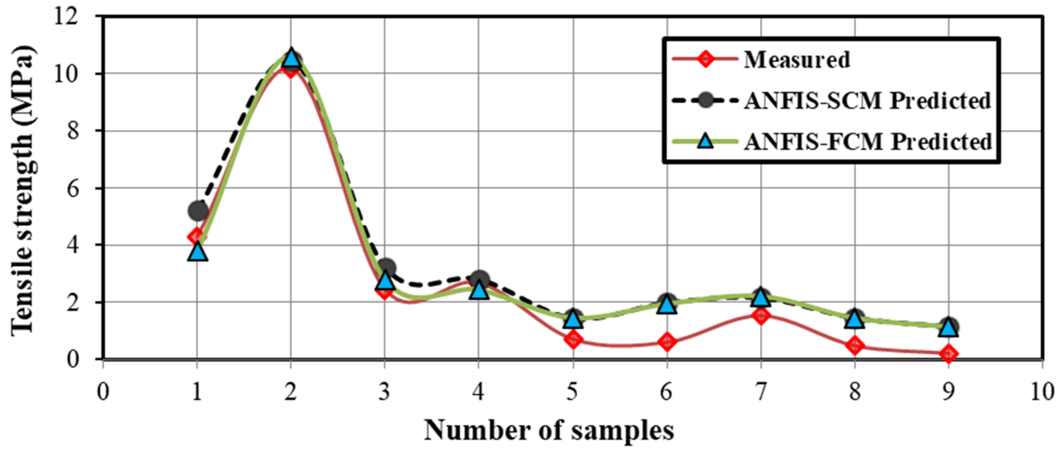


Figure 6. Comparison between the measured and predicted σ_t by the ANFIS models for the testing datasets.

6. Conclusions

In this work, the indirect estimation of σ_t was investigated using two ANFIS models (ANFIS-SCM and ANFIS-FCM), and the following conclusions could be drawn:

- Porosity, specific gravity, dry unit weight, and saturated unit weight were incorporated for the indirect estimation of σ_t of rocks.
- A comparison was made between two ANFIS models (ANFIS-SCM and ANFIS-FCM) using 29 data samples and based upon the performance indices MSE and R^2. ANFIS–SCM with MSE = 0.016 and R^2 = 0.9887 was selected as the best predictive model.
- The generalized Gaussian MFs were used in the present models. MFs were tested. It is important to mention that the rules used are generally based on the model and variables that are dependent on the user's experience and the trial-and-error method. Furthermore, the shape of MFs depends on the parameters involved, and changing these parameters will change the shape of MF.
- Consequently, one may conclude that ANFIS–SCM is a reliable system modeling technique for predicting σ_t of rocks with a highly acceptable degree of accuracy and robustness.
- This work shows that the ANFIS approach can be applied as a powerful tool for modeling some of the problems involved in mining and civil engineering.

References

[1]. Baykasoğlu, A., Güllü, H., Çanakçı, H. and Özbakır, L. (2008). Prediction of compressive and tensile strength of limestone via genetic programming. Expert Syst Appl 35 (1):111-123.

[2]. Ceryan, N., Okkan, U. and Kesimal, A. (2012). Application of generalized regression neural networks in predicting the unconfined compressive strength of carbonate rocks. Rock Mech Rock Eng 45 (6):1055-1072.

[3]. Kahraman, S. and Yeken, T. (2010). Electrical resistivity measurement to predict uniaxial compressive and tensile strength of igneous rocks. B Mater Sci 33 (6):731-735.

[4]. Singh, V. and Singh, D. and Singh, T. (2001). Prediction of strength properties of some schistose rocks from petrographic properties using artificial neural networks. Int J Rock Mech Min Sci 38 (2):269-284.

[5]. Cai, M. (2010). Practical estimates of tensile strength and Hoek–Brown strength parameter m i of brittle rocks. Rock Mech Rock Eng 43 (2):167-184.

[6]. Karakus, M. (2011). Function identification for the intrinsic strength and elastic properties of granitic rocks via genetic programming (GP). Comput Geosci 37 (9):1318-1323.

[7]. Chen, G., Jia, Z. and Ke, J. (1997). Probabilistic analysis of underground excavation stability. Int J Rock Mech Min Sci 34 (3-4):51. e51-51. e16.

[8]. Heidari, M., Khanlari, G., Torabi Kaveh, M. and Kargarian, S. (2012). Predicting the uniaxial compressive and tensile strengths of gypsum rock by point load testing. Rock Mech Rock Eng 45 (2):265-273.

[9]. Abolhosseini, H., Hashemi, M. and Ajalloeian, R. (2020). Evaluation of geotechnical parameters affecting

the penetration rate of TBM using neural network (case study). Arab J Geosci 13 (4):183.

[10]. Iphar, M., Yavuz, M. and Ak, H. (2008). Prediction of ground vibrations resulting from the blasting operations in an open-pit mine by adaptive neuro-fuzzy inference system. Environ Geol 56 (1):97-107.

[11]. Sezer, E.A., Nefeslioglu, H.A. and Gokceoglu, C. (2014). An assessment on producing synthetic samples by fuzzy C-means for limited number of data in prediction models. Appl Soft Comput 24:126-134.

[12]. Fattahi, H. (2016). Adaptive neuro fuzzy inference system based on fuzzy C–means clustering algorithm, a technique for estimation of TBM peneteration rate. Int J Optim Civil Eng 6 (2):159-171.

[13]. Fattahi, H .(2016). Indirect estimation of deformation modulus of an in situ rock mass: an ANFIS model based on grid partitioning, fuzzy c-means clustering and subtractive clustering. J Geosci 20 (5):681–690.

[14]. Karimpouli, S. and Fattahi, H. (2018), Estimation of P-and S-wave impedances using Bayesian inversion and adaptive neuro-fuzzy inference system from a carbonate reservoir in Iran. Neural Comput Appl 29 (11):1059-1072.

[15]. Fattahi, H. and Karimpouli, S. (2016). Prediction of porosity and water saturation using pre-stack seismic attributes: a comparison of Bayesian inversion and computational intelligence methods. Computat Geosci 20 (5):1075-1094.

[16]. Jang, J-S .(1993). ANFIS: adaptive-network-based fuzzy inference system. IEEE T Syst Man Cyb 23 (3):665-685.

[17]. Weiling C, Lee J Fuzzy Logic for the Applications to Complex Systems. In: Proceedings of the International Joint Conference of CFSA/IFIS/SOFT on Fuzzy Theory and Applications. Singapore et al.: World Scientific, 1995.

[18]. Chiu, S.L. (1994). Fuzzy model identification based on cluster estimation. J Intell Fuzzy Syst 2 (3):267-278

[19]. Bezdek, J.C. (1973). Fuzzy mathematics in pattern classification. Cornell university, Ithaca.

[20]. Ghobadi, M.H., Mousavi, S., Heidari, M. and Rafie, B. (2015). The Prediction of the Tensile Strength of Sandstones from their petrographical properties using regression analysis and artificial neural network. Geopersia 5 (2):177-187.

[21]. Gholami, R., Moradzadeh, A., Maleki, S., Amiri, S. and Hanachi, J. (2014). Applications of artificial intelligence methods in prediction of permeability in hydrocarbon reservoirs. J Pet Sci Eng 122:643-656.

[22]. Jayalakshmi, T. and Santhakumaran, A. (2011). Statistical normalization and back propagation for classification. International Journal of Computer Theory and Engineering 3 (1):1793-8201.

[23]. Fattahi, H. (2016). Application of improved support vector regression model for prediction of deformation modulus of a rock mass. Eng Comput 32 (4):567-580.

[24]. Fattahi, H. (2017). Applying soft computing methods to predict the uniaxial compressive strength of rocks from schmidt hammer rebound values. Computat Geosci 21 (4):665-681. doi:10.1007/s10596-017-9642-3.

[25]. Fattahi, H. and Bazdar, H. (2017). Applying improved artificial neural network models to evaluate drilling rate index. Tunn Undergr Sp Tech 70:114-124.

[26]. Karimpouli, S. and Fattahi, H .(2018). Estimation of P-and S-wave impedances using Bayesian inversion and adaptive neuro-fuzzy inference system from a carbonate reservoir in Iran. Neural Comput Appl 29 (11):1059–1072.

یک روش جدید برای پیش بینی غیرمستقیم مقاومت کششی نمونه‌های سنگی ماسه سنگ

هادی فتاحی

گروه مهندسی ژئومکانیک، دانشکده مهندسی علوم زمین، دانشگاه صنعتی اراک، ایران

ارسال 2020/05/10، پذیرش 2020/05/22

* نویسنده مسئول مکاتبات: h.fattahi@arakut.ac.ir

چکیده:

مقاومت کششی یک سنگ نقش مهمی در ساخت ایمن سازه‌های مختلف عمرانی مانند پی سد، انواع تونل‌ها و حفاری‌ها دارد. از طرفی برای پروژه‌های خاص تعیین مقاومت کششی در آزمایشگاه می‌تواند پرهزینه، مشکل و زمانبر باشد. بعلت مسائل بیان شده، در پروسه‌ی آزمایشگاهی معمولاً مقاومت کششی به صورت غیرمستقیم ارزیابی می‌شود. در این تحقیق از سیستم استنتاج تطبیقی نرو-فازی (انفیس) برای ساخت مدل جهت پیش‌بینی غیرمستقیم مقاومت کششی نمونه‌های سنگی سنداستون به کمک خواص فیزیکی استفاده شده است. در این تحقیق دو مدل انفیس به نام های انفیس-خوشه‌بندی کاهشی و انفیس-سی مینز فازی ساخته شدند. مدل‌های انفیس مذکور بر روی داده‌های موجود از منابع قابل در دسترس بکارگرفته شدند. در این مدل‌ها، تخلخل، وزن مخصوص، وزن مخصوص خشک و وزن مخصوص اشباع به عنوان ورودی و مقاومت کششی بعنوان خروجی مورد استفاده قرار گرفت. برای ارزیابی عملکرد مدل‌ها از دو شاخص میانگین خطای مربعات و ضریب تعیین استفاده شد. نتایج بدست آمده از این تحقیق نشان می‌دهد که انفیس-خوشه‌بندی کاهشی یک روش قابل اعتماد و با درجه دقت بالا برای پیش بینی مقاومت کششی است.

کلمات کلیدی: مقاومت کششی، خواص فیزیکی، سیستم استنتاج تطبیقی نرو-فازی ، روش خوشه‌بندی کاهشی، روش سی مینز فازی.