Optimal Earthmoving Fleet Size for Minimising Emissions and Cost

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Abstract

Traditionally, the earthmoving operations have been developed based on the minimum cost per production criterion. Nowadays, due to the negative impacts of the emissions on the environment, there is an increasing public awareness to reduce the emissions from the earthmoving operations. Different management strategies can be employed to reduce emissions, amongst other things, which can also result in a reduction in the operational costs. This paper aims to examine the cost and emissions related to the earthmoving equipment from an operational standpoint. The queue theory is used in order to demonstrate that the optimum cost per production fleet size and the optimum emissions per production coincide. The linear and non-linear server utilization functions are employed to present a general optimization proof independent from any specific case study. The findings of this research work provide a better understanding of the relationship between the emissions and cost and how the under-trucking and over-trucking conditions affect the productivity and environmental affairs in the earthmoving operations.

Keywords
Earthmoving
Loader-truck operation
Surface mining
Fleet size
Emissions

1. Introduction

The earthmoving operations generally include the off-road equipment working in a load and dump point and a fleet of trucks cycling between these points. A fleet of equipment is generally required to transport the materials from the extracted site to the dumping site. The amount of material produced in a unit time is directly dependent on the number of equipment employed in the earthmoving system. Employing less than the optimal fleet size results in the underutilization of the haulage system. It is noteworthy that the material handling can contribute to about 50% of the production costs in the open-pit mines [1]. Therefore, the size of fleet equipment can significantly affect the earthmoving productivity. In order to reach the minimum production cost and maximum productivity, it is of great importance to determine the optimal fleet size in each operation [2].

Since 1960s, much effort has been made to develop the operational research techniques in order to determine the optimal fleet size. O’Shea et al. [3] and Griffis [4] were amongst the first who determined the optimal number of fleet size using the mathematical models and the queuing theory. Since then, several techniques have been employed to determine the optimal fleet size in specific operations. The queuing theory, which has been widely used in the previous studies, divides the cycle queue into different phases of equipment loading, loaded travel, dumping, and empty travel. The average time in each phase is estimated by calculation of the utilization of the expected time and the number of trucks at each phase. Then the production and unit cost are calculated followed by variation in the haulage units to determine the optimal cost and productivity [5, 6]. More details regarding the queuing theory are provided in the following section. This technique has also been employed on the other urban applications such as
determining the optimal fleet size of electric car sharing systems based on the total revenue [7].

The queuing theory can be employed in conjunction with the algorithms such as linear programming, where after defining different phases, the estimated queue length, and waiting time and utilization, the problem is discretised to the linear programming optimization problems having dual or several functions such as the reducing cost, maximizing production, and equipment utilization [7-9].

Dynamic programming is another fleet size optimization approach in which the changes in the system and policies can be incorporated, unlike most implicit approaches. These changes can include the equipment and labour costs as well as the extracted material price and demand. The application of this approach based on the regeneration sequence and non-linear programming can be seen in the previous studies [10, 11].

Discrete event simulation (DES) is a computer-based technique that provides modelling, simulation, and analysis of systems in a sequence of discrete events based on what if analysis. In this approach, the state variables are changed at the discrete time standpoint in which certain events occur [12]. The analyst is guided by the conceptual frameworks to select an appropriate framework based on the system characteristics and specified model objectives. SIMAN, GPSS, and SLAM are the most common programs used for the DES analysis [2]. The DES approach has the advantage of stochastic optimization, where the traditional deterministic approaches are not able to guarantee an optimal solution in these cases [1]. The DES approaches are usually able to iterate on a family of solutions, employing the current state not the past solutions as well as imitating non-linear programming. Some efforts have been made to use DES to enhance the system productivity as well as reducing the environmental impact of the haulage system [13]. Ahn et al. [14] have used a DES algorithm in order to estimate the different components of the cycle times in a case study operation.

In addition to the above-mentioned methods, other methods such as the genetic algorithms [15-17], inventory theory [18], demand Pivot method [19], knowledge-based expert systems [20, 21], multi-criteria decision-making techniques [22], neural networks [23, 24], element build up with modifying factors [25], multiple regression [26, 27], match factor [28], analytical hierarchy process [29], mixed integer programming [30], and machine repair modelling [31] have been employed in the previous studies in order to determine the optimal fleet size.

A review of the previous studies shows that the computer simulations based on the DES approach are the most common approach to predict the cycle times in the earthmoving operations. Terrazas Prado et al. [32] have developed a custom-made truck cycle and delay automated data collection system (TCD-ADCS) in a surface coal mining. However, there are questions about the accuracy of the simulation methods (for example, [33, 34]). Moreover, it has been shown that the computer programs such as the truck and loader productivity and cost (TALPAC) underestimate the cycle times for shorter haul routes and overestimate them for the longer haul routes [27]. Chanda and Gardiner [27] by comparing the actual cycle times from a mining operation with the estimated times from the three different methods of neural networks, multiple regression, and TALPAC simulations have demonstrated that the cycle times from neural networks and multiple regression methods are more reliable than the TALPAC simulations. Despite the advantages of different fleet size optimization methods, most of these approaches are numerically-based. Only the queuing theory permits a general analytical proof, and it provides an analytical tractability where the operational analysis is constructed based on the finite source queuing theory.

Despite the well-established methods to obtain the optimum equipment fleet size based on the minimum production cost [35], a limited number of studies have been carried out considering the emission of earthmoving operations. One environmental aspect of the earthmoving operations is related to the exhaust emissions from vehicles and equipment including carbon dioxide, carbon monoxide, hydrocarbons, nitrogen oxides, and particulate matter [36].

The off-road vehicles are a significant source of air pollution, and produce a large amount of emissions compared to the on-road vehicles such as automobiles. For example, the amount of particulate matter for a bulldozer with a 175 hp engine is nearly 500 times more than that of a new automobile [37]. Reducing this pollution will contribute to a healthier lifestyle and decreases the environmental problems. As the public awareness on this issue has been raised, efforts are being directed to minimize the level of pollutants produced by the earthmoving vehicles and equipment. Government regulations, fuel specifications, engine modifications, and vehicle
fleets and fleet management are some of the approaches adopted to decrease pollution [37].

The United States Environmental Protection Agency [38] and California Environmental Protection Agency Air Resources Board [39] have provided models for determining emissions from the off-road equipment. These models can be used in the overall design and planning of earthmoving operations, and in particular, in the selection of the appropriate combination of loading and hauling units. However, these models are not precise enough for specific work cycles as they typically give emissions per year. Such models, due to the use of the average load factors that are not based on the job specific conditions, provide a very general estimation of the emissions.

Emissions from the off-road vehicles presented in the regulations and standards are usually quantified based on the steady-state engine dynamometer tests [40], and hence, may not be representative of the actual emissions in the field. Furthermore, the undertaken research works show that the exhaust emissions are dependent on the equipment type and the tasks that they are performing [41, 42]. This points to the need for further data-based research works on the actual vehicle activities to consider the operational field conditions.

Few studies have been undertaken to investigate the effects of the operating parameters on emissions based on field measurements. Frey et al. [40] have carried out field measurements and compared the emissions of fuel type B20 versus petroleum diesel. Their study provides a good insight about the effect of fuel type and engine tier on the emission rates; however, the measurements were made for periods of several hours, which may be different from the annual averages. Hansen [43] has assessed the performance of fuel biodiesel blends in the off-road vehicles on the front-end loader performance. He has found that on one hand, the biodiesel decreases the emission rate of some pollutants but on the other hand, it increases the NOx rate. Therefore, the type of fuel influences the emitted pollutants, and consequently, evaluation of alternative fuels is necessary before using in engines. As mentioned earlier, the EPA emission values are based on the steady state engine dynamometer tests and so they can differ from the actual equipment emissions in the field. Field emissions of the diesel-powered off-road vehicles have been measured and evaluated by Gautum [44] in a study performed for the California Air Resources Board and the California Environmental Protection Agency. It was found that exhaust emissions were dependent on the vehicle type. Thus, in order to have a proper emission data for modelling purposes, a range of vehicle types and models should be tested. The off-road vehicles used by Gautum were a street sweeper, a rubber-tired loader, an excavator, and a bulldozer. Therefore, there is a need for supplementary studies to cover the earthmoving activities in a more comprehensive way.

Lewis [41] has presented a new approach for determination of emissions for specific work cycles of the construction equipment. A portable emission monitoring system (PEMS) was used to collect the fuel consumption and emission data of seven types of equipment while they were working. The data obtained was collected from 8 backhoes, 6 bulldozers, 3 excavators, 6 motor graders, 3 off-road trucks, 3 track loaders, and 5-wheel loaders. Lewis divided the engine load into 10 different modes and used the average fuel consumption (modal fuel use) and emissions (modal emissions) in each mode to determine the emissions of different work cycles. Based on these measurements, the equations were presented to estimate the fuel consumption in different engine modes. Using these equations and the fraction of time equipment spent in different engine modes, the emission values can be calculated for a variety of engine powers and engine tiers.

The Lewis’s emission model, however, is not capable of determining the idling and non-idling emissions of the equipment. Therefore, Carmichael et al. [45] have developed a model in order to estimate the idling and non-idling emissions of the loaders and trucks. They evaluated the accuracy of this model by comparing the results obtained with the ones from the model presented by Lewis et al. [46]. The emissions model developed by Carmichael et al. [45, 47] was used in this work to estimate the idling and non-idling emissions of the equipment.

With the main focus on the conventional loader-truck earthmoving operations, in this work, we examined different equipment configurations in terms of emissions and costs in order to understand their interrelationship. First, the earthmoving operation was optimized based on the minimum emissions per production (EPP) and minimum cost per production (CPP) criteria. The queuing analysis was used to estimate the change in emissions and cost from the altering fleet sizes due to its analytical tractability. The linear $\eta$ functions were then used in the under-trucked and over-trucked scenarios in the server utilization in order to demonstrate that the truck fleet size increment in the under-trucked
scenario decreased the unit emissions and cost, while it increased the unit cost and emissions in an over-trucked scenario. This means that there is an optimal truck fleet size based on the unit emissions and the unit cost in the transition between an operation being under-trucked and over-trucked. Finally, the non-linear $\eta$ functions were presented in the transition parts to prove that the optimum fleet size in terms of unit cost coincided with that for the unit emissions, independent from any specific case study numbers.

2. Case Study and analytical background

In order to examine the optimum fleet size in terms of the unit cost and unit emissions, a real-world operation must be studied. The existing operation in this work is a coal mining operation in Australia including a loader (Komatsu WA470) and a fleet of trucks (Komatsu HD325) that transport the overburden removal from the loading area to a dump in order to get to the minerals. The average results for the same operation (same grade, payload, and haul distance) were used for this analysis, and the average truck cycle component times were measured in a period of approximately 4 hours, which are summarized as follow:

- Manoeuvre at loader = 0.36 min
- Load = 3 min
- Loaded travel = 10.3 min
- Manoeuvre at crusher = 0.17 min
- Dump = 0.22 min
- Empty travel = 10.77 min

As noted earlier, the finite source queuing theory was used in this work in order to calculate the different components of the truck cycle times. In order to analyse the operation, the service time (denoted as $1/\mu$), the back-cycle time (denoted as $1/\lambda$), the number of loaders ($c$), and the number of trucks ($K$) were required, which can be measured in site. The service time is defined as the sum of the truck manoeuvre time and load time, while the back-cycle time is defined as the loaded haul time plus the dump time and the return time. The average waiting time in queue for different truck fleet sizes is:

$$W_q = \frac{K}{\mu \eta} - \frac{1}{\mu} - \frac{1}{\lambda}$$

where $\eta$ is the server utilization, and shows the proportion of time that the server is busy. The server utilization ($\eta$) can be determined based on the service factor, defined as $\lambda/\mu$. According to Carmichael [48], the average of the finite source $(D/D/c)/K^*$ and $(M/M/c)/K$ server utilizations can be used in earthmoving, quarrying, and mining operations. The exponential probability distribution is applied for describing the back-cycle and service times in the $(M/M/c)/K$ model, whereas a constant probability distribution is used in the $(D/D/c)/K$ model. Production can then be obtained as follows:

$$\text{Production} = \mu \eta \text{CAP}$$

where $\text{CAP}$ is the capacity of a truck ($m^3$), and $T$ is the time period when production is being measured.

The issue considered in this section is to determine and compare the optimal truck fleet size for the given parameters of the case study in terms of the unit cost and unit emissions. The objective functions are then cost per production (CPP) and emissions per production (EPP).

2.1. Cost per production (CPP)

The queue theory was used to calculate the different components of the cycle times. The following assumptions were made to facilitate the analysis:

- Loader is the server and loader utilisation corresponds to the server utilization. This means that the loader starts working when the truck starts manoeuvring at the server.

- For trucks, non-idling is assumed to be equal to the back-cycle time excluding the waiting time in dump because the waiting time in the dump is negligible.

For an operation with a loader and a fleet of trucks, $\text{Cost} = C_1 + KC_2$

where $C_1$ is the hourly operating cost of a loader, and $C_2$ is the hourly operating cost of a truck [48, 49]. Therefore, CPP is calculated as:

$$\text{Cost/production} = \text{CPP} = \frac{C_1 + KC_2}{\mu \eta \text{CAP}}$$

For the current case study, $C_1 = $110/h, $C_2 = $160/h, $c = 1$, and $\text{CAP} = 36.5 m^3$.

servers; and $f$ is the calling population or input source. $M$ refers to the exponential case; $D$ the constant case; and $K$ the finite source size of customers.
2.2. Emissions per production (EPP)

Lewis [41] has presented an emission model in order to determine the exhaust emissions of the off-road equipment. Using this model, the fuel use rates can be estimated in different engine modes. These rates can then be multiplied by the proportion of time equipment spent in different engine modes in order to determine the modal weighted fuel use. Summing these values gives the total fuel consumption rate for the activities such as moving soil and loading a truck. Multiplying this value (gal/h) by the weighted average emission rate leads to the weighted average emissions rate (g/h) for a specific activity. The notations used for the equipment emission values are as follow:

\[ N_L : \text{Loader non-idling emissions} \]
\[ I_L : \text{Loader idling emissions} \]
\[ N_T : \text{Truck non-idling emissions} \]
\[ I_T : \text{Truck idling emissions} \]

The cycle time can be defined as

\[ \text{Cycle time} = \frac{1}{\mu} + \frac{1}{\lambda} + W_q = \frac{K}{\mu \eta} \]

Proportion of the loader non-idling time =

\[ \frac{K}{\mu} \]

Proportion of the loader idling time = \( 1 - \eta \)

Total loader emissions = \( N_L (\eta) + I_L (1 - \eta) \)

Proportion of the truck non-idling time =

\[ \frac{1}{\lambda} \]

Proportion of the truck idling time = \( 1 - \frac{\mu \eta}{K \lambda} \)

Total truck emissions (K trucks) = \( N_T \left( \frac{\mu \eta}{K \lambda} \right) K + I_T \left( 1 - \frac{\mu \eta}{K \lambda} \right) K \)

Therefore, EPP of the operation is:

\[
EPP = \frac{N_L - I_L + N_T \rho - I_T / \rho}{\mu \eta \text{CAP}} + \frac{1 + I_T K / \mu \eta \text{CAP}}{\rho}
\]

where \( \eta_T \) and \( (1 - \eta_T) \) are the proportion of time that the trucks spend travelling and idle (waiting and loading). These values can be observed in the field or estimated via the methods such as the simulation and queuing theory. The idling and non-idling emissions of loader and truck \( (I_L \text{ and } N_L) \) and \( (I_T \text{ and } N_T) \) can be estimated by emissions model in Carmichael et al. [45].

Analysing different truck fleet sizes, \( \text{CO}_2 \) per production, and CPP values are plotted versus the fleet size in Figure 1. The trend of EPP diagram for \( \text{CO}_2 \) is similar to that for the other emissions such as nitrogen oxides (\( \text{NO}_x \)), hydrocarbons (HC), carbon monoxide (CO), and particulate matter (PM). Comparing the \( \text{CO}_2 \)/production (upper plot) with the CPP plot (lower plot) shows the optimum fleet size in terms of the unit emissions coinciding with that for the unit cost.

![Figure 1. CO₂ EPP and CPP versus fleet size.](image)

The case study only discusses a single operation, and hence, is not representative of all operations. Therefore, it is important to evaluate the effects of altering the design parameters such as the back-cycle time and service time on the results. To this end, a sensitivity analysis was performed for changing the back-cycle time and service time of this operation. The analyses show that the results obtained remain similar for doubling the service time and back-cycle time, as shown in Figures 2 and 3.
As illustrated, the trend of the EPP and CPP diagrams remains similar when changing the design parameters; this means that the optimum fleet size in terms of the unit cost is the same as that for unit emissions. The queue analysing was conducted in order to evaluate whether the two minima still coincided for a range of $C_1/C_2$, and a range of $I_L/I_T$. To this end, 636 cases with different values of cost and emissions were analysed, as shown in Table 1. As it can be seen, the optimal fleet size may change for different values of $C_1/C_2$ and $I_L/I_T$: however, the minimum CPP configuration coincides with the minimum EPP configuration. Regarding this information, Figure 4 can be used to determine the optimum $K (K^*)$ regions based on the servicing factor ($\rho$) and the ratio of the loader and truck’s idling emissions ($I_L/I_T$). For example, the shaded region shows the area with an optimum truck’s number of 5. Thus, the truck fleet size for the minimum CPP can be chosen using Figure 4.
Table 1. Optimum fleet size (K*) for different ratios of \( C_i/C_2 \) and \( I_T/I_L \).

<table>
<thead>
<tr>
<th>Cost and emission ratio</th>
<th>Service factor (( \rho ))</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>0.497</td>
</tr>
<tr>
<td>( C_i/C_2 = 4 )</td>
<td>3</td>
</tr>
<tr>
<td>( I_T/I_L = 4 )</td>
<td>2</td>
</tr>
<tr>
<td>( C_i/C_2 = 3.5 )</td>
<td>3</td>
</tr>
<tr>
<td>( I_T/I_L = 3.5 )</td>
<td>2</td>
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<td>( C_i/C_2 = 3 )</td>
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<td>( I_T/I_L = 3 )</td>
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<td>( C_i/C_2 = 2.5 )</td>
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<td>( C_i/C_2 = 1.5 )</td>
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<td>( I_T/I_L = 1.5 )</td>
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<tr>
<td>( C_i/C_2 = 1 )</td>
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<td>( I_T/I_L = 1 )</td>
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<tr>
<td>( C_i/C_2 = 0.5 )</td>
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<tr>
<td>( I_T/I_L = 0.5 )</td>
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</table>

Figure 5 demonstrates the optimum truck fleet size regions based on the servicing factor (\( \rho \)) and the ratio of the loader and truck’s cost (\( C_i/C_2 \)). As it can be seen, the optimum regions based on the minimum CPP is similar to that for the minimum EPP. However, there are some inconsistencies that are mostly due to the drawing assumptions (a polynomial function of degree 3 trendline was fitted to the points in the border of area with similar optimum fleet size to define the K* regions).

The analyses show that the optima fleet size based on the minimum unit emissions and unit cost criteria coincides for all the earthmoving, quarrying, and open-cut mining operations. However, a general proof can be provided to confirm it mathematically.
3. General Proof

A general optimisation proof, independent from any specific case study numbers, can be given by using a general $\eta$ function. The linear and non-linear $\eta$ functions were used in this work to prove the hypothesis.

3.1. Linear $\eta$ function

Consider a typical plot of the server utilization (equivalent production) versus the fleet size, as shown in Figure 6. The truck queue times in the initial part of the plot (under-trucked) are small or negligible and the loader is underutilized. When the plot reaches a plateau (over-trucked), the truck queue lengths grow as more trucks are added to the operation and the loader works nearly all the time.

As it can be seen in Figure 6, the analysis can be broken into two scenarios corresponding to the two main parts of the server utilization plot. For each scenario, a linear approximation is made to the utilization plot such that for small fleet sizes (small $K$), $\eta = \alpha_1 + \beta_1 K$, and for the larger fleet sizes (large $K$), $\eta = \alpha_2 + \beta_2 K$, where $\beta_1$ and $\beta_2$ are constants. In order to decrease the number of independent parameters and to facilitate the calculations, it is assumed that $\alpha_1$ is negligible. The values for $\beta_1$ and $\beta_2$ are also substituted with $\alpha$ and $\beta$, respectively. Therefore, the linear approximation to the utilization plot for small fleet sizes (small $K$) is $\eta \approx \alpha K$, and for larger fleet sizes (large $K$), it is $\eta \approx \alpha K + \beta (K - K_s)$, where $K_s$ is the value of $K$ at the point of intersection of these two parts.
Different analyses were conducted to find the $\alpha$ and $\beta$ values, which resulted in the occurrence of $K_s$ near the optimum fleet size. In order to have a more accurate $K_s$, the area under linear $\eta$ functions in under-trucked and transition parts should be equal to the area under the actual server utilization graph in this area. Moreover, $\alpha$ should be a function of the servicing factor $\rho$, as follows:

$$\alpha = -0.39 \rho^2 + 0.81 \rho + 0.013$$

Using the linear $\eta$ functions, the additional CPP and additional EPP were calculated from increasing the fleet size for the under-trucked and over-trucked scenarios. A summary of the results is highlighted in the following sections.

### 3.1.1. Additional cost per production

The slope of the CPP curve or additional CPP, $\hat{N}_c^p$, for increasing the fleet size for the under-trucked and over-trucked scenarios is as follows:

$$\hat{N}_c^p = \left( \frac{\text{Cost}}{\text{Production}} \right)_{K+1} - \left( \frac{\text{Cost}}{\text{Production}} \right)_K$$

$$= \frac{C_1 + (K+1)C_2}{\mu \eta_{k+1} \text{CAP}} - \frac{C_1 + KC_2}{\mu \eta_k \text{CAP}}$$

Therefore,

$$\hat{N}_c^p = \frac{\eta_k (C_1 + (K+1)C_2) - \eta_{k+1} (C_1 + KC_2)}{\mu \eta_{k+1} \eta_k \text{CAP}}$$

Consider the under-trucked and over-trucked cases in turn;

For $K < K_s$ (under-trucked):

$$\hat{N}_c^p = \frac{KC_1 + (K+1)C_2 - (K+1)(C_1 + KC_2)}{\mu \alpha K(K+1) \text{CAP}}$$

(8)

$$\hat{N}_c^p = \frac{-C_1}{\mu \alpha K(K+1) \text{CAP}}$$

The numerator in Equation (8) is negative, while its denominator is always positive; therefore, the value for $\hat{N}_c^p$ is positive in the under-trucked scenario ($\beta$ is smaller than $\alpha$ and $\alpha K_s$), while it is negative in the under-trucked scenario. This means that increasing the fleet size, and accordingly increasing the production, for the under-trucked case, leads to a reduction in CPP in the operations.

For $K > K_s$ (over-trucked):

$$\hat{N}_c^p = \frac{(\alpha - \beta) K_s C_2 - \beta C_1}{\mu \alpha K_s (\alpha K_s + \beta (K+1-K_s)) (\alpha K_s + \beta (K-K_s))}$$

(9)
The value for $\beta$ is small compared to $\alpha$ and $\alpha K_s$; therefore $\hat{N}_{E}^p$ is positive. Thus larger fleet sizes result in more CPP in the under-trucked case.

The value for $\hat{N}_{E}^p$ is positive in the over-trucked scenario, while it is negative in the under-trucked scenario. Thus the greater fleet sizes result in a reduced CPP in the under-trucked scenario and an increased CPP in the over-trucked scenario.

### 3.1.2. Additional Emissions per production

The slope of the EPP curve or the additional EPP, $\hat{N}_{E}^p$, for increasing the fleet size for under-trucked and over-trucked scenarios is as follows:

$$\hat{N}_{E}^p = \left(\frac{\text{Emissions}}{\text{Production}}\right)_{K+1} - \left(\frac{\text{Emissions}}{\text{Production}}\right)_{K} \quad (10)$$

$$\hat{N}_{E}^p = \left(\frac{I_L + (K+1)I_T}{\mu(\alpha K_S + \beta(K+1-K_S))\text{CAP}}\right) - \left(\frac{I_L + K I_T}{\mu(\alpha K_S + \beta(K-K_S))\text{CAP}}\right)$$

$$\hat{N}_{E}^p = \frac{(\alpha - \beta) K_S I_T - \beta I_L}{\mu\text{CAP}(\alpha K_S + \beta(K+1-K_S))(\alpha K_S + \beta(K-K_S))} \quad (12)$$

The value for $\beta$ is small compared to $\alpha$ and $\alpha K_s$; therefore, $\hat{N}_{E}^p$ is positive. Thus larger fleet sizes lead to a more EPP.

Similar to the additional CPP, $\hat{N}_{E}^p$ is negative in the under-trucked scenario, while it is positive in the over-trucked scenario. Thus the greater fleet sizes lead to fewer EPP in the under-trucked scenario and more EPP in the over-trucked scenario.

Therefore, additional cost and EPP from increasing the truck fleet size are negative in the under-trucked part of the server utilisation, while they are positive in the over-trucked part. It means that the minimum EPP and the minimum CPP take place approximately in the intersection of the two linear $\eta$ functions, namely $K_s$. However, $K_s$ is not placed on the $\eta$ curve, and it can be a source of error in the estimations.

### 3.2. Non-linear $\eta$ function

Taking a derivative of CPP and EPP with respect to $K$ and setting to zero can show the coincidence of the $K$ values for a minimum CPP and minimum EPP. A $\eta$ function in terms of $K$ is required, which is applicable for a range of $\eta$ in either sides of the optimum point. Due to the variation and different trends of the $\eta$ curves, as shown in Figure 7, it is impossible to have a certain $\eta$ function that fits all $\eta$ values. Therefore, breaking $\eta$ and $\rho$ is necessary to get more accurate equations for different areas.

![Figure 7. Different $\eta$ functions versus fleet size.](image)

A piecewise $\eta$ function can be used to obtain more accurate values. However, the optima mostly take place in the transition part from operation being under-trucked to over-trucked. Developing $\eta$
functions for other regions is of secondary importance and are discussed in this paper. A piecewise non-linear \( \eta \) function for (i) small \( \eta \), (ii) transition part, and (iii) large \( \eta \) can be used to overcome the shortcomings of the linear \( \eta \) function. The function needs to be a good fit near the region where the optima lie; elsewhere, the fit is not as important, and the goodness of fit can be sacrificed. Therefore, the \( \eta \) functions for the crucial region near the optima are discussed in this section.

The finite source queue server utilisation was used in order to find the best fit mathematical function to the \( \eta \) curves. The function gives \( \eta \) in terms of the truck fleet size (\( K \)) and servicing factor (\( \rho \)), \( \eta = f(K, \rho) \), where \( \rho \) is defined as \( \lambda / \mu \). The servicing factor takes place at specific values for different earthmoving configurations. The best fit \( \eta \) functions are derived into the four different regions of the servicing factor (\( 0.03 \leq \rho \leq 0.06 \), \( 0.06 < \rho \leq 0.13 \), \( 0.13 < \rho \leq 0.25 \), \( 0.25 < \rho \leq 0.72 \)) to get more accurate results.

For \( 0.03 \leq \rho \leq 0.06 \),
\[ \eta = (2.5\rho^2 - 0.333\rho + 0.0047)K^2 + (-120\rho^2 + 11.676\rho - 0.016)K + (54.509\rho - 4.863) \]

For \( 0.06 < \rho \leq 0.13 \),
\[ \eta = (-0.89\rho^2 + 0.031\rho - 0.0054)K^2 + (11.398\rho^2 - 0.936\rho + 0.273)K + (-138.7\rho^2 + 36.111\rho - 3.186) \]

For \( 0.13 < \rho \leq 0.25 \),
\[ \eta = (-0.25\rho^2 + 0.032\rho - 0.015)K^2 + (2.99\rho^2 - 1.028\rho + 0.397)K + (-6.93\rho^2 + 6.558\rho - 1.373) \]

For \( 0.25 < \rho \leq 0.72 \),
\[ \eta = (0.203\rho^2 - 0.281\rho + 0.039)K^2 + (-1.2\rho^2 + 1.536\rho)K + (0.38\rho - 0.273) \]

\( \eta \) curves from these functions are compared with those from the queue theory for \( \rho = 0.09 \), \( 0.15 \) and \( 0.30 \) in Figure 8. The results of these two approaches provide similar results, which show that the best fit \( \eta \) functions can be employed for calculating the productivity of operations, especially in the critical parts.

Table 2 compares several \( \eta \) values from these functions with those from the queue theory. As it can be seen, the differences between the results from these two methods are less than 3%. This comparison shows the accuracy of the best fit functions; hence, they can be used in correspondence with the queue theory for the analysis.

![Figure 8. \( \eta \) curves from functions and queue theory.](image)
Substituting the best fit $\eta$ functions into the CPP expression and setting the first derivative of this expression with respect to $K$ equal to zero gives the stationary value ($\hat{K}$), which is the minima, as the second derivative of the expression is always positive. The nearest integer number to this value is the optimum number of trucks in terms of the unit cost. The same procedure can be applied for EPP, which gives the minimum EPP fleet size. Comparing the results obtained shows that the optimum number of trucks in terms of the unit cost coincides with that for the unit emissions.

### 3.2.1. Cost per production

For any given operation, $\rho$ and $\mu$ are constants. The CPP numerator is a linear function of $K$, and its denominator is a function of $\eta$. As discussed, the $\eta$ functions from the queue theory can be converted to quadratic functions of $K$ as follows:

$$\eta = A K^2 + B K + C$$  \hspace{1cm} (14)

where $A$, $B$, and $C$ are the functions of $\rho$ that take different values for the nominated boundaries of $\rho$. These parameters were used instead of the actual functions to facilitate the estimations.

In order to find the minimum CPP fleet size, the derivative of CPP with respect to $K$ is set to zero as follows:

$$\frac{d\text{CPP}}{dK} = \frac{2AC_2K^2 + 2AC_1K + (BC_1 - CC_2)}{\mu\text{CAP}(AK^2 + BK + C)^3} = 0$$  \hspace{1cm} (15)

Therefore,

$$AC_2K^2 + 2AC_1K + (BC_1 - CC_2) = 0$$

The values for $\hat{K}$ satisfying this equation are the stationary values for CPP.

$$\hat{K} = \frac{C_1 \pm \sqrt{(C_1)^2 - (B/A)(C_1)(C_2) + (C/A)(C_2)^2}}{-C_2}$$  \hspace{1cm} (16)

The sign of the second derivative is required to be examined to establish whether such values are minima, maxima or points of inflection.

### 3.2.2. Emissions per production

In order to find the $K$ values for the minimum EPP (Eq. 4), the derivative of this expression with respect to $K$ is set to zero as follows:

$$\frac{d\text{EPP}}{dK} = \frac{AI_1K^2 + 2AI_1K + (Bl_t - Cl_t)}{\mu\text{CAP}(AK^2 + BK + C)^3} = 0$$  \hspace{1cm} (18)

Therefore,

$$AI_1K^2 + 2AI_1K + (Bl_t - Cl_t) = 0$$

$$\hat{K} = \frac{I_1 \pm \sqrt{(I_1)^2 - (B/A)(I_1)(I_2) + (C/A)(I_2)^2}}{-I_2}$$  \hspace{1cm} (19)

This equation is similar to that for CPP, except that $C_1$ and $C_2$ are replaced with $I_1$ and $I_2$, respectively. As discussed in Section 4.2.1., the stationary values for $\hat{K}$ are the minima; thus by substituting $A$, $B$, and $C$ in this equation, the $\hat{K}$ values for the minimum EPP can be calculated for different operations.

Table 3 compares the $\hat{K}$ values for the minimum EPP with those from minimum CPP for $\rho$ values equal to 0.055, 0.09, 0.15, 0.26, and 0.5. The optimum numbers of trucks for the minimum CPP and minimum EPP are also compared in Table 3.

---

**Table 2. $\eta$ values from functions and queue theory.**

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$K$</th>
<th>H</th>
<th>Best fit functions</th>
<th>Queue analysis</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.05</td>
<td>18</td>
<td>0.8360</td>
<td>0.8179</td>
<td></td>
</tr>
<tr>
<td>0.10</td>
<td>18</td>
<td>0.8591</td>
<td>0.8441</td>
<td></td>
</tr>
<tr>
<td>0.20</td>
<td>7</td>
<td>0.9270</td>
<td>0.9397</td>
<td></td>
</tr>
<tr>
<td>0.30</td>
<td>5</td>
<td>0.9292</td>
<td>0.9304</td>
<td></td>
</tr>
<tr>
<td>0.50</td>
<td>4</td>
<td>0.9770</td>
<td>0.9524</td>
<td></td>
</tr>
</tbody>
</table>
Table 3. Optimum number of trucks for the minimum CPP and EPP.

<table>
<thead>
<tr>
<th>Region</th>
<th>ρ</th>
<th>K̂</th>
<th>K*= Optimum K</th>
</tr>
</thead>
<tbody>
<tr>
<td>ρ ≤ 0.06</td>
<td>0.055</td>
<td>17.71</td>
<td>17.69</td>
</tr>
<tr>
<td>0.06 &lt; ρ ≤ 0.11</td>
<td>0.09</td>
<td>10.63</td>
<td>10.59</td>
</tr>
<tr>
<td>0.11 &lt; ρ ≤ 0.25</td>
<td>0.15</td>
<td>6.27</td>
<td>6.22</td>
</tr>
<tr>
<td>0.25 &lt; ρ ≤ 0.72</td>
<td>0.26</td>
<td>3.76</td>
<td>3.66</td>
</tr>
<tr>
<td>0.25 &lt; ρ ≤ 0.72</td>
<td>0.5</td>
<td>2.22</td>
<td>2.13</td>
</tr>
</tbody>
</table>

K̂ = Stationary points

According to Table 3, the optimum numbers of trucks for CPP and EPP are the same. However, there are some contradictions. For example, assume an earthmoving operation with ρ = 0.21 (1/λ = 203, 1/μ = 960), and CT= $150; other characteristics are the same as the case study. Using the relevant η equation, the K̂ values for the minimum CPP and EPP are 4.56 and 4.46, respectively. There is a small difference between these values but the nearest integer number to the former is 5, while it is 4 for the latter.

Table 4 presents the values for CPP and CO₂/production for a variety of fleet sizes in this example. As it can be seen, the difference between EPP for the 4 and 5 trucks is too small, while it is more significant for the 3 and 6 trucks. Similar results exist for EPP. Therefore, both values of K can be accepted as the optimum number of trucks.

Table 4. CO₂ per production and CPP for different fleet sizes.

<table>
<thead>
<tr>
<th>K</th>
<th>CO₂/production (g/m³)</th>
<th>CPP</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>908.63</td>
<td>2.304</td>
</tr>
<tr>
<td>2</td>
<td>856.66</td>
<td>1.844</td>
</tr>
<tr>
<td>3</td>
<td>842.04</td>
<td>1.711</td>
</tr>
<tr>
<td>4*</td>
<td>837.41</td>
<td>1.666</td>
</tr>
<tr>
<td>5*</td>
<td>837.45</td>
<td>1.663</td>
</tr>
<tr>
<td>6</td>
<td>846.21</td>
<td>1.723</td>
</tr>
<tr>
<td>7</td>
<td>872.82</td>
<td>1.897</td>
</tr>
<tr>
<td>8</td>
<td>908.63</td>
<td>2.087</td>
</tr>
</tbody>
</table>

As illustrated, the K̂ values in terms of cost and emission match in the most regions of 0.03 ≤ ρ ≤ 0.72. It can be said that the minimum CPP fleet size coincides with that for the minimum EPP. However, the real values are compared in Figure 9, while the natural numbers should be used for the truck fleet size (for example, using the best fit η function method in an earthmoving operation with ρ = 0.4 (the other parameters are kept the same as the case study), the optima are about 3 and 2 trucks in terms of cost and emissions, respectively, while the queue theory gives the optimum of 3 for both of them). Therefore, the optima might be different for the minimum unit emissions and minimum unit cost, as shown in Figure 10. The CPP and EPP values are too close for different optima.
Table 5 compares the optimal fleet sizes ($K^*$ values) for the minimum CPP and minimum EPP. The $\rho$ values are selected from four different regions of $\rho$. As it can be seen, the optima are the same for CPP and EPP.

This paper addresses the operational costs and emissions for the existing equipment and earthmoving set-up. However, the parameters such as the introduction of new technology or modifying existing equipment to bring about lower emissions, equipment age, operator ability, reconfigurations operations to get absolute minimum emissions (such as the super-elevating haul road bends) were not studied in this research work. However, based on the analysis given in this thesis, the coincident unit cost and unit emission solutions will be reached irrespective of the equipment age, operator ability, reconfigurations or any equipment technology modifications; all such practices do is to increase/decrease the absolute emissions but do not alter the coincident result; they can be confirmed in the future studies. The operator skill and engine performance will affect the fuel use, and hence, emissions. If the purpose of the study is to focus on emissions, it is suggested that, rather than measuring the fuel use as done in this paper and estimating the emissions based on that, portable emission measurement systems (PEMS) could be equipped to the trucks to measure the actual emissions.

<table>
<thead>
<tr>
<th>$\rho$</th>
<th>$K_\hat{}$</th>
<th>$K^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPP</td>
<td>EPP</td>
<td>CPP</td>
</tr>
<tr>
<td>0.06</td>
<td>16.10</td>
<td>16</td>
</tr>
<tr>
<td>0.11</td>
<td>8.64</td>
<td>9</td>
</tr>
<tr>
<td>0.22</td>
<td>4.27</td>
<td>4</td>
</tr>
<tr>
<td>0.33</td>
<td>2.97</td>
<td>3</td>
</tr>
<tr>
<td>0.66</td>
<td>1.90</td>
<td>2</td>
</tr>
</tbody>
</table>

$K_\hat{}$ : Value of $K$ satisfying the first derivative of CPP and EPP equal to zero (stationary value)

$K^*$ : Optimum $K$

4. Conclusions

In this paper, we discussed the optimum cost and emissions units in the earthmoving operations using the linear and non-linear $\eta$ functions in order to determine whether the optimum fleet size based on emissions coincides with that for cost. The finite source queuing theory was used for this analysis, and a model was developed to estimate the idle and non-idle emissions. A case study of the earthmoving operations was evaluated for the effects of varying truck fleet sizes on the cost and EPP. The sensitivity analysis was undertaken by doubling the loading and back-cycle times to demonstrate that the optimum fleet size in terms of the minimum unit cost coincides with that for the minimum unit emissions irrespective of the design parameters.

The undertaken linear server utilizations analyses for the under-trucked and over-trucked scenarios show that increasing the fleet size decreases the cost and EPP in the under-trucked case and increases the cost and EPP in the over-trucked case. It means that there is an optimal truck fleet size based on the unit emissions and the unit cost in the transition between an operation being under-
trucked and over-trucked. The optima take place approximately in the intersection of the two linear \( \eta \) functions. Finally, the undertaken analytical analysis using the non-linear \( \eta \) functions showed that the optimum number of trucks in terms of emission coincided with that in terms of cost, independent of any specific case study numbers.

References


[37] EPA. Clean construction USA. U.S. Environmental Protection Agency; 2005.


تعیین تعداد بهینه تجهیزات برداشت مواد معدنی با حداقل کردن میزان آلایندگی و هزینه

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چکیده:
غالباً طراحی تجهیزات برداشت مواد معدنی ناشی از فناوری و تکنولوژی آینده بر محیط زیست، نلش عمومی بر کاهش میزان آلایندگی ناشی از تغییرات طبیعی از دست نمی‌دهد. استراتژی‌های مدیریتی مختلفی برای کاهش میزان آلایندگی می‌توان در نظر گرفت، اما در این مبان روشنی که باعث کاهش میزان آلایندگی عملیاتی نیز شود، در آینده قرار می‌گیرد. هدف این مقاله بررسی آلایندگی آلایندگی مربوط به تجهیزات برداشت مواد معدنی با دیدگاه عملیاتی می‌باشد. در این تحقیق از تئوری، شرایط عملیاتی و شرایط موردی میزان تعداد بهینه ماشین آلایندگی بر اساس آلایندگی تهیه شده است. این تحقیق از کاربرد آزمون‌های جدید، توانایی تولید و بهره‌وری بهره‌وری تهیه شده است. در این تحقیق از تولید و بهره‌وری بهره‌وری تهیه شده است. 

کلمات کلیدی: برداشت مواد معدنی، عملیات لودر - کامیون، مدیرکاری سطحی، تعداد بهینه کامیون، آلایندگی، هزینه.