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## Evaluation of Underground Mineable Reserve in Presence of Grade and Commodity Price Uncertainties

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### Abstract

The uncertainty-based mine evaluation and optimization have been regarded as a critical issue. However, it has received less attention in the underground mines than in the open-pit mines due to the diversity of the underground mining methods, and the underground mining parameters' complexity. The grade and commodity price uncertainties play essential roles in mining projects. Mine planning by not incorporating these uncertainties is accompanied by risks. The evaluation and risk assessment of the mine plans is possible through evaluating the mineable reserve in the presence of such uncertainties. In the present work, we evaluate the effects of grade and commodity price uncertainties on the underground mining stope optimization and the resultant mineable reserve. In this regard, the stope boundary is studied both deterministically and stochastically in the presence of the grade and price uncertainties. For this purpose, in this work, we implement the conditional simulation in order to generate equally probable ore reserve models. Furthermore, we optimize the stope boundary using the floating-stope algorithm in each realization. Several decision support criteria including the 'mineable reserve,' 'metal-content,' 'profit,' and 'value-at-risk' are defined to assist the decision-maker in uncertain conditions. Finally, a procedure is defined in order to consider two types of uncertainty sources simultaneously in underground mining. It will guide the decision-maker toward the most appropriate stope boundary that best fits the mining company's requirements. The procedure is implemented in a bauxite mine, and the optimal stope boundary is determined concerning the different criteria.

## 1. Introduction

The mining methods are divided into the surface and underground methods. Generally, the use of these methods depends on the depth of the mineral. Underground mining is appropriate for deep deposits in environmentally sensitive areas [1]. Optimization is considered essential for both the surface and underground mine design and production scheduling [2].

A proper stope design plays a significant role in the profit and safety of the operation. It requires several data including the ore model and some geotechnical data. An ore model is usually obtained by estimation or simulation using the geostatistical tools. The geotechnical condition

controls the hanging wall and footwall angles, stope dimensions, *in situ* stress tensor, rock strength, and local geological structures. Deciding the stope's size and location will affect the maximum profit. The number of algorithms in the pit limit optimization exceeds the number in the underground methods. The true optimum solution is guaranteed for optimizing the pit limit, and several computer packages are available for the industry. However, only a small number of algorithms have been developed for optimizing the ultimate stope boundaries in underground mining [3].

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Various approaches have been suggested for the stope optimization [3-5]. Dynamic programming [6] and the branch and bound technique [7] have been utilized in order to optimize a stope in 2D problems. Also, regarding the stope optimization in 3D problems, the mathematical morphology tools [8, 9], floating-stope [10], maximum value neighborhood method [11], and octree division [12] have been implemented. Further, Manchuk and Deutsch (2008) have presented a simulated annealing-based algorithm [13], and Bai *et al.* (2013) have developed a stope optimizer based on graph theory [14].

The risk is an important issue for mine evaluation, and it is available in all facets of mining and is categorized into the technical, financial, and environmental factors. All the uncertainty sources should be considered in the feasibility study of mining projects. The geological uncertainty is regarded as one of the main technical uncertainties in mining, known as a primary risk source. It is recognized as a significant factor in mining failures. In some research works, the geological uncertainty modeling has been obtained using conditional simulation [15, 16]. The economic uncertainties are significant in mining, and commodity price uncertainty is the primary uncertainty source along with the mining operations [17]. The unpredictability of the raw mineral prices is considerably more severe than that of the other industrial products. For example, as illustrated in Figure 1, the aluminum price has been highly volatile during 2000-20. Therefore, the price uncertainty plays a significant role in achieving the production plan's monetary goals [18]. The researchers have recently studied the economic uncertainties such as a combination of the commodity price and operational cost uncertainties [17] or the commodity price uncertainty and exchange rate uncertainty [19].



**Figure 1. Fluctuations in aluminum price (\$/t) during 2000-20 [20].**

In designing the underground mines, evaluation and optimization have had less applications than the open-pit mines due to the diversity of the underground mining methods. All these approaches have failed to consider the uncertainty, and accordingly, have assumed the inputs as certain. A limited number of studies have reported integrating simulated ore bodies and grade risk models through the conventional optimizers. In this regard, some geological risk-based approaches have been introduced for optimizing the stope layout [21, 22], paving the way for developing a risk-based underground mine design. Recently, a method has been introduced for evaluating the block-cave mine production scheduling in the presence of delays from hang-ups and grade uncertainty [23]. More recently, two significant studies have been conducted in order to investigate the uncertainty in the underground mines. In one of them, the sequential Gaussian conditional simulation has been applied to design an underground mine of Iran under the grade uncertainty [24]. In the other one, the dilution risk in underground metal mines has been investigated [25]. However, the current studies have focused more on the grade uncertainty. Thus, the present work aimed to evaluate the effects of the grade and price uncertainties simultaneously on the stope optimization and underground mine evaluation.

## 2. Materials and methods

In this work, the floating-stope algorithm as the stope boundary optimizer and the conditional simulation as the probable reserve generator were applied. The floating-stope is a technique to determine the optimal boundary for the mineable reserve. The floating-stope approach's general concept was raised in 1995 as a heuristic approach, compared to the moving cone method for the pit limit optimization. Floating stope is taken from floating a minimum stope size through the ore body in order to evaluate the stope grades for any stope position. Accordingly, two envelopes are created from this process including the maximum envelope as the union of all possible economic stope positions and the minimum envelope as the union of all the best grade stope positions for every ore block in the ore body. The envelopes provide a limit for the final stope positions, and it is recommended that the minimum envelope be the best option for further analysis. This algorithm is an underground boundary optimizer available in a commercial software [26].

The geological uncertainty modeling is obtained through using conditional simulation. It generates detailed models for an orebody that considers the orebody's spatial and statistical specifications. Based on the conditional simulation, the simulated models can be developed at very tight-spaced geographical positions by covering the whole ore body, in addition to the sampled section. The simulated models regenerate the real variability (histogram) and spatial continuity (variogram) of interest attributes. They are used as a measure of uncertainty and variability related to the evaluation [27].

### 3. Model construction

In this work, the Underground Stope Boundary (USB) is evaluated in 2 different conditions. First, USB is evaluated in the deterministic conditions, and then it is studied in the presence of grade and price uncertainties. These uncertainties will influence the total income of the mine. However, it should be noted that the grade uncertainty is an intrinsic uncertainty, while the price is an extrinsic uncertainty and parametric analysis is the industry standard for price deviations. Moreover, decision-making in the deterministic conditions is somehow straight forward. However, several criteria are defined to assist the decision-maker in uncertain situations. For this purpose, "mineable reserve," "metal-content," "profit," and "value-at-risk" were considered as the decision support criteria. For the sake of comparison, the stope boundary and the resulting mineable reserve is evaluated in 3 distinct conditions. Section 3.1 explains the steps of deterministic stope boundary optimization. Section 3.2 explains the stope boundary optimization in the presence of the grade uncertainty. Finally, Section 3.3 explains the stope boundary optimization in the presence of the price uncertainty.

#### 3.1. Stope optimization without considering uncertainty

The floating-stope is a heuristic size optimization algorithm to determine the optimal (boundary) limit of an ore reserve for the aim of underground mining [28]. The general concept of floating-stope, raised in 1995, is similar to the moving cone algorithm for the pit limit optimization. The floating-stope is taken from floating a minimum stope size through the ore body in order to evaluate the stope grades for any stope position. Therefore, three factors are required for the implementation of the floating-stope. For this purpose, based on the economic estimations, a cut-off grade is calculated

in order to determine whether a block is an ore or a waste. This process also requires an initial stope shape or geometry determined based on the required minimum stope dimension.

Additionally, a target grade (referred to as head grade) is specified in order to evaluate the generated stopes. In each iteration, the algorithm generates several stope options for a given ore block, and checks for the average grade of material inside each stope. If the average grade of blocks inside the stope is equal or greater than the head grade, they are flagged as the mineable blocks. Finally, the mineable blocks' union will generate the underground mine limits (referred to as the mine envelope). The algorithm follows two strategies for the determination of mine envelopes. The first one is called the 'inner envelope' or 'minimum envelope,' and it is constructed by adding the blocks having a grade more than the cut-off grade. The second envelope, called the 'outer envelope' or 'maximum envelope,' is built concerning all the possible stope union for each ore block. Accordingly, two envelopes are created from this process including the maximum envelope, as the union of all the possible economic stopes, and the minimum envelope, as the union of all best grade stope positions for every ore block in the ore body. These envelopes provide a limit for the final stope positions, and it is recommended that the final stope design fits the minimum envelope as close as possible. In order to determine the stope boundary deterministically, the following steps are required:

**Step 1:** Reserve estimation using the exploration data.

**Step 2:** Stope optimization using the floating-stope algorithm.

**Step 3:** Determine the minable reserve (Equation 1).

$$R_m = \sum_{i \in B} R_i x_i \quad (1)$$

where  $R_m$  is the mineable reserve,  $R_i$  is the tonnage of reserve in block  $i$  (defined in step 1),  $B$  is the set of blocks in the block model, and  $x_i$  is a decision variable (defined in step 2) that determines whether block  $i$  should be mined (assumes the value 1) or not (assumes the value 0).

**Step 4:** Determination of the metal-content (Equation 2).

$$M = \sum_{i \in B} R_i g_i x_i \quad (2)$$

where  $M$  is the metal content and  $g_i$  is the grade of block  $i$  (defined in step 1).

**Step 5:** Determination of profit of the optimized stopes (Equation 3).

$$P = \sum_{i \in B} (vR_i g_i r - c)x_i \tag{3}$$

where  $P$  is the profit,  $v$  is the metal price,  $r$  is the metal recovery, and  $c$  is the metal extraction cost.

### 3.2. Stope optimization by considering grade uncertainty

In this section, the grade uncertainty and its effect on the stope boundary is evaluated. For this purpose, the geological uncertainty modeling is obtained using the conditional simulation. The simulated models regenerate the real variability (histogram) and the spatial continuity (variogram) of the attributes of interest. They are used in order to solve the uncertainty and variability related to the evaluation [29, 30]. The following steps are required to determine the stope boundary in the presence of the grade uncertainty.

**Step 1:** Generate the equally probable reserve model realization by applying the conditional simulation technique. In this step, a number of reserve realizations will be generated, and they are stored in set  $E$ .

**Step 2:** Optimize the stope using the floating-stope algorithm in each realization. In this step, a number of  $n$  stope boundaries will be determined. These boundaries are referred to as the scenarios that we are going to select the best one, and they are treated in set  $S$ .

**Step 3:** Evaluate each scenario with respect to the available reserve realizations in  $E$ , and determine the minable reserve in each scenario. Then select the scenario with the maximum expected minable reserve (Eqs. 4 and 5).

$$R_{ms} = EV \left( \sum_{i \in B} \sum_{e \in E} R_{ie} x_{is} \right) \tag{4}$$

$$Z_1 = Max (R_{ms} \forall s \in S) \tag{5}$$

where  $Z_1$  is the best scenario with respect to the goal of maximum expected minable reserve,  $EV(\ )$  refers to the expected value,  $R_{ms}$  is the expected mineable reserve of scenario  $s$ ,  $x_{is}$  is the decision variable that determines whether block  $i$  should be mined or not under scenario  $s$ ,  $R_{ie}$  is the tonnage of reserve in block  $i$  within the realization  $e$ ,  $S$  is the set of stope scenarios, and  $E$  is the set of available reserve realizations.

**Step 4:** Determine the metal content for each scenario, and select the scenario with respect to different strategies (Eqs. 6-9),

$$M_s = \sum_{i \in B} R_{ie} g_{ie} x_{is} \tag{6}$$

$$Z_2 = Max (Max\{M_s, \forall e \in E\} \forall s \in S) \tag{7}$$

$$Z_2 = Max (Min\{M_s, \forall e \in E\} \forall s \in S) \tag{8}$$

$$Z_2 = Max (Ave\{M_s, \forall e \in E\} \forall s \in S) \tag{9}$$

where  $M_s$  is the metal content of scenario  $s$ ,  $Z_2$  is the best scenario with respect to the goal of maximum metal-content.  $Z_2$  that can be calculated with respect to different strategies (Eqs. 7-9),  $g_{ie}$  is the grade of block  $i$  in simulation  $e$ , and  $E$  is the set of reserve realizations.

According to Eqs. 7-9,  $Z_2$  can be calculated concerning different strategies. The Max-Max strategy (Equation 7) yields the ‘best of the best’ outcome. It is often referred to as an aggressive or optimistic strategy. The Max-Min strategy (Equation 8) yields the ‘best of the worst’ outcome. It is also referred to as a pessimistic or conservative strategy. The Max-Ave strategy (Equation 9) yields the ‘best of the average’ outcome. Decision-making in the uncertain condition is somehow a subjective task, and each decision-maker may choose a strategy based on his risk-taking behavior.

**Step 5:** Determine the project profit in each scenario, and select the scenario with the maximum profit (Eqs. 10-13).

$$P_s = \sum_{i \in B} (vR_{ie} g_{ie} r - c)x_{is} \tag{10}$$

$$Z_3 = Max (Max\{P_s, \forall e \in E\} \forall s \in S) \tag{11}$$

$$Z_3 = Max (Min\{P_s, \forall e \in E\} \forall s \in S) \tag{12}$$

$$Z_3 = Max (Ave\{P_s, \forall e \in E\} \forall s \in S) \tag{13}$$

where  $P_s$  is the profit of scenario  $s$  and  $Z_3$  is the best scenario with respect to the goal of maximum profit.  $Z_3$  can be calculated with respect to different strategies similar to the metal-content (Eqs. 11-13), and each decision-maker may choose a strategy based on his/her risk-taking behavior,  $v$  is the metal price,  $r$  is the metal recovery that is assumed to be constant, and  $c$  is the metal extraction cost.

**Step 6:** Determine the Value-at-Risk (VaR) for profit, and select the lowest risk scenario (Equation 14).

$$VaR_\alpha(P) = inf\{P' | Pr(P > P') > \alpha\} \tag{14}$$

where  $\alpha \in (0,1)$  is the confidence level,  $VaR_\alpha(P)$  is the value at risk of  $P$  (i.e. profit) at a confidence level of  $\alpha$ ,  $Pr(\ )$  is the probability, and  $inf(A)$  refers to the greatest number that is less than or equal to all elements of set  $A$ .

VaR is a well-known risk management tool, which measures the worst expected damage in the normal conditions, and it is computed for a particular time period at a certain confidence level. By definition, VaR(q) finds the amount one can lose over a pre-set horizon with a probability of q% [30].

### 3.3. Stope optimization considering both grade and prices uncertainties

In this section, the combined effect of the price and grade uncertainties on the stope boundary is evaluated. For this purpose, the geological uncertainty is obtained using the conditional simulation. There are various methods available for generating the price scenarios such as the bootstrapping, geometric Brownian motion, and mean reverting. In this work, the bootstrapping method was applied in order to model the price variations. Bootstrapping is a kind of sampling method. In order to predict the future prices by bootstrapping, the individual returns should be calculated for each period. The main advantage of this method is that it does not require any assumptions about the distribution of returns. According to bootstrapping the price forecasts, the minimum and maximum values of price were considered during the last years. These prices were used in order to evaluate the ore reserve value.

**Step 1:** Generate the price scenarios. Put the price scenarios in set  $K$ .

**Step 2:** Calculate the expected profit of each scenario considering the price variations (Equation 15).

$$P_{sk} = EV \left( \sum_{i \in B} \sum_{e \in E} \sum_{k \in K} (v_k R_{ie} g_{ie} r - c) x_{is} \right) \quad (15)$$

where  $P_{sk}$  is the expected profit of scenario  $s$  in the case of different price and ore reserve realizations, and  $K$  is the set of prices.

**Step 3:** Inspection of different scenarios in different price ranges (Equation 16).

$$Z_4 = \text{Max}(P_{sk}, s \in S) \quad (16)$$

**Step 4:** Determine VaR of the profit, and select the lowest risk scenario (Equation 17).

$$\text{VaR}_\alpha(P_{sk}) = \inf\{P' | \text{Pr}(P_{sk} > P') > \alpha\} \quad (17)$$

## 4. Results and discussion

A flow diagram of the presented underground mine evaluation model is introduced in this section for a better understanding. It was used for the Golbini bauxite mine of Iran. In the flow diagram presented in Figure 2, an attempt was made in order to summarize the mineable reserves assessment process in the presence of the mentioned uncertainties. This mine is planned to operate using the “cut and fill” mining method. According to the initial assessments, the local mining cost is about \$28.5 per ton of bauxite ore, and the selling price is \$300 per ton of alumina. Approximately 3 tons of bauxite are required to produce 1 ton of alumina, and to produce 1 ton of aluminum, 2 tons of alumina is required. In this section, the stope optimization and minable reserve evaluation are conducted according to the steps explained in Section 2.

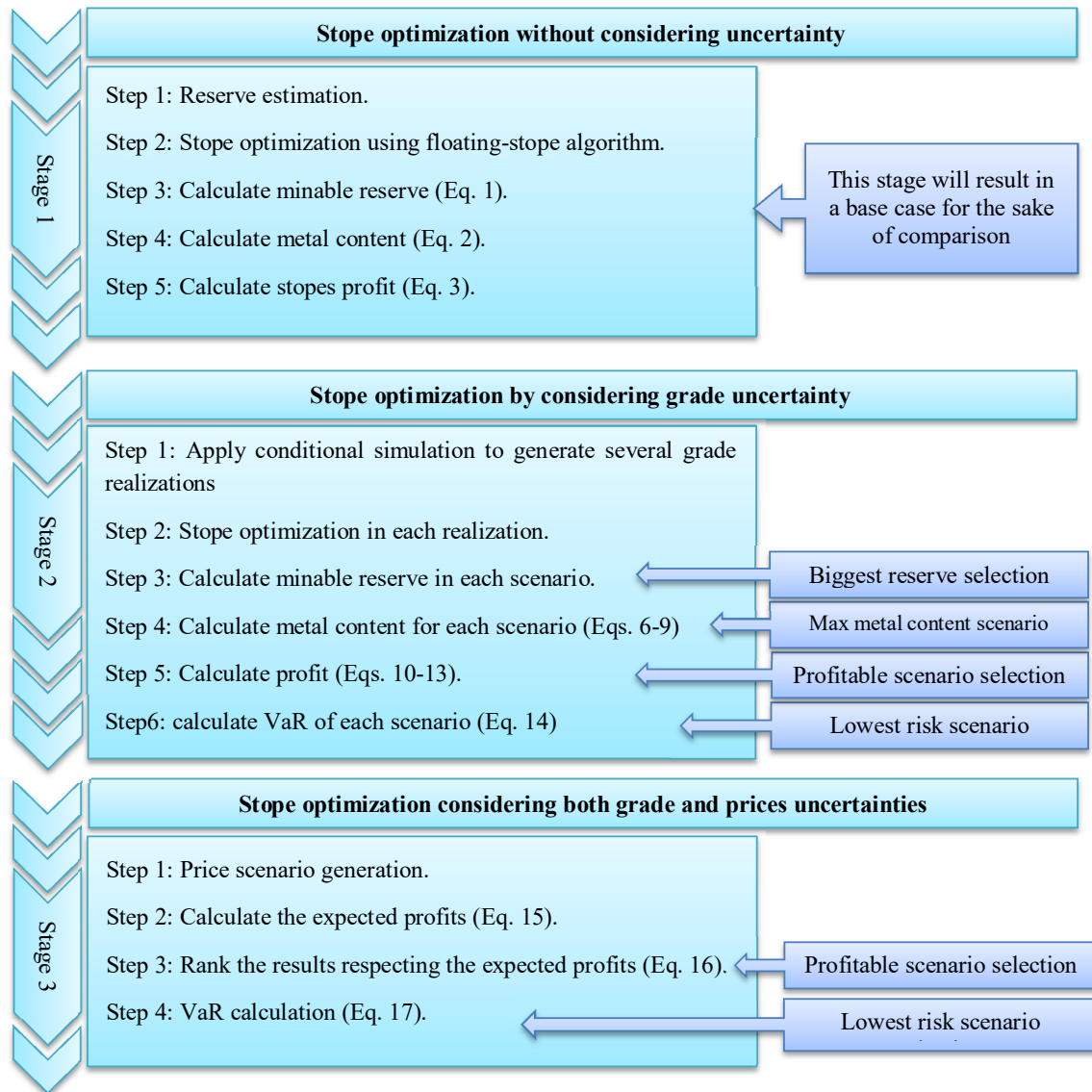


Figure 2. A flow diagram of the presented underground mine evaluation model.

#### 4.1. Golbini mine stope optimization without considering uncertainty

**Step 1:** The geologic model is generated on the basis of exploratory boreholes, and the grades are estimated by the Kriging method. According to the model, the mine reserve is estimated at about 3.5 Mt with average grade of  $\text{Al}_2\text{O}_3$  at 46.3% and  $\text{SiO}_2$  at 12.8%. Figure 3 shows the block model of the deposit. The silica content is far below the maximum acceptable level, and therefore, its deviations and its effects on the mine design are not considered here.

**Step 2:** Using the floating-stope algorithm, the optimal stope layout is determined for the deposit (Figure 4). In this case, the minimum stope

dimension is  $10 \times 5 \times 1$  cubic meter, the cut-off grade of  $\text{Al}_2\text{O}_3$  is 40%, and the head grade is assumed to be 45% according to the run of mine requirements.

**Step 3:** Based on the resulting stope layout, and using Equation 1, the mineable reserve within the stope boundary is 1.5 Mt.

**Step 4:** Using Equation 2, the metal-content and the average grade are calculated. The metal contained within the stope boundary is about 720,000 tons.

**Step 5:** Using Equation 3 and based on the estimated mineable reserve, the metal-content, the bauxite ore price, and the mining costs, the profit of the operation is about 8.9 M\$.

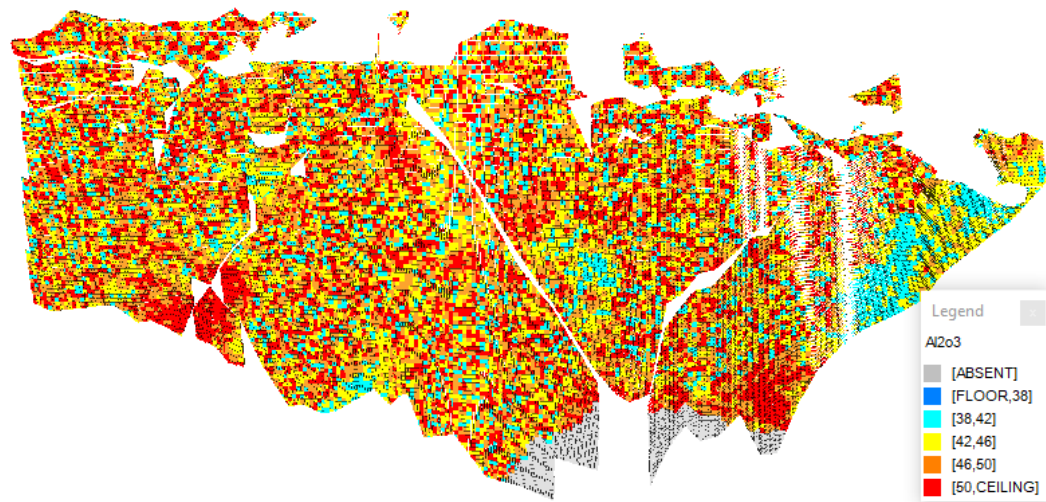


Figure 3. Block model of the deposit.



Figure 4. Deterministic stope layout.

#### 4.2. Golbini mine stope optimization considering grade uncertainty

**Step 1:** The block model consists of 266,937 blocks with the dimensions of  $1 \times 1 \times 1$  cubic meter. Using the conditional simulation, and the exploration boreholes, 10 simulated reserves were generated. Table 1 shows the average grade of each scenario.

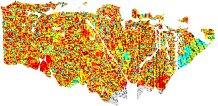
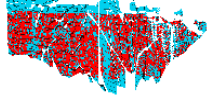
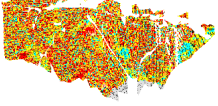
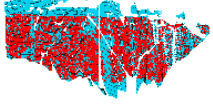
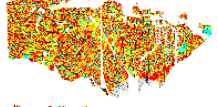


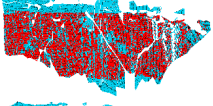

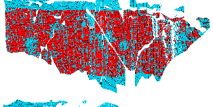
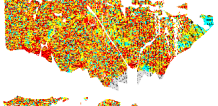
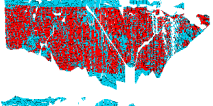
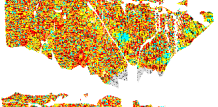




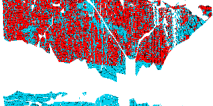
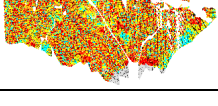
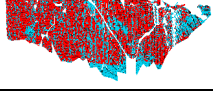
**Step 2:** In this step, the optimal stope layout was determined for each one of the generated block models using the floating-stope algorithm assuming that the minimum stope dimension was

$10 \times 5 \times 1$  cubic meter and  $\text{Al}_2\text{O}_3$  cutoff grade was 40%. Also the head grade was assumed to be 45% according to the run of mine requirements.

**Step 3:** Using Equation 4, the mineable reserve of each scenario and their average grade (Table 1) were calculated based on the results of the optimal stope layout. If the goal was to maximize the amount of mineable reserve, then according to Equation 5, the 4<sup>th</sup> scenario (i.e. mine 4) was founded as the optimum one, where the amount of mineable reserve was the maximum compared to the other scenarios.



**Table 1. Average grade of generated scenarios.**

Label	Grade variation	Average grade (%)	Stope boundary	Mineable reserve (kt)	Mine flow Ave. grade (%)
Mine 1		47.07		1,543	47.51
Mine 2		46.92		1,586	47.50
Mine 3		47.07		1,568	47.53
Mine 4		46.90		1,590	47.50
Mine 5		46.98		1,560	47.52
Mine 6		46.94		1,581	47.52
Mine 7		46.97		1,556	47.50
Mine 8		47.01		1,555	47.52
Mine 9		47.03		1,552	47.51
Mine 10		46.88		1,586	47.51

**Step 4:** In this step, using Eqs. 6-9 and based on the tonnage and average grade of each scenario, the metal-content of each scenario was calculated. According to Figure 5, in all the three modes of Max-Min, Max-Max, and Max-Ave. the 4<sup>th</sup> scenario (i.e. mine 4) was the best option. Thus if the goal is to maximize the amount of metal-content, again the 4<sup>th</sup> scenario will be the optimal option.

**Step 5:** According to Eqs. 10-13 and the sale figures, the profit of each scenario was calculated (Figure 6). Using this figure, the most profitable scenario could be selected, and accordingly, the most profitable minable reserve could be estimated. Considering Figure 6, in the modes of Max-Min and Max-Ave, the 4<sup>th</sup> scenario (i.e. mine 4), and in the mode of Max-Max, the 6<sup>th</sup> (i.e. mine 6)

scenario is the best option. The 6<sup>th</sup> scenario is the most optimistic option but due to the high variance of profit in the 6<sup>th</sup> scenario, selecting this option is risky. Thus if the goal is to maximize the profit of the mineable reserve, the 4<sup>th</sup> scenario is again the optimal scenario.

**Step 6:** In this step, VaR of the profit was determined, and the lowest risk scenario was selected. According to the simulation results and Equation 14, VaR (10%) of each scenario was calculated (Table 2). The results obtained show that the 4<sup>th</sup> scenario (i.e. mine 4) is the low-risk option. This means that by completing the 4<sup>th</sup> scenario, the probability of achieving a profit of 10.765 M\$ is 90%, which is the maximum achievable profit compared to the others.



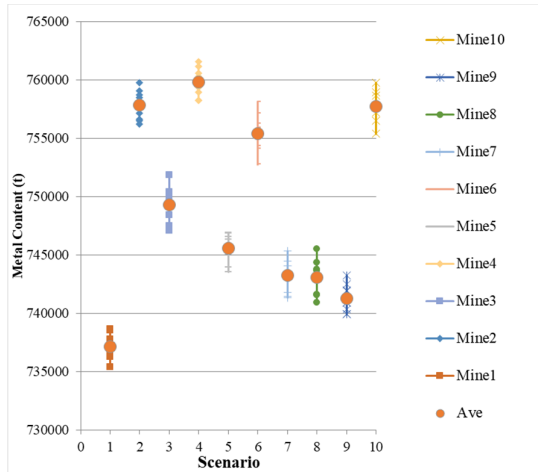


Figure 5. Metal-content of each scenario.

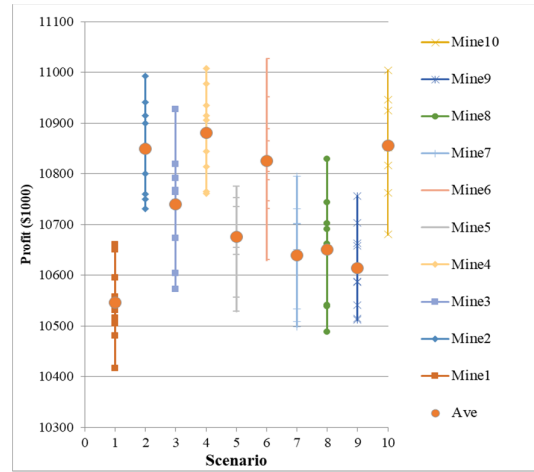


Figure 6. Profit of each scenario.

Table 2. VaR (10%) of profit for scenarios (\$1000).

Mine	VaR (10%)
Mine 1	10480
Mine 2	10750
Mine 3	10604
<b>Mine 4</b>	<b>10765</b>
Mine 5	10556
Mine 6	10731
Mine 7	10508
Mine 8	10538
Mine 9	10514
Mine 10	10762

**4.3. Golbini mine stope optimization considering both grade and prices uncertainties**

*Step 1:* Here, the minimum and maximum values of the bauxite price were considered during the past three years. The minimum price minus 20% and the maximum price plus 20% were considered for the minimum and maximum price range of the scenarios, and then the price scenarios were generated between these two values.

*Step 2:* According to Equation 15, using the bauxite price scenarios and the local mining cost (28.5 \$/t), the profit of each scenario was calculated. The results obtained are given in Table 3.

*Step 3:* According to the results obtained and using Equation 16, the best scenario with respect to maximization of profit in variable prices can be selected. As shown in Table 3, from price 65 up to 84 the best scenario is Mine 1, from price 84 up to 87 the best scenario is Mine 3, from the price 88 up to 89, the best scenario is mine 6, and from above the price 89, the best scenario is mine 4. It should be noted that the project is unfeasible if the price is lower than 86 \$/t.

*Step 4:* In this step, in order to evaluate the profit of the candidate stope boundaries, more price scenarios are generated. In this work, the mine-life is 3 years; therefore, the price forecasts are generated with respect to the price variations in the previous three years. In this step, the bootstrapping method is applied for this purpose. The main advantage of the bootstrapping method is that it does not require any assumptions about the distribution of the returns. Assuming a given set of (yearly or monthly) price data, the yearly or monthly returns are calculated. The return is equal to the natural logarithm of the division of two consecutive prices (Equation 18).

$$\mu = \ln \left( \frac{S_t}{S_{t-1}} \right) \tag{18}$$

where  $\mu$  is the return, and  $S_t, S_{t-1}$  are the mineral price in periods t and t-1.

Using the historical price data, and applying the bootstrapping method, 50 price scenarios are generated. After that, the average of the generated prices in each scenario is calculated. Then each stope's profit is simulated using the price scenarios, and then VaR of the profit is determined. According to the simulation results and Equation 17, VaR (10%) of each scenario is calculated and presented in Table 4. Based on the results obtained, the 4th scenario (i.e. mine 4) is the low-risk option in this work. It means that by completing the 4th scenario, the probability of achieving a profit of 10.501 M\$ in variable prices is 90%, which is the maximum achievable profit compared to the others.

**Table 3. Simulation result of profit (\$1000); the optimal scenarios for variable prices are colored.**

Price	Mine1	Mine 2	Mine 3	Mine 4	Mine 5	Mine 6	Mine 7	Mine 8	Mine 9	Mine 10
65	-10625	-10927	-10785	-10955	-10740	-10880	-10716	-10702	-10681	-10920
...										
84	-870	-902	-870	-903	-875	-886	-883	-872	-872	-897
85	-357	-374	-348	-374	-356	-360	-365	-354	-356	-369
86	156	152	172	154	162	165	151	162	159	158
87	669	680	694	683	681	691	669	679	676	685
88	1182	1208	1216	1212	1200	1217	1186	1197	1192	1213
89	1696	1735	1738	1741	1720	1743	1704	1714	1708	1740
90	2209	2263	2260	2270	2239	2269	2221	2232	2224	2268
...										
142	28905	29700	29394	29780	29237	29621	29132	29135	29069	29701

**Table 4. VaR (10%) of profit for scenarios in variable prices (\$1000).**

Mine	VaR (10%)
Mine 1	10196
Mine 2	10472
Mine 3	10378
<b>Mine 4</b>	<b>10501</b>
Mine 5	10316
Mine 6	10452
Mine 7	10273
Mine 8	10281
Mine 9	10256
Mine 10	10475

## 5. Conclusions

The optimization of the stope boundary and minable reserve determination is an essential issue in the underground mine design and planning. Mining is inherited with uncertainty. The grade and price uncertainties are considered as the main sources of uncertainty. These uncertainties will affect the amount of mineable reserve and the operation's profitability. In this work, in order to evaluate the underground mining projects, a risk-based procedure was developed to determine the optimum stope layout in the presence of the grade and price uncertainties. For this purpose, several reserve realizations and various price scenarios were generated in order to evaluate the effect of the price uncertainty. By applying the procedure, different designs were regarded as the candidate stope boundaries. Several decision support criteria were defined in order to assist the decision-maker in uncertain conditions. The results obtained indicated that the mineable reserve evaluation depended on the decision-maker's strategy. The procedure presented in this paper uses two types of uncertainties simultaneously in underground mining. It will guide the decision-maker toward the most appropriate stope boundary that best fits the mining company's requirements. The procedure was implemented in a bauxite mine. According to

the VaR results, the 4th scenario (i.e. mine 4) is recommended as the optimal stope boundary. This scenario has the lowest risk compared to the other scenarios, and the total profit is increased by 18% compared to the deterministic stope boundary.

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## ارزیابی ذخایر معادن زیرزمینی در شرایط عدم قطعیت عیار و قیمت مواد معدنی

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### چکیده:

ارزیابی و بهینه سازی مبتنی بر عدم قطعیت در معادن مسئله مهمی است که در معادن زیرزمینی کمتر از معادن روباز مورد توجه قرار گرفته است و دلیل این امر تنوع در روش های استخراج زیرزمینی و پیچیدگی پارامترهای آنها است. عدم قطعیت عیار و قیمت نقش بسیار مهمی را در پروژه های معدنی بازی می کنند. به طوری که طراحی و برنامه ریزی معادن بدون در نظر گرفتن این عدم قطعیت ها پروژه معدنی را با ریسک همراه خواهد کرد. ارزیابی ذخایر معدنی در حضور این گونه عدم قطعیت ها، ارزیابی و برآورد ریسک پروژه های معدنی را ممکن می سازد. در این تحقیق اثرات عدم قطعیت عیار و قیمت بر روی بهینه سازی کارگاه های استخراج معادن زیرزمینی و ذخیره حاصل از آن، ارزیابی شده است. در این راستا، محدوده نهایی کارگاه های استخراج معادن زیرزمینی به دو صورت قطعی و تصادفی و در حضور عدم قطعیت عیار و قیمت مطالعه شده است. برای این منظور، در این تحقیق ابتدا شبیه سازی شرطی به منظور تولید مدل های ذخیره تصادفی برای معدن استفاده سپس محدوده نهایی بهینه کارگاه های استخراج معدن زیرزمینی با استفاده از الگوریتم کارگاه شناور برای ذخایر شبیه سازی شده محاسبه شده است. در شرایط حضور عدم قطعیت، معیارهای مختلفی برای کمک به تصمیم گیری استفاده شده اند که از آن جمله می توان به معیارهایی مانند ذخیره قابل معدنکاری، فلز محتوا، سود و ارزش در معرض ریسک اشاره کرد. نهایتاً در این تحقیق شیوه ای برای در نظر گرفتن عدم قطعیت های عیار و قیمت در استخراج زیرزمینی تعریف شده است که تصمیم گیرنده را به سمت مناسب ترین محدوده نهایی متناسب با نیازهای معدن راهنمایی می کند. این شیوه در یک معدن بوکسیت استفاده و محدوده نهایی در آن با توجه به معیارهای تصمیم گیری مختلف، بهینه سازی شده است.

**کلمات کلیدی:** ارزیابی معدنکاری زیرزمینی، بهینه سازی کارگاه های استخراج، عدم قطعیت عیار، عدم قطعیت قیمت مواد معدنی، آنالیز ریسک.