Compressive Failure Analyses of Rock-Like Materials by Experimental and Numerical Methods

Mohammad Davood Yavari1, Hadi Haeri2, Vahab Sarfarazi3, Mohammad Fatehi Marji4* and Hossein Ali Lazemi1

1. Department of Mining Engineering, Bafgh Branch, Islamic Azad University, Bafgh, Iran
2. State Key Laboratory for Deep GeoMechanics and Underground Engineering, Beijing, China
3. Department of Mining Engineering, Hamedan University of Technology, Hamedan, Iran
4. Mine Exploitation Engineering Department, Faculty of Mining and Metallurgy, University of Yazd, Yazd, Iran

Abstract

Investigating the crack propagation mechanism is of paramount importance in analyzing the failure process of most materials. This process may be exposed during each kind of loading on the materials. In this work, the cracking mechanism in rock-like materials is studied using the numerical methods and compared with the experimental test results. However, the mechanism of crack growth in brittle materials such as rocks is influenced by different parameters. This research work focuses on the effect of the initial crack angles on the crack growth paths of these materials. Some cubic samples containing pre-existing cracks are tested in compression by considering different flaw orientations. The specimens are made of cement, water, and sand. Moreover, the mentioned process is numerically simulated using three different methods: the finite difference method for discontinuous bodies or discrete element method, the displacement discontinuity method, and the versatile finite element method. The micro-parameters for simulation are gained by the trial-and-error procedure for the discrete element method. Eventually, the crack growth paths observed in the experiments are compared with the numerically simulated models. The results obtained show that these central cracks propagate in two ways, which are dependent on their initial angle. By increasing the initial crack angle to greater than 30° (α > 30°), the wing crack path moves further away from the initial crack, and by decreasing α to smaller than 30° (α < 30°), only the shear cracks are initiated. Therefore, the validity and accuracy of the results are manifested by comparing all the corresponding results obtained by different methods. Based on these results, it can generally be concluded that the strength of the cubic (rock material) specimens increases with increase in the crack angles with respect to the applied loading direction.

Keywords
Cracking mechanism
Fracture mechanics
Cubic specimen
Physical modeling
FEM

1. Introduction

Some brittle materials consisting of the civil and rock structures have various micro-cracks in their constructions. Based on the Griffith theory, new fractures can be created from the extension of the pre-existing cracks in the concretes and rocks. For example, in rock-like material specimens, the pre-existing cracks mainly propagate in two manners under compression, i.e. the tensile (wing) cracks propagate first, and then the shear (secondary) cracks are extended. Specifically, the tensile cracks...
start their propagation first, and continue their extension from the tip of the pre-existing cracks, and then the shear cracks start their initiation near the crack tip but continue to propagate in the direction of the maximum shear stress [1-4]. Thus the wing cracks may occur more possible than the secondary cracks in rock-like materials because the Mode I (tensile mode) fracture toughness of the specimens is lower than the toughness in shear loading. Therefore, it is significant to investigate the failure mechanism of wing and secondary cracks of brittle materials [1]. Already, several analytical, experimental, and numerical studies have been accomplished on the fracture mechanics and crack propagation process of rock-like materials [5-8].

However, several experimental works have been performed considering the mechanism of crack extensions in brittle material samples containing one or more cracks, and some valuable laboratory results have been provided in the literature. For example, the mechanism of crack growth and coalescence in three-dimensions for the rock-like material samples with a spherical pore [9-11] and two pre-existing cracks [12] have been experimentally studied under biaxial [13] and tri-axial [14] compression. The fracture mechanics of the rock-like material samples with pre-existing cracks under compression have been studied by several researchers [15-18]. On the other hand, in [19-22], some experiments have been carried out in order to investigate the distribution of the fractures in the rock-like samples under compression loading. The interaction between the pre-existing holes in the geo-materials under uniaxial compression test has been studied [23, 24]. Also some uniaxial tests have been performed in order to investigate the fracturing mechanism and failure behavior of the rock-like material samples with a large-opening crack [14]. Moreover, Sun et al. [25] have investigated the interaction between the central and edge cracks in the specimens of rock-like materials.

In addition to the experimental studies, several types of research works have focused on the fracture mechanics, process of cracks extension, and crack coalescence in the geo-materials specimens using the numerical and experimental-numerical methods [26]. For example, the crack propagation process between the cracks and holes in the specimens of rock-like material under uniaxial compression has been studied using the Discrete Element Method (DEM) [14, 23, 24, 27-29]. Moreover, a Boundary-Finite Element Method (BEM-FEM) has been presented for modeling the wing crack propagation in the pre-existing crack problems [30]. Crack growth from a spherical pore in brittle materials has been simulated by the Finite Element Method (FEM) [11, 15]. Some crack growth simulations in geo-materials have been performed using the particle flow codes (PFCs) in two and three dimensions [31, 32].

In addition to the static problems, there are some quasi-static and dynamic research works in the literature considering the dynamic crack growth mechanism in brittle geo-materials. These works have mainly focused on the dynamic crack propagation in rock specimens due to the Hopkinson pressure test [14, 33, 34] and the numerical dynamic loadings [35, 36]. Moreover, dynamic crack growth due to rock blasting has also been investigated using both the numerical analyses and experimental tests [37-40].

Therefore, the solutions to the rock fracture mechanics problems related to the initiation, propagation, extension paths of the pre-existing cracks in geo-materials are important from the theoretical and practical viewpoint for many engineers dealing with the surface and underground rock structures. Most of the previous studies have focused on a special problem with specific crack geometry and specific material properties using one or two methods. In this research work, the cubic type of rock-like materials containing cracks are specially prepared in the laboratory and tested under compressive loading. The mechanism of crack growth paths is investigated. The observed experimental results showing the formation of the wing and secondary cracks are compared with the modeled results obtained using FEM, BEM, and DEM. Various types of specimens considering different crack inclination angles are studied in this research work. The process of rock fracturing for different scenarios are considered, and the corresponding results of each method are compared with one another, which provide the validity and accuracy of these different results.

2. Experimentally observed crack growth mechanism for rock-like specimens

In this work, the physical modeling and laboratory tests were conducted in order to experimentally analyze the mechanism of the crack growth process in brittle materials. The cube specimens were provided by mixing 15% of Portland Pozzolana cement (PPC), 75% of fine sands, and 10% water, and dried at room temperature. The specimen dimensions were 15 ×
$15 \times 15$ cm. Then the propagation mechanism of the wing and shear (secondary) cracks initiating from the tips of a central slant crack were studied experimentally by carrying out several laboratory tests. A schematic view of the prepared specimen with a central slant crack and the orientation angle ($\alpha$) is shown in Fig. 1. Five specimens with slant cracks of $\alpha = 0^\circ$, $15^\circ$, $30^\circ$, $45^\circ$, and $55^\circ$ were constructed as shown in Figs. 2(a) to 2(e).

The uniaxial compression tests were carried out on the cubic specimens prepared from the concrete materials (Fig. 3). In these tests, a constant loading rate of $0.02$ MPa/s was uniformly applied on the cubic type pre-cracked specimens.

![Figure 1: A schematic view of the setup of specimens.](image1)

![Figure 2: Constructed specimens with a central crack considering different crack inclination angles.](image2)
problems in engineering and science [43-45]. Recently, the higher order elements have been used in order to solve many rock fracture problems. In the semi-analytical BEM, the problem boundaries are discretized into a finite number of boundary elements. In the analytical part of the solution, the potential variables such as the displacement and traction vectors are specified to the nodes of these elements in the form of boundary integral equations (BIEs). Then a set of simultaneous algebraic equations are numerically arranged relating the solved boundary integral equations to one another. Using the proper boundary conditions (and the initial conditions for dynamic problems), the tractions, displacements or intermediate unknown variables such as fictitious stresses or displacement discontinuities at all boundary nodes are obtained as the numerical solution of these algebraic equations [41]. In the modified form of DDM, i.e. the higher order displacement discontinuity method (HODDDM), some higher order displacement discontinuity (HDD) elements are specified to each boundary element and/or crack element. This code is specially modified to solve the 2D fracture mechanics problems [42-44].

In this section, the modified form of the displacement discontinuity (DD) method [45] is used for crack analyses. The higher order quadratic DD elements and the special crack tip elements are specified at the boundaries and crack tips, respectively. This computer code is known as the Two-dimensional Displacement Discontinuity method using Quadratic elements for Crack analyses (TDDQCR). The crack extension process in rock-like specimens is simulated by the proposed method [10, 46-49].

Figure 4a shows the 2D displacement discontinuity distribution at quadratic collocation point \( m \). The quadratic displacement discontinuity variations along the boundary elements can be expressed as:

\[
D_j(\xi) = \sum A_m(\xi) D_j^m \quad (1)
\]

\((j=x,y; \ m=1,2,3)\)

where \( D_j^f \), \( D_j^s \), and \( D_j^c \) are the nodal shear and normal displacement discontinuities at the \( j^{th} \) element in the \( x \) and \( y \) directions. How to determine these parameters and the shape functions of DDM has already been explained in [26].
Figure 4: Discretizing each boundary element into a quadratic element to specify its displacement discontinuities [26], b) Enriched nodes in XFEM [53], c) contact for a PFC model [60].

In the TDDQCR code, the crack tip singularity effects of the displacement discontinuities can be minimized by providing some accurate estimations of the stress distribution near the crack ends. This procedure increases the accuracy of the displacement discontinuity components. However, a special treatment of the crack (at its tip only) has been used in this computer code [26, 50]. On the other hand, the Mode I and Mode II (mixed mode) stress intensity factors (denoted as $K_I$ and $K_{II}$) have been numerically estimated based on the famous concepts of crack opening displacement (COD) and crack sliding displacement (CSD), respectively. In this approach, COD and CSD represent the normal and shear components of displacement discontinuities, respectively. Based on the general concept of linear elastic fracture mechanics (LEFM), the formulations for $K_I$ and $K_{II}$ are given as [51, 52]:

$$K_I = \frac{\mu}{4(1-\nu)} \left(\frac{2\pi}{c}\right)^{1/2} D_y(c)$$  \hspace{1cm} (2)

$$K_{II} = \frac{\mu}{4(1-\nu)} \left(\frac{2\pi}{c}\right)^{1/2} D_x(c)$$

where $\mu$ and $\nu$ are the shear modulus and Poisson ratio, respectively, and $c$ is the crack’s half-length. Based on the above-mentioned concepts, the problems shown in Figs. 1 and 2 have been numerically treated for the five central crack orientations $0°$, $15°$, $30°$, $45°$, and $55°$.

3.2 Finite element modeling of problem

Finite Element Method (FEM) has already been exploited for solving the rock-like material problems [11]. Moreover, the modified version of FEM called the Extended Finite Element Method (XFEM) has been improved for the analyses of crack problems, and widely used for solving different types of problems in fracture mechanics [53-56].

The crack propagation process in this section has been numerically simulated using the Abaqus software. This computer code is based on XFEM, and is designed as a flexible tool for finite element modeling of most engineering problems (Fig. 4b). The main flexibility aspect of XFEM in Abacus is the steps that are used to divide the problem history, which is more convenient for each history phase. For example, in a creeping hold, a dynamic transient or a thermal transient, many classes of stress analysis can be solved with XFEM (Abaqus) [57]. Based on the above-mentioned concepts, the problem shown in Figures 1 and 2 has been numerically modeled in all the five central crack orientations $0°$, $15°$, $30°$, $45°$, and $55°$. 
3.3 Discrete element modeling

The numerical simulation of crack analyses in rock-like materials can be accomplished by the versatile Discrete Element Method (DEM) [29, 40, 58]. Also the application of extended DEM for crack growth under uniaxial compression in geomaterial is explained, and the numerical results are compared with their experimental counterparts [27].

In this work, the crack analyses in rock-like material samples were conducted by the 2D PFC. The discrete element modeling of the fracturing mechanism in the modeled samples is performed by simulating the samples in the form of particle assemblies [59]. The standard method for assembling a PFC2D model for the solution of the problems related to this research work is fully described in reference [29]. The main aspects of the solution process include the generation of particles for a proper simulation of the material sample, the packing of these particles to form a particle assembly of the modelled specimen, the initialization of isotropic stress conditions within the assembly, removing the floating particles from the assembly, and installing the bonds to make a solidified modeled sample for the simulated geomaterial specimen. In this modeling procedure, a contact modelling scheme is adopted to simulate the constitutive equations related to the material behavior (Fig. 4c). In the parallel bond model, a set of elastic springs is assumed to be distributed over a rectangular contact plane centered at the contact point. The contact point springs are acting parallel to these bond springs. An assembly of rounded particles are free to move in the shear and normal directions but interacting one another via shear and normal springs. They can also rotate within the particle assembly [60]. A proper value for damping factor was estimated as 0.7 in this numerical simulation procedure. Then the modelled specimens shown in Figs. 1 and 2 simulated numerically the rock-like material samples containing central cracks of the five crack inclination angles 0°, 15°, 30°, 45°, and 55°.

4. Results and discussion

Results of the crack analyses as obtained from both the laboratory tests and numerical simulation of the pre-cracked samples of the brittle materials (rock-like specimens) are reported and discussed in this section. Also these results are compared to each other and discussed.

As mentioned earlier, the mechanism of crack extension in the specially prepared pre-cracked specimens was investigated. The uniaxial compressive tests were conducted on these specimens in the laboratory. Figure 5 shows the results of the uniaxial loading of the specimens with the central cracks of different inclinations. It was observed experimentally that by changing the angle of the central crack, the crack initiation and propagation patterns also changed. In the experimental modeling, two different types of induced cracks could be observed in the concrete specimens: wing (primary or tensile) and secondary (shear) cracks. If the inclination angle of the central crack is less than 30°, there are only secondary cracks during the loading. Moreover, by increasing the original crack angle, α (i.e. for α ≥ 30°), the wing crack path moves towards the other side of the crack. Therefore, in this case, the primary wing cracks are more dominant during the failure of rock materials, while the secondary cracks are of less importance [6].

In these experiments, the uniaxial compressive strength (UCS) and the crack propagation patterns for each specimen were measured experimentally. Fig. 6 shows the variations in UCS of the specimens in the uniaxial compressive test for different pre-existing crack inclination angles. The intact specimen strength (i.e. specimen without a pre-existing crack) is 23 MPa. As it can be seen in Fig. 6, as the crack angle (α) increases, the strength of the specimen also increases. Furthermore, a trend line can be fit to these results, and the relationship between the flaw inclination (tanα) and UCS (in MPa) is obtained as follows:

\[ \text{UCS} = 2.5882 \tan \alpha + 14.05 \]  
(3)
Moreover, crack propagation in the specimens was numerically simulated using BEM. In this case, the specimens were modeled using a 2D displacement discontinuity code (TDDQCR), which was utilized from the quadratic displacement discontinuity elements. The process of crack propagation was simulated under the uniaxial compressive test. The results obtained from modeling of five different specimens are shown in Fig. 7. The results of the displacement discontinuity modeling show that if the inclination angle of the central crack is less than 30° (\(\alpha < 30°\)), only the secondary cracks will appear during the uniaxial compressive loading. Moreover, by increasing the central crack angle (for \(\alpha \geq 30°\)), the wing crack path moves towards the other side of the crack. Therefore, in the boundary element modeling (displacement discontinuity modeling), the wing cracks appear for the angles greater than 30° (\(\alpha \geq 30°\)), similar to experimental modeling, and the wing crack propagation is generally agreed with the experimental results. The limitation of TDDQCR is that it is incapable in the simulation of
porosity and grain shape. For the crack angles of less than 30°, the wing cracks are not dominant or may not be observed as they may not get a chance to initiate and propagate. This phenomenon may be related to the crack’s tendency, which adjusts its propagation parallel to the direction of the applied compressive loading. The crack tip’s shear zone may be formed after the formation of primary cracks due to the stress concentration increase at their tips.

Figure 7: Displacement discontinuity modeling of crack propagation patterns in the modelled specimens with different inclination angles (α) for the pre-existing crack (TDDQCR code).

As mentioned earlier, a finite element modeling was carried out for simulation of the pre-cracked samples. The numerical modelling results of the crack propagation patterns in these five specimens using the Abaqus software are given in Fig. 8. In these results, it is shown that if the inclination angle of the central crack is less than 45° (α < 45°), the secondary cracks are dominant during the uniaxial compressive loading condition. On the other hand, as the inclination angle is greater than 45° (α ≥ 45°), the wing cracks tend to be dominant and replace the secondary cracks. However, the same results were also observed in the physical models for the angles greater than 30° (α ≥ 30°). The limitation of the Abaqus software is that it is incapable in the simulation of porosity and grain shape. In this analysis, when the inclination angles of cracks in the modelled specimens are less than 30°, the observation of wing cracks diminishes, and the secondary or shear cracks appear in the samples. This is mainly due to the tendency of the cracks to propagate parallel to the direction of the applied load. However, the concentration of stresses near the crack tips increases, and the shear zones are produced at their vicinities.
Figure 8: Crack propagation in finite element analyses of the modelled specimens with different angles.

The crack propagation process of the tested samples was also modelled by PFC2D, and the results obtained were shown in Fig. 9. In this modeling procedure, the pre-existing cracks of different inclination angles (α) are considered (i.e. α = 0°, 15°, 30°, 45°, 55°). As shown in Fig. 9, the simulated crack propagation process by PFC2D is in a good agreement with the corresponding experimental results already given in Fig. 5. The results of discrete element modeling also show that for the central crack angle of less than 30° (α < 30°), only the secondary cracks are visible after the uniaxial compressive loading is applied. Moreover, by increasing the central crack angle (for α ≥ 30°), the wing crack path moves towards the other side of the crack. The main limitations of PFC for crack analyses in geo-materials include: (a) The flaws are closely related to the grain size, and therefore, the grain-size effect should be considered; (b) Some cross-effects may be produced due to the difference between the shape and size of the particles (elements) compared to those of the real grains; (c) It is somewhat difficult to establish a proper relation between the microscopic and macroscopic parameters used in the modelled and physical specimens, respectively, for the geo-material samples.
4.1. Calculation of stress intensity factor

The Mode I and Mode II stress intensity factors (SIFs) for the crack tips were calculated using both the analytical and numerical formulations. The analytical formulations to calculate the Mode I and Mode II SIFs (or Mode I and Mode II fracture toughnesses) for the center crack in an infinite space are given as:

\[ K_I = \sigma \sqrt{\pi c \varepsilon_I} \]  
\[ K_{II} = \sigma \sqrt{\pi c \varepsilon_{II}} \]  

where \( K_I \) and \( K_{II} \) are the Mode I and Mode II SIFs, respectively, \( \sigma \) is the compressive stress, \( c \) is the half-length of center crack. The two dimensionless coefficients \( \varepsilon_I \) and \( \varepsilon_{II} \) are functions of the inclination angle (\( \alpha \)), and can be obtained from the following equations:

\[ \varepsilon_I = \frac{1 + \cos 2\alpha}{2} \]  
\[ \varepsilon_{II} = \frac{\sin 2\alpha}{2} \]  

With more notion in the above equations, it can be understood that SIF is related to the specimen’s geometry and crack length.

Moreover, the stress intensity factors are calculated using the DD method, as mentioned earlier in Equation (2). This method has been developed for the analysis of the crack tip element in TDDQCR [42]. However, based on Equation (2) and the analytical Equation (4) and (5), the normalized SIFs, \( K^N_I \) and \( K^N_{II} \), are expressed as:

\[ sK^N_I = \frac{K_I}{\sigma \sqrt{\pi c}} \]  
\[ K^N_{II} = \frac{K_{II}}{\sigma \sqrt{\pi c}} \]  

The Mode I and Mode II SIFs under the uniaxial compressive test are shown in Fig. 10 for both the numerical (using HODDM) and experimental tests. These results obtained show the relatively high accuracy of the numerical values of SIFs as compared to their measured values obtained in the laboratory.
5. Concluding remarks

The effect of the crack inclination angles for a rock-like specimen with central pre-existing crack on the mechanism of crack propagation within the material sample was studied experimentally and numerically. The specially made specimens of rock-like materials containing central cracks with different inclination angles were tested experimentally in the laboratory under the uniaxial loading condition. The crack propagation patterns observed are evident by formation of the wing and shear cracks in the failed specimens. Furthermore, the effect of the inclination angles of the central pre-existing cracks on the wing and secondary cracks propagation paths was studied numerically. For this purpose, some numerical modeling schemes were also performed using DDM (as one of the sub-methods of BEM), FEM, and DEM. Some of the main conclusions of this research work may be given as follow:

- According to the experimental and numerical results, there are two different crack propagation patterns in the specimens as tensile (wing or primary) cracks and shear (secondary).
- If the inclination angle of the central crack is less than 30° ($\alpha < 30^\circ$), only the secondary cracks will appear during the uniaxial compressive loading. This is observed in all the experimental and numerical models (except for a small difference in the finite element modeling).
- Based on the results obtained from the experimental and numerical models, by increasing the central crack angle greater than 30° ($\alpha \geq 30^\circ$), the wing cracks appear and move towards the other side of the crack (except for a small difference in the finite element modeling).
- The finite element modeling results show that if the central crack’s inclination angle is less than 45° ($\alpha < 45^\circ$), only the secondary cracks will be observed during the uniaxial compressive loading. Therefore, in the finite element modeling, the wing cracks appear for the angles greater than 45° ($\alpha \geq 45^\circ$).
- The results of the experimental and numerical simulations of the crack propagation patterns in the rock-like material samples due to the uniaxial compressive loading conditions are similar to each other. Thus there are good agreements between the experimental and all the three numerical modeling methods.
- In addition to the crack propagation patterns, the uniaxial compressive strength (UCS) for the specimens containing central cracks with various inclination angles was measured. The strength of the pre-cracked specimens increases with increase in the flaw’s angle. Furthermore, a relationship between the flaw’s angle ($\alpha$) and UCS was obtained.
- The Mode I and Mode II stress intensity factors (SIFs) were calculated numerically and compared with their corresponding experimentally measured values.

Comparing the results obtained with the
experimental tests, the analytical and numerical methods show that there is a proper agreement between the corresponding analytical and numerical values of SIFs. This confirms the validity of the various methods used in this work and the accuracy of the results obtained herewith.

References


تجزیه و تحلیل شکست فشاری مواد شبه سنگی با استفاده از روش های آزمایشگاهی و عدیدی

مهدی حاجی‌فرد ۱، وهاب سربارازی ۲، منصور فتحی مرجی ۳، حسین علی‌یزدی ۴

چکیده:

بررسی مکانیزم انتشار ترک از اهمیت بالایی در تحلیل فرآیند شکست بیشتر مواد برشدار است. این فرآیند ممکن است در هنگام بازکردن هر نوع مواد در معرض آن قرار گیرد. در این تحقیق، مکانیزم ترک کردنی در مواد شبه سنگ با استفاده از روش‌های عدیدی بررسی شده و با تأثیر آزمایشگاهی مقایسه می‌شود. با این حال، مکانیزم رشد ترک در مواد شکننده همانند سنگ تحت تأثیر پارامترهای مختلف است بنابراین کل تحقیقاتی بر تأثیر زاویه‌های ترک اولیه در مسیر رشد ترک این مواد متمرکز است. برخی از نمونه‌های مکانیمی که شامل ترک‌های در پیش‌دوره یا نظری در نظر گرفته‌اند، می‌توانند تحت بازگذاری فشاری آزمایش‌های شونده شونده با آب و سیمان، از این سو به شکست هم‌زمان نیز می‌شود. نتایج آزمایش‌های عدیدی نشان داد که این ترک‌های محوری در باره‌های اختلافی با داشتن دوره‌های کوچک‌تر از ۳۰ درجه (0<α<30) و بزرگ‌تر از ۳۰ درجه (α>30) از همیشه و کارگیری پیش‌دوره (FEM) به بهترین روش کار باعث شدند. بنابراین، اعتبار و صحت نتایج با مقایسه نام‌زده‌ی نتایج مختلف مواد کشیده مشابه همان‌ریخت با روش‌های اختلافی آمده‌ی شونده‌ی مواد کشیده با اساس نتایج بیشتر از طریق کلی می‌توان نتیجه گرفت که مقاومت نمونه‌های مکانیکی (مواد شبه سنگی) با افزایش در زاویه‌های ترک به نواحی به جهت پذیرش محیطی اعمال شده افزایش می‌یابد.

کلمات کلیدی: مکانیزم ترک خوردنی، مکانیزم شکست، نمونه مکانیکی، مدل سازی فیزیکی.