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Determining an Appropriate range for the Number of Cuboid Lifters in Ball Mills using DEM

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Abstract

The number of lifters in the liner of ball mills and the mill rotation speed are among the most significant factors affecting the behavior of grinding charge (balls) and their motion trajectory, and consequently, the comminution mechanism in these mills. In this research, in order to find a suitable range for the number of lifters in the liner of ball mills, the DEM method is utilized. Initially, a pilot-scale ball mill with dimensions of $2.0 \text{ m} \times 1.11 \text{ m}$ without any lifter is simulated. Afterwards, by adding, respectively, 1, 2, 4, 8, 16, 20, 26, 30, and 32 cuboid lifter(s) with dimensions of $2 \text{ m} \times 5 \text{ cm} \times 5 \text{ cm}$, nine other separate simulations are performed. The influences of the number of cuboid lifters on the two new factors introduced here, namely 'head height' (HH) and 'impact zone length' (IZL) at various mill speeds, that is, 70% and 80% of its critical speed (CS) are investigated. The results indicate that in order to find a suitable range for the number of lifters in the liner of ball mills, it is necessary to consider these two parameters simultaneously as the criteria for selecting the appropriate range, That is, liners that simultaneously produce both a higher HH and a greater IZL are more suitable for use in the industry. The results also demonstrate that the suitable range for the number of cuboid lifters in the liner of ball mills is between 16 and 32, which field research on the ball mills of three different plants in the industry confirms the accuracy of the results obtained in this research. Unlike the previous research works, it has now been shown that the number of ball mill lifters does not only depend on the diameter of the mill but also depends on the width, height, angle of the lifter, and generally on the type of lifter.

1. Introduction

The power draw of ball mills is one of the most important factors involved to include in their design because it specifies their economic efficiency. Power draw is mostly specified by the charge filling level, lifter height, number of lifters, and mill rotation speed. Nevertheless, almost all the classical theories used for computing the power draw of ball mills ignore the influence of the number and type of the lifters, and focus only on the mill rotation speed, the charge filling level, and also the size and shape of the milling medium. As

a result, it may cause errors [1]. Thus, it is important to obtain a more efficient milling in ball mills, as they have a low rate of efficiency, in part, due to the shortage of cascading and cataracting motions for the balls as well as the improper shoulder and toe points [2]. For controlling, optimizing, and reducing the power draw of ball mills, the plant engineers must obtain sufficient information about their operating conditions. One of the most efficient methods is to take advantage of the computer simulation. Computer simulations

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that take advantage of the methods like the discrete element method (DEM) may be efficient in finding the optimal speed of ball mills, and thus creating the right shoulder and toe points inside them, and also making the cascading and cataracting motions.

The ball mill liners and lifters protect its shell against abrasion and raise balls and ore particles to a specific height. Hence, they should be able to withstand intense impact charges during the grinding operation. The rate of wear in these parts is very high, and they usually fracture or encounter with break due to wear, which may drastically affects the ball mill process efficiency. Therefore, research on the liner shape and the ball mill configuration is critical for enhancing the production throughput [1]. In a large quantity of the previous research works, the important role of lifters on charge trajectory in tumbling mills has been taken into consideration. Cleary took advantage of DEM to investigate and predict the lifter wear and the power consumption of ball mills [3, 4]. Kalala et al. have scrutinized the wear rate of lifter profiles at dry coal grinding mills [5, 6]. Banisi and Hadizadeh have made a mechanical device for measuring the wear and mass reduction of liners and lifters in the SAG mills [7]. A modification of the shape of the SAG mill liners has been performed by Yahyaei et al. based on the wear profile measurements of the 3D liners [8]. Examples of the 3D SAG mill models with detailed forecasts of power consumption, wear rates, and stresses of liners as well as energy spectra have been provided by Cleary [9]. Mishra and Rajamani have studied the trajectory of the grinding charge (balls) in industrial ball mills using DEM [10]. Powell and McBride later depicted medium motion and milling regions “head, departure shoulder, center of circulation, equilibrium surface, bulk toe, and impact toe” [11]. The definitions given by them are as follows: “head = apex of particle trajectory, bulk toe = point of intersection of tumbling (cascading) charge with mill shell, and impact toe = region, where the cataracting charge impacts shell or bulk charge” (Figure 1). DEM has been taken into account by Djordjevic et al. in order to model the influences of lifter height (5 to 25 cm) and mill speed (50 to 90% of CS) on the power consumption and frequency distribution of specific energy (J/kg) against the normal impacts in a 5-m-diameter SAG mill [12]. In another study, Djordjevic has examined the effect of lifters on the power consumption of tumbling mills using DEM [13]. The validation of the results of DEM simulations was performed by comparing them with charge motion in a transparent laboratory mill

by Hlungwani et al. [14]. They scrutinized the effects of liner profiles and mill speed on the energy efficiency and mill capacity. Two types of square and trapezoidal lifter profiles were used to investigate the mill power and charge behavior. Rosales-Marín et al. [15] have evaluated the effect of face angle and wear of lifters as well as mill speed on the power draw and breakage rate of tumbling mills. By gently increasing the SAG mill length and exploring the charge trajectory and deformation under certain operating conditions, namely “ball filling, mill speed, and liner type”, the effect of the role of the end wall on the charge path by Maleki-Moghaddam et al. was examined [16]. The link between the shape characteristics of the charge and the filling level, and the lifter height inside a SAG mill was attained by Owen and Cleary [17]. The model development relating the charge shape and power consumption to the operating parameters of SAG mills and their usage in deciding on the mill operating strategies to calculate the wear of liners was done by Cleary and Owen [18]. The aspect of end liners in dry SAG mills was studied experimentally by Hasankhoei et al. in order to gain a full insight of the influences of end liner design on the charge trajectory and performance of SAG mills [19]. The laboratory works were performed in a smaller scale one-meter-diameter mill. It was demonstrated that the liners did not experience any deformation in the initial and final 20% of the mill length, which were protected by deflecting liners [19]. Usman [20] and Usman et al. [21] have examined the effects of lifter configurations and mill speeds and other operational parameters on the efficiency, power draw, and performance of ball mills. Rezaeizadeh et al. [22] have shown that in order to achieve a higher impact frequency and amount of impact, which can lead to a higher overall efficiency, the mineral processing plant engineer must increase the number of lifters, lifter height, and mill speed but must decrease the mill filling. This study also showed that the milling power had a linear relationship with the height and distance of the lifters (S/H) and the milling speed. Recently, Jahani Chegeni and Kolahi [23] have investigated the effect of seven different types of liners on the performance of industrial scale SAG mills by DEM. They concluded that the Osborn liner could be suggested as the best liner for SAG mills due to its appropriate number, height, and width of the lifter and its angularity [23]. They showed that the type of liners, which was a function of the angularity of the lifters, the width of the lifters, and especially their height, strongly affected the

performance of SAG mills. In another study, similar to the present study, Kolahi and Jahani Chegeni [24] examined the influence of the number of lifters on the performance of SAG mills by the DEM method. They concluded that the optimal number of lifters for SAG mills on a semi-industrial scale was between 16 and 32, and for the industrial SAG mills was between 32 and 64 [24]. In the research related to the SAG mill, 15% of the mill volume was filled with balls, while in the present article, 34% of the mill volume was filled with balls. Also, the geometry of the mills, the size of the balls, and the effect of the lifters on them as well as the results were completely different. Yahyaei et al. [25] have extended a method to design liners for performance through investigating the effect of relining efficiency in an industrial case. Xu et al. [26] have investigated the effect of particle shape on liner wear in tumbling mills using DEM. Based on the results of the particle motions, it was revealed that the sliding of the particles on the lifters had a large effect on wear. In another study, Xu et al. [27] conducted a multi-level DEM study on liner wear in tumbling mills for an engineering level approach. Chimwan and Bwalya [28] have used DEM to investigate how shell liner can induce ball segregation in a ball mill. They found that varying axial liner profile configuration could affect ball segregation, particularly for the mill running at 75% of the critical speed (CS).

From previous research, there are simple relationships to determine the number of lifters in different mills, all of which are covered in this section. Banisi [29] has proposed the following formula for the number of lifters of SAG mills:

$$\text{Number of lifters} = 2 \times \text{diameter of mill in feet} \quad (1)$$

However, in the same reference, there are two examples that violate this formula. The SAG mill of concentration plant No. 2 of Sarcheshmeh Copper Complex with a diameter of 9.7 m (31.8 feet) has 120 lifters, while according to the formula, it should have 64 lifters. The diameter of the SAG mill of Los Pelambres mine is also 9.1 m (29.9 feet) in diameter but it has 72 lifters, while according to the formula, it should have 60 lifters. These examples show that the above formula cannot be reliable. Also in Gupta and Yan [30], there are the following relations to determine the number of lifters for double-wave and single-wave liners, in which D is the diameter of the mill:

$$\begin{aligned} \text{Number of lifters} &\approx \\ 3.3 \pi D \text{ (m) for double wave liners} &\approx \quad (2) \\ 6.6 D \text{ (m) for single wave liners} & \end{aligned}$$

As it can be seen, these relationships are only proposed for double-wave and single-wave liners, and are not valid for other types of liners. Also, in Mular et al. [31], the following relationships are presented for rod and ball mills (for double-wave and single-wave liners):

In rod mills, the number of lifters is approximately equal to:

$$\begin{aligned} \text{Number of lifters} &\approx 6.6 \times D \text{ (m)} \\ \text{Number of lifters} &\approx 2 \times D \text{ (ft)} \end{aligned} \quad (3)$$

In ball mills, with double-wave liners, the number of lifters is equal to:

$$\begin{aligned} \text{Number of lifters} &= 13.1D \text{ (m)} \\ \text{Number of lifters} &= 13.1D \text{ (ft)}/3.3 \end{aligned} \quad (4)$$

In ball mills, with single-wave liners, the number of lifters is equal to:

$$\begin{aligned} \text{Number of lifters} &= 6.6D \text{ (m)} + 2 \\ \text{Number of lifters} &= 2D \text{ (ft)} + 2 \end{aligned} \quad (5)$$

As it can be seen, the formulas presented here are only valid for double-wave and single-wave liners, and are not applicable to other types of liners. In Kawatra [32], the following equations are provided for the number of lifters:

In large mills:

$$36 + D \text{ (ft)} \quad (6)$$

For HiLo lifters with smooth cover:

$$2 \times D \text{ (ft)} \quad (7)$$

As it can be seen, Equation 6 is more valid for SAG mills with large diameters, and Equation 7 is only valid for certain HiLo liners, and is not applicable to other types of liners. Sherman and Rajamani [33] in a 1999 study showed that the number of lifters required for a mill with a diameter D (in feet) was only D lifters, instead of the existing 2D rule. As it can be seen, the 2D relation is very old and the relation presented by Sherman and Rajamani is also incorrect in the opinion of the authors of this article. For example, for the mill studied in the present study, which has a diameter of 1.11 m, equivalent to 3.64 ft, Sherman and Rajamani suggest 4 lifters, which is very insufficient according to the simulations performed, and these formulas seem to be somewhat valid only for very large SAG mills (over 10 m in diameter). In the Mineral processing and Extractive Metallurgy Handbook [34], the following equation is provided for the relation between the height of the lifters and the distance between them:

$$B = (1 - FCs) \times A \quad (8)$$

where B is the lifter height, A is the distance between the lifters, and FCs is the fraction of CS of the mill. This equation shows that the height and width of the lifter as well as the rotation speed of the mill affect the number of lifters. According to formula 8 and the research work done by Rezaeizadeh *et al.* [22] as well as the results of the simulations performed in this research, it can be said that determining the number of lifters is not only correct through the diameter of the mill and it does not have high accuracy, because the number of lifters, in addition to the diameter of the mill, depends on the type of lifters, that is, the height, width, and angularity or the waveform or smoothness of the lifter and also the rotation speed of the mill. DEM simulation can be a good way to determine an appropriate range for the number of different liner lifters by considering all the effective factors.

In this research, DEM is utilized to simulate the milling operation of semi-industrial ball mills using a powerful open-source software called LIGGGHTS. Initially, a semi-industrial ball mill with dimensions of 2.0 m × 1.11 m without any lifter is simulated. Afterwards, by insetting 1, 2, 4, 8, 16, 20, 26, 30, and 32 cuboid lifters with dimensions of 2 m × 5 cm × 5 cm, respectively, nine other independent simulations have been performed. Also in this study, based on the definitions given by Powell and McBride [11] for head, shoulder, bulk toe and impact toe points (Figure 1a) to explore the impact mechanism, and consequently, improve the mill performance, for the first time two new parameters presented by the authors, that is, 'head height' (HH) and 'impact zone length' (IZL) are regarded as the basis for selecting the appropriate range for the number of cuboid lifters in a semi-industrial ball mill. 'HH' of the charge means the distance from the lowest inner part of the mill cylinder to the point of the head, and 'IZL' means the linear distance between the bulk toe and the impact toe (Figure 1b)). Additionally, the effects of the number of lifters on creating cascading, cataracting, and centrifugal motions for balls at two different mill speeds, that is, 70 and 80% of its CS have been appraised. On the other hand, for validating the simulation results, a laboratory-scale ball mill with dimensions of 0.16 cm × 57.3 cm has been simulated.

In summary, in this study, in order to find a suitable range for the number of lifters in the liner of ball mills, unlike the previous works that are based only the diameter of the mill on finding the

number of lifters, other parameters such as the height and width of the lifter, angularity of the lifter, and, in general, the shape of the lifter are also considered in the simulation of the discrete element method (DEM), and it has been shown that the diameter of the mill alone cannot be a suitable basis for determining the number of lifters in ball mills, and the effective factors mentioned above should also be considered. On the other hand, it has been shown that DEM simulation, taking into account all these effective factors, can be a suitable way to determine an appropriate range for the number of lifters of different liners. Also, unlike all the previous studies that considered the shoulder height and bulk toe as the basis for proper mill performance analysis, 'HH' and 'IZL' are considered as the basis of mill performance. It has been shown that the shoulder height is not a good basis, and 'HH' should be considered instead. It is also shown that the bulk toe alone cannot be the basis of analysis but its distance from the impact toe, i.e. 'IZL' should be the basis of analysis. In other words, the criteria for our measurements are HH and IZL. It has been claimed that the longer the IZL and the higher the HH, the better the comminution. Because if the HH increases, the balls gain more potential energy, and when they fall into the impact zone, they will have more kinetic energy, and consequently, more speed, and can cause the particles to break due to the impact mechanism. Also, if IZL increases, the probability of particles hitting this area will increase, and as a result, the probability of more effective comminution will increase.

2. Equipment and methods

2.1. DEM for predicting particle flow

In this study, the LIGGGHTS DEM solver was used to perform the ball mill simulations. The theory of DEM, the introduction of the LIGGGHTS open-source software, the relations, and the physical equations used in it have been detailed in the previous articles by the authors [23, 24, and 35]. Therefore, their repetition is avoided here. In summary, LIGGGHTS uses the Hertz–Mindlin's contact force law, which performs DEM simulations based on the soft particle method. Regarding that, the shear modulus (N/m²) can be computed using the Young's modulus and Poisson ratio, the following factors are used in the Hertz–Mindlin contact model [35]: Young's modulus (N/m²), Poisson ratio, coefficient of sliding friction, coefficient of rolling friction, and coefficient of restitution.

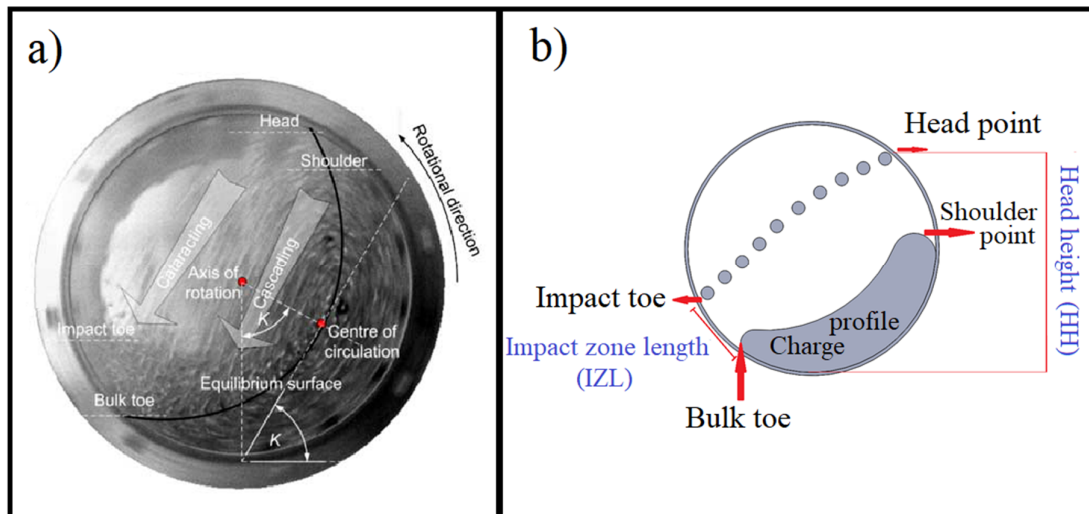


Figure 1. a) Medium motion and milling zones (head, departure shoulder, center of circulation, equilibrium surface, bulk toe, and impact toe) introduced by Powell and McBride [11]; b) Head, shoulder, bulk toe, and impact toe points as well as 'HH' and 'IZL' used and defined in this research.

2.2. Ball mill configuration

In this research, a semi-industrial ball mill with dimensions of 2.0×1.11 m without any lifter was simulated by DEM. Later, by insetting, respectively, one, two, four, eight, sixteen, and thirty-two (2 to the power of n and $n = 0, 1, 2, 3, 4, 5$) cuboid lifters with dimensions of 2 meters \times 5 cm \times 5 cm, six other separate simulations were run (Figure 2). Of course, it is already clear that one or two lifters and even four lifters are not enough for a semi-industrial ball mill. However, in order for the number of lifters to have a logical formula, this was done. This was also done in order to explain the continuity of the motion of the balls inside the mill. As it will be seen in Section 3, the liners with eight or more lifters will produce cataracting motions at all times. However, the liners with one, two, and four lifters only create cataracting motions when the lifter is in the right place; in other words, there is no continuity of ball motion in these liners.

2.2.1. Field research on industrial ball mills

According to the field research in several mineral processing plants active in the industry, the number of lifters used in their liners were twenty, twenty-six, thirty, and thirty-six lifters, respectively. Therefore, four more simulations were performed for this number of lifters (Figure 2). However, the simulation with 36 lifters was not successful, and caused the mill geometry to rupture. The reason for the rupture of the mill geometry when using 36 lifters is that the particles fall from the insertion

face at gravity acceleration and have a high velocity, and their kinetic energy is extremely high when they hit the wall of a moving mill, and due to the reduction of the distance between the lifters and the reduction of the volume of the mill, the kinetic energy becomes approximately ten times of the normal state, which due to the material of the balls and their high density, this speed can cause the geometry of the mill to rupture (Figure 3). By increasing the time step, it is possible to make the balls enter the geometry of the mill with a smaller number (reduction of tonnage and flow rate of the mill) but since the aim is to compare the number of lifters, the simulation conditions must be the same for all of them. For example, one simulation cannot be done with 60,000 time steps and another with 600,000 time steps, and compare their data. Also, in the simulations, it has been tried that the input feed flow is the same in all mills, and is close to the operating state. In total, it is possible to perform the simulations with 36 or more lifters without tearing the geometry but if this is done, the possibility of comparison is eliminated. Also, according to the width of the cuboid lifters, a maximum of 64 lifters can be installed in the liner of the mentioned ball mill. Adding 64 lifters makes the entire inner wall of the mill fill with lifters, and the lifters act as liners, making the mill look like a smooth mill (without lifters) with a thicker liner and less internal volume. Therefore, the maximum number of lifters was considered to be 32 (Figure 2). Then, using DEM, the effects of the number of lifters on the shoulder, bulk toe, impact toe, and charge head points as well as on 'HH' and 'IZL' and, in general, on the comminution mechanism was studied.

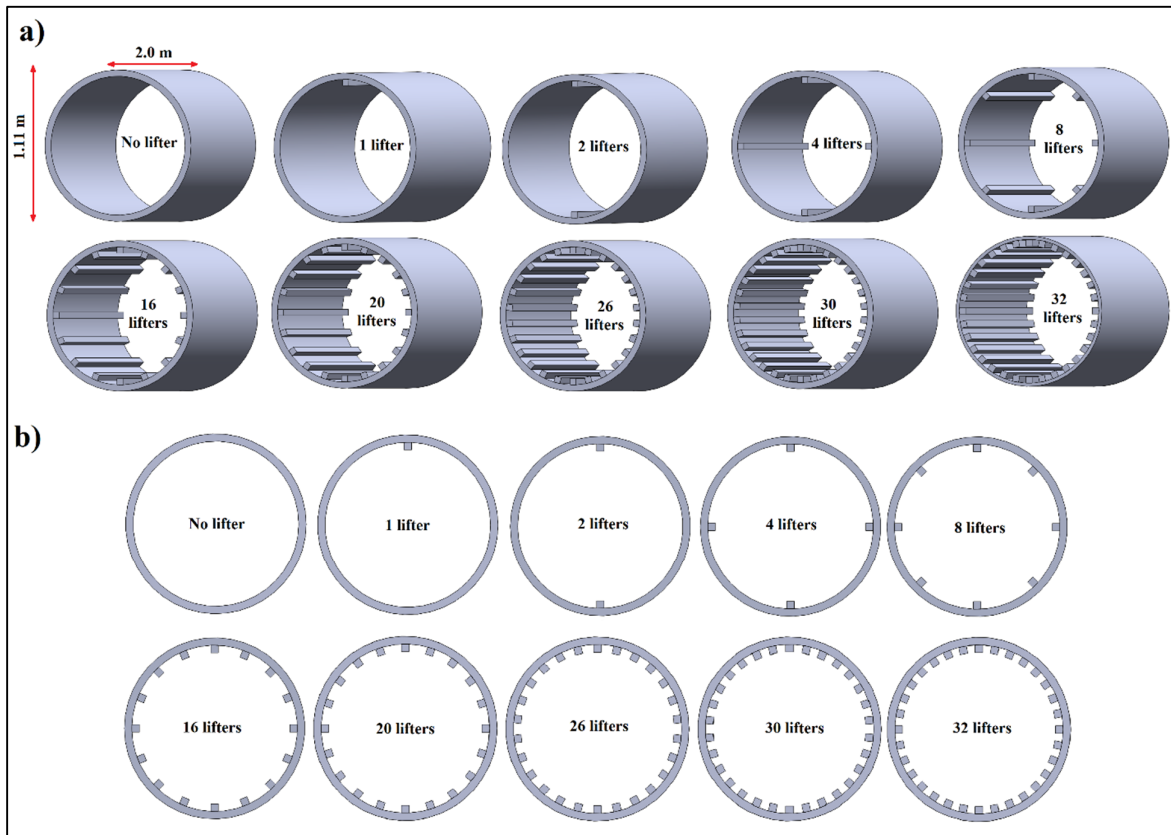


Figure 2. a) 3D and b) 2D geometries of semi-industrial ball mills with zero to thirty-two lifters.

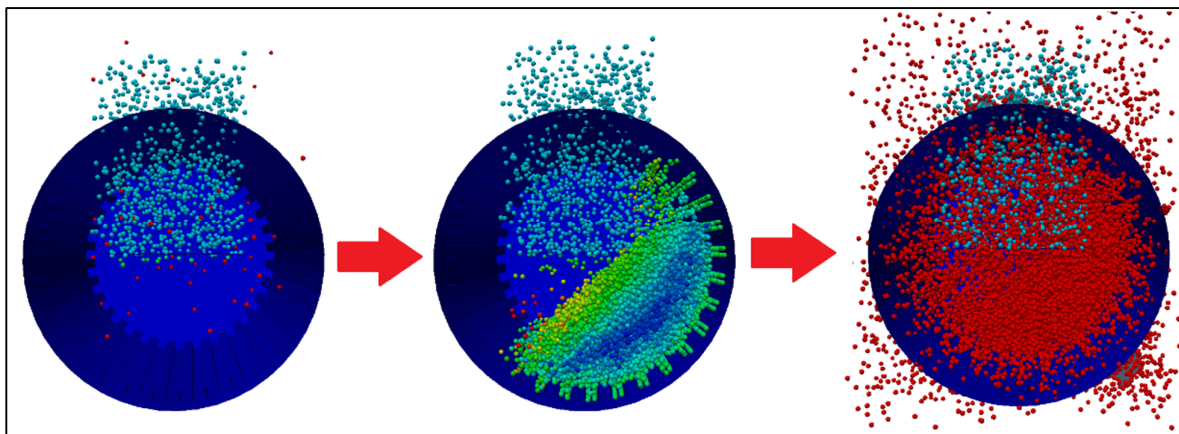


Figure 3. Unsuccessful simulation of a semi-industrial scale ball mill with thirty-six lifters, geometry disruption due to excessive kinetic energy of the balls.

2.2.2. Operating and geometric conditions, calculations, and parameters of DEM simulations

The detailed operating and geometric conditions, material properties, and calculations for these semi-industrial ball mills are tabulated in Tables 1 to 3. In this study, all balls have the same diameter (3 cm). The reason for keeping the diameter of the balls constant is to prevent the effect of changing their size on the charge HH and the IZL.

Optimizing the size distribution of balls for all liners used in this study is the aim of future research works by the authors. In Table 2, the particle interaction distance (neighborhood), i.e. the distance that the particles exert a vertical and shear force on each other, is calculated as follows: One-twentieth (5%) radius of the smallest particle (15 mm). It is noteworthy to mention that the material of the balls and walls of the mills used in these simulations is stainless steel. The parameters used

in Table 3 such as the ball density, ball sliding coefficient, ball rolling coefficient, Poisson ratio, Young's modulus, and ball restitution coefficient belong to stainless steel, and obtained from the reputable internet websites. However, as we know, the Young's modulus of stainless steel is between 190 and 210 GPa. What is in the table is half hundredth (1/200) of this value. Unfortunately, at the moment, the LIGGGHTS software considers the maximum Young's modulus to be 1 GPa, and does not accept higher values. But in a near future, this amount will definitely increase, as if by 2015 the maximum Young's modulus was considered to be 1 MPa. Therefore, since this software is an open source, it requires validation.

Table 2. Calculations and specifications of DEM balls.

Ball diameter (mm)	30
Volume of one ball (m ³)	1.4137×10^{-5}
Filling of mill ball charge (%)	34
Volume of all balls (m ³)	$17\% \times 1.9393 = 0.3297$
Number of balls	$0.3297 / (1.4137 \times 10^{-5}) = 23319$
Ball density (kg/m ³)	8050
Total mass of mill charge (kg)	2653.87
Total mass of balls (kg)	$8050 \times 0.3297 = 2653.87$
Mass rate (kg)	2653.9 (10 stages)
Particle interaction distance (m)	7.5×10^{-4}

Table 3 - Parameters of DEM simulations

DEM model details	Value
DEM spring constant (kg/m)	10^6
Sliding friction coefficient of balls	0.5
Rolling friction coefficient of balls	0.0015
Poisons ratio	0.285
Young's modulus (N/m ²)	10^9
Restitution coefficient of balls	0.817

3. Results and Discussion

3.1. DEM simulations of semi-industrial ball mills

Figure 4 shows the 3D simulation snapshots (front view) of semi-industrial ball mills using DEM with different numbers of lifters, respectively: with zero, one, two, four, eight, sixteen, and thirty-two lifters when the mill rotates

Table 1. Dimensions and velocities of pilot-scale ball mill.

Semi-industrial ball mill	Value
Mill shell thickness (cm)	2 – 5
Mill length (m)	2.0
Mill diameter (m)	1.11
Mill volume (m ³)	1.9393
CS (rpm)	40.40
Mill rotation direction	Counter-clockwise
Cuboid lifter length (m)	2
Cuboid lifter height (cm)	5
Cuboid lifter width (cm)	5

a) at 70% and b) at 80% of its CS. According to the obtained results and observations, the best number of lifters for ball mills is between 16 and 32. The vastness of this range causes the accuracy of the results reduce. As a result, in order to validate the results obtained in this stage and also to further investigate them, field research and other simulations were performed. According to the field research in several mineral processing plants, the number of lifters used in their liners were twenty, twenty-six, thirty, and thirty-six lifters, respectively. The number of lifters used in the ball mill of Baharieh Kaolin mine located in Kashmar, Khorasan Razavi is twenty lifters, in the ball mill of Esmiran Iron Ore Complex located in Sirjan, Kerman is twenty-six lifters, in that of Opal Parsian Sangan Industrial & Mining (OPSIM) company located in Sangan, Khorasan Razavi is thirty lifters and also in that of the Asphalt Tous company located in Khaf, Khorasan Razavi is thirty-six lifters. Therefore, four other simulations were performed with the number of lifters: twenty, twenty-six, thirty, and thirty-six lifters, respectively (Figure 4). As mentioned in the simulation of a semi-industrial ball mill with thirty-six lifters, the geometry of the mill is torn, the reason is the excessive increase of kinetic energy of the balls and their collision with the mill wall (Figure 3). Therefore, the maximum number of lifters was considered to be thirty-two (Figure 4).

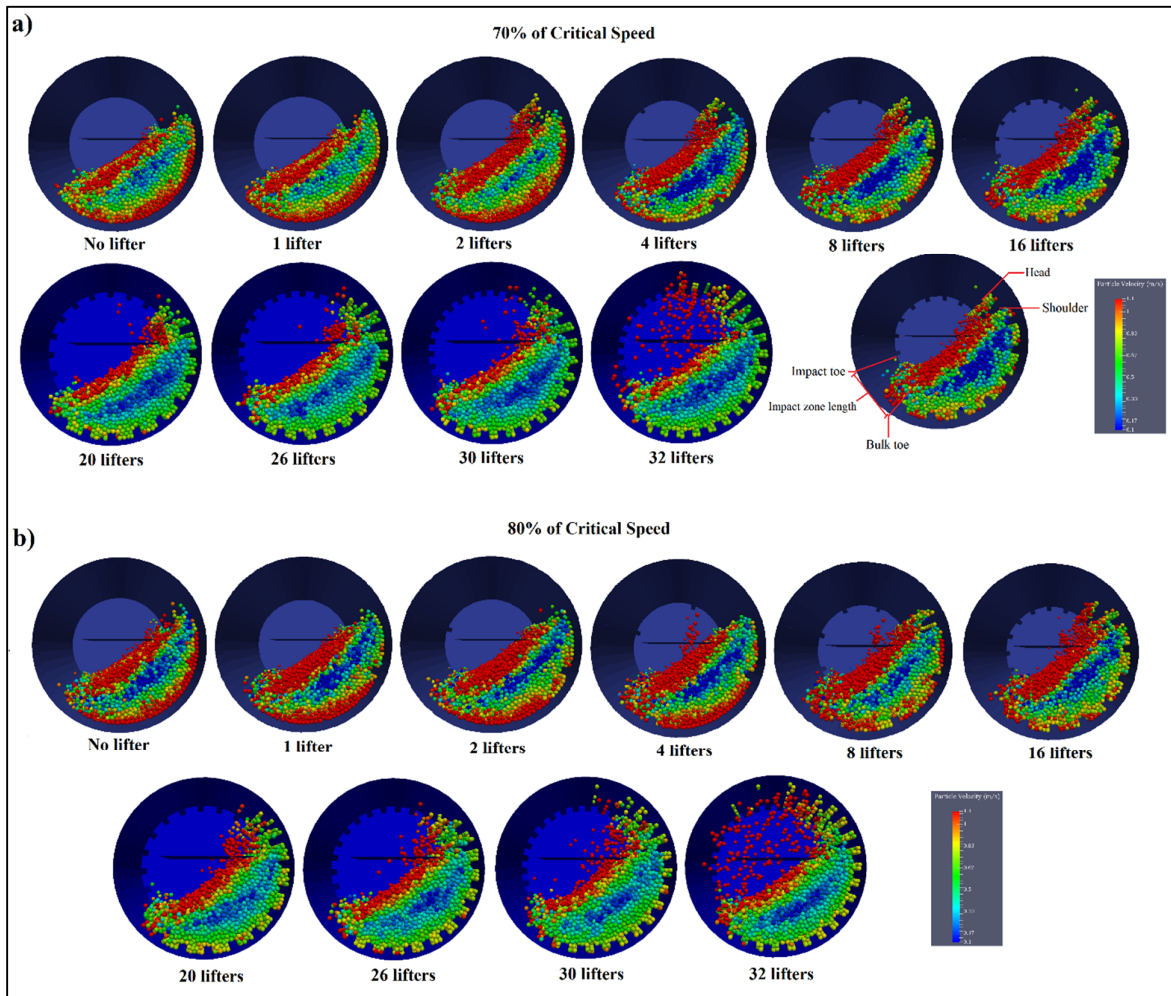


Figure 4. 3D simulation snapshots (front view) of semi-industrial ball mills with different numbers of lifters a) at 70% and b) at 80% of CS.

The results of the simulations and their related video files as well as the corresponding snapshots in Figure 4 show that a ball mill with even one cuboid edged lifter can create cataracting motions in the balls. However, the question that arises is that in Figure 4, in the case of one lifter, no cataracting motion is observed to confirm this result. As mentioned earlier, in the mills with one lifter, two lifters, and even four lifters, there is no continuity of ball motion, That is, the cataracting motion is observed only when the lifter is in the proper position. In other words, the liners with eight or more lifters produce cataracting motions at all times, although the number of balls with cataracting motions may be less in some, but there is anyway. Conversely, the liners with one, two, and four lifters only produce cataracting motions when the lifter is in the right place. In other words, there is no continuity of ball motion in these liners. Also, since the aim of this study was to compare the number of lifters, this comparison should be

done at a specific time (end of simulation time) (for example, at the time step of 60,000 and after 6 seconds from the beginning of the simulation). Therefore, when the simulation image or snapshot is taken, in some cases, especially when the number of lifters is less than eight, the lifter may not be in the right place, which is why the cataracting motion in some of the snapshots in Figure 4 is not visible. However, in the previous steps and in the simulation videos, this motion has been observed. Increasing the number of lifters in the mill (to an appropriate extent) causes more balls to participate in the comminution operation, which improves the performance of the mill and its impact mechanism. As shown in Figure 4 a), the two, four, eight, and sixteen lifter mills all produce the appropriate HH and cataract motion. However, in the two- and four-lifter modes, due to the small number of lifters, cataract motion occurs only when the lifters affect the charge, and at other times, their performance is similar to a no-lifter

mill. This figure also shows that increasing the number of lifters from four to sixteen does not have a significant effect on the magnitude of the cataract motion, and only increases the number of times this motion is created (continuity of operation), which, in turn, is very effective in improving the comminution operation and impact mechanism in the mill. However, in increasing the number of lifters from sixteen to thirty-two, it can be seen that in this increasing trend, in addition to increasing the number of creation of cataract motions, this motion itself has become larger and more impressive, and as a result has a greater and more effective impact on the comminution operation in the mill. It is worth noting that the cascade motion starts from the charge shoulder and ends at the bulk toe. But cataract motion means from the head point to the impact toe. Also, since both the head point and the impact toe are not fixed points and move depending on the rotation speed of the mill and the number of lifters, here this means that HH has increased. As a result, the beginning of the cataract motion has been from a higher height, and has caused the cataract motion or the cataract height to be larger. In a ball mill with thirty-two cuboid lifters, also the largest participation of balls in the impact mechanism and the largest cataracting motions is observed.

3.2. Effect of mill rotation speed

Figure 4b shows that by increasing the rotation speed of the mill by 10%, in all mills of four to thirty-two lifters, a cataract motion and suitable HH is created. The higher the HH, the higher the potential energy of the balls (particles). Consequently, according to the principle of energy conservation, when this energy is converted into kinetic energy at the impact toe, it can cause a better comminution. It is noteworthy that, unlike part a), no cataracting motion is observed in the two-lifter mode in the corresponding snapshot. This is because the number of lifters is low, and they were not in the right place when taking the simulation snapshot. The issue of lack of number of lifters is also true for the four-lifter mode. However, in the case of eight to thirty-two lifters, due to the sufficient number of lifters, the appropriate HH and cataracting motions are created at all times. Also, this increase in speed has caused a large number of balls to participate in the comminution operation, and cataracting motions have been created much more and with appropriate heights. In general, the best cataracting motions, impact mechanism, and the largest number of balls

that participate in the impact mechanism belong to the thirty-two lifter mill. However, as it can be seen, in this mill, a number of balls hit the wall and body of the mill, which is very inappropriate, and causes damage to the mill wall. Therefore, more research is required to get the best performance. It is worth noting that one, two or four lifters cannot play an effective role for the impact mechanism in a semi-industrial ball mill. Having a suitable number of lifters in a mill (according to the dimensions of the mill and its lifter design) and creating a suitable rotation speed for it creates the best performance, maximum comminution and impact mechanism. The best performance is achieved when both 'HH' and 'IZL' are increased. As HH increases, the potential energy of the balls increases, and more potential energy is converted into kinetic energy. As a result, when the balls hit the impact toe, they apply more energy to it and cause the impact mechanism to prevail in the comminution, which is desirable for us. Also, when the distance between the bulk toe and the impact toe increases (increasing IZL), more balls hit this zone (the probability of the balls hitting this zone increases) and cause the particles to break. As a result, the impact mechanism will prevail again, and comminution will be more effective. It should also be noted that the increase in IZL is due to the lowering of the bulk toe. If the balls hit above the impact point, they may damage the mill wall. Also, if the balls cannot reach the bulk toe due to low height or low speed, we will not have a good comminution.

3.3. Using an online protractor to determine heights and angles

Figure 5 shows how to determine the head point angle, and the angle between the bulk toe and the impact toe (in degrees) as well as HH and IZL (in centimeters) using an online protractor (https://www.gini-fab.com/feeds/angle_measurement/) [36] in semi-industrial ball mills with zero, one, two, four, eight, sixteen, twenty, twenty-six, thirty, and thirty-two lifters when the mill rotates at a) 70% and b) 80% of its CS using DEM. The above website allows you to use the simulation snapshots and adjust the online protractor on the snapshots to get the exact angle of the desired points. Here, to obtain the HH and the IZL, first, the angle of the head, and the difference between the angle of the bulk toe and the impact toe are obtained by the online protractor, and then these two parameters are calculated using the trigonometric relations (Table 4).

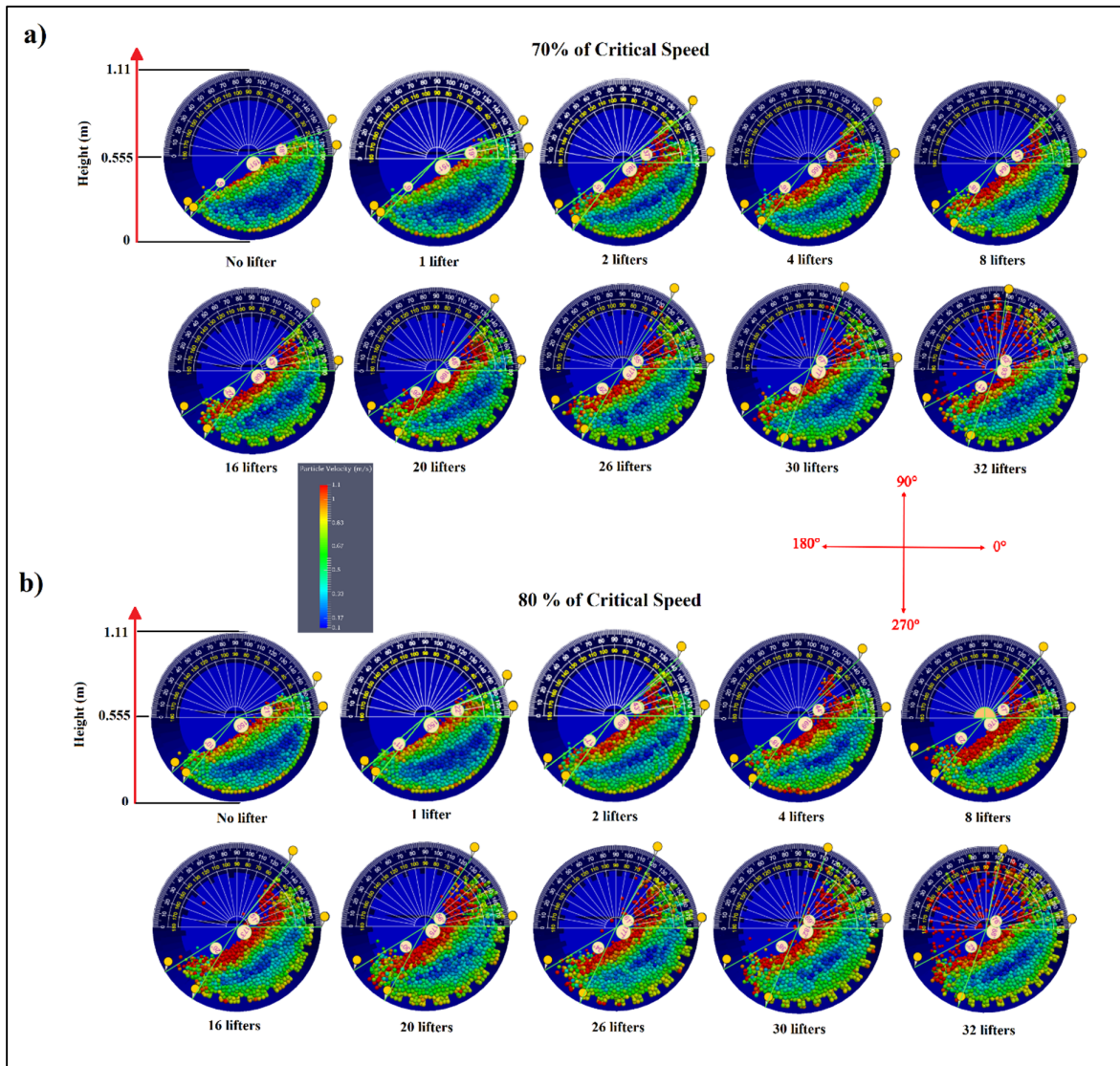


Figure 5. Online protractor to measure height (cm) and angle (degree) of head point and length of impact zone (cm) in semi-industrial scale ball mills with different numbers of lifters in a) 70% and b) 80% CS using DEM.

It is noteworthy that given that the exact coordinates of the individual particles and thus the head points, bulk toe and impact toe in DEM simulations are precisely known and measurable, and also due to the complete accuracy of the online protractor described above, the obtained angles are quite accurate, and as a result, their variance and standard deviation is zero, and is not required to be measured more than once.

3.4. Interpretation of simulation results

Figure 5 and Table 4 show the following results: cascading motion is observed in all liners at both mill rotation speeds. Also in all liners, except for the no-lifter and one-lifter liners, the cataracting motion is also generated at both 70% and 80% of

CS. In the two-lifter mode, as described earlier, only at 70% of CS, a cataracting motion is observed in the figure. Also, centrifugal motions are observed only in the thirty-two lifter mills. It should be noted that centrifugal motions (i.e. the balls sticking to the mill wall and not participating in the comminution operation) are different from the centrifugal force. There is a centrifugal force in all cases but centrifugal motions can be caused by the high number of lifters or the high speed of the mill. In the 32-lifter mill, due to the large number of lifters, some balls have centrifugal motions, which this issue can be solved by reduction of the mill speed to 65% or even 60% of CS. In the 32-lifter mode, the liner gives a high kinetic and potential energy to the balls, and according to the simulations, the dispersion of particles is higher. In

this case, the mill speed can be reduced to prevent the balls from hitting the wall of the mill and damaging it. In other words, the balls hit the wall of the mill at a higher height than the impact toe and are outside the impact zone. A 10% increase in mill speed has increased the charge HH and the IZL in all mill liners. At both CSs, the minimum of the charge HH and the IZL are related to the no-lifter mill. Also, in 70% and 80% of CS, the highest charge HH is related to the thirty-two-lifter mills by 110.46 and 109.79 cm, respectively, and also the longest IZL is related to the thirty-two-lifter mills by 38.87 and 39.78 cm, respectively. In general, the highest charge HH is created in the thirty-two-lifter mill, at 70% of CS and the longest

IZL is created in the thirty-two-lifter mill, at 80% of CS. For the range of 20 to 30 industrial lifters, Figure 5 and Table 4 show the following results: cascading and cataracting motions are observed in all mills at both mill rotation speeds. At 70% of CS, the maximum IZL is 33.38 cm for the thirty-lifter mode, and also at 80% of CS, the maximum IZL is 36.14 cm for the thirty-lifter mill. The highest charge HH is related to the thirty-lifter mill, which is equal to 106.59 cm at 70% of CS and 107.31 cm at 80% of CS. It can be concluded that the thirty-lifter mill will be the optimal choice, and can prevail the impact mechanism better than the other two mills, and therefore, has the best performance in comminution.

Table 4. Angle values and height of head point as well as angle difference between bulk toe and impact toe and length of the ball impact zone for semi-industrial ball mills with different numbers of lifters at 70% and 80% of mill CS.

Number of lifters	CS (%)	Observing cataract motion in simulation snapshots	HH (cm)	IZL (cm)	Head angle (degree)	Angle difference between bulk toe and impact toe (degree)
0	70	No	72.41	4.81	18	5
1	70	No	72.56	4.92	19	6
2	70	Yes	86.42	15.31	37	15
4	70	Yes	89.67	15.45	38	16
8	70	Yes	91.91	17.36	41	18
16	70	Yes	93.35	21.18	43	22
20	70	Yes	96.74	24.97	48	26
26	70	Yes	100.96	27.79	55	29
30	70	Yes	106.59	33.38	67	35
32	70	Yes	110.46	38.87	82	41
0	80	No	75.39	9.67	21	10
1	80	No	79.42	9.72	22	11
2	80	No	93.35	17.24	43	17
4	80	Yes	94.05	17.36	44	18
8	80	Yes	96.09	22.13	47	23
16	80	Yes	98.63	24.97	51	26
20	80	Yes	101.51	28.73	56	30
26	80	Yes	104.04	32.45	61	34
30	80	Yes	107.31	36.14	69	38
32	80	Yes	109.79	39.78	78	42

3.4.1. Investigation of values of HH and IZL

The effect of the number of shell lifters of the semi-industrial scale ball mill on the charge HH and the IZL at different mill speeds is shown in Figure 6.

As shown in Figure 6, at 70% of CS, the highest charge HH is related to the ball mill with thirty-two lifters. As a result, the cataracting motions and impact mechanism in this liner are more pronounced than in other liners, which improves mill performance. The charge HH of the other liners is also significantly increased compared to the no-lifter and one-lifter liners. Especially from the number of eight lifters upwards, the increase in

the charge HH has had a significant trend. In general, it is observed that increasing the number of lifters has enhanced the charge HH, and there is a direct relationship between them. Also in this figure, the influence of the number of mill shell lifters on IZL at 70% of CS is shown. According to the diagram, IZL of all liners has increased compared to the no-lifter and one-lifter liners. This means that the trend of the influence of the number of lifters on the IZL is ascending. However, the growth of IZL has been more dramatic in the 32-lifter mills. The 32-lifter liner has also had the most positive effect. As mentioned earlier, IZL is the distance between the bulk toe and the impact toe;

the longer the distance, the better. However, if the balls hit the points above the impact toe, in other words, they hit the mill wall outside the impact zone, as is the case with the 32-lifter liner; this is undesirable and can cause the liner to break and damage it. Figure 6 also shows the effect of the number of mill shell lifters on the charge HH at 80% of CS. All liners have an ascending trend relative to each other. The diagram shows that the addition of the lifters at 80% of CS has a great effect on the charge HH. This figure also shows the effect of the number of mill shell lifters on IZL at 80% of CS. As observed, increasing the number of lifters at 80% of CS has increased IZL. In general, it can be concluded that increasing the number of cuboid lifters in a ball mill (according to the dimensions of the mill and the design of the geometric shape of the lifter) improves the performance of the mill and its comminution operation. It is suitable if the balls hit the impact zone but if they hit the mill wall outside the impact zone, it is inappropriate, and will cause damage to the mill body or break the liners. In general, as the distance between the bulk toe and the impact toe increases (increasing IZL), more balls hit this area (the probability of the balls hitting this area increases) and cause the particles to break. As a result, the impact mechanism will prevail, and comminution will be more effective and the efficiency of the mill will be higher. Usually IZL is

increased due to the lowering of the bulk toe and the charge profile (Figure 1). If the balls do not reach the bulk toe and fall on top of each other (due to the lack of the number of lifters or low mill speed), the efficiency of the mill will decrease. On the other hand, if the balls hit the points above the impact toe (due to the high number of lifters or the high speed of the mill), the efficiency of the mill will still decrease, and the balls can cause a serious damage to the mill wall. However, if the balls hit the impact zone, the efficiency of the mill will increase because their high potential energy has been completely and optimally converted into kinetic energy, and causes the balls to break the particles by the impact mechanism. According to all the simulations performed in this study, it can be concluded that the trend of the effect of the number of lifters on the performance and comminution rate of ball mills is ascending up to a certain number, considering the dimensions of the mill itself, design of lifter dimensions, mill rotational speed, and the size of the balls used in it, and from then on, due to all the factors affecting this, it causes the mill shell to wear out, the lifters to break, and as a result, to create turbulence in the mill. For each type of ball mill, by performing various simulations, its optimal state can be obtained, and it can be used the most and most effectively in the industry.

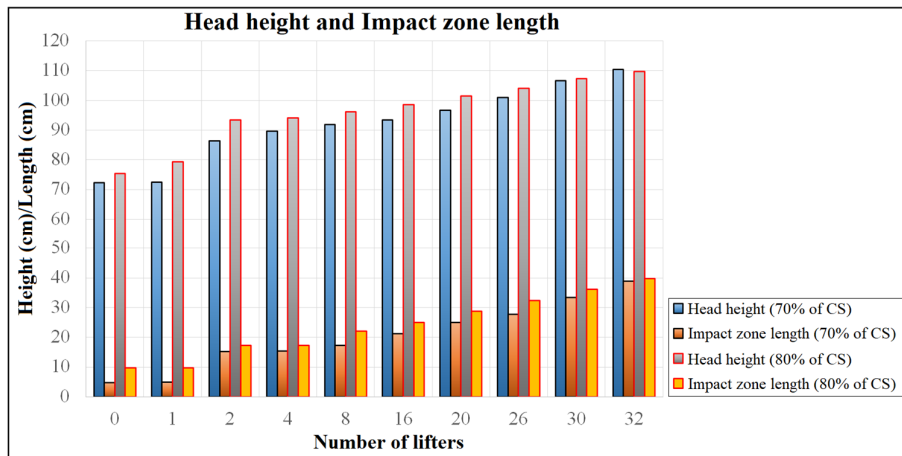


Figure 6. Influence of the number of lifters on HH and IZL at 70% and 80% of CS.

3.4.2. Effect of mill rotation speed on HH and IZL

Besides, Figure 6 demonstrates the effect of the mill rotation speed on the charge HH and IZL for different numbers of mill shell lifters. The effect of a 10% increase in the mill rotation speed on the charge HH from no-lifter liner to thirty-lifter liner

is an upward trend and has continued but in the thirty-two-lifter liner, it has also reduced the charge HH. Also, the effect of a 10% increase in the mill rotation speed on IZL shows that the effect of the increase in speed initially had a rapid upward trend, and increased IZL but the rate of this increase has gradually diminished. As a result, it emphasizes that increasing CS in a particular dimension of the

ball mill in all the number of lifters does not have a positive effect on the charge HH and the IZL, and if exceeded to some extent, has a negative effect on the comminution operation, which is due to the influence of other factors. It is noteworthy that in the thirty-two-lifter liner, a 10% increase in the mill speed has caused the number of balls to participate in the impact mechanism and comminution operation to be much higher. However, this has caused a large number of balls to hit the mill wall and cause damage to it, which cannot be ignored, and it is recommended to reduce the mill speed during these cases. As mentioned earlier, it is desirable if collisions occur in the impact zone, and the longer the impact zone, the greater the likelihood of such collisions. However, it is undesirable if the collisions occur outside the impact zone (points higher than the impact toe), i.e. the mill wall. In the present study, for a semi-industrial scale ball mill with thirty-two lifters, 60% to 65% CS is recommended because in this case, it has the best performance and comminution compared to all the other liners. In general, it can be concluded that in a ball mill, according to the design of the geometric shape of the lifter, the use of 16 to 32 lifters improves the performance of the mill, the impact mechanism, and the comminution operation in it. Figure 6 emphasizes that the best number of lifters for a ball mill is 16 to 32 lifters; this number of lifters has also performed well for the ball mills available in the industry, and indicates that the proposed number of lifters is correct and practical.

3.5. Validation

As we know, the new approach in DEM is calibration, not validation. Calibration means a purposeful change of the simulation parameters in

order to achieve consistency with the reality. However, this is only true for a commercial software such as PFC3D and EDEM because the commercial softwares are validated, and their results are completely reliable. For example, to account for the particle shape or particle repose angle in the spherical particle simulations, sliding and rolling friction coefficients can be increased to account for the effect of particle non-sphericity, and thus calibration can be done. However, in an open-source software such as LIGGGHTS, validation is inevitable. For example, as mentioned earlier, the Young's modulus of stainless steel is about 200 GPa, while the LIGGGHTS software does not allow the user to enter more than 1 GPa; this can cause simulation errors. Therefore, validation must be done for open-source softwares. On the other hand, since in this study the mill balls were completely spherical, no calibration was required. However, if we want to simulate the motion of non-spherical particles with the LIGGGHTS software, calibration must be done. It is worth noting that the simulation of non-spherical particles in ball mills is the subject of future research works by the authors.

In this study, a laboratory-scale ball mill was simulated in order to validate the simulation results (Figure 7). The reason the mill is larger in diameter than its length is that it is easier to photograph due to its shallower depth than conventional ball mills. Also, head, shoulder, bulk toe, and impact toe points are more easily recognizable than the conventional ball mills. This laboratory-scale ball mill has dimensions of 57.3×16 cm. The detailed operating and geometric conditions and material properties for this mill are available in the previous articles written by the authors [23, 24]. Therefore, their repetition was avoided here.

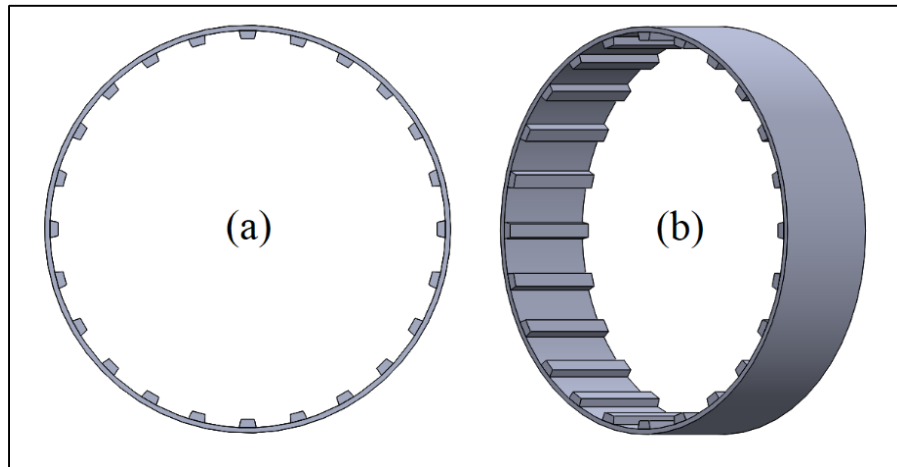


Figure 7. Laboratory-scale ball mill a) 2D geometry; b) 3D geometry.

In order to validate the results obtained as well as the open-source software used in this research (LIGGGHTS DEM solver), the charge HH and the IZL of the simulations performed with this software for the laboratory scale ball mill were

compared to the charge HH and the IZL of the experimental results under similar conditions (Figures 8–10 and Table 5). The high correlation between the results shows their validity as well as the validity of the DEM software (LIGGGHTS).

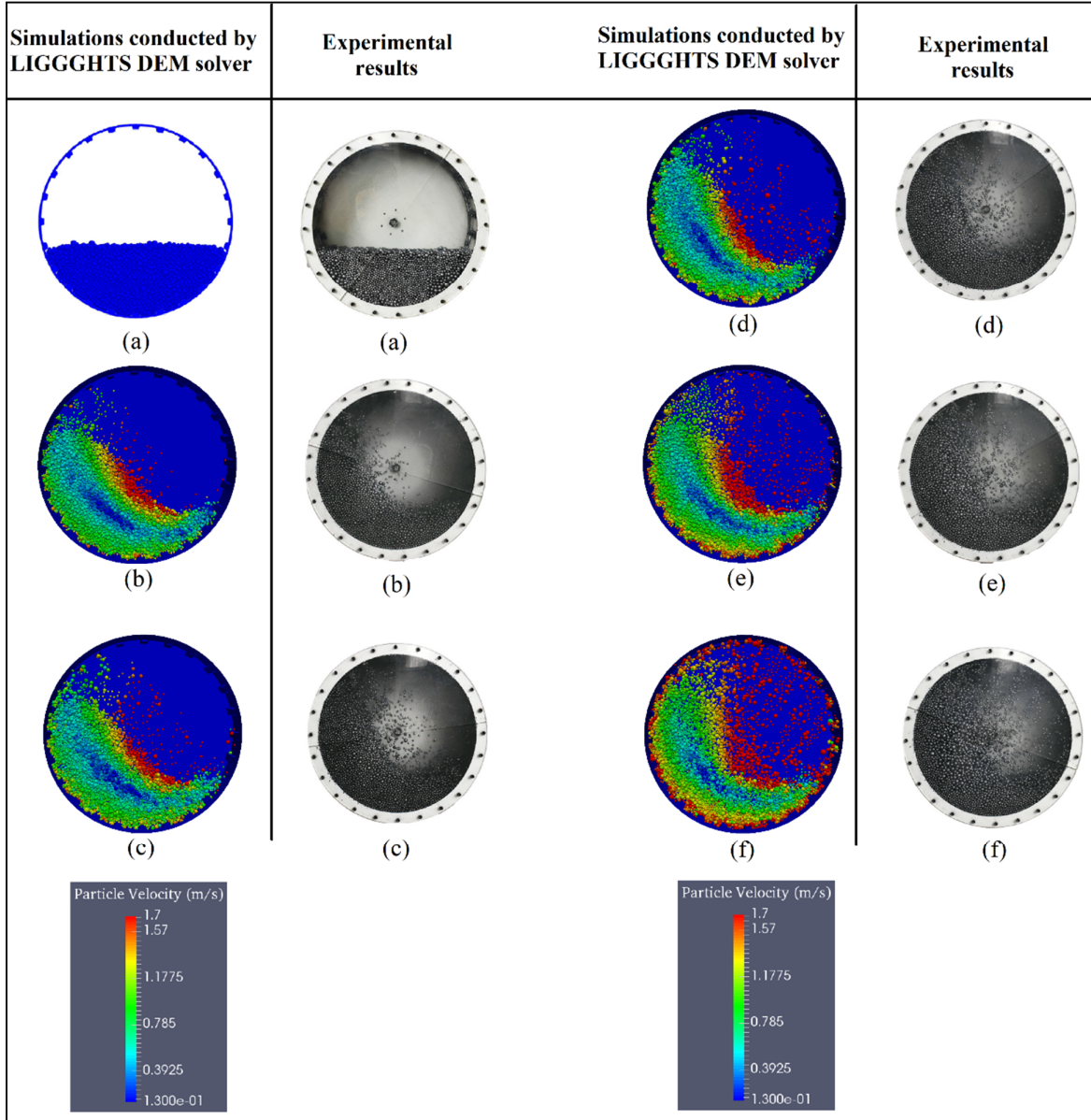


Figure 8. Comparison between simulation and experimental results at similar operating conditions when mill rotates at (a) 0%; (b) 60%; (c) 70%; (d) 75%; (e) 80%; and (f) 90% of CS.

Table 5. Comparison between HH (cm) and IZL (cm) of simulations of laboratory-scale ball mill and real images of experimental results at the same operating conditions.

Mill rotation speed (rpm)	Simulation results		Experimental results	
	HH (cm)	IZL (cm)	HH (cm)	IZL (cm)
32.79	51.23	4.50	50.92	4.99
38.26	52.68	7.97	52.68	7.48
40.99	54.18	9.46	55.57	7.97
43.72	55.90	12.40	56.57	11.91
49.19	56.95	15.79	57.14	18.18

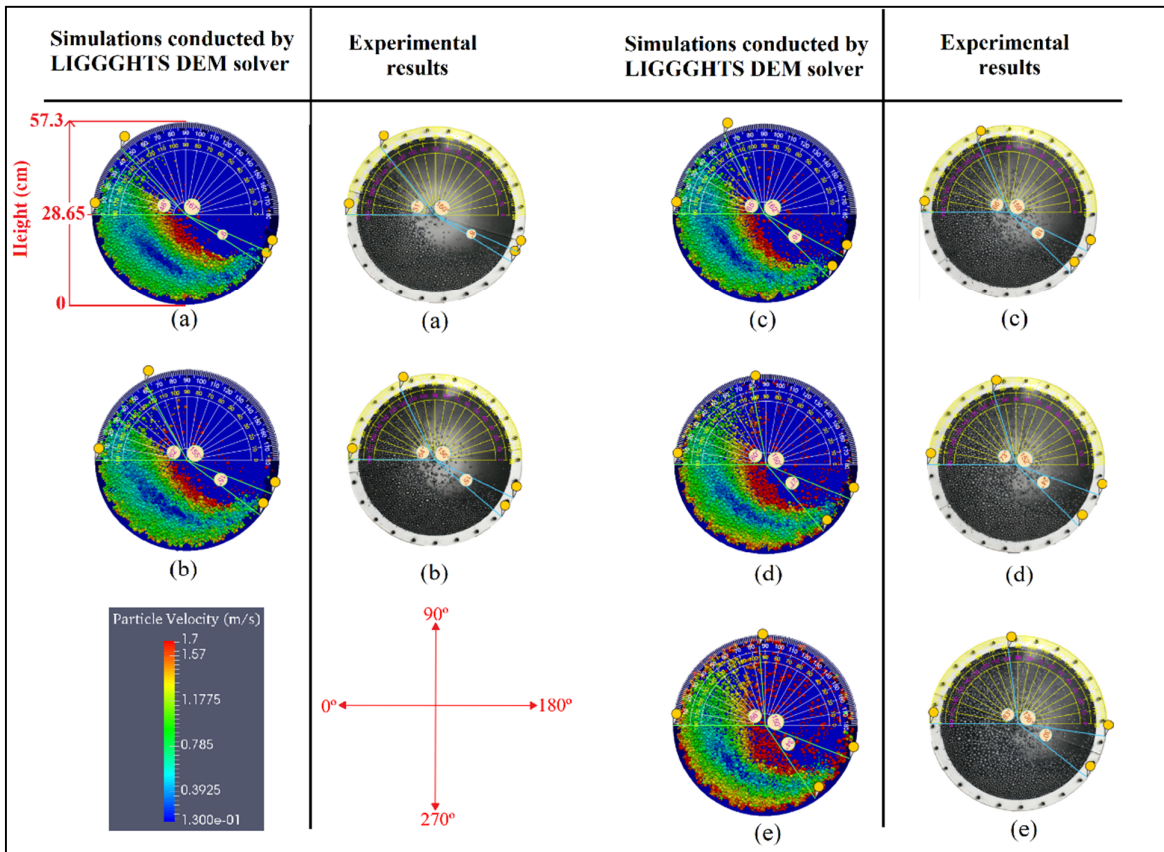


Figure 9. Online protractor for measuring and comparing HH (cm) and IZL (cm) of simulation snapshots of laboratory-scale ball mill and real images of experimental results at the same operating conditions when mill rotates at (a) 60%; (b) 70%; (c) 75%; (d) 80%; and (e) 90% of CS.

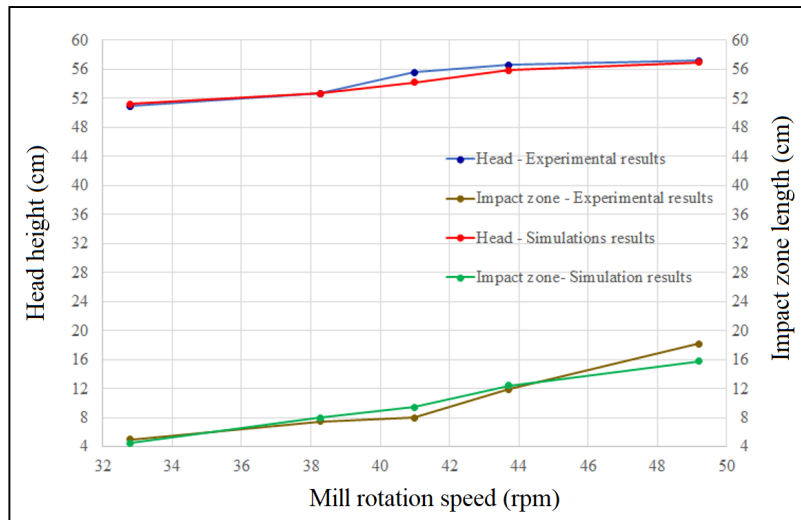


Figure 10. Comparison between the simulation and experimental results.

4. Conclusions

In this study, for the first time, a method was proposed by which an appropriate range of the number of cuboid lifters for ball mill liner could be determined. The proposed method can be done and

expanded for all types of lifters. In this method, by measuring ‘HH’ and ‘IZL’, an appropriate range of the number of lifters can be determined. Unlike the previous research works, it has been shown that the number of ball mill lifters does not only depend on the diameter of the mill but also depends on the

width, height, angle of the lifter, and generally on the type of lifter. In particular, in this study, considering the dimensions of the mill, the size of the balls, and the dimensions of the cuboid lifters, the proposed range was between 16 and 32 lifters. In other words, the parameters mentioned above and the mill speed were all considered in the input file of the DEM software so DEM could be an appropriate method and tool to determine the number of lifters. As shown in Figure 1, the head point and bulk toe have already been used by the others. However, HH from which the highest potential energy of particles can be calculated and the impact toe and its distance from the bulk toe, i.e. IZL, which indicates the conversion of particle potential energy to useful kinetic energy, have been introduced by the authors. Also, the online protractor was first used by the authors to measure the angles. With the increase in the number of lifters from 36 to 64, due to the reduction of their distance, their role practically diminishes, and the mill operates the same as a no-lifter mill, which has a smaller volume than the original mill. In other words, the volume of the mill is reduced by the total volume of the lifters. Therefore, there are optimum values for HH and IZL. For example, in our simulations, the 32-lifter mill had the maximum HH and IZL. If the number of lifters increases to be within the optimal range of 16 to 32 lifters, the distance between the bulk toe and the impact toe, i.e. IZL, increases, and if the balls hit the impact zone when they hit the mill wall, the comminution will take place optimally. However, if the balls fall on the charge profile due to the low number of lifters or the low speed of the mill, i.e. they do not reach the bulk toe, the comminution will not be desirable. On the other hand, if due to the excessive number of lifters or the high speed of the mill, the balls hit higher points of the impact toe, i.e. outside the impact zone, the comminution will not be done properly, and the energy of the balls may damage the mill wall and break the liners. The mill rotation speed rate and its number of lifters affect the performance of the mill, and there is an inverse relationship between them. Therefore, in order to improve the performance of the mill, a special proportion and balance must be established between them. Increasing the mill rotation speed costs less than increasing the number of lifters so a mill with a smaller number of lifters and a higher rotation speed can be used to improve the comminution performance, energy consumption, and lower costs. It is recommended that all the previous proposed relations to determine the number of different mill lifters

obtained by the previous researchers and scientists based solely on the mill diameter be re-examined using DEM in order to verify their accuracy or inaccuracy.

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تعیین محدوده مناسب برای تعداد بالابرهاى مكعب مستطیلی در آسیاهای گلوله‌ای با استفاده از روش اجزای گسسته

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چکیده:

تعداد بالابرها در آستر آسیاهای گلوله‌ای و سرعت چرخش آسیا از مهمترین پارامترهایی هستند که رفتار بار خردکننده (گلوله‌ها) و مسیر حرکت آنها و در نتیجه مکانیزم خردایش را در این آسیاها تحت تأثیر قرار می‌دهند. در این پژوهش به منظور یافتن محدوده مناسب برای تعداد بالابرها در آستر آسیاهای گلوله‌ای، روش اجزای گسسته (راگ) مورد استفاده قرار گرفته است. ابتدا یک آسیای گلوله‌ای نیمه‌صنعتی با ابعاد $2/0$ متر \times $1/11$ متر بدون هیچ بالابری شبیه‌سازی شده است. سپس با افزودن به ترتیب ۱، ۲، ۴، ۸، ۱۶، ۲۰، ۲۶، ۳۰ و ۳۲ بالابر مکعب مستطیلی با ابعاد 2 متر \times 5 سانتی‌متر \times 5 سانتی‌متر، نه شبیه‌سازی مستقل دیگر انجام شده‌اند. تأثیرات تعداد بالابرها بر دو پارامتر جدید معرفی شده در اینجا یعنی «ارتفاع هد» (HH) و «طول زون ضربه» (IZL) در سرعت‌های مختلف آسیا یعنی ۷۰٪ و ۸۰٪ سرعت بحرانی (CS) آن مورد ارزیابی قرار گرفته‌اند. نتایج به دست آمده نشان می‌دهند که منظور یافتن محدوده مناسب برای تعداد بالابرها در آستر آسیاهای گلوله‌ای لازم است این دو پارامتر به طور همزمان به عنوان معیارهای انتخاب محدوده مناسب در نظر گرفته شوند. یعنی آسترهایی که همزمان هم HH بالاتر و هم IZL بیشتری ایجاد می‌کنند برای استفاده در صنعت مناسب‌تر هستند. همچنین نتایج نشان می‌دهند محدوده مناسب برای تعداد بالابرهاى مكعب مستطیلی در آستر آسیاهای گلوله‌ای بین ۱۶ تا ۳۲ است، که تحقیقات میدانی بر روی آسیاهای گلوله‌ای سه کارخانه مختلف موجود در صنعت صحت نتیجه به دست آمده در این تحقیق را تأیید می‌نماید. بر خلاف تحقیقات گذشته، نشان داده شده است، تعداد بالابرهاى آسیای گلوله‌ای تنها به قطر آسیا بستگی ندارد بلکه به عرض، ارتفاع، زاویه بالابر و به طور کلی به نوع بالابر نیز بستگی دارد.

کلمات کلیدی: شبیه‌سازی روش اجزای گسسته، آسیاهای گلوله‌ای، تعداد بالابرها، ارتفاع هد (HH)، طول زون ضربه (IZL)، سرعت بحرانی (CS).