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# A parametric model for predicting cut point of hydraulic classifiers

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**Abstract**

A new parametric model was developed to predict the cut point of hydraulic classifiers. The model directly uses operating parameters including pulp flowrate, feed particle size characteristics, pulp solids content, solid density and particles retention time in the classification chamber and also covers uncontrollable errors using calibration constants. The model applicability was first verified using a bench scale classifier and then validated at industrial scale for a coal classifier. The results showed that the new model can predict the cut point more precisely compared to the conventional Masliyah model, i.e. the accuracy values of 80% and 37% for the new and Masliyah models, respectively. Sensitivity study showed that the model was extremely sensitive to the particle size distribution of feed while being least sensitive to the particles retention time.

**Keywords:** *modeling, operating parameters, cut point, hydraulic classifier.*

## 1. Introduction

Classification is a method of separating mixtures of minerals into two or more products on the basis of velocity with which grains fall through a fluid medium. In mineral processing, this is usually water, and wet classification is generally applied to mineral particles which are considered too fine to be efficiently sorted by screening [1]. Classifiers are used in many mineral applications such as removal of clay fines from siliceous sands, particle size control in closed circuits with mills, fine control in taconite pellet washing, dewatering coal tailings prior to centrifugation, silica removal from iron ores, cement purification, etc. [2–7].

The commonest method of representing classifier efficiency is by a performance or partition curve, which relates the mass fraction for each particle size class in feed which reports to the same particle size class in underflow, to the particle size. Cut point, or separation size, of the classifier is defined as the size for which 50% of the particles in the feed report to the underflow, i.e. particles of this size have an equal chance of going either with the overflow or underflow. This point is usually referred to as the $d_{50}$ size [1].

Despite the design consideration and application, all classifiers follow the rules governing the motion of solid particles in a fluid environment. When a solid particle falls into a viscous medium, such as water, it is subjected to a resistance. When equilibrium is attained between the gravitational and fluid resistances forces, the solid particle reaches its terminal velocity and thereafter moves at a uniform rate. Two general models which are applied in classification are Stokes' and Newton's laws.

Stokes' law assumes the drag force on a spherical particle to be entirely due to viscous resistance, thus:

$$v = \frac{gd^2(\sigma_s - \sigma_f)}{18\mu} \tag{1}$$

Newton's law assumes that the drag force is entirely related to turbulent resistance, and therefore:

$$v = \left[\frac{3gd(\sigma_s - \sigma_f)}{\sigma_f}\right]^{\frac{1}{2}} \tag{2}$$

where $v$ is the terminal velocity, $g$ is the acceleration due to gravity, $d$ is particle size, $\sigma$ is the phase density (solid, s, and fluid , f), and $\mu$ is the fluid viscosity. Then, cut point can be predicted from Stokes' and Newton's models, respectively, as follows:

$$\frac{d_{50}}{d_x} = \left[\frac{\sigma_x - \sigma_p}{\sigma_{50} - \sigma_p}\right]^{\frac{1}{2}} \tag{3}$$

$$\frac{d_{50}}{d_x} = \frac{\sigma_x - \sigma_p}{\sigma_{50} - \sigma_p} \tag{4}$$

Stokes' law is valid for particles below 50 µm in diameter. The upper size limit is determined by the dimensionless Reynolds number ($N_R$). Newton law holds for particles larger than 0.5 cm in diameter. There is, therefore, an intermediate range of particle size in which neither law fits the experimental data. This particle size corresponds to the range in which most wet classification is performed [1].

For evaluating classifier performance in the practical range of particle size (generally between 40 µm and 800 µm), Masliyah developed a model which is actually derived from Stokes' law as follows [8]:

$$v = \frac{d^2(\sigma_s - \sigma_f)g}{18\mu(0.0015 + 0.0107 N_R^{0.252})} \tag{5}$$

The corrected cut point can be estimated as follows:

$$d_{50} = \sqrt{\frac{18u\mu_o(0.0015 + 0.0107 N_R^{0.252})}{980.7(\sigma_s - \sigma_{p_o})}} \tag{6}$$

where $d_{50}$ is the corrected cut point (cm), $\mu_o$ is the apparent viscosity of overflow (g/s cm), and $u$ is the pulp velocity in overflow outlet (cm/s). In addition,

$$\mu_o = \exp(0.338\varphi_o) \tag{7}$$

$$N_R = \frac{d_{50} u \sigma_{po}}{\mu_o} \tag{8}$$

$$\varphi_o = \frac{V_s}{V_s + V_w} \tag{9}$$

$$u = \frac{Q_o}{A} \tag{10}$$

where $\varphi_o$ is the volumetric fraction of solid in overflow stream, $\sigma_{po}$ is the overflow pulp density (g/cm), $Q_o$ is the overflow rate (cm$^3$/s), and $A$ is the area of overflow discharge gate (cm$^2$).

Although Masliyah model covers the effects of various operating parameters on the cut point, two points emerge from the equation as its disadvantages:

- The models are developed to predict effects of the input variable on output response(s) while Masliyah model has been based on output factors; i.e. overflow properties.
- Cut point directly depends on Reynolds number and vice versa. This input/output dependency will sharply increase prediction error since the equation should be solved using a try-and-error approach.

The aim of this paper is to introduce a new parametric model which applies the design and operating parameters to predict their effects on the cut point values.

## 2. Model development and verification

### 2.1. Selecting the model variables

The model variables were selected following the literature review. Then, the relationship between each factor and cut point was organized and stated as an equation using mathematical laws.

### 2.2. Model validation using bench scale studies

The applicability of the model was evaluated using the data obtained from a laboratory hydraulic classifier (Figure 1). Bench classifier included several overflow outlets at different heights, which enabled adjusting volume of the classifier. Numerous tests were run in different operating conditions and the samples were analyzed to determine the data required for model calibration. Then, new experiments were conducted to evaluate the model accuracy.

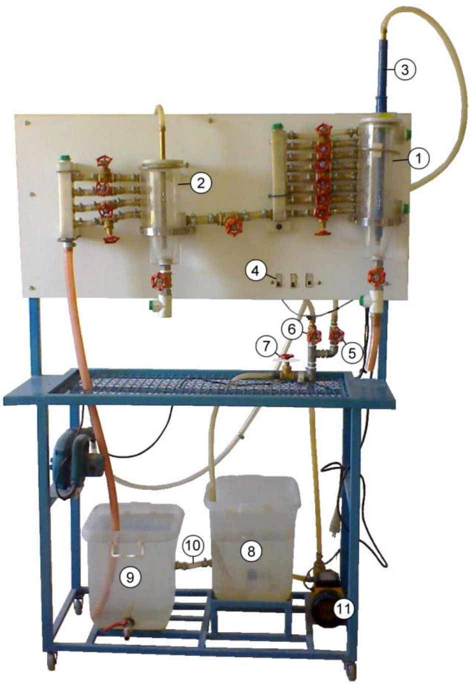


**Figure 1. Laboratory classifier used for validation studies: (1) classification column, (2) cleaner column, (3) pulp inlet, (4) safety fuses, (5-7) flowrate controling valves, (8-10) water tanks, (11) pump.**

### 2.3. Model validation in an industrial environment

Similar to the bench scale studies, accuracy of the model was assessed for a hydraulic classifier in Zarand Coal Washing Plant (Kerman, Iran). The required data were measured from the samples collected following a sampling program. In this regard, about 40 representative samples were collected during a three-month period. To prepare each representative sample, 5 sub-samples were collected from feed and products' streams in each operating shift and then mixed and divided to obtain appropriate weight of the sample for particle size analysis. Other parameters including pulp flowrate and solids content were also measured during the sample collection.

## 3. Results and discussion

### 3.1. Model development

Many investigators well recognize the effects of various operating parameters [9-27]. The relationship between each factor and cut point can be stated as follows:

- The cut point is directly influenced by feed particle size; thus, considering characteristic size ($F_{80}$) and imperfection coefficient ($I$) as representative characteristics of feed size distribution (in cumulative percent passing form), $d_{50} \propto F_{80}$ and $d_{50} \propto I$ ;
- As solids content increases, the cut point increases due to the effect of hindered-settling condition, thus $d_{50} \propto X$ ;
- Density difference between the particles has a pronounced effect on classification, especially in coarser size ranges. Eqs. (3) and (4) clearly show that the cut point decreases as the density of the particles increases, so:
- $d_{50} \propto \dfrac{1}{\sigma_s}$ or $d_{50} \propto \dfrac{\sigma_p}{\sigma_s - \sigma_p}$ .
- The cut point increases by increasing hydraulic (feed) flowrate, i.e. $d_{50} \propto Q$;
- At constant feed flowrate, two phenomena lead to decreased cut point by increasing volume of the classifier. First, large classifiers provide a sorting column with less turbulent regime inside the classifier. Second, the retention time of the particles decreases with an increase of volume. Both of these effects improve the efficiency of classification process; therefore, the cut point is inversely related to volume of the classifier, $d_{50} \propto \dfrac{1}{V}$ ;

Referring to the subscribe law, the cumulative correlation between cut point and selected factors can be stated as follows:

$$d_{50} \propto F_{80}\, I\; X\; Q \cdot \frac{1}{V} \cdot \frac{\sigma_p}{\sigma_s - \sigma_p} \qquad (11)$$

Since there is a critical upper limit for each factor, over which the system will saturate or show reverse response, Eq. (11) can be corrected by considering a power:

$$d_{50} \propto \left( F_{80}\, I\; X\; Q \cdot \frac{1}{V} \cdot \frac{\sigma_p}{\sigma_s - \sigma_p} \right)^m \qquad (12)$$

This power is expected to take positive values of less than unit. To convert proportion sign to equal, right hand of the equation should be multiplied by a constant number:

$$d_{50} = n \cdot \left( F_{80}\, I\; X\; Q \cdot \frac{1}{V} \cdot \frac{\sigma_p}{\sigma_s - \sigma_p} \right)^m \qquad (13)$$

Variables Q and $\dfrac{1}{V}$ show the inverse relation between the cut point and particles retention time since $t$ is equal to the volume divided by volumetric flowrate; i.e. $d_{50} \propto \dfrac{Q}{V} = \dfrac{1}{t}$. Therefore, Eq. (13) can be rearranged as follows:

$$d_{50} = n \cdot \left( \frac{F_{80} I X}{t} \cdot \frac{\sigma_p}{\sigma_s - \sigma_p} \right)^m \qquad (14)$$

where,

$d_{50}$: Corrected cut point (μm)
$F_{80}$: Characteristic top size of feed materials (μm)
$I$: Imperfection coefficient of feed particle size plot ($\dfrac{d_{75,\text{feed}} - d_{25,\text{feed}}}{2d_{50,\text{feed}}}$)
$X$: Pulp solids content (%wt)
$t$: Retention time of particles (s)
$\sigma_s$: Particles density (kg/m$^3$)
$\sigma_p$: Feed pulp density (kg/m$^3$)
$m$ and $n$: Calibration constants

To use this model, calibration coefficients should be first estimated from the practical data obtained from real classifier using data fitting and least-squared error method.

### 3.2. Model validation at laboratory scale

Constants $m$ and $n$ were estimated using fitting experimental cut point values versus values calculated by the model (Eq. (14)) using try-and-error method and least squared error (Solver tool, Microsoft Office Excel®). Calibration calculations resulted in $m$ = 0.0821 and $n$ = 59.2069. These values were then applied for predicting the cut point. The modeling results are listed in Table 1. Model error was calculated using Eq. (15):

$$E_{\mathrm{R}}(\%)=\frac{1}{2}\left(\left|\frac{d_{50,\mathrm{mod}}-d_{50,\mathrm{exp}}}{d_{50,\mathrm{exp}}}\right|+\left|\frac{d_{50,\mathrm{mod}}-d_{50,\mathrm{exp}}}{d_{50,\mathrm{mod}}}\right|\right)\times 100 \quad (15)$$

Model error was found to be 8.93% for the verification data, which indicates that the new model is more reliable than Masliyah model with error of 28% reported by Poozesh [8].

**Table 1. Bench scale verification results of the model calibrated by *m* = 0.0821 and *n* = 59.21**

| Run | Q ($\times10^{-5}$ m$^3$/s) | V ($\times10^{-5}$ $^3$) | t (s) | X (%) | $F_{80}$ (µm) | I | Density ratio | Corrected cut point (µm) | | Error (%) |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | Exp. | Mod. | |
| 1 | 7.14 | 93 | 13.02 | 6.98 | 390.2 | 0.8251 | 0.56 | 78.5 | 86.12 | 9.28 |
| 2 | 20 | 93 | 4.65 | 6.98 | 390.2 | 0.8251 | 0.56 | 88.8 | 93.72 | 5.40 |
| 3 | 7.14 | 93 | 13.02 | 6.98 | 798 | 0.9863 | 0.56 | 116 | 92.68 | 22.63 |
| 4 | 20 | 93 | 4.65 | 6.98 | 798 | 0.9863 | 0.56 | 93.5 | 100.86 | 7.58 |
| 5 | 7.14 | 93 | 13.02 | 13.63 | 390.2 | 0.8251 | 0.56 | 75 | 90.99 | 19.44 |
| 6 | 20 | 93 | 4.65 | 13.63 | 390.2 | 0.8251 | 0.56 | 97.6 | 99.02 | 1.44 |
| 7 | 7.14 | 93 | 13.02 | 13.63 | 798 | 0.9863 | 0.56 | 98 | 97.92 | 0.08 |
| 8 | 20 | 93 | 4.65 | 13.63 | 798 | 0.9863 | 0.56 | 114 | 106.56 | 6.76 |
| 9 | 7.14 | 204 | 28.56 | 6.98 | 390.2 | 0.8251 | 0.56 | 77.6 | 80.74 | 3.97 |
| 10 | 20 | 204 | 10.20 | 6.98 | 390.2 | 0.8251 | 0.56 | 99.5 | 87.87 | 12.47 |
| 11 | 7.14 | 204 | 28.56 | 6.98 | 798 | 0.9863 | 0.56 | 96.5 | 86.89 | 10.51 |
| 12 | 20 | 204 | 10.20 | 6.98 | 798 | 0.9863 | 0.56 | 99 | 94.56 | 4.59 |
| 13 | 7.14 | 204 | 28.56 | 13.63 | 390.2 | 0.8251 | 0.56 | 90 | 85.30 | 5.36 |
| 14 | 20 | 204 | 10.20 | 13.63 | 390.2 | 0.8251 | 0.56 | 96.5 | 92.83 | 3.88 |
| 15 | 7.14 | 204 | 28.56 | 13.63 | 798 | 0.9863 | 0.56 | 71.3 | 91.80 | 25.54 |
| 16 | 20 | 204 | 10.20 | 13.63 | 798 | 0.9863 | 0.56 | 96 | 99.90 | 3.98 |
| | | | | | Ave. Error (%) | | | | | 8.93 |

### 3.3. Model verification at industrial scale

A series of samples collected from a coal classifier in Zarand Coal Washing Plant was used for the model calibration. These calculations presented calibration coefficients as $m$ = 0.0121 and $n$ = 117.9841. These constants were then applied to the model to predict the cut point of classifier in different operating conditions (Table 2). For comparative purposes, the cut points were also predicted by Masliyah model and error was calculated. The modeling results listed in Table 2 show that the new model can predict the cut point with accuracy of about 80%. When compared to Masliyah model with accuracy of 37%, it can be claimed that the new model is more reliable than Masliyah model. In addition to higher precision, the new model is completely independent from output parameter, i.e. those corresponding to overflow product as considered by Masliyah model. This enables us to speculate the effect of any operating manipulation on the performance response of classifier. Besides, constant coefficients can cover the effects of any other factors which can not be directly measured, i.e., factors such as errors due to personnel, analysis and sample losses. There are other parameters specified for any classifier design which can be considered in the parametric model. Some classifiers, for example, are equipped with a baffle to modify the turbulent effect inside the sorting chamber. Constant coefficients can also cover effect of such specific factors.

**Table 2. Industrial scale validation results of the model calibrated by *m* = 0.0121 and *n* = 117.98**

| Run | Q ($m^3$/s) | V ($m^3$) | t (s) | X (%) | $F_{80}$ (µm) | I | Density ratio | Exp. $d_{50}$ (µm) | New model | | Masliyah model | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | $d_{50}$ (µm) | $E_R$ (%) | $d_{50}$ (µm) | $E_R$ (%) |
| 1 | 0.1587 | 307.5 | 1937.62 | 15.57 | 509.5 | 2.6167 | 4.7377 | 135.9 | 123.76 | 9.37 | 235.18 | 57.64 |
| 2 | 0.0997 | 307.5 | 3084.25 | 16.89 | 511.1 | 2.6341 | 4.8129 | 173.9 | 123.23 | 35.13 | 237.24 | 31.56 |
| 3 | 0.2056 | 307.5 | 1495.71 | 14.47 | 491.4 | 2.5311 | 4.6767 | 156.8 | 123.92 | 23.75 | 242.83 | 45.15 |
| 4 | 0.2482 | 307.5 | 1238.92 | 15.06 | 456.5 | 2.5383 | 4.7092 | 146.4 | 124.17 | 16.55 | 249.40 | 55.83 |
| 5 | 0.1004 | 307.5 | 3062.75 | 14.26 | 455 | 2.5898 | 4.6653 | 159.8 | 122.74 | 26.70 | 289.88 | 63.14 |
| 6 | 0.1010 | 307.5 | 3044.55 | 15.23 | 467.4 | 2.6879 | 4.7185 | 129.9 | 122.96 | 5.50 | 291.40 | 89.87 |
| 7 | 0.1786 | 307.5 | 1721.72 | 13.65 | 399 | 2.6257 | 4.6323 | 98.6 | 123.35 | 22.58 | 287.66 | 128.73 |
| 8 | 0.1510 | 307.5 | 2036.42 | 15.27 | 424.9 | 2.6208 | 4.7209 | 123.5 | 123.38 | 0.09 | 285.03 | 93.73 |
| 9 | 0.1024 | 307.5 | 3002.93 | 9.28 | 417.7 | 2.5397 | 4.4092 | 111.8 | 121.88 | 8.65 | 161.73 | 37.76 |
| 10 | 0.0983 | 307.5 | 3128.18 | 12.77 | 421.1 | 2.6 | 4.5856 | 131.1 | 122.40 | 6.87 | 220.22 | 54.22 |
| 11 | 0.0672 | 307.5 | 4575.89 | 14.77 | 424.4 | 2.6168 | 4.6932 | 109.4 | 122.11 | 11.01 | 207.80 | 68.65 |
| 12 | 0.1293 | 307.5 | 2378.19 | 14.76 | 402.9 | 2.6364 | 4.6929 | 116.7 | 123.02 | 5.28 | 211.63 | 63.10 |
| 13 | 0.2003 | 307.5 | 1535.13 | 26.41 | 482.2 | 2.3545 | 5.4355 | 99.1 | 124.88 | 23.33 | 286.85 | 127.45 |
| 14 | 0.3139 | 307.5 | 979.61 | 21.14 | 482.7 | 2.3542 | 5.0723 | 187.1 | 125.12 | 41.33 | 267.14 | 36.37 |
| 15 | 0.2325 | 307.5 | 1322.58 | 21.86 | 507.8 | 2.4220 | 5.1190 | 173.2 | 124.85 | 33.32 | 270.32 | 46.00 |
| 16 | 0.2083 | 307.5 | 1476.54 | 23.17 | 507.4 | 2.4200 | 5.2062 | 183.4 | 124.79 | 39.46 | 275.46 | 41.81 |
| 17 | 0.2066 | 307.5 | 1488.26 | 21.75 | 504.5 | 2.4331 | 5.1121 | 181.2 | 124.66 | 38.28 | 267.79 | 40.06 |
| | | | | Ave. Error (%) | | | | | | 20.42 | | 63.59 |

## 3.4. Sensitivity study of the model

In order to verify which of the operating parameters affect the cut point estimation more significantly, a sensitivity analysis was followed similar to what has been presented for flotation recovery sensitivity [1]. Sensitivity of the cut point relative to each operating variable can be estimated using partial differentiation approach. The squares of the partial derivatives are referred to as sensitivity coefficients [19].

If Eq. (14) is partially differentiated with respect to $d_{80}$, $I$, $X$, $t$ and $\sigma_s$ respectively, then:

$$\frac{\partial d_{50}}{\partial F_{80}} = n \cdot \left( \frac{IX}{t} \cdot \frac{\sigma_p}{\sigma_s - \sigma_p} \right)^m \cdot m F_{80}^{m-1} \qquad (16)$$

$$\frac{\partial d_{50}}{\partial I} = n \cdot \left( \frac{F_{80} X}{t} \cdot \frac{\sigma_p}{\sigma_s - \sigma_p} \right)^m \cdot m I^{m-1} \qquad (17)$$

$$\frac{\partial d_{50}}{\partial X} = n \cdot \left( \frac{F_{80} I}{t} \cdot \frac{\sigma_p}{\sigma_s - \sigma_p} \right)^m \cdot m X^{m-1} \qquad (18)$$

$$\frac{\partial d_{50}}{\partial t} = n \cdot \left( F_{80} IX \cdot \frac{\sigma_p}{\sigma_s - \sigma_p} \right)^m \cdot \frac{-m}{t^{m+1}} \qquad (19)$$

$$\frac{\partial d_{50}}{\partial \sigma_s} = n \cdot \left( \frac{F_{80} IX}{t} \cdot \sigma_p \right)^m \cdot \frac{-m}{(\sigma_s - \sigma_p)^{m+1}} \qquad (20)$$

Since the variance of a function can be found from its derivatives, therefore:

$$V_{d_{50}} = \left( \frac{\partial d_{50}}{\partial F_{80}} \right)^2 V_{F_{80}} + \left( \frac{\partial d_{50}}{\partial I} \right)^2 V_I + \left( \frac{\partial d_{50}}{\partial X} \right)^2 V_X + \left( \frac{\partial d_{50}}{\partial t} \right)^2 V_t + \left( \frac{\partial d_{50}}{\partial \sigma_s} \right)^2 V_{\sigma_s} \qquad (21)$$

where $V_{d_{50}}$, $V_{F_{80}}$, $V_I$, $V_X$, $V_t$, and $V_{\sigma_s}$ are the variances in $d_{50}$, $F_{80}$, $I$, $X$, $t$, and $\sigma_s$ respectively. Eq. (21) is useful in assessing the error that can be expected in the calculated value of cut point due to errors in the measurement of each variable.

Differential values for the studied classifier were calculated using industrial validation data (Table 2) and the results are given in Tables 3. Eq. (21) can be stated for Zarand classifier as follows:

$$V_{d_{50}} = (0.0033)^2 V_{F_{80}} + (0.5932)^2 V_I + (0.095)^2 V_X + (-0.0008)^2 V_t + (-0.007)^2 V_{\sigma_s} \qquad (22)$$

It is apparent that the calculated value of cut point is most sensitive to the variations of the imperfection coefficient and is least sensitive to the particles retention time. The cut point's sensitivity to the solids content of feed pulp is also significant.

**Table 3. Model differential values calculated using industrial validation data**

| Run | $(\partial d_{50}/\partial F_{80})$ | $(\partial d_{50}/\partial I)$ | $(\partial d_{50}/\partial X)$ | $(\partial d_{50}/\partial t)$ | $(\partial d_{50}/\partial \sigma_s)$ |
|---|---|---|---|---|---|
| 1 | 0.0030 | 0.5760 | 0.0968 | -0.0008 | -0.0069 |
| 2 | 0.0029 | 0.5697 | 0.0888 | -0.0005 | -0.0070 |
| 3 | 0.0031 | 0.5962 | 0.1043 | -0.0010 | -0.0069 |
| 4 | 0.0033 | 0.5957 | 0.1004 | -0.0012 | -0.0069 |
| 5 | 0.0033 | 0.5771 | 0.1048 | -0.0005 | -0.0068 |
| 6 | 0.0032 | 0.5571 | 0.0983 | -0.0005 | -0.0069 |
| 7 | 0.0038 | 0.5721 | 0.1100 | -0.0009 | -0.0068 |
| 8 | 0.0035 | 0.5733 | 0.0984 | -0.0007 | -0.0069 |
| 9 | 0.0036 | 0.5844 | 0.1599 | -0.0005 | -0.0064 |
| 10 | 0.0035 | 0.5733 | 0.1167 | -0.0005 | -0.0067 |
| 11 | 0.0035 | 0.5683 | 0.1007 | -0.0003 | -0.0068 |
| 12 | 0.0037 | 0.5683 | 0.1015 | -0.0006 | -0.0068 |
| 13 | 0.0032 | 0.6459 | 0.0576 | -0.0010 | -0.0078 |
| 14 | 0.0032 | 0.6472 | 0.0721 | -0.0016 | -0.0074 |
| 15 | 0.0030 | 0.6277 | 0.0696 | -0.0011 | -0.0074 |
| 16 | 0.0030 | 0.6280 | 0.0656 | -0.0010 | -0.0075 |
| 17 | 0.0030 | 0.6239 | 0.0698 | -0.0010 | -0.0074 |
| Average | 0.0033 | 0.5932 | 0.0950 | -0.0008 | -0.0070 |

Imperfection coefficient is the slope of the linear section of particle size plots (cumulative undersize vs. particle size). This part of plot covers many particle sizes of a wide size distributed feed. Cut point is mostly included in this section; therefore, it could be expected that imperfection coefficient as the representative of feed size distribution is the most effective parameter for the cut point value in classification processes. Instead, characteristic size ($F_{80}$), a size in the upper coarse limit of linear section, has less influence on cut point values. In other words, it cannot take the cut point position.

Solids content of pulp directly control viscosity and density of classification environment. The two mechanisms involved in hydraulic classification are:

- Free settling which refers to the sinking of particles in a volume of fluid which is large with respect to the total volume of particles; hence, particle crowding is negligible. The free settling condition is dominant when the percentage of solids by weight is less than about 15.
- As the proportion of solids in the pulp increases, the effect of particle crowding becomes more apparent and the falling rate of the particles begins to decrease. The system begins to behave as a heavy liquid the density of is that of the pulp rather than that of the carrier liquid; hindered-settling conditions now prevail. In effect, hindered-settling reduces the effect of size while increasing the effect of density on classification.

During the sampling program in Zarand Coal Washing Plant, solids content of the classifier feed ranged from 12.15% to 19.58% (i.e. 15.86±3.72%). The values in this range provide both free and hindered-settling conditions for classification process. That is why cut point is very sensitive to solids content (under the studied conditions).

Zarand classifier has been designed to sort a coal feed with relatively constant density (ash content of 30±2%). Therefore, even at high solids content pulps and hindered-settling conditions, cut point would not be sensitive to the particles density. Solid density of 1250±5 kg/m$^3$ was measured during the sampling program.

The least sensitivity is related to the particles retention time inside the classification chamber. Retention time in this model is calculated from feed flowrate and classifier volume. This negligible sensitivity can be attributed to the large volume of Zarand classifier providing enough time required for a perfect classification at operating flowrates measured in this study.

## 4. Conclusions

The main operating parameters influencing performance of hydraulic classifiers were identified from the literature review. These factors were used to develop a parametric model for predicting the cut point as a response of efficiency of hydraulic classifiers. Using an industrial coal classifier, the model validation gave acceptable precision in comparison to Masliyah model. Key advantages of the new model are its independency from output parameters and applying calibration constants. In this study, industrial verification of the model was done using a coal classifier available to the author. Further studies are required to confirm the model applicability to non-coal practices.

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