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Comparison of golden section search method and imperialist competitive algorithm for optimization cut-off grade- case study: Mine No. 1 of Golgohar

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Abstract

Optimization of the exploitation operation is one of the most important issues facing the mining engineers. Since several technical and economic parameters depend on the cut-off grade, optimization of this parameter is of particular importance. The aim of this optimization is to maximize the net present value (NPV). Since the objective function of this problem is non-linear, three methods can be used to solve it: analytical, numerical, and meta-heuristic. In this study, the Golden Section Search (GSS) method and the Imperialist Competitive Algorithm (ICA) are used to optimize the cut-off grade in mine No. 1 of the Golgohar iron mine. Then the results obtained are compared. Consecuently, the optimum cut-off grades using both methods are calculated between 40.5% to 47.5%. The NPVs obtained using the GSS method and ICA were 18487 and 18142 billion Rials, respectively. Thus the value for GSS was higher. The annual number of iterations in the GSS method was equal to 18, and that for ICA was less than 18. Also computing and programming the process of golden section search method were easier than those for ICA. Therefore, the GSS method studied in this work is of a higher priority.

Keywords: *Optimization, Cut-off Grade, Golden Section Search (GSS) Method, Imperilist Competitive Algorithm (ICA), Mine No. 1 of Golgohar.*

1. Introduction

Optimal exploitation of mineral reserves has always been considered by the designers and engineers. The most important objective of this operation is to maximize the net present value (NPV). Since 1954, optimizing the cut-off grade, upon which several operational and economical parameters depend, has been considered by several researchers. The basic algorithm used to determine the cut-off grades, which maximizes NPV of an operation in a one-metal deposit, subject to mining, milling, and refining capacities, has been proposed by Lane [1]. His theory takes into account the costs and capacity associated with these stages. Mine capacity is the maximum rate of mining the deposit, mill capacity is the maximum rate of processing ore, and refinery capacity is the maximum rate of production of the final product. Determination of the cut-off grade is based upon the fact that either one of these stages alone limits the total capacity of operation or a pair of stages may limit the entire operation. The optimum cut-off grade theory introduced by Lane determines the annual cut-off grades [2].

Ataei and Osanloo have developed a method to find out the optimum cut-off grade for multiple metal deposits. First, they defined the objective function for multiple metal deposits, and then, they used the golden section search (GSS) method and its equivalent factor to solve this optimizing problem [2, 3]. Among recent researches, the major contribution belongs to the Asad's efforts. He first modified the Lane's algorithm for the cut-off grade optimization of two-mineral deposits with an option to stockpile. Then he presented a

model by combining the impacts for economical parameters, escalation, and stockpiling options into the cut-off grade optimization model [4, 5]. Bascetin and Nieto have proposed a new method for determination of the cut-off grade strategy based on the Lane's algorithm by adding an optimization factor to the generalized reduced gradient algorithm in order to maximize NPV [6]. In 2008, Rashidinejad and co-workers presented a model for the optimum cut-off grade that not only relies on the economical aspect but also minimizes the form of acid mine drainage elimination or mitigation against the approach of postponing the restoration/reclamation activities at the end of the project life [7]. In 2009, he and others proposed a method to determine the cut-off grade based on the genetic-neural optimization for crude ore [8]. In 2012, Barr used the stochastic dynamic method to define the objective function for determining the optimum cut-off grade for single-metal and multi-metal deposit underprice uncertainty [9]. Abdolahisharif modified the Lane's method in order to incorporate variable processing capacities in the algorithm [10]. Azimi utilized the multi-criteria ranking system to select the cut-off grade strategy under the metal price and geological uncertainties [11].
In this work, the performance of two different methods was studied for determination of the optimum cut-off grades. For this purpose, at first, the objective function for determination of the optimum cut-off grade was defined based on maximizing NPV for future cash flow for mine No. 1 in the Golgohar iron mine. Then the GSS method and the ICA were used to find out the optimum cut-off grade strategy, amount of material that must be send to each unit, amount of selling product, profit, and NPV of five-year production plans for the iron mine.

2. Objective function

Figure 1 shows the operation process for mine No. 1 of Golgohar. As it can be seen from this figure, the mine is capable of putting on the market three types of products including sizing (the size 0-6, 6-12 and 12-25 mm), concentrate, and pellet. Since the capacity of each unit including mining, concentrating, and pelletizing can constrain the operation, consequently, three objective functions can be defined based upon these constraints. Table 1 shows the parameters used to define the objective functions, and these functions are shown in Table 2.

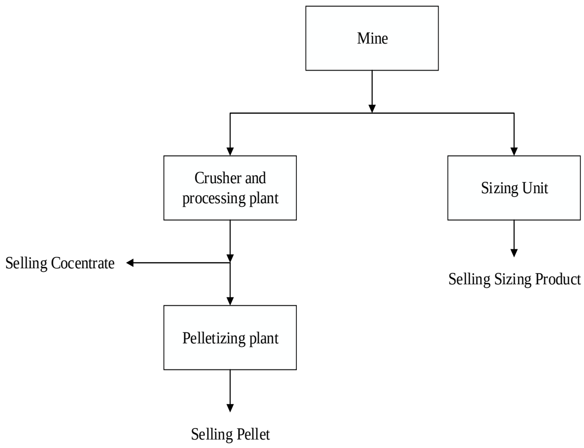


Figure 1. Operation process in mine No. 1 of Golgohar.

Table 1. Notations used for objective functions.

Symbol	Definition	Unit
Q_m	Material mined	tone
Q_c	Ore processed	tone
Q_{con}	Concentrate produced	tone
Q_p	Pellet produced	tone
Q_{gr}	Ore sizing produced	tone
M	Mining capacity	tone/year
C	Milling capacity	tone/year
V_p	Palletizing capacity	tone/year
α	A part of ore that sent to concentrate plant	-
β	A part of concentrate that sent to pelletizing plant	-
P_p	Pellet price	Rial/tone
P_{con}	Concentrate price	Rial/tone
P_{gr}	Ore sizing price	Rial/tone
m	Mining cost	Rial/tone
c	Processing cost	Rial/tone
p	Pelletizing cost	Rial/tone
C_{gr}	Ore sizing cost	Rial/tone
f	Fixed cost	Rial
T	Years of production	year
y_c	Recovery of processing	%
d	Discount rate	%

Table 2. Objective functions.

Limiting capacity	Objective function
Mining	$v_m = (P_p - p)Q_p + [(P_{gr} - C_{gr})(1-\alpha) - c\alpha]Q_c + (1-\beta)P_c Q_{con} - \left(m + \frac{f+Vd}{M}\right)Q_m$
Concentrator	$v_c = (P_p - p) + (1-\beta)P_c Q_{con} + \left[(P_{gr} - C_{gr})(1-\alpha) - c\alpha - \frac{f+Vd}{C}\right]Q_c - mQ_m$
Pelletizing	$v_p = \left(P_p - p - \frac{f+Vd}{V_p}\right)Q_p + (1-\beta)P_c Q_{con} + [(P_{gr} - C_{gr})(1-\alpha) - \alpha c]Q_c - mQ_m$

3. Process of problem solving

The optimum cut-off grade depends upon NPV, which cannot be found out until the optimum cut-off grades have been determined. The solution to this inter-dependent problem involves the iterative process. Therefore, a computer program was developed to solve the problem. The input data for this program is grade-tonnage distribution and economical and operational parameters shown in Tables 3 and 4, respectively.

Table 3. Grade-tonnage distribution in pushback.

Grade (%)	Tonnage (tone)	Average grade (%)
40.5 - 45	6137335	43.75
45 - 49.5	27346643	47.53
49.5 - 54	33254956	51.52
54 - 58.5	11258398	55.34
58.5 - 63	438098	58.89
Total ore (tone)	78435430	
Total waste (tone)	109305000	
Total material (tone)	187740430	

Table 4. Values for economical and operational parameters.

Parameter	Unit	Value
Mining capacity	tone/year	40,000,000
Milling capacity	tone/year	12,000,000
Palletizing capacity	tone/year	4,200,000
Mining cost	Rial/tone	32,000
Processing cost	Rial/tone	212,000
Pelletizing cost	Rial/tone	400,000
Ore sizing cost	Rial/tone	50,000
Fixed cost	Rial/year	400,000,000,000
Pellet price	Rial/tone	2,600,000
Concentrate price	Rial/tone	874,000
Ore sizing price	Rial/tone	2,575,000
Recovery	%	67
Discount rate	%	21

4. Optimization by GSS method

One of the fastest methods to calculate the optimum point of unimodal functions is the elimination method. In the first step of this method, the uncertainty space of the problem is guessed. In the next step, by selecting the test points in the uncertainty space, and evaluating and comparing the objective functions at these test points, a part of the uncertainty space is eliminated. This reducing procedure is repeated until the uncertainty interval in each direction is less than a small specified value ε, where ε is the desirable accuracy for determining the optimum cut-off grades [2].

This method is described in the following steps [12]:

1. Start with an initial guess point, say x_1.

2. Find $f_1 = f(x_1)$.

3. Assuming a step size s, find $x_2 = x_1+s$.

4. Find $f_2 = f(x_2)$.

5. If $f_2<f_1$, and if the problem is one of minimization, the assumption of unimodality indicates that the desired minimum cannot lie at $x<x_1$. Hence the search can be continued further along the points x_3, x_4 ,… using the unimodality assumption, while testing each pair of experiments. This procedure is continued until a point, $x_i = x_1+(i-1)s$, shows an increase in the function value.

6. The search is terminated at xi, and either x_{i-1} or xi can be taken as the optimum point.

7. Originally, if $f_2>f_1$, then the search should be carried out in the reverse direction at points x_{-2}, x_{-3},…, where $x_{-j} = x_1-(j-1)s$.

8. If $f_2=f_1$, then the desired minimum lies between x_1 and x_2, and the minimum point can be taken as either x_1 or x_2.

9. If it happens that both f_2 and f_{-2} are greater than f_1, this implies that the desired minimum lies in the double interval $x_{-2}<x<x_2$.

The ratio of the remaining length, after the elimination process, to the initial length in each

dimension is called the reduction ratio. Among the elimination methods, the reduction ratio for the GSS method is optimum and equal to 0.618. (This number is called the golden number.) [2] Hence, this method has the widest application.
Figure 2 shows the GSS method for a one-dimensional function. In the first step, assume (*L*, *U*) to be the initial interval of uncertainty, and note that the initial interval includes the optimum point. Then select two test points, $g1$ and $g2$, are calculate them as follows [2]:

$$g_1 = L + (U - L) \times 0.382$$
$$g_2 = L + (U - L) \times 0.618 \quad (1)$$

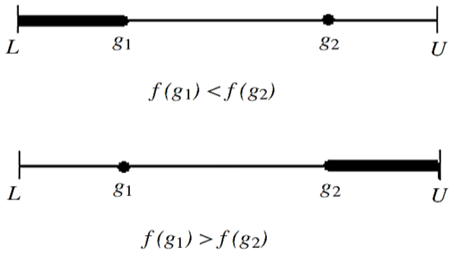


Figure 2. GSS method for one-dimensional function [2].

In the next step, the objective functions are evaluated in the g_1 and g_2 points. Depending upon the objective function values for these points, the length of the new uncertainty interval is successively reduced in each iteration. By considering this process for a maximizing problem, the results obtained for the objective function evaluation and reducing the interval of Figure 3 are as follow:

$$\begin{array}{llll} if & f(g_1) < f(g_2) & \Rightarrow L = g_1 & U = U \\ if & f(g_1) > f(g_2) & \Rightarrow L = L & U = g_2 \end{array} \quad (2)$$

In this study, the desirable accuracy and the interval uncertainty were assumed to be 0.01% and 40.5%-58.5%, respectively. By running the program, the annual optimum cut-off grade in 18 iterations was calculated. Table 5 shows the results obtained.

Table 5. Optimum cut-off grade for different years of mine life.

Year	Optimum cut-off grade (%)	Material mined (tone)	Ore sent to concentrator (tone)	Ore sent to sizing unit (tone)	Salable concentrate (tone)	Pellet produced (tone)	Benefit (billion Rial)	NPV (billion Rial)
1	47.48	40000000	11974565	650565	3011651	4200000	7659	18487
2	47.28	39937732	12000000	928027	2188152	4200000	6874	14710
3	47.02	39943516	12000000	900361	1345421	4200000	6067	10926
4	45	34169784	12000000	739198	669563	4200000	5254	7153
5	40.5	33689398	11993351	672744	0	4016084	4115	3401

5. Optimization by ICA

Different meta-heuristic algorithms have been proposed for solving an optimization problem. Most of these methods are inspired by modeling natural processes. In 2007, for first time, ICA was proposed by Atashpaz-Gargari and Lucas, and was inspired by the imperialist competition. Contrary to the conventional evolutionary methods, this algorithm is not based upon any phenomenon from the nature. ICA uses the socio-political evolution of human as a source of inspiration for developing a strong optimization strategy. In particular, this algorithm considers imperialism as a level of human social evolution, and by mathematically modeling this complicated political and historical process, it arrives at a tool for an evolutionary optimization [13].
Figure 3 shows the flowchart of ICA. Like other evolutionary ones, ICA starts with an initial population. Each individual of the population is called a country, in which some having the least cost are established as the imperialists, and the rest are the colonies of these imperialists.

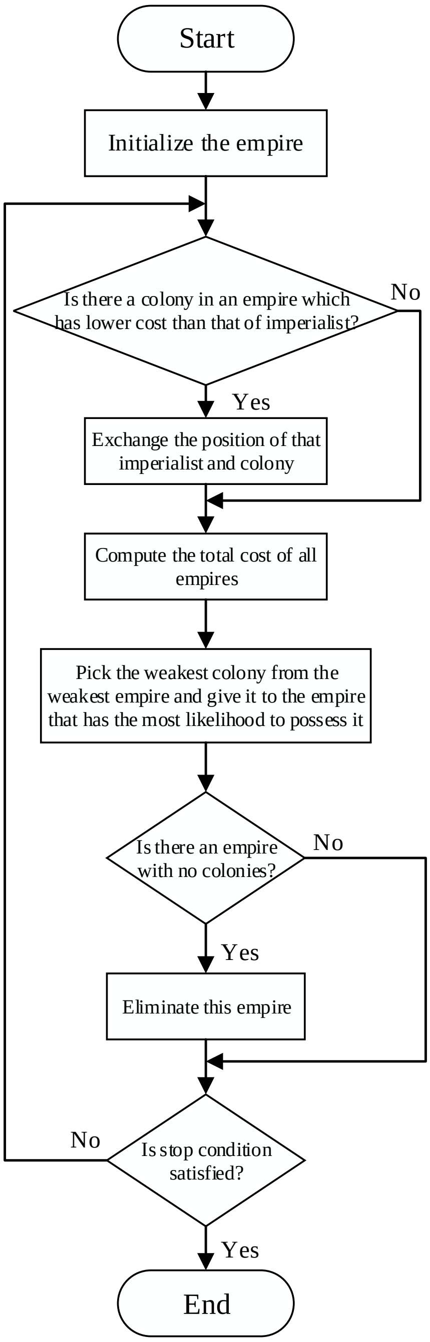


Figure 3. Flowchart of ICA [14].

Division of all the colonies of the initial countries is based upon the power of the imperialist. For this, at first, it is necessary to define the normalized cost of an imperialist, by:

$$C_n = \max_i\{c_i\} - c_n \tag{3}$$

where c_n is the cost of the n_{th} imperialist, and C_n is its normalized cost. Having the normalized cost of all imperialists, the normalized power of each imperialist is defined as:

$$p_n = \left| \frac{C_n}{\sum_{i=1}^{N_{imp}} C_i} \right| \tag{4}$$

From a different point of view, the normalized power of an imperialist is the portion of colonies that should be possessed by that imperialist. Then the initial number of colonies of an empire would be:

$$N.C._n = round\{p_n.(N_{col})\} \tag{5}$$

where $N.C._n$ is the initial number of colonies of the n_{th} empire, and N_{col} is he number of all colonies. To divide the colonies, for each imperialist, $N.C._n$ was chosen randomly [14].

The colonies in each one of the empires start moving towards their imperialist, based on the assimilation policy. Figure 4 shows the movement of a colony towards the imperialist. In this movement, θ and x are arbitrary numbers, which are generated uniformly as $x \sim U(0, \beta \times d), \theta \sim U(-\gamma, \gamma)$. Here, d is the distance between colony and imperialist, and β must be greater than 1. This constraint causes the colonies to get closer to the imperialist state from both sides. Moreover, γ is a parameter that adopts the deviation from the main direction. Although β and γ are random numbers, most of the times, the best fitted values for β and γ are approximately 2 and $\pi/4$ (Rad), respectively [15].

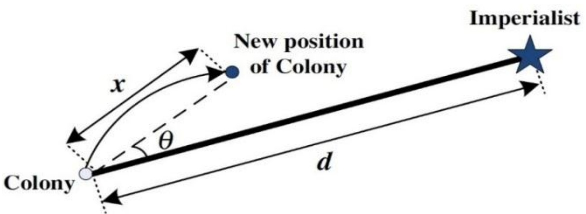


Figure 4. Movement of colonies toward their relevant imperialist [14].

The total power of an empire is defined by the imperialist power and percentage of the colony power. Thus the total cost is defined by:

$$T.C._n = Cost(imperialist_n) + \xi\, mean\{Cost(colonies\ of\ empire_n)\} \tag{6}$$

where $T.C._n$ is the total cost of the nth empire, and ξ is a positive number, which is considered to be less than 1.

By defining the above equations, the imperialist competition begins. All the empires try to take the colonies of other empires under their control. The imperialistic competition gradually results in an increase in the power of powerful empires and a decrease in the power of weaker empires. This results in the collapse of weak empires. To start the competition, first one must find the possession probability of each empire based on its total power. The normalized total cost is simply obtained by:

$$N.T.C._n = \max_i \{T.C._i\} - T.C._n \tag{7}$$

where $N.T.C._n$ is the normalized cost of the n_{th} empire. Having the normalized total cost, the possession probability of each empire is given by:

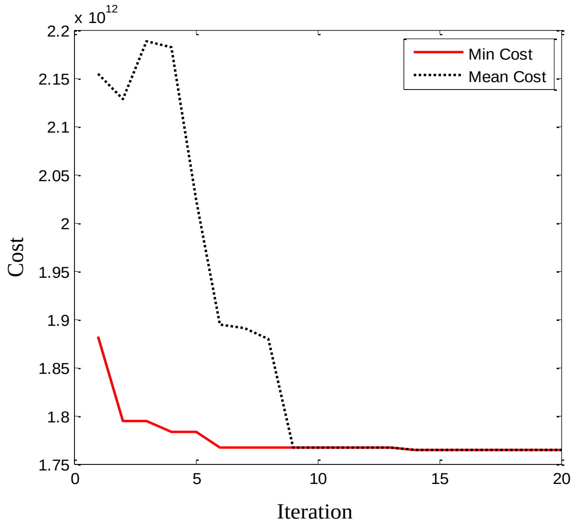
$$p_{p_n} = \left| \frac{N.T.C._n}{\sum_{i=1}^{N_{imp}} N.T.C._i} \right| \tag{8}$$

Finally, these processes successfully cause all the countries to converge to a situation in which there exists only one empire in the world, and all the other countries are colonies of that empire that have the same position and power as the imperialist [16]. The main steps in the algorithm are summarized in the pseudo-code shown in Figure 5.

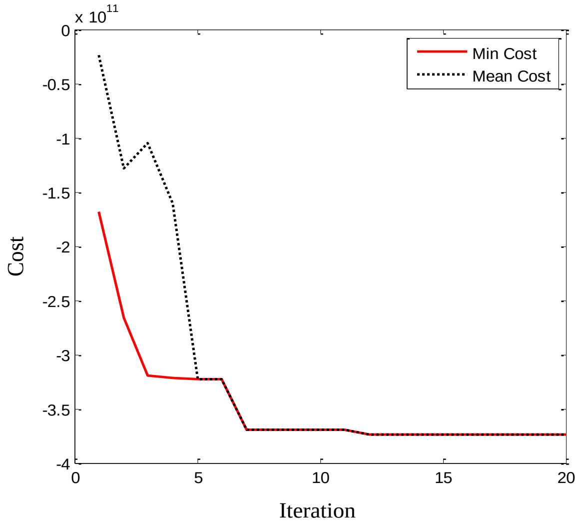
1) Select some random points on the function, and initialize the empires.
2) Move the colonies toward their relevant imperialists (Assimilation).
3) If there is a colony in an empire which has the lowest cost than that of the imperialist, exchange the positions of that colony and the imperialist.
4) Compute the total cost of all empires (related to the power of both the imperialist and its colonies).
5) Pick the weakest colony (colonies) from the weakest empire, and give it (them) to the empire that has the most likelihood to possess it (Imperialist competition).
6) Eliminate the powerless empires.
7) If there is just one empire, stop, if not go to 2.

Figure 5. Pseudo-code of ICA [14].

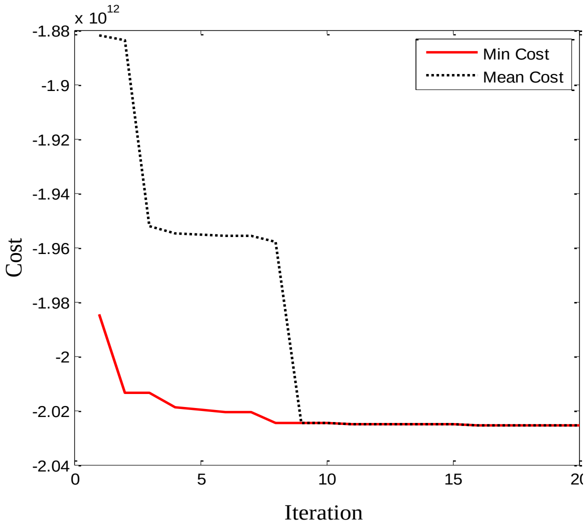
It should be noted that each candidate grade is a country in ICA, and the objective function of the problem is the cost function of this algorithm. Also the annual final empire for ICA is the optimum cut-off grade. Figure shows the minimum and mean costs of all the empires *vs.* iteration for each year. Since ICA is designed for the minimization problem, the objective function was used in its negative form. Therefore, in Figure 6, NPV is negative. Table 6 shows the results obtained for this optimization method.



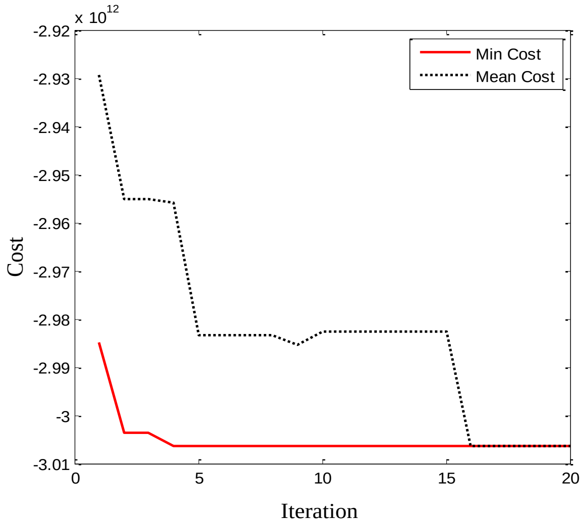
a. 1st year



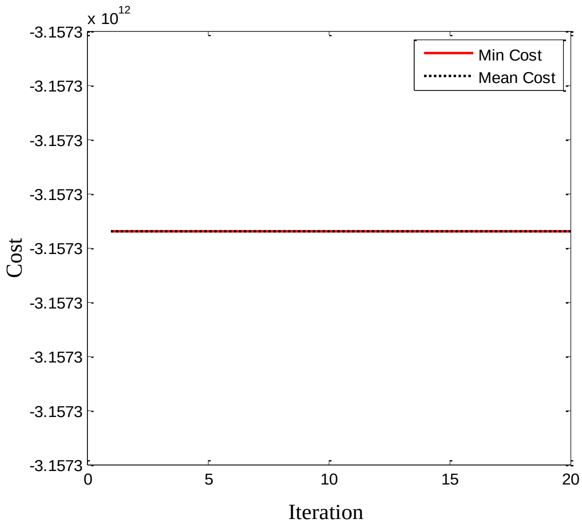
b. 2nd year



c. 3rd year



d. 4th year



e. 5th year

Figure 6. Minimum and mean costs of all imperialists *vs.* iteration for each year.

Table 6. Optimum cut-off grade for different years of mine life.

Year	Optimum cut-off grade (%)	Material mined (tone)	Ore sent to concentrator (tone)	Ore sent to sizing unit (tone)	Salable concentrate (tone)	Pellet produced (tone)	Benefit (billion Rial)	NPV (billion Rial)
1	47.46	39981211	12000000	950009	3027388	4200000	7661	18142
2	47.28	39988755	12000000	929231	2185268	4200000	6873	14291
3	47.03	40000000	11998962	901609	1341771	4200000	6066	10420
4	46.61	39987460	12000000	859080	496849	4200000	5220	6542
5	40.5	27783008	9743843	550968	0	3261176	3262	2696

6. Conclusions

In this work, we investigated the performance of two different methods to find out the optimum cut-off grades in the mine No. 1 of Golgohar. For this purpose, in the first step, the objective function was developed by considering three types of salable products in this iron ore mine. In order to do so, at first, the Lane's method was modified, and the objective function for determination of the optimum cut-off grade based on maximizing the net present value (NPV) for future cash flows was defined. Then the golden section search (GSS) method and imperialist competitive algorithm (ICA) were used. Conscuently, the optimum cut-off grades were calculated between 40.5% and 47.5% using both of these methods. NPVs obtained by the GSS method and ICA were 18487 and 18142 billion Rials, respectively, and thus the value for the GSS method is higher. To solve the problem, the number of iterations in the GSS method for each year was equal to 18, and that in ICA was less than 18. Also the process of programming and computing in GSS was very easier than that in ICA. Thus, in this problem, GSS had a priority higher than ICA.

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مقایسه روش‌های جستجوی نسبت طلایی و الگوریتم رقابت استعماری در بهینه‌سازی عیار حد- مطالعه موردی: معدن شماره ۱ گل‌گهر

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چکیده:

یکی از مهم‌ترین مسائل پیش روی مهندسین معدن، بهینه‌سازی عملیات استخراجی است. از این میان بهینه‌سازی عیار حد به دلیل وابستگی پارامترهای متعدد فنی و اقتصادی به آن، دارای اهمیت ویژه‌ای است. هدف از این بهینه‌سازی افزایش ارزش خالص فعلی است. از آنجایی‌که این مسئله از نوع غیرخطی است، روش‌های تحلیلی، عددی و فرا ابتکاری را برای حل آن می‌توان بکار برد. ازاین‌رو، در پژوهش حاضر روش عددی جستجوی نسبت طلایی و الگوریتم فرا ابتکاری رقابت استعماری برای بهینه‌سازی عیار حد معدن شماره ۱ گل‌گهر مورد استفاده قرار گرفته و نتایج حاصل از آن‌ها با یکدیگر مقایسه شده است. بر این اساس، مقادیر عیارهای حد در هر دو روش در بازه ۴۰/۵٪ تا ۴۷/۵٪ محاسبه شده است. همچنین، مقدار ارزش خالص فعلی حاصل از روش جستجوی نسبت طلایی برابر با ۱۸۴۸۷ میلیارد ریال و برای الگوریتم رقابت استعماری برابر با ۱۸۱۴۲ میلیارد ریال به‌دست‌آمده که نشان‌دهنده مقدار بیشتر برای روش جستجوی نسبت طلایی است. برای حل مسئله تعداد دورهای محاسباتی برای همه‌ی سال‌های طرح در روش جستجوی نسبت طلایی برابر با ۱۸ و در روش رقابت استعماری کمتر از ۱۸ بوده است. از سوی دیگر، روش جستجوی نسبت طلایی فرآیند محاسباتی و برنامه‌نویسی آسان‌تری از الگوریتم فرا ابتکاری رقابت استعماری داشته است؛ بنابراین به‌طورکلی، در این مطالعه روش جستجوی نسبت طلایی دارای برتری و اولویت نسبت به الگوریتم رقابت استعماری بوده است.

کلمات کلیدی: بهینه‌سازی، عیار حد، روش جستجوی نسبت طلایی، الگوریتم رقابت استعماری، معدن شماره ۱ گل‌گهر.