

A new approach for obtaining settling velocity in a thickener using statistical regression: A case study

M. Hosseini Nasab*

Department of Mining Engineering, University of Sistan and Baluchestan, Zahedan, Iran

Received 8 January 2015; received in revised form 29 May 2015; accepted 8 June 2015

*Corresponding author: hosseininasab@eng.usb.ac.ir (M. Hosseini Nasab).

Abstract

In this research work, the parameters affecting the settling velocity within the thickeners were studied by introducing an equivalent shape factor. Several thickener feed samples of different densities including copper, lead and zinc, and coal were prepared. The settling tests were performed on the samples, and the corresponding settling curves were plotted. Using the linear regression analysis, the Chein's equation was fitted to the experimental data in order to obtain the equivalent shape factors for the different minerals. Moreover, the relations between the equivalent shape factors and the settling parameters were investigated. The R-squared values for the fits proved the capability of the Chein's equation to fit well on the experimental data ($0.96 < R^2 < 0.99$ for the copper sample, $0.96 < R^2 < 0.98$ for the lead and zinc sample, and $0.93 < R^2 < 0.97$ for the coal sample). The results obtained showed that the equivalent shape factor generally decreased with increase in the initial solid concentration. The equivalent shape factor for the coal sample in the samples with concentrations higher than 25% was negative but close to zero. This unusual behavior was explained by discussing the effects of the flocculants on the formation of the network structure in these samples. Determination of the equivalent shape factors would be an invaluable achievement in mineral processing. The researchers can simply put these factors into the Chein's equation to calculate the settling velocity of the minerals instead of performing expensive and time-consuming experiments.

Keywords: Settling Velocity, Equivalent Shape Factor, Chein's Equation, Statistical Regression.

1. Introduction

Regarding the scarcity of water resources and hazardous minerals in the waste water of thickeners, recycling is an inseparable part of the mineral processing, which prevents environmental impacts [1-3]. A major part of water recovery is performed in the thickeners, where the higher density of solid particles, compared to water, causes the solid particles to be settled by the gravity force. Increasing the settling capacity of the thickeners with the aim of optimizing the residual moisture content and water recovery has been widely emphasized [1, 4, 5].

The settling capacity for a sedimentation unit is characterized by the settling velocity in the free settling zone, where the solid concentration of the

suspension is low and the particles are at a considerable distance from each other [5-10]. While the factors affecting the settling velocity in a thickener under a continuous operation has remained relatively poorly understood due to some technical impediments [11], the research efforts have been mainly focused on the laboratory experiments [3, 4, 11-13] and modeling techniques [1, 14-16]. Simultaneously including all the effective parameters to study the settling velocity of the mineral particles makes the problem highly complicated. Even when all the major factors can be controlled under the practical conditions, with only a few parameters that are independently variable, the task is still difficult to do [17]. Therefore, previous studies have been mostly concentrated on the effect of one

parameter on the settling velocity, while others remained constant [18-20]. For example, Muhanned (2013) has reported that the settling velocity of the solid particles is greatly affected by the particle path during settling. In his study, the spherical particles followed the vertical path during settling, while the irregularly-shaped particles followed different paths and orientations (like springing, circular, oscillating, and unstable paths), which could decrease their settling velocity. The particle size also has a great effect on the settling velocity, as increase in the particle diameter or volume, resulting in increase in the settling velocity, which is in agreement with the previous studies [21-24].

The influences of the particle size and shape on the particle terminal velocity have been investigated in detail for some regular geometries such as spheres, disks, cylinders, and isometric particles [4, 20]. While the floc structure is one of the factors greatly affecting settling the velocity of mineral particles [21, 25], few studies have been concentrated on these irregularly-shaped particles [12, 13, 21, 22]. Flocs are highly porous and irregularly-shaped aggregates formed by bridged particles within a suspension. Adding flocculants (high molecular weight water-soluble polymers) to a dilute stable suspension bridges the particles together [26-28], and leads to the formation of flocs [12]. The floc structure depends on the nature of several factors including the solid (surface chemistry, size, size distribution, shape, and density), the liquid (viscosity and dielectric constant), the suspension (solid loading, pH, ionic strength, and temperature), and the flocculant (chemical nature of the backbone and side chains, molecular weight, molecular weight distribution, charge, and charge density).

Berres et al. (2013) have tried to describe the dewatering dynamics using two material-specific models for the local solid concentrations [1]. In a similar attempt, Garmsiri and Haji-Amin-Shirazi (2012) have performed many experiments for various solid concentrations and different types and dosages of chemical aids. Based on their experimental data, they suggested a mathematical model for the analysis of the settling curves obtained [13].

Regardless of the methods (experimental, analytical, and numerical), most of the previous studies have been suffering over simplification, limited number of samples, and insufficient included variables. Moreover, their highly time-consuming and expensive procedures cannot be neglected.

The aim of this research work was to calculate the settling velocity for mineral particles using the Chein's equation. While this equation has been widely utilized for calculating the settling velocity of non-mineral particles [23, 24], it has not been applied to mineral particles yet. Since mineral particles do not have regular geometric shapes, and their shapes have been changing during the settling process, an equivalent shape factor for the floc was introduced for each sample, which can be determined by fitting the Chein's equation on the experimental settling scatters. Moreover, the effects of using the flocculant in the coal samples were investigated.

Determination of these equivalent shape factors would be an invaluable achievement in mineral processing and water recycling. Researchers can simply put these factors into the Chein's equation to calculate the settling velocity of minerals (or suspended particles in water), which is necessary for the design of new thickeners (or sedimentation tanks) with improved functionality. This significantly decreases the current level of time and money invested in the experimental and modeling studies related to the mineral sedimentation and water purification units.

2. Materials and method

2.1. Experimental details

2.1.1. Materials

In this study, samples of three different materials were used for the settling experiments, as follow:

- Coal refuse (average density: 1.6 g/cm^3) from the thickener feed of the Interkarbon Coal Preparation Plant (located in Zarand, Iran), with 80% of the particles smaller than $35\mu\text{m}$.
- Copper ore (average density: 2.7 g/cm^3), with 80% of the particles smaller than $80\mu\text{m}$.
- Lead and zinc ore (average density: 3.7 g/cm^3), with 80% of the particles smaller than $55\mu\text{m}$.

To make the observation of the mud line easier, polyacrylamide (A65) was used for the coal samples. By dissolving the solid flocculant in water, a flocculant solution with a 0.05% concentration was prepared. It should be noted that, according to the ISO 10.86 standard, the flocculant was used only up to 24 hours after preparation.

2.1.2. Settling test method

The settling tests were carried out in glass cylinders of 10mm diameter and 1000mL volume.

For each experiment, the cylinder was filled with a certain amount of one of the three materials, and water was added to the cylinder until the total volume reached 1000mL. If needed, the required amount of flocculant was added to the cylinder, and the content was mixed well using a mixer. Finally, the mud line height versus time was recorded.

To avoid the loss of a portion of the sample, unlike the traditional method that includes inverting the test cylinder for several times, we used a drilled disk mixer to achieve a homogenous solution. Stirring is supposed to do three functions: 1- to break up the flocculant bonds, which keep the particles apart; 2- to allow the fines to move into the voids between coarse particles; and 3- to facilitate the escape of liquid from the settling bed of solids [29].

2.2. Experimental design

The settling tests were carried out on the samples of coal refuse, copper, and lead and zinc ores in order to determine the equivalent shape factors.

For the coal sample, the settling tests were started by changing the initial concentrations from 2% to 10% with 2% intervals, and from 15% to 30% with 5% intervals. The flocculant concentrations were 15, 25, and 35 g/t. To determine the settling experiment errors, the tests with solid percentages of 2, 4, and 8% were repeated for three times, and the relative standard deviation for the settling velocity was calculated as an index for the experimental error. The average standard deviation for these experiments was 0.13.

The settling tests for the copper and lead & zinc samples without using the flocculant were performed by changing the initial concentration from 5% to 40% with 5% intervals. Due to the ease of mud line observation, the test was only repeated once. The experimental details are reported in Table 1.

Table1. Different experimental conditions.

Pulp type	Initial solid concentration (%)
Copper (without flocculant)	5 to 40, with 5 intervals
Lead & Zinc (without flocculant)	5 to 40, with 5 intervals
Coal (with 15, 25, and 35g/t of flocculant)	2 to 10, with 2 intervals 15 to 30, with 5 intervals

2.3. Introducing equivalent shape factors for minerals

All the thickener surface calculation methods are based upon the settling velocity in the individual settling zone [30, 31]. Up to the present time, this velocity has been mostly measured by performing the time-consuming settling experiments, whose results could be used only for the test conditions. This means that the thickener designers should perform new experiments whenever they wish to design a new thickener, which demands lots of money and time. Thus it would be very valuable to have an equation to calculate the settling velocity based on the other settling parameters involved. Such an equation has been widely employed in several fields of science and engineering, and is known as the Chein's settling velocity equation [18, 24]:

$$V_P^2 + 4.458e^{(5.03\varphi)} \left(\frac{\mu_e}{d_P \rho_f} \right) V_P - 19.45e^{(5.03\varphi)} d_P \left(\frac{\rho_p}{\rho_f} - 1 \right) = 0 \quad (1)$$

Where:

V_P : Settling velocity of particle

μ_e : Effective viscosity of fluid

d_P : Average particle diameter

ρ_f : Fluid density

ρ_p : Particle density

φ : Shape factor

The settling mineral particles have often irregular shapes, especially when the flocculant is used to increase the settling velocity [18]. Therefore, the Chein's equation should be modified in a way to be applicable in mineral processing. We suggested the use of an equivalent shape factor, which is not related to individual solid particles but to their networks and flocs. It should also be clarified that, contrary to the routine shape factor, which is defined within the range of 0-1[19], there is no obligation on the equivalent shape factor value.

A major impediment for using this equation in mineral processing originates from the fact that mineral particles do not have stable and simple geometries. Thus we cannot assign a constant shape factor to each mineral sample. As a result of the changes in the shape of the minerals during settling, the shape factor is predicted to vary from its initial value during the process. This may prevent the use of Chein's equation in mineral processing, although defining an average equivalent shape factor can solve the problem.

Since the settling tests performed in this study were performed in aqueous solutions, we took:

$$\rho_f = 980 \frac{kg}{m^3}, \mu_e = 0.81 \text{ mpa.s}$$

Properties of the solid particles were set as reported in sections 2.1 and 3.1. Slope of the settling curves, which represent mud line height versus time, was considered as the settling velocity (V_p). The only variable that remained undefined in the Chein's equation was the equivalent shape factor (c). Using a linear regression method, the Chein's equation was fitted on the settling curves for all the samples, and the equivalent shape factors were determined utilizing the R statistical software [32].

3. Results and discussion

3.1. Settling test without flocculant

Unlike coal, the mud line for the copper and lead & zinc samples could easily be observed without using the flocculant. The settling velocities for these two samples with different initial concentrations (from 5% to 40%) were obtained. The settling curves for the experiments carried out

on the copper and lead & zinc samples are presented in Figures 1 and 2, respectively. As a common trend, all of these graphs were linear at the beginning, behaved non-linearly afterward, and finally, became horizontal. A tangent drawn at any point on these graphs gives the settling rate of the solids in the vicinity of the interface for that point. As the graphs reveal, the settling rates are higher at the beginning but later become considerably lower. This decrease in the settling rates is due to the change in the settling conditions that occurs over the time [29]. The zone and compressive settling regimes were further characterized by the sharp boundary that existed between the settling solids and supernatant liquid. The clarification or particulate settling regimes, on the other hand, did not show a distinct interface (see Figures 3 and 4).

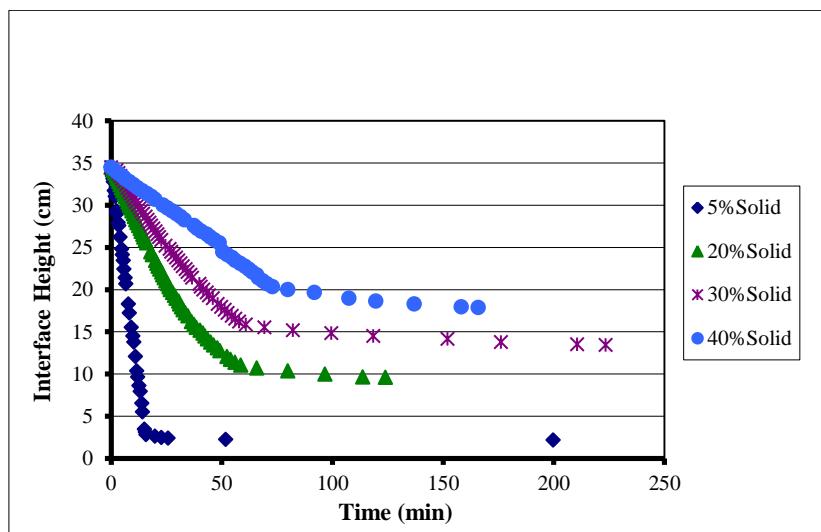


Figure 1. Copper settling rate for different initial solid percentages without flocculant.

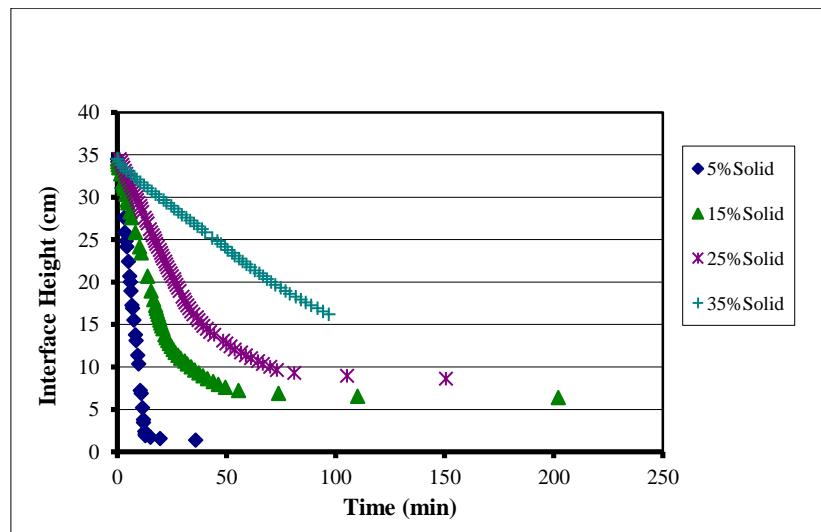


Figure 2. Lead & zinc settling rate for different initial solid percentages without flocculant.

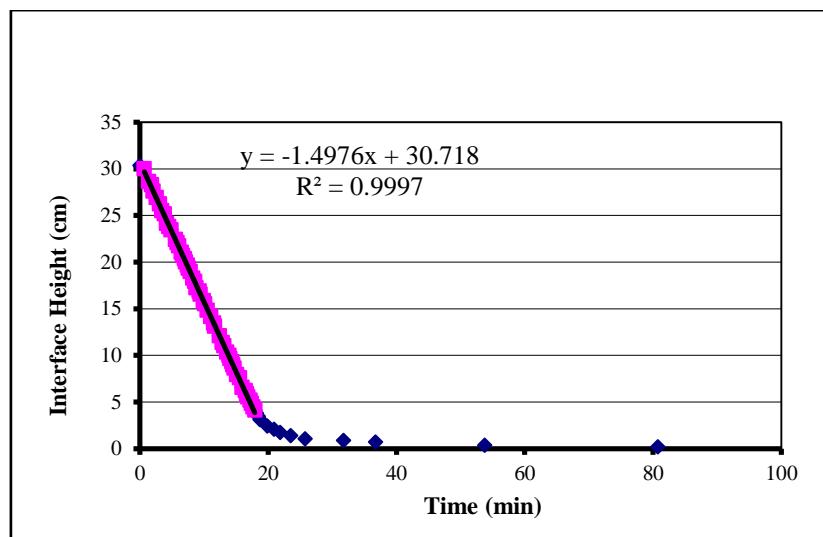


Figure 3. Velocity determination in particulate setting zone for copper sample with 10% of solids in feed without flocculant.

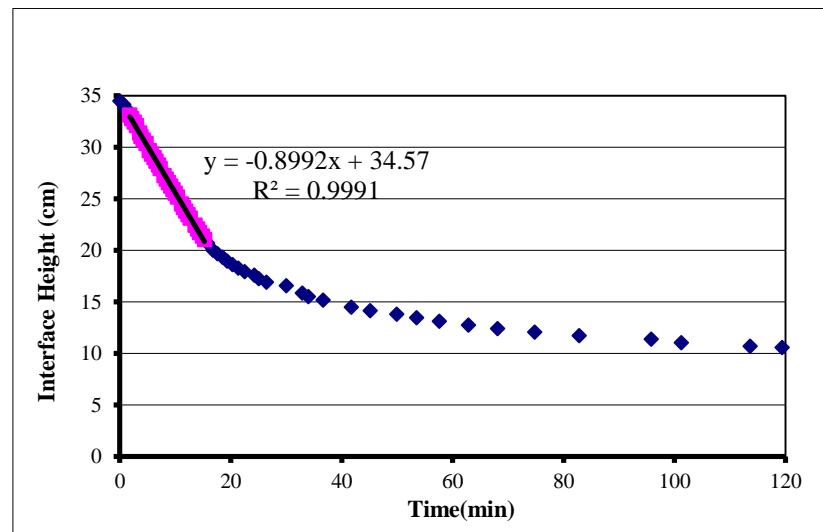


Figure 4. Velocity determination in particulate setting zone for coal sample with 10% of solids in feed and 35 g/t flocculant dosage.

3.2. Settling test using flocculant

Due to the low settling velocity of the coal sample, it was difficult to see the mud line. Therefore, these tests were carried out with the flocculant. The settling velocities for this sample were obtained for different initial concentration values (from 2 to 10%). As mentioned earlier, each experiment was repeated for three times, and an average value was reported for the settling velocity.

Figure 5 shows the results obtained for the coal sample. As it can be seen in this figure, the floc formation increases the settling velocity, compared to the samples without flocculant. However, adding the flocculant more than its optimal amount not only does not improve the

settling velocity but also produces looser flocs, which consequently reduces the compressibility of solids during compaction, and decreases the final pulp density. Indeed, increasing the flocculant dosage reduces the overflow clarification since, compared to a thick pulp, the flocculant used in a diluted pulp produces larger flocs. As the amount of feed solid concentration goes up, the settling rate increases even by using lower amounts of the flocculant. Under the low-pulp-density conditions, sedimentation of individual flocs is possible, and, therefore, the settling rate of the suspension becomes higher. At higher pulp densities, however, the flocs form a network structure that reduces the sedimentation rate.

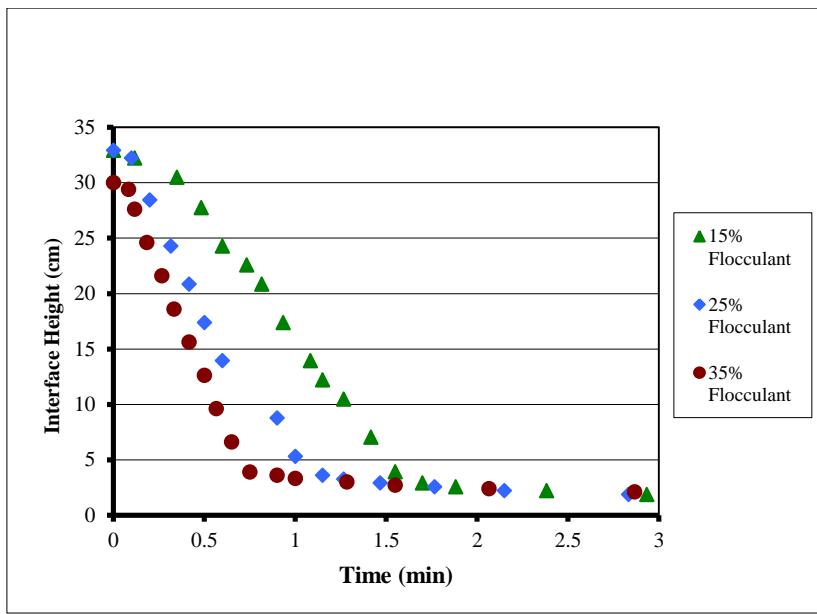


Figure 5. Effect of flocculant dosage on settling velocity for coal sample with 2% of solids.

3.3. Relationship between equivalent shape factor and settling velocity

The equivalent shape factors were computed for the different initial solid concentrations fitting the Chein's equation on the settling curves. The R-squared values were different for the different samples and test conditions ($0.96 < R^2 < 0.99$ for the copper sample, $0.96 < R^2 < 0.98$ for the lead and zinc sample, and $0.93 < R^2 < 0.97$ for the coal sample).

As it can be seen in Figure 6, the equivalent shape factor increases as the settling rate rises. This fact was confirmed by the observations made on the three samples.

Figures 7 and 8 show the equivalent shape factor versus initial solid concentration for the copper and lead & zinc samples, respectively.

Figures 9-11 show the equivalent shape factor versus initial solid concentration for the coal sample when different flocculant dosages were used.

Figures 7-11 indicate that as the initial solid concentration increases, the equivalent shape factor (degree of sphericity) decreases. The reason may be the formation of larger flocs due to higher pressures applied by other particles in the samples with higher initial solid contents. In other words, by increasing the initial solid concentration, it takes longer to achieve the desired underflow density in a compressive zone. As Figures 9-11 show, for the coal sample, changes in the

equivalent shape factor due to addition of flocculant are remarkable. Comparing Figures 7-11 revealsthat the flocs with compact structures have equivalent shape factors in the range of 0-1. Negative shape factors were calculated for the coal samples with initial solid contents higher than 25%. This can be discussed by the considerable changes in the floc's structure when the flocculant is used. Actually, the pressure induced by the upper layers to the solids located at the bottom layers causes formation of the network structures in the samples with higher concentrations. This pressure facilitates formation of network structures, and causes a significant decrease in the settling velocity, and, therefore, in the equivalent shape factor.

Unlike the expected decreasing trend, the settling velocity, and hence, the equivalent shape factor for 25% initial solids in the copper sample and 20% initial solids in the lead & zinc sample increased significantly. This means that the difference in the velocity up for withdrawing water from the flocs and the settling velocity due to the gravity force is increased. This behavior is often a result of channeling or short-circuit in the fluid that occurs at high concentrations. This fact was estimated to be related to the cracks in the structure of the solids in the presence of inter-particle forces.

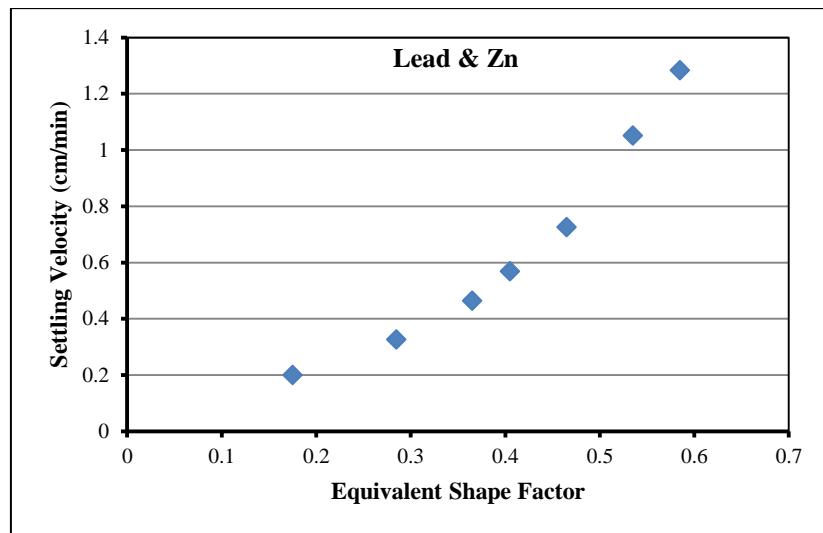


Figure 6. Settling velocity versus equivalent shape factor for lead & zinc sample.

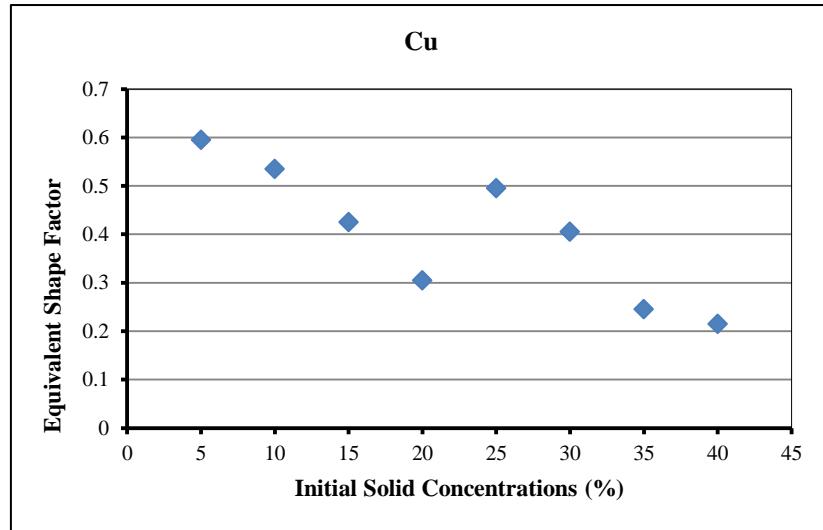


Figure 7. Equivalent shape factor versus initial solid concentration for copper sample.

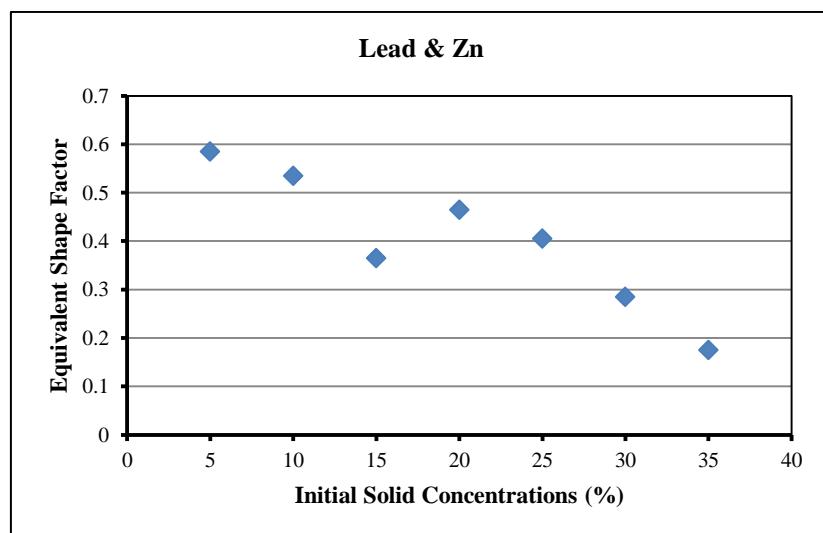


Figure 8. Equivalent shape factor versus initial solid concentration for lead & zinc sample.

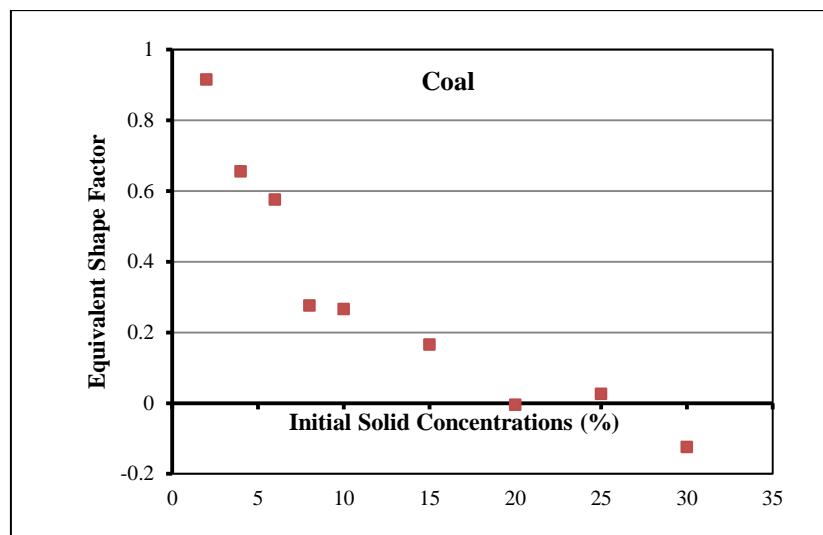


Figure 9. Equivalent shape factor versus initial solid concentration for coal sample with 15 g/t flocculant dosage.

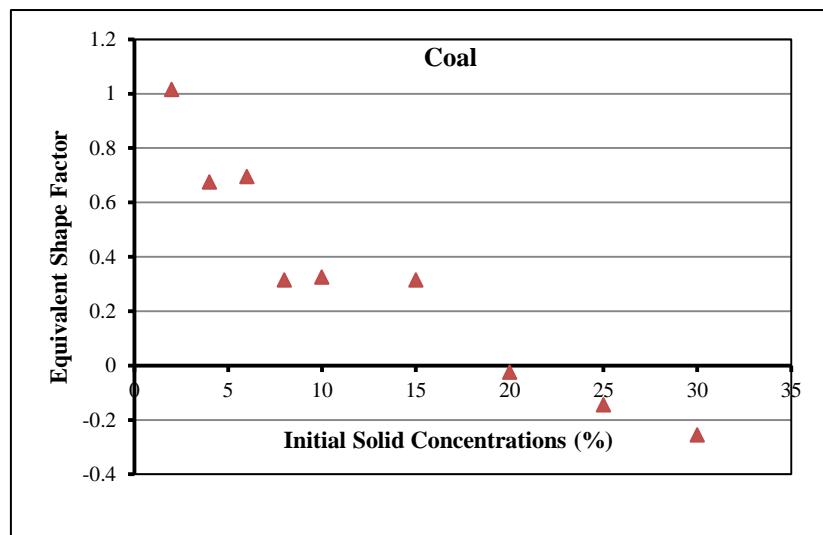


Figure 10. Equivalent shape factor versus initial solid concentration for coal sample with 25 g/t flocculant dosage.

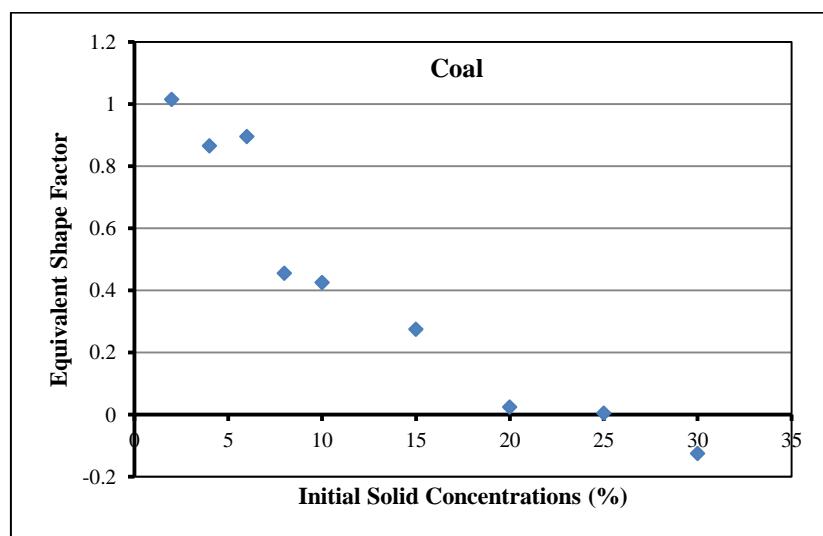


Figure 11. Equivalent shape factor versus initial solid concentration for coal sample with 35 g/t flocculant dosage.

4. Conclusions

In general, the Chein's equation was well-capable to fit on the experimental data regarding the R-squared values ($0.96 < R^2 < 0.99$ for the copper sample, $0.96 < R^2 < 0.98$ for the lead and zinc sample, and $0.93 < R^2 < 0.97$ for the coal sample). The copper and lead & zinc flocs had equivalent shape factors in the range of 0-1 under all the test conditions but the coal flocs had small negative values in higher concentrations.

Under the low-pulp-density conditions, sedimentation of individual flocs is common, and, therefore, the settling rate of the suspension is higher. The equivalent shape factor generally decreases as the initial solid concentration increases. For the coal sample, decrease in the settling velocity as a result of increase in the pressure on the below particles and formation of larger flocs reduces the equivalent shape factor.

Unlike the other samples, the equivalent shape factor for the coal sample was not within the 0-1 range. The use of flocculant and formation of network structures in the samples with higher initial concentrations could be responsible for this observation.

Contrary to the predicted decreasing trend, the equivalent shape factor for 25% of the initial solids in the copper sample and 20% of the initial solids in the lead & zinc sample increases significantly. This behavior is often a result of channeling or short-circuit in the fluid that occurs at high concentrations, and may be related to the cracks in the structure of solids in the presence of inter-particle forces.

Determination of the equivalent shape factors for the mineral materials is an important achievement in mineral processing. Researchers, especially those who wish to design thickeners, can simply put these factors into the Chein's equation to calculate the settling velocity instead of performing expensive and time-consuming experiments and modeling tasks. Although this article introduced this new method, few minerals were investigated, with limited variations in the settling parameters involved. Further studies on different minerals under various settling conditions are required to clarify the merits and demerits of the method.

References

- [1]. Berres, S., Garcés, R. and Usher, S.P. (2013). Characterizing Mineral Slurry Dewatering through Laboratory Centrifugation, 20th International Congress on Modeling and Simulation, Adelaide, Australia.
- [2]. Parsapour, G.A., Arghavani, E., Mosavi, S.M. and Banisi, S. (2014). Designing the feed well of the GOL-E-GOHAR iron ore company thickener, International Journal of Current Life Sciences. 4 (2): 684-687.
- [3]. Unesi, M., Noaparast, M., Shafaei, S.Z. and Jorjani, E. (2014). Modeling the effects of ore properties on water recovery in the thickening process, International Journal of Minerals, Metallurgy and Materials. (21): 9: 851-861.
- [4]. Behrouzi, K., VafaeiFard, M., Raeiszadeh, A. and Faeghinia, A. (2011). Water Recycling at processing plants in water scarce regions- a case study of thickener design for the Mansour Abad processing plant, Proceeding Tailings and Mine Waste, Vancouver, BC.
- [5]. Weston, V. (2013). The Application of Mathematics, Physics, Chemistry and Engineering to Evaluate Solutions in Process, Environmental, and Mineral applications for Separating Suspended matter and Soluble Constituents from an Aqueous Phase, Salt Lake Community College Science, Math and Engineering Symposium.
- [6]. Fitch, B. (1962). Sedimentation Process Fundamentals. Trans, Society of Mining Engineering. AIME, 223: 129-137.
- [7]. Joel, B.C. (1994). Improve Clarifier and Thickener Design and Operation, Chemical engineering progress. 50-56.
- [8]. Schoenbrunn, F., Hales, L. and Bedell, D. (2002).Strategies for instrumentation and control of thickener and other solid-liquid separation circuits, Mineral processing plant design. 2: 2164-2173.
- [9]. Fuerstenau, M.C. and Han, K.N. (2003). Principles of mineral processing. 584 P.
- [10]. Eswaraiah, C., Biswal, S.K. and Mishra, B.K. (2012). Settling characteristics of ultrafine iron ore slimes, International Journal of Minerals, Metallurgy and Materials, 19 (2): 95-99.
- [11]. Vietti, A.J. and Dunn, F. (2014). A description of the sedimentation process during dynamic thickener operation, Australian Centre for Geo-mechanics, pp. 1-10.
- [12]. Parsapour, Gh. A., Hossininasab, M., Yahyaei, M. and Banisi, S. (2014). Effect of settling test procedure on sizing thickeners, Separation and Purification Technology. 122: 87-95.
- [13]. Garmsiri, M.R. and Haji Amin Shirazi, H. (2012). A new approach to define batch settling curves for analyzing the sedimentation characteristics, Journal of Mining & Environment. 3 (2): 103-111.
- [14]. Shi, Z., Zhou, H.J., Eittreim, S.L. and Winterwerp, J.C. (2003). Settling velocities of fine suspended particles in the Changjiang Estuary, China, Journal of Asian Earth Sciences. 22: 245–251.

- [15]. Tan, C.K., Setiawan, R., Lei, Q.Y., Bao, J. and Bickert, G. (2013). Dynamic Modelling and Analysis of Sedimentation-Consolidation Model in a Paste Thickener, Chemeca 2013: Challenging Tomorrow. Barton, ACT: Engineers Australia. Pp. 397-402.
- [16]. Bruce, H. K. (2003). Modeling of hindered-settling column separations, A Thesis in Mineral Processing, Submitted in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy.
- [17]. Moudgil, B.M. and Shah, B.D. (1986). Selection of flocculants for solid-liquid Separation Processes in: H.S., Muralidihara (Ed.), Advances in Solid-Liquid Separation, Ohio. pp. 191-202.
- [18]. Mutsvangwa, C. (2011). Application of Stokes & Newton's laws for settling of discrete particles, Water & Environmental Management. No. WT1/11.
- [19]. Cheng, N.S. (1997). A simplified settling velocity formula for sediment particle, Journal of Hydraulic Engineering, ASCE. 123(2): 149-152.
- [20]. Tsakalakis, K.G. and Stamboltzis, G.A. (2001). Prediction of the settling velocity of irregularly shaped particles.
- [21]. Skei, J.M. and Syvitski J.P.M. (2013). Natural flocculation of mineral particles in seawater- influence on mine tailings sea disposal and particle dispersal, Mineralproduksjon. 3: 1-10.
- [22]. Muhammed, A.R. (2013), Studying the Factors Affecting the Settling Velocity of Solid Particles in Non-Newtonian Fluids, College of Engineering Journal (NUCEJ). 16 (1): 41-50.
- [23]. Eltahir Eltilib, R.A.E., Al Kayiem, H.H. and Azuraien, J. (2011). Investigation on the particle settling velocity in Non- Newtonian fluids, Journal of Applied Sciences. 11 (9): 1528-1535.
- [24]. Goenka, A., Bhunia, K., Chandra Shukla, S. and Kundu, G. (2010). Effect of particle shape on settling characteristics. Ind. Engng. Chem.
- [25]. Dahlstrom, D.A. (1986). Selection of Solid-Liquid Separation Equipment, In Advances in Solid-Liquid Separation, Edited by Muralidihara, H.S., Ohio. pp. 205-239.
- [26]. Owen, A.T., Nguyen, T.V. and Fawell, P.D. (2009). The effect of flocculant solution transport and addition conditions on feed well performance in gravity thickeners, International Journal of Mineral Processing. 93: 115-127.
- [27]. Usher, S.P., Spehar, R. and Scales, P.J. (2009). Theoretical analysis of aggregate densification: Impact on thickener performance. Chemical Engineering Journal. 151: 202-208.
- [28]. Pearse, M.J. (2003). Historical use and future development of chemicals for solid-liquid separation in the mineral processing industry, Minerals Engineering. 16: 103-108.
- [29]. Yalcin, T. (1988). Sedimentation characteristics of Cu-Ni mill tailings and thickener size estimation, Mineral processing, CIM Bulletin. 81 (910): 69-75.
- [30]. Coe, H.S. and Clevenger, G.H. (1917). Methods for determining the capacities of slime settling Tanks, Trans. AIME. 55: 356-384.
- [31]. Bonnier, A.C. (1989). Practical Liquid / Solids Thickeners, CIM Bulletin. 82 (922): 75-76.
- [32]. A language and environment for statistical computing, R foundation for statistical computing, Vienna, Austria, ISBN 3-900051-07-0, URL <http://www.R-project.org/>.

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* نویسنده مسئول مکاتبات: hosseininasab@eng.usb.ac.ir

چکیده:

در این تحقیق، پارامترهایی که سرعت تهشینی را در درون تیکنر تحت تأثیر قرار می‌دهند با معرفی یک فاکتور شکل معادل معرفی شده است. چندین نمونه از خوارک تیکنر با دانسیته‌های متفاوت شامل مس، سرب، روی و زغال آماده شدند. آزمایش‌های تهشینی بر روی نمونه‌ها انجام و منحنی‌های تهشینی متناظر با آن‌ها کشیده شدند. با استفاده از تحلیل رگرسیون خطی، معادله‌ی چین به داده‌های آزمایشگاهی برازش داده شد تا فاکتور شکل معادل برای مواد معدنی مختلف به دست آید. بعلاوه ارتباط بین فاکتورهای شکل معادل و پارامترهای تهشینی بررسی شده است. مقادیر R^2 به دست آمده، قابلیت رابطه‌ی چین در برازش مناسب به داده‌های آزمایشگاهی را تائید می‌کند (برای نمونه مس: $R^2 = 0.99$ ، برای نمونه سرب و روی: $R^2 = 0.96$ و برای نمونه زغال: $R^2 = 0.97$). نتایج به دست آمده نشان داد که فاکتور شکل معادل به طور کلی با افزایش غلظت جامد اولیه در پالپ کاهش می‌یابد. فاکتور شکل معادل برای نمونه‌های زغال با غلظت اولیه بیشتر از ۲۵ درصد، مقادیر منفی اما نزدیک به صفر داشت. این رفتار غیر عادی با بیان تأثیر فلوكولانت در تشکیل ساختار شبکه توضیح داده شده است. تعیین فاکتور شکل معادل، دستیابی ارزشمندی در صنعت فرآوری مواد معدنی خواهد بود. محققین می‌توانند بجای انجام آزمایش‌های وقت‌گیر و هزینه‌بر برای تعیین سرعت تهشینی مواد معدنی، به سادگی این فاکتورهای شکل معادل را در درون معادله‌ی چین قرار دهند.

کلمات کلیدی: سرعت تهشینی، فاکتور شکل معادل، معادله‌ی چین، رگرسیون آماری.