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Technical Note

The effect of price changes on optimum cut-off grade of different open-pit mines

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Abstract
Maximizing economic earnings is the most common goal in cut-off grade optimization of open-pit mining operations. When this is the case, the price of the product has a critical effect on optimum value of cut-off grade. This paper investigates the relationship between optimum cut-off grade and price to maximize total cash flow and net percent value (NPV) of operation. In order to visualize this relationship, two hypothetical mines were employed. To determine the optimum value of cut-off grade in different cases, two nonlinear programming models were formulated, and then, all models were solved using *Solver* in *Excel*. The results show that the optimum cut-off grade would always be a descending function of price when we intend to maximize total cash flow. On the other hand, this function may be descending or ascending when we intend to maximize NPV. This result also reveals that both maximum cash flow and maximum NPV always increase and decrease, respectively when the price of product increases or decreases.

Keywords: *Cut-off grade, Open-pit Mine, Net Present Value, Opportunity Cost, Cut-off grade-price relationship.*

1. Introduction

One of the most critical parameters in mining operation is cut-off grade. Taylor defined cut-off grade as ‘‘any grade that, for any specific reason, is used to separate two courses of action, e.g. to mine or to leave, to mill or to dump . . .’’. (Taylor, 1972; Taylor, 1985)

In another research cut-off grade is defined as the grade that is used to discriminate between ore and waste within a given ore body. (Dagdelen, 1992) The material in a deposit with grade higher than cutoff grade is ore, which is sent to the processing plants. The material below the cutoff grade is sent to the waste dumps (Dagdelen, 1992; Dagdelen, 1993). However, it is critical that the material classified as waste today could become economical to be processed in future (Asad, 2005).

In practice different cut-off grades are defined for different purposes within one operation (Bradley, 1980). Processing plant cut-off grade is an important parameter, which is determined after designing ultimate pit limits. Processing plant cut-off grade is defined as a grade that discriminates between ore and waste blocks within a given pit. If block grade in the pit is above cut-off grade it is classified as ore and if block grade is below cut-off grade it is classified as waste. Ore being the economical portion of the mineral deposit is sent to the mill or processing plant for crushing, grinding, and up-gradation of metal content (Asad, 2007).

As cut-off grade provides a basis for the determination of tons of ore and tons of waste, it directly affects the cash flows of a mining operation, based on the fact, that higher cut-off grade leads to higher grades per ton of ore, hence, higher net present value (NPV) is realized depending upon the grade distribution of the mineral deposit. (Dagdelen, 1993)

Most researchers have used break-even cut-off grade criteria to define ore as a material that will just pay mining and processing costs. These methods are not

optimum but the mine planner often seeks to optimize the cut-off grade of ore to maximize the net present value (NPV). (Osanloo and Ataei, 2003)
The choice of the cutoff grade in mining influences the profitability and life of individual mines and thereby the quantity of a resource that is available to society. The optimal cut-off grade depends on all the salient technological features of mining, such as the capacity of extraction and of milling, the geometry and geology of the ore body, and the optimal grade of concentrate to send to the smelter (Cairns and Shinkuma, 2004).
One important issue in the management of mining firms is how the cut-off grade, the lowest grade of extracted ore, should change in response to a change in the price of the metal. Associated with this issue, there exists "a rule of thumb" for mines. It requires that the cut-off grade should decrease/increase when the rate of metal price increase is greater/smaller than the rate of discount (Shinkuma and Nishiyama, 2000; Shinkuma, 2000).
Price fluctuations are a common feature in metals markets. In the case of some metals, particularly gold and silver, there is empirical evidence that firms reduce the average quality of extracted ore and sometimes reduce the production of metal in response to an increase in the metal price (Farrow and Krautkraemer, 1989).
None of the theoretically derived rules is consistent with empirical regularities in mineral-industry extraction profiles. For example, in the 20th century, most mineral industries experienced a secular decline in both present-value price and average grade of ores mined. This pattern implies a positive correlation between grade and price. The anomaly is that a nominal price increase/ decrease is observed to be accompanied by a decline/ increase in the average grade of ores mined, which implies a negative correlation between grade and price (Slade,1988).
In some instances, the inverse grade-price relationship is the result of conscious national policy concerning a mining industry. For example, the South African government constrains companies to mine to their average grade of gold reserves. Because reserves are defined as material that is profitable under current price and cost conditions, when price increases reserves increase and higher cost ores are mined. The South African policy is often thought to be at variance with private profit-maximizing decisions on the part of firms. However, it is shown that similar behavior results when all decision makers are private companies (in the U.S. copper industry, for example) (Slade, 1988).
When an open pit mine is operating, cut-off grades are used to distinguish economical ore from non-economical ore. If the ore grade is higher than the operating cut-off grade, the mined material is sent to the mill, otherwise it is sent to a dump as waste. However, under a dynamic economic system, this traditional concept of cut-off grades fails to optimize the profit for an open pit mine. This means some valuable ore is dumped as waste (Ren and Sturgul, 1999).
It is impossible to achieve true maximization of present value with a cut-off grade that is constant over time (Taylor, 1972). The practical requirements for this maximization depend strongly on one condition: "Can rejected ore be retrieved subsequently?" and "Provided that ore does not deteriorate in open storage, material below this cut-off, but of foreseeable positive residual value, can with advantage be stockpiled for treatment in the future" (Ren and Sturgul, 1999).
Under this condition, the cut-off should increase as price rises and decreased as price falls. In a period of depressed prices, processing and refining may have room to deal with the stockpiled low grade ore which no longer has mining cost at that time (Ren and Sturgul, 1999).
Cut-off grade optimization can be performed considering different objectives. Maximizing net present value (NPV) is the most applicable objective. Work undertaken in the field of cut-off grade optimization has not advanced much beyond the work undertaken by Lane, which began in 1964 (Lane, 1964) and completed in 1988 (Lane, 1988). His definitive work is based on the calculus of the NPV criterion, which is the most widely understood, consistent, and appropriate method by which sequential cash flows arising from the extraction of mineral reserves from an exhaustible resource can be represented(Minnitt, 2004).

2. Problem definition

As mentioned, the problem under discussion in this paper is the effect of price variation on the optimum cut-off grade. For analyzing this subject two hypothetical mines are examined as examples (Mine I and Mine II). The amounts of mineralized materials in two pits are the same, but their grade-tonnage distributions are different. Also, it is assumed that economic and operational parameters (capacities, price, costs, and so on) for the two mines are the same.

2.1. Defining Model Parameters and Decision Variables

Parameters and decision variables that are used in this research are as follow:

Q_m: Tonnage of total material in the pit, which is a constant amount.

g: Cut-off grade that is the main decision variable of model.

y: Recovery (yield) that is constant.

Q_h: Tonnage of total ore in the pit that is increased as cut-off grade is decreased and vise versa. So Q_h is a descending function of g.

$$Q_h = f_Q(g) \tag{1}$$

$\bar{g}$: Average grade of ore that is increased as cut-off grade is increased and vise versa. So $\bar{g}$ is an ascending function of g.

$\bar{g} = f_g(g)$ (2)

Q_k: Tonnage of total product, which depends on amount of ore and its average grade. Like Q_h, Q_k is a descending function of g.

$Q_k = y.\ \bar{g}\ .Q_h = y.\ f_g(g).f_Q(g)$ (3)

T: Length of production period. This is one of the decision variables.

C_F: Total cash flow results from operation through production period.

$C_F = (p - k)Q_k - mQ_m - hQ_h - fT$ (4)

where p is price of metal, k is smelting, refining, and selling costs per ton of metal, m is mining cost per ton of mined material, h is cost of processing per ton of processed ore, and f is fixed costs per year.

2.2. Data of hypothetical mines

Mineral inventory of two mines is shown in Table 1, and assumed economic and operational parameters of production are shown in Table 2.

Table 1. Mineral inventory for hypothetical mines

Grade (%)	Mine I		Mine II	
	Quantity (10^3tons)	$\bar{g}$ (%)	Quantity (10^3tons)	$\bar{g}$ (%)
< 0.1	395.32	0.05	361.19	0.04
0.1-0.2	37.96	0.17	128.71	0.14
0.2-0.3	57.60	0.24	102.78	0.25
0.3-0.4	77.24	0.33	82.07	0.36
0.4-0.5	96.88	0.44	65.53	0.47
0.5-0.6	116.52	0.58	52.33	0.58
0.6-0.7	98.89	0.64	41.79	0.64
0.7-0.8	51.39	0.75	33.37	0.77
0.8-0.9	28.35	0.85	26.64	0.85
0.9-1.0	14.98	0.95	21.28	0.95
1.0-1.1	7.39	1.05	16.99	1.06
1.1-1.2	5.23	1.15	13.57	1.15
1.2-1.3	3.51	1.25	10.83	1.27
1.3-1.4	2.93	1.35	14.43	1.34
> 1.4	5.81	1.88	28.50	2.50

Table 2. Economic and operational information for hypothetical mines

Recovery (y)	85%
Mining Cost (m)	1.2 \$/t(Rock)
Processing Cost (h)	3 \$/t(Ore)
Smelt, Refine & Sell Cost (k)	750 \$/t(Metal)
Fixed Cost (f)	7000 \$/year
Price (p)	p \$/t(Metal)
Mining Capacity (M)	50000 t/year
Plant Capacity (H)	15000 t/year
Market (or smelting) Capacity (K)	100 t/year
Discount Rate (d)	10%

3. Formulating and Solving the Problem

Problem of investigating relationship between optimum cut-off grade and price is formulated and solved considering two objectives, namely maximizing total cash flow and net present value. Optimization of cut-off grade in each case is performed with different prices, assuming that other parameters remain unchanged. For calculating optimal cut-off grade two non-linear programming (NLP) models, one model for each case, are developed. The constraints for these models are the same, but the objective functions are different according to the objective of optimization. Finally, models have been solved by *Solver* in *Microsoft Excel* environment, considering different prices. Cumulative tons and average grade of ore within two mines as a function of cut-off grade is shown in Table 3. As can be seen there is one million ton mineralized material within each of two mines, and average grade of mineral in mine I and mine II are 0.35% and 0.38%, respectively.

Table 3. Cumulative tons and average grade of ore within hypothetical mines as a function of cut-off grade

g (%)	Mine I		Mine II	
	Q_h (10^3tons)	$\bar{g}$ (%)	Q_h (10^3tons)	$\bar{g}$ (%)
0.0	1000.00	0.35	1000.00	0.38
0.1	604.68	0.54	638.81	0.58
0.2	566.72	0.57	510.10	0.69
0.3	509.12	0.61	407.33	0.80
0.4	431.88	0.66	325.26	0.91
0.5	335.00	0.72	259.72	1.02
0.6	218.48	0.79	207.39	1.13
0.7	119.59	0.92	165.61	1.25
0.8	68.20	1.05	132.24	1.37
0.9	39.85	1.19	105.60	1.50
1.0	24.87	1.33	84.32	1.63
1.1	17.48	1.45	67.33	1.78
1.2	12.25	1.57	53.76	1.94
1.3	8.74	1.70	42.93	2.11
1.4	5.81	1.88	28.50	2.50

3.1. Formulating and solving the problem in order to maximize total cash flow of operation

Considering maximizing total cash flow as objective of cut-off grade optimization, means ignoring time value of money. In this case time of cash flow realization is not important, so traditional relationship between cut-off grade and price becomes the prevailing factor in determining optimum cut-off grade. Therefore, cut-off grade decreases when price increases and vice versa.

Decision variables of this model are g, $\bar{g}$, T, Q_h, and Q_k . Constraints (a), (b), and (c) are related to capacity of mining, processing, and market activities. Equations (d), (e), and (f) are the same equations (1), (2), and (3) discussed in section 2.1. By substituting Q_h, Q_k, and $\bar{g}$ from the last three constraints in the objective function and the first three constraints, number of decision variables will be reduced to two variables, i.e. g and T.

In the model Figure 1 all terms, except functions $f_g(g)$ and $f_Q(g)$, are known or defined. Information in Table 3 can be used to define those two functions. In this table, setting regression between the second and the first columns for $f_Q(h)$, and between the third and the first columns for $f_g(g)$, in the range 0.1%<g<0.6%, results:

Mine I:
$$Q_h = -982g^2 - 85g + 623$$
$$\bar{g} = 0.61g^2 + 0.07g + 0.53$$

Mine II:
$$Q_h = 800e^{-2.25g}$$
$$\bar{g} = 1.1g + 0.465$$

Coefficient of Determination for all of these equations (R^2) is equal to 1.0.

By substituting these equations and amount of parameters from Table 2 in Model I, final model for Mines I and II would be as Figures 2 and 3, respectively. Optimum solutions of these models for different values of p are shown in Table 4.

$$\begin{array}{ll}
\max C_F = (p-750)Q_k - 1.2Q_m - 3Q_h - 7000T & \\
s.t. \quad T \geq 20 & \\
\quad 15000T - Q_h \geq 0 & \\
\quad 100T - Q_k \geq 0 & \\
\quad Q_h + 982g_c^2 + 85g_c - 623 = 0 & \\
\quad \bar{g} - 0.61g_c^2 - 0.07g_c - 0.53 = 0 & \\
\quad Q_k - 0.0085\bar{g}Q_h = 0 & \\
\quad g_c,\ \bar{g},\ Q_h,\ Q_k \geq 0 &
\end{array}$$

Figure 2. Model for cut-off optimization aiming maximization total cash flow (Mine I).

$$\begin{array}{ll}
\max C_F = (p-750)Q_k - 1.2Q_m - 3Q_h - 7000T & \\
s.t. \quad T \geq 20 & \\
\quad 15000T - Q_h \geq 0 & \\
\quad 100T - Q_k \geq 0 & \\
\quad Q_h - 800e^{-2.25g_c} = 0 & \\
\quad \bar{g} - 1.1g_c - 0.465 = 0 & \\
\quad Q_k - 0.0085\bar{g}Q_h = 0 & \\
\quad g_c,\ \bar{g},\ Q_h,\ Q_k \geq 0 &
\end{array}$$

Figure 3. Model for cut-off optimization aiming maximization total cash flow (Mine II).

Table 4. Optimum cut-off grade of two mines for different prices

p($/t)	Mine I		Mine II	
	g_{opt} (%)	C_F ($10^3$$)	g_{opt} (%)	C_F ($10^3$$)
1500	0.53	-825.09	0.49	-455.38
1750	0.42	-287.73	0.37	154.73
2000	0.35	322.38	0.29	826.18
2500	0.25	1634.00	0.23	2239.60
2600	0.23	1903.99	0.22	2531.06
2700	0.22	2175.55	0.21	2824.74
2800	0.20	2448.43	0.20	3120.37
2900	0.19	2722.42	0.19	3417.69
3000	0.18	2997.35	0.19	3716.53
3100	0.17	3273.09	0.18	4016.70
3200	0.16	3549.50	0.17	4318.06
3300	0.15	3826.50	0.17	4620.50
3400	0.14	4103.99	0.16	4923.89
3500	0.13	4381.91	0.16	5228.14
4000	0.09	5775.93	0.14	6759.84

Figure 4 reveals variation of optimum cut-off grade versus price in mines I and II. Variation of maximal values of total cash flow for these mines is shown in Figure 5.

In this case, as mentioned previously, cash flows are indifferent with respect to realization time and, as has been shown in Fig. 4, higher prices that increase worth of less valuable materials, drive optimum cut-off grade toward lower values, and vice versa.

Also, as can be seen in Fig. 5, maximum amount of total cash flow is an ascending function of price.

3.2. Formulating and solving problem in order to maximize NPV of operation

Maximizing NPV is the most applicable objective in cut-off grade optimization. Setting NPV maximization as objective function of the model, involves time value of money in calculations.

Models with NPV maximization objective, commonly apply opportunity cost concept. The opportunity cost involves lost opportunities due to postponing income realization, and is a function of present value of unrealized incomes related to materials that have not been mined and remained within the pit. Higher opportunity costs result in upper optimum cut-off grade (Lane, 1988).

More non-mined material means more potential present value, and so higher opportunity cost. Therefore, at the beginning of mining operation that no material is mined, amount of opportunity cost is in the highest level. Through the operation period with increasing amount of mined material, the opportunity cost is decreased and simultaneously optimum cut-off grade is lowered (Lane, 1988). In other words, to achieve true maximization of present value, optimum cut-off grade have to change trough the operation period. In the models using opportunity cost for maximizing present value, objective function is as follow:

$$Z = (p-k)Q_k - mQ_m - hQ_h - (F+f)T \qquad (5)$$

where F is the opportunity cost.

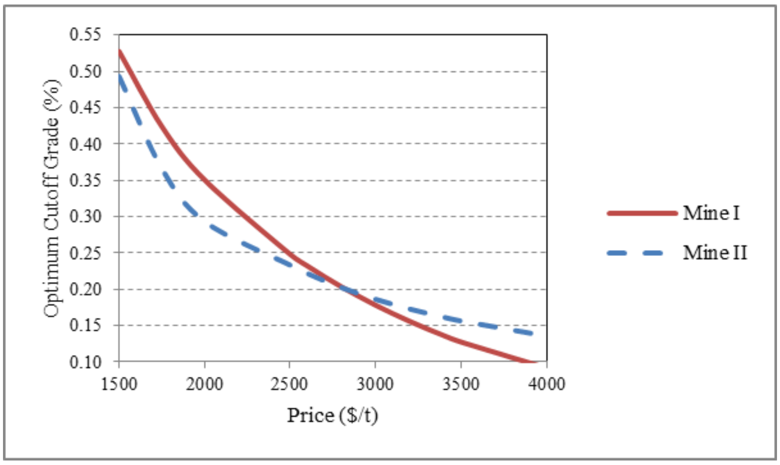

Figure 4.Optimum cut-off grade versus price (Total cash flow maximization).

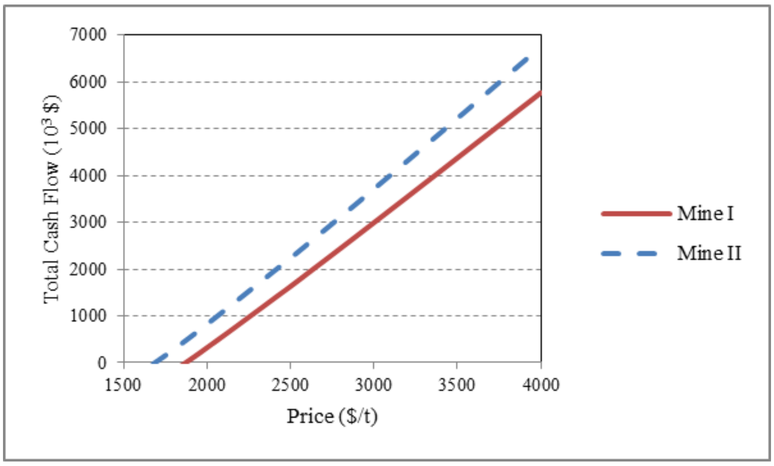

Figure 5.Maximumvalue of total cash flow versus price.

For simplicity, this paper maximizes present value assuming constant optimum cut-off grade through the time instead of using opportunity cost that causes change of optimum cut-off grades in different years. Even though this approach does not result in true maximum value of NPV, and the amount of objective function is a little less, firstly, the difference between two NPVs in two models is not significant, secondly, the target of investigation i.e. entering time value of money in calculations is achieved, and finally, considering constant cut-off grade simplifies formulation of problem and solving it. In this approach objective function of model will be as follows:

$$V = \frac{C_F}{T} \cdot \frac{(1+d)^T - 1}{d\,(1+d)^T} \qquad (6)$$

where d is the discount rate.

Formulation of cut-off grade optimization models in mines I and II are shown in Figures 6 and 7.

Optimum solutions of these models for different prices are shown in Table 5.

Figure 8 reveals variation of optimum cut-off grade versus price changes in mines I and II. Variation of maximum value of net present value for these mines is shown in Figure 9. As can be seen, in this case curve of optimum cut-off grade versus price in mine I is ascending, while in mine II this trend is descending. This behavior is due to the fact that when optimization aims at maximizing NPV, cut-off grade will be subject to two opposite forces.

1. Higher prices increase worth of less valuable material and so decrease cut-off grade,
2. Increase the value of material remained in the pit, so cause higher opportunity cost that drive cut-off grades toward upper values.

Therefore, when price rises, the resultant of above two opposite forces determines the direction of optimum cut-off grade movement. In mine I driving force resulted from opportunity cost has overcome other force, and in mine II vice versa. Figure 9 shows that for both mines the variation of maximum NPV versus price is ascending.

$$\begin{aligned}
&\max V = \frac{C_F}{T}\cdot\frac{1.10^T-1}{0.10\times 1.10^T}\\
&s.t. \quad T \ge 20\\
&\qquad 15000T - Q_h \ge 0\\
&\qquad 100T - Q_k \ge 0\\
&\qquad Q_h + 982g_c^2 + 85g_c - 623 = 0\\
&\qquad \bar{g} - 0.61g_c^2 - 0.07g_c - 0.53 = 0\\
&\qquad Q_k - 0.0085\,\bar{g}\,Q_h = 0\\
&\qquad C_F = (p-750)Q_k - 1.2Q_m - 3Q_h - 7000T\\
&\qquad g_c,\ \bar{g},\ Q_h,\ Q_k \ge 0
\end{aligned}$$

Figure 6.Model for cut-off optimization aiming maximization net present value (Mine I).

$$\begin{aligned}
&\max V = \frac{C_F}{T}\cdot\frac{1.10^T-1}{0.10\times 1.10^T}\\
&s.t. \quad T \ge 20\\
&\qquad 15000T - Q_h \ge 0\\
&\qquad 100T - Q_k \ge 0\\
&\qquad Q_h - 800e^{-2.25g_c} = 0\\
&\qquad \bar{g} - 1.1g_c - 0.465 = 0\\
&\qquad Q_k - 0.0085\,\bar{g}\,Q_h = 0\\
&\qquad C_F = (p-750)Q_k - 1.2Q_m - 3Q_h - 7000T\\
&\qquad g_c,\ \bar{g},\ Q_h,\ Q_k \ge 0
\end{aligned}$$

Figure 7.Model for cut-off optimization aiming maximization net present value (Mine II).

4. Conclusions

In a designed open-pit mine, when cut-off grade optimization aims at maximizing economic earnings, price has an essential effect on optimum cut-off grade. This paper investigated the relationship between optimum cut-off grade and price for total cash flow and net present value maximization. In order to visualize this relationship, two hypothetical mines were examined. For calculating optimum cut-off grade in different cases two nonlinear programs were formulated and solved by *Solver* in *Microsoft Excel*.

Table 5. Optimum cut-off grade of two mines for different amounts of price

p($/t)	Mine I		Mine II	
	g_{opt} (%)	C_F ($10^3$$)	g_{opt} (%)	C_F ($10^3$$)
1500	0.30	-282.58	0.41	-172.00
1750	0.38	-93.21	0.38	55.09
2000	0.40	101.14	0.36	283.51
2500	0.44	495.96	0.33	743.25
2600	0.44	575.47	0.32	835.56
2700	0.44	655.09	0.32	927.97
2800	0.45	734.81	0.31	1020.48
2900	0.45	814.61	0.31	1113.08
3000	0.45	894.49	0.30	1205.76
3100	0.46	974.43	0.30	1298.52
3200	0.46	1054.42	0.29	1391.35
3300	0.46	1134.46	0.29	1484.24
3400	0.46	1214.55	0.29	1577.15
3500	0.46	1294.67	0.29	1670.05
4000	0.47	1695.74	0.29	2134.56

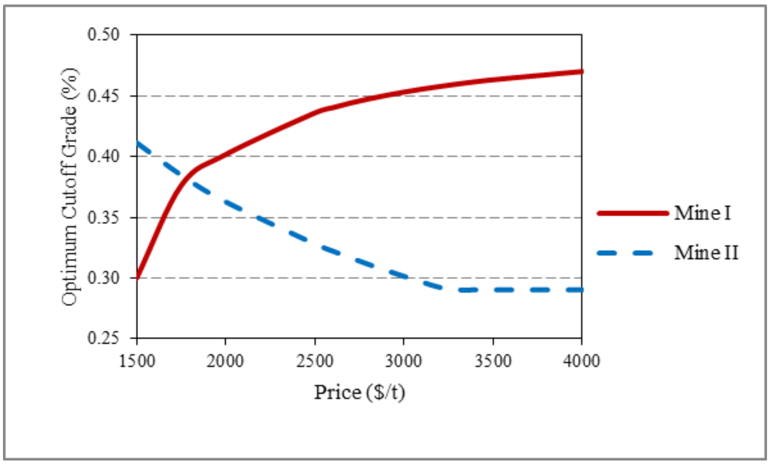

Figure 8.Optimum cut-off grade versus price (Present Value maximization).

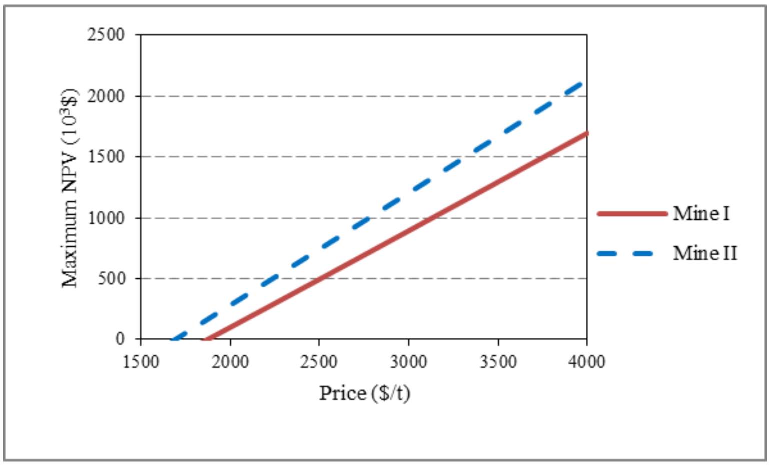

Figure 9. Maximum present value versus price.

Outcomes of this paper can be summarized as follow,

1- If objective of cut-off grade optimization is maximizing total cash flow, traditional relationship between cut-off grade and price would be prevailed, i.e. higher prices result in lower cut-off grades.

2- When objective of cut-off grade optimization is maximizing net present value, higher prices may lead to higher or lower cut-off grades.

3- Variation of both maximum cash flow and maximum net present value versus price are ascending.

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