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Robust production scheduling in open-pit mining under uncertainty: a box counterpart approach

A. Alipour[1*], A.A. Khodaiari[1], A. Jafari[1] and R. Tavakkoli-Moghaddam[2, 3]

1. School of Mining Engineering, College of Engineering, University of Tehran, Tehran, Iran
2. School of Industrial Engineering, College of Engineering, University of Tehran, Tehran, Iran
3. LCFC, Arts et Métier ParisTech, Centre de Metz, France

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**Corresponding author: aref.alipour@gmail.com (A. Alipour).*

Abstract

Open-Pit Production Scheduling (OPPS) problem focuses on determining a block sequencing and scheduling to maximize Net Present Value (NPV) of the venture under constraints. The scheduling model is critically sensitive to the economic value volatility of block, block weight, and operational capacity. In order to deal with the OPPS uncertainties, various approaches can be recommended. Robust optimization is one of the most applicable methods in this area used in this study. Robust optimization based on the box counterpart formulation is applied to deal with the OPPS problem. To have a comparison between the solutions of the box counterpart optimization model and the deterministic model, a Two-Dimensional (2D) numerical study of a hypothetical open-pit mine is conducted followed by additional computations on the actual large-scale instances (Marvin orebody). This investigation shows that the different features of the robust planning under uncertainty can be scheduled. Also the price of robustness is obtained in different levels of conservatism.

Keywords: *Open-Pit Mine Production Scheduling, Robust Counterpart Optimization, Uncertainty, Block Economic Value*.

1. Introduction

Open-pit mines are typically represented by discretization of the orebody consisting of equal-sized units known as blocks. Geological attributes such as weight, grade, rock description, and location are assigned to individual blocks [1-3]. OPPS concentrates on determining a block extraction sequence in a way that maximizes NPV of the venture under access, mining capacity, processing capacity constraints, and some other criteria such as blending constraints (extracted ore grade) [4].

The optimization of OPPS has a long history, and numerous studies have addressed different features of the deterministic OPPS problem. In 1967, the graph theory and network flow-based approaches were applied [5]. Tolwinski and Underwood [6] have developed a dynamic programing combined with a heuristic method. Caccetta and Hill [7] have presented a Mixed-Integer Programming (MIP) model and introduced a branch-and-cut algorithm to solve the problem. Lambert and Newman [8] have employed tailored Lagrangian relaxation in the OPPS formulation. Detailed information on the OPPS formulation can be found in a tutorial of fundamental OPPS mathematical formulation models by Lambertet al. [9]. Shishvan and Sattarvand [10] have applied an ant colony optimization to untangle an extended OPPS problem for a real-world mine. Liu and Kozan [11] have developed two different graph-based algorithms to tackle with large-scale benchmark OPPS instances from Mine Lib. To find more related research works in the area of OPPS problem, the readers are referred to an article by Kozan and Liu [12].

Production scheduling problem solutions are critically sensitive to price volatility, ore grade uncertainty, operational capacity, etc. Hence, the scheduling process involves a significant degree of uncertainty. In order to deal with uncertainties, various approaches can be recommended, some of which include the chance-constrained programming, stochastic programing with recourse and robust stochastic optimization, fuzzy programing, and robust optimization programing. Due to the inherent limitations and compatibility of chance-constrained and fuzzy programing methods, their application is limited in mine production scheduling problems. In the scenario-based optimization methods such as stochastic programing with resource framework, considering conditional simulation onerous procedure and large number of blocks, a large-scale optimization problem is resulted. (The readers are referred to references [13-15].) To see more related research works in the area of uncertain OPPS problem, the readers are referred to the references [16-20].

The robust optimization method is an applicable option in dealing with mine production scheduling problem uncertainties. Data uncertainties may lead to quality, optimality, and feasibility problems when deterministic models are used. Hence, it is required to generate a solution immune to data uncertainty. In other words, the solution should be robust [21]. The robust optimization theory provides a framework to handle the uncertainty of parameters in the optimization problems that could immunize the optimal solution for any realization of the uncertainty in a given bounded uncertainty set [22-24].

The variation in response is derived from uncertainties in the design variables and/or design parameters [21, 25]. The purpose of global robust optimization is to find a design with the target response and the smallest variation. Different types of robust framework have been developed. Soyster [26] have considered simple perturbations in the data aiming to find a framework of robust counterpart optimization such that the resulting solutions are feasible under all possible perturbations.

The robust counterpart optimization techniques are broadly used in engineering optimization problems. Set-induced robust counterpart optimization techniques include interval set, combined interval and ellipsoidal, adjustable box, pure ellipsoidal, pure polyhedral, combined interval, ellipsoidal, and polyhedral set [21]. Lin et al. [27] have introduced mixed integer linear optimization (MILP) robust optimization formulation. Verderame and Floudas have applied both the continuous and discrete uncertainty distributions to extend the robust optimization framework [28]. The degree of solution conservatism has been considered in Bertsimas and Sim [23]. A combined interval and polyhedral uncertainty set with coefficient uncertainty has been presented for robust linear programming. Afterward, Bertsimas and co-workers [29] have applied a robust optimization framework in the fields of discrete programming. Averbakh [22] has suggested a general approach to find min-max regret solutions for a class of combinatorial problems with interval uncertain objective function coefficients based on reducing the problem with uncertainty to a set of deterministic problems. Equivalency of set-based robust optimization formulations and conditional value-at-risk (CVaR) bound-based approximations to individual chance constraints has been demonstrated in Chen et al.'s work [30].

Mulvey et al. [31] and Yu and Li [32] have developed a stochastic model called RSO to capture the randomness of the uncertain parameters. The aim of RSO is not only to maximize/minimize the objective function but also to obtain a robust solution. In other words, RSO attempts to generate a solution that is insensitive to different realizations of input data. Ghaoui et al. [33] have used the worst-case probability distributions to extend worst-case value-at-risk (VaR) bounds for a robust linear optimization.

Price, costs, and discount rate used in OPPS problem depend upon a series of unknown future events, and are modelled by stochastic processes. There have been more investigations conducted regarding ore grade (geological) and price uncertainty in open-pit mine stochastic production scheduling problems [4, 19, 34, 35].

The application of the three different types of robust optimization in OPPS problem has been reported in different research works. Kumral [19] has presented a stochastic robust optimization model to deal with uncertainty in block grades, price, mining, and processing costs. Espinoza et al. [36] have implemented an uncertainty-based robust optimization method to consider the volatility of metal. Lagos et al. [37] have compared Value-at-Risk, Conditional Value-at-Risk, and a proposed Modulated Convex-Hull robust optimization model for optimization under ore-grade uncertainty of each block. In this work, robust counterpart optimization formulation based on the box counterpart was applied to handle the OPPS problem.

2. Robust counterpart optimization

Robust optimization is an approach used for modeling optimization problems under uncertainty, where the modeler aims to find optimal decisions for the worst-case realization of the uncertainties within a given set. Typically, the original uncertain optimization problem is converted into an equivalent deterministic form (called the robust counterpart) using strong duality arguments and is solved using standard optimization algorithms [21].

In set induced robust optimization, the uncertain data is assumed to vary in a given uncertainty set, and the aim is to choose the best solution among those "immunized" against data uncertainty, i.e. candidate solutions that remain feasible for all realizations of the data from the uncertainty set. In general, consider the following linear optimization problem with uncertainty in the left hand side (LHS) constraint coefficients, right hand side (RHS), and objective function coefficients:

$$\begin{aligned} &\max \sum_j \tilde{c}_j x_j \\ &s.t. \\ &\sum_j \tilde{a}_{ij} x_j \le \tilde{b}_i, \forall i \end{aligned} \tag{1}$$

where x_j can be either a continuous or an integer variable. Note that the objective and RHS uncertainty can be transformed into LHS uncertainty as follows:

$$\begin{aligned} &\max z \\ &s.t. \\ &z - \sum_j \tilde{c}_j x_j \le 0 \\ &\tilde{b}_i x_0 + \sum_j \tilde{a}_{ij} x_j \le 0, \forall i \\ &x_{0=-1} \end{aligned} \tag{2}$$

Thus without loss of generality, we focused on the following general *ith* constraint of a (mixed integer) linear optimization problem considering only the LHS uncertainty:

$$\sum_j \tilde{a}_{ij} x_j \le b_i \tag{3}$$

and $\tilde{a}$ is subject to uncertainty. Define the uncertainty as follows:

$$\tilde{a}_{ij} = a_{ij} + \xi_{ij} \hat{a}_{ij}, \forall j \in J_i, \tag{4}$$

where a_{ij} represents the nominal value of the parameters, $\hat{a}_{ij}$ represents positive constant perturbations, ξ_{ij} represents independent random variables that are subject to uncertainty, and J_i represents the index subset that contains the variables whose coefficients are subject to uncertainty. Constraint (3) can be re-written by grouping the deterministic part and the uncertain part for the LHS of (3) as follows:

$$\sum_j a_{ij} x_j + \sum_{j \in J_i} \xi_{ij} \hat{a}_{ij} x_j \le b_i \tag{5}$$

In the set induced robust optimization method, the aim is to find solutions that remain feasible for any ξ_{ij} in the given uncertainty set U so as to immunize against infeasibility, i.e.

$$\sum_j a_{ij} x_j + \max_{\xi \in U} \left\{ \sum_{j \in J_i} \xi_{ij} \hat{a}_{ij} x_j \right\} \le b_i \tag{6}$$

2.1. Uncertainty Sets

The formulation of the robust counterpart optimization model is related to the selection of the uncertainty set U. In this section, three general types of uncertainty sets are introduced. For the sake of simplicity, we eliminated the constraint index i in the random vector ξ.

$$\begin{aligned} &\text{"Box "US} \\ &U_\infty = \left\{ \xi \middle| \|\xi\|_\infty \le \Psi \right\} = \left\{ \xi \middle| |\xi_j| \le \psi, \forall j \in J_i \right\} \end{aligned} \tag{7}$$

$$\begin{aligned} &\text{"Polyhedral"US} \\ &U_1 = \left\{ \xi \middle| \|\xi\|_1 \le \Gamma \right\} = \left\{ \xi \middle| \sum_{j \in J_i} |\xi_j| \le \Gamma \right\} \end{aligned} \tag{8}$$

$$\begin{aligned} &\text{"Box + Polyhedral"US} \\ &U_{1\infty} = \left\{ \xi \middle| \sum_{j \in J_i} |\xi_j| \le \Gamma, |\xi_j| \le \Psi, \forall j \in J_i \right\} \end{aligned} \tag{9}$$

where Ψ and Γ are the adjustable parameters controlling the size of the uncertainty sets. Different advanced frameworks of uncertainty sets are introduced in references [21, 38].

2.2. Box counterpart optimization formulation

For constraint (5), its robust counterpart optimization formulation (6) was derived for different uncertainty sets introduced above as follows; if the set U is the box uncertainty set (7), the corresponding robust counterpart constraint (6) will be equivalent to the following constraints:

$$\sum_j a_{ij}x_j + \Psi \sum_{j\in J_i} \hat{a}_{ij}u_j \le b_i$$
$$-u_j \le x_j \le u_j \qquad (10)$$

The box uncertainty set can be described using the $\infty-norm$ of the uncertain data vector, as follows: $U_\infty = \left\{\xi \middle| \|\xi\|_\infty \le \Psi\right\} = \left\{\xi \middle| |\xi_j| \le \Psi, \forall j \in J_i\right\}$, where Ψ is the adjustable parameter controlling the size of the uncertainty set. Figure 1 illustrates the box uncertainty set for parameter $\tilde{a}_j$ defined by $\tilde{a} = a_j + \xi_{j\hat{a}j}, j = 1,2$, where $\tilde{a}_j$ denotes the true value of the parameter, a_j denotes the nominal value of the parameter, ξ_j denotes the uncertainty, and $\tilde{a}_j$ represents a constant perturbation. If the uncertain parameters are known to be bounded in given intervals $\tilde{a}_{ij} \in \left[a_{ij} - \tilde{a}_{ij}, a_{ij} + \tilde{a}_{ij}\right] \forall j \in J_i$, the uncertainty can be represented by $\tilde{a}_{ij} = a_{ij} + \xi_{j\hat{a}ij}$, and this results in the interval uncertainty set, which is a special case of box uncertainty set when $\Psi = 1 (i.e., U_\infty = \left\{\xi \middle| |\xi_j| \le 1, \forall j \in J_i\right\})$ [24]. For the first time, Soyster [26] used the "interval uncertainty set" to denote the box set with Ψ= 1.
To attain robust solutions, we looked for solutions feasible for any realization of the uncertain data in a predefined uncertainty set. More details about linear robust optimization accompanied with a simple numerical example are presented in the appendix (see reference [21]).

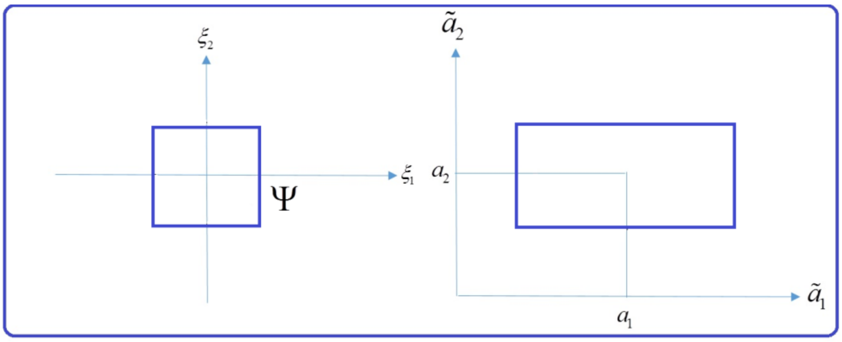

Figure 1. Illustration of box uncertainty set [21].

3. Computational study for mine robust production scheduling

3.1. Implementation and evaluation with a simple example

Mine production scheduling focuses on determining a block mining in order to maximize NPV under sequencing and capacity constraints. The OPPS problem in deterministic form can be presented using the following formulations [19]:

Let x_{ij} be the decision variable and T be the number of mining periods. N denotes the number of blocks, V_{ij} is the present value of block j in period i, d_j is the ore mass in block j, A, A' show the processing capacity (maximum and minimum capacity, respectively), v_j is the waste mass in block j, and C, C' are mining capacity. Our goal was to find:

$$\max \sum_{i=1}^{T}\sum_{j=1}^{N} V_{ij}x_{ij}, (1)$$
$$s.t.$$
$$\sum_{i=1}^{T} x_{ki} \ge \sum_{i=1}^{T} x_{ji},\ j\ blocks\ overlying\ block\ k, (2)$$
$$C' \le \sum_{j=1}^{N}(d_j + v_j)x_{ij} \le C, \forall i = 1,...,T\ :Constraint\ of\ mining\ capacity\ (3) \qquad (11)$$
$$A' \le \sum_{j=1}^{N} d_j x_{ij} \le A, \forall i = 1,...,T\ :Constraint\ of\ processing\ capacity\ (4)$$
$$\sum_{i=1}^{T} x_{ij} \le 1, \forall j = 1,...,N, (5)$$

The sequencing constraint (11.2) means that in order to (?) extract a block, the overlying blocks should be mined either earlier or in the same period.
The mining capacity constraint (11.3) is chosen according to the economic and operational considerations.
Processing capacity constraint (11.4): In the open-pit mining, both ore and waste blocks are mined due to the sequencing constraint. While waste blocks are sent to the waste dumps, ore blocks are transferred to the mineral processing plant. The amount of ore blocks should be in commensurate with the processing capacity.
Block conservation constraint (11.5): This constraint guarantees that a block can be mined just for one time.
Deterministic OPPS approaches do not consider data uncertainties. The implementations based on deterministic solutions may lead to significant NPV losses/rewards and/or capacity utilization problems. Now we introduce the complete box uncertainty set that induces robust counterpart formulation for OPPS with constraints. It can be expressed as follows:
where V_{ij} denotes the nominal values of the parameters, Ψ denotes the uncertainty, and $\hat{V}_{ij}, \hat{r}_j, \hat{v}_j, \hat{C}, \hat{C}', \hat{A}, \hat{A}', \hat{d}$ represent constant perturbation of the objective function and constraint coefficients.
A hypothetical copper deposit with relevant data for developing economic block model was used to explain the details of the robust OPPS method implementation. Besides, the open-pit mine was typically represented by 2D blocks in this investigation. The effective parameters for calculating the economic values of blocks are presented in Table 1.

$$
\begin{aligned}
&\max \sum_{i=1}^{T}\sum_{j=1}^{N} V_{ij}x_{ij} - \Psi\left(\sum_{i=1}^{T}\sum_{j=1}^{N}\hat{V}_{ij}x_{ij}\right)\\
&s.t.\\
&\sum_{i=1}^{T} x_{ki} \geq \sum_{i=1}^{T} x_{ji}\\
&\sum_{j=1}^{N} r_j x_{ij} + \Psi\left(\sum_{j=1}^{N}\hat{r}_j x_{ij} + \hat{C}\right) \leq C, \forall i=1,\dots,T, d_j + v_j = r_j : Constraint\ of\ mining\ capacity\\
&C' \leq \sum_{j=1}^{N} r_j x_{ij} + \Psi\left(\sum_{j=1}^{N}\hat{r}_j x_{ij} + \hat{C}'\right), \forall i=1,\dots,T, d_j + v_j = r_j : Constraint\ of\ mining\ capacity\\
&\sum_{j=1}^{N} d_j x_{ij} + \Psi\left(\sum_{j=1}^{N}\hat{d}_j x_{ij} + \hat{A}\right) \leq A, \forall i=1,\dots,T : Constraint\ of\ processing\ capacity\\
&A' \leq \sum_{j=1}^{N} d_j x_{ij} + \Psi\left(\sum_{j=1}^{N}\hat{d}_j x_{ij} + \hat{A}'\right), \forall i=1,\dots,T : Constraint\ of\ processing\ capacity\\
&\sum_{i=1}^{T} x_{ij} \leq 1, \forall j=1,\dots,N
\end{aligned}
\tag{12}
$$

Table 1. Parameters used for calculating economic values of blocks.

Item	Description
Recovery	0.88
Selling price ($/t)	5500
Selling cost ($/t)	1000
Processing cost ($/t)	4
Mining cost ($/t)	1
Discount rate (%)	4
Dimension of blocks in X(m)	10
Dimension of blocks in Y(m)	10

Each block must have associated economic values related to its possible destination. Each field related to the economic value (economic value Process/Waste) has to represent the value of blocks as a function of its destination, grades, recovery, cost of mining, transport, treatment, selling price, etc. The economic parameters of blocks for block value calculations are presented in Table 2. In this table, the maximum value of economic process and economic waste of each block is considered as the block value. The economic value of a block is equal to its selling price minus the extraction costs. For example, for a copper mine, the economic value of a block is calculated as follows in Equations (13) to (15).
For further explanation, the block economic value calculation method is presented in Table 2.
A hypothetical copper deposit with geological block model containing 200 blocks was assumed and subjected to scheduling to explain the details of the proposed algorithm implementation (Figure

2). Table 1 illustrates the technical and economic parameters used for the economic block model. Figure 3 demonstrates the economic block model of the hypothetical copper deposit prepared to be imported into the OPPS problem. The mining operation was considered to continue for 4 years, and the maximum and minimum mining capacities were assumed to be 24 and 18 blocks per year, respectively. The maximum and minimum processing capacities were considered to be 15 and 9 ore blocks per year, and the discount rate was assumed to be 4%.

Box counterpart robust optimization was systematically applied in this section. Also only block economic value (coefficients of objective function) was considered as an uncertain parameter. Perturbation rate of value can be represented as follows:

$$\hat{V}_{i=1,2j} = 0.1,$$
$$\hat{V}_{i=3,4j} = 0.2,$$
$$\hat{V}_{i=5,6j} = 0.3, .$$
$$\hat{V}_{i=7,8j} = 0.4,$$
$$\hat{V}_{i=9,10j} = 0.5$$

Considering the objective function uncertainty, first the objective uncertainty was transformed into constraint uncertainty, and then the uncertainty set-induced robust counterparts were derived based on the LHS uncertainty.

Generalization of the Soyster method as a part of box counterpart robust optimization was systematically applied in this section. This method is a worst-case scenario for the mine production scheduling problem. The worst case scenario solution means that the uncertainty set covers the whole uncertainty space. According to the CPLX solver results, for instance, the NPV's of deterministic and the robust box counterpart known as "interval set" (the box set with Ψ= 1) were obtained to be 41565 and 31421$, respectively, in this research work.

Bertsimas and Sim [23] have presented the concept of ''price of robustness'', which considers how ''heavily'' the objective function value is penalized when we are guarded against objective underperformance and/or constraint violation. Implicitly, this is the difference between the robust solution and the objective function value of deterministic state.

The variation in conservatism level (Ψ) versus NPV is illustrated in Figure 4. It can be observed that the NPV value decreases by increasing Ψ (i.e. the size of uncertainty set increases). The results obtained for NPV and price of robustness are demonstrated in Table 3. The price of robustness is a proportion of NPV values by the robust optimization and deterministic method.

Mine block scheduling and sequencing by exact CPLEX solver in two different cases $(\Psi = 0, \Psi = 4)$ are demonstrated in Figure 5. This figure shows that the robust production scheduling is different from the deterministic state; according to the price of robustness specific production, scheduling was obtained.

$$MassCu(t) = BlockVolume * BlockDensity * g_{Cu} / 100 \tag{13}$$

$$\begin{aligned} &EconomicValue\,\mathrm{Pr}ocess(\$) = \\ &BlockVolume * BlockDensity * g_{Cu} / 100 * \mathrm{Recov}ery * (Selling\,\mathrm{Pr}ice - Selling\,\mathrm{Cos}t) \\ &EconomicValue\,\mathrm{Pr}ocess(\$) = MassCu * \mathrm{Recov}ery * (Selling\,\mathrm{Pr}ice - Selling\,\mathrm{Cos}t) \end{aligned} \tag{14}$$

$$EconomicValueWaste(\$) = -BlockVolume * BlockDensity * Mining\,\mathrm{Cos}t \tag{15}$$

Table 2. Example of an economic block model chart prepared to be applied into the model.

IX	IY	Cu grade (%)	Density	Price ($)	m*m	Block mass (ton)	Mass Cu (ton)	Econ. Process	Econ. Waste	Block Value ($)
1	1	0.04	2.27	5500	100	227	0.0908	-775	-227	-227
2	2	0.17	2.27	5500	100	227	0.3859	393	-227	393
3	1	0.04	2.27	5500	100	227	0.0908	-775	-227	-227
4	12	0.12	2.27	5500	100	227	0.2743	-56.	-227	-56
9	8	0.2	2.27	5500	100	227	0.454	663	-227	663

Y j

X i

0.04	0.05	0.06	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.02
0.04	0.17	0.15	0.14	0.2	0.17	0.17	0.12	0.19	0.22	0.1	0.18	0.05	0.04	0.25	0.17	0.31	0.2	0.16	0.02
0.04	0.17	0.11	0.12	0.2	0.2	0.15	0.14	0.28	0.07	0.19	0.28	0.1	0.25	0.25	0.18	0.01	0.38	0.32	0.01
0.01	0.13	0.2	0.24	0.39	0.25	0.12	0.14	0.02	0.25	0.29	0.09	0.25	0.23	0.31	0.25	0.02	0.19	0.02	0.02
0.01	0.23	0.11	0.26	0.22	0.02	0.13	0.2	0.14	0.17	0.3	0.16	0.13	0.28	0.11	0.22	0.2	0.15	0.03	0.03
0.01	0.04	0.2	0.21	0.02	0.29	0.15	0.17	0.3	0.29	0.26	0.32	0.04	0.26	0.41	0.17	0.19	0.21	0.25	0.01
0.02	0.17	0.18	0.27	0.24	0.33	0.21	0.08	0.35	0.36	0.17	0.22	0.23	0.38	0.23	0.25	0.18	0.09	0.22	0.01
0.03	0.3	0.37	0.18	0.42	0.1	0.25	0.15	0.18	0.16	0.13	0.18	0.16	0.25	0.15	0.15	0.03	0.2	0.09	0.02
0.02	0.34	0.12	0.24	0.29	0.38	0.21	0.2	0.16	0.15	0.16	0.16	0.13	0.32	0.27	0.14	0.15	0.17	0.23	0.02
0.17	0.12	0.27	0.19	0.23	0.16	0.07	0.04	0.13	0.17	0.18	0.06	0.14	0.2	0.34	0.16	0.32	0.18	0.02	0.02

Figure 2. Geological block model of hypothetical copper mine.

Y j

X i

-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227	-227
-227	393	213	123	663	393	393	-56	573	843	-227	483	-227	-227	1112	393	1652	663	303	-227
-227	393	-146	-56	663	663	213	123	1382	-227	573	1382	-227	1112	1112	483	-227	2281	1742	-227
-227	34	663	1022	2371	1112	-56	123	-227	1112	1472	-227	1112	933	1652	1112	-227	573	-227	-227
-227	933	-146	1202	843	-227	34	663	123	393	1562	303	34	1382	-146	843	663	213	-227	-227
-227	-227	663	753	-227	1472	213	393	1562	1472	1202	1742	-227	1202	2551	393	573	753	1112	-227
-227	393	483	1292	1022	1831	753	-227	2011	2101	393	843	933	2281	933	1112	483	-227	843	-227
-227	1562	2191	483	2640	-227	1112	213	483	303	34	483	303	1112	213	213	-227	663	-227	-227
-227	1921	-56	1022	1472	2281	753	663	303	213	303	303	34	1742	1292	123	213	393	933	-227
393	-56	1292	573	933	303	-227	-227	34	393	483	-227	123	663	1921	303	1742	483	-227	-227

Figure 3. Economic block model of hypothetical copper mine.

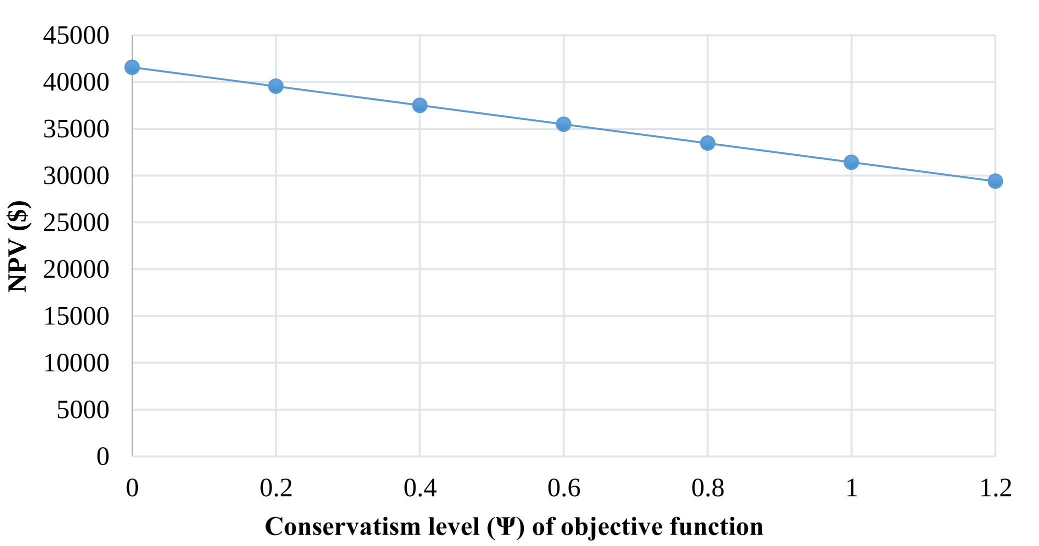

Figure 4. NPV versus different box counterpart optimization level, Ψ.

Table 3. Summary of optimization solutions for box counterpart model (2D model).

Conservatism level (Ψ) of objective function	NPV ($)	Price of robustness
0, Deterministic model	41565	1
0.2	39537	0.95
0.4	37508	0.90
0.6	35480	0.85
0.8	33451	0.80
1, Soyster model (interval set)	31421	0.75
1.2	29390	0.71

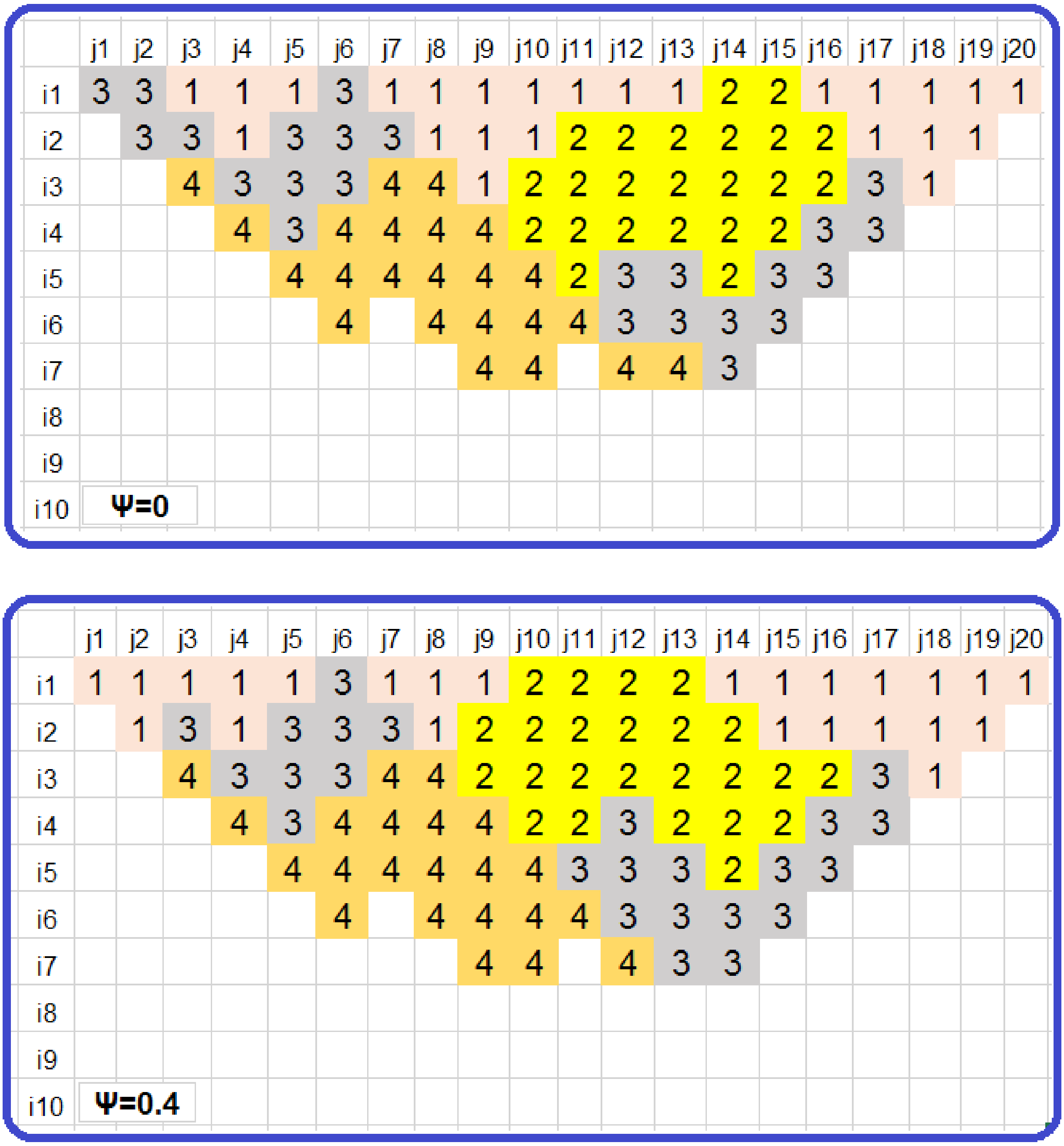

Figure 5. Production scheduling solutions in deterministic and robust form $(\Psi = 0, \Psi = 4)$.

3.2. Case study: Marvin orebody

The data set used in this section came from the work of Whittle Challenge [39, 40]. The economic and technical parameters in Marvin oreboy are summarized in Table 4.

Table 4. Summary of input distributions and parameters.

Parameter	Value
Recovery	0.88
Selling price ($)	5280
Selling cost ($)	1500
Processing cost ($/t)	4
Mining cost ($/t)	1
Discount rate (%)	4
Dimension of the blocks in X(m)	30
Dimension of the blocks in Y(m)	30
Dimension of the blocks in Z(m)	30
Maximum total mining capacities, 10^6 (t/year)	70
Maximum processing capacities, 10^6 (t/year)	40
Maximum dumping capacities, 10^6 (t/year)	30

In reality, OPPS is known as a NP-hard problem. The global solution to an NP-hard optimization problem cannot be achieved within an admissible time. Thus heuristics (approximation techniques) have to be used in order to work out these problems. In this section, a Genetic Algorithm (GA) was used to solve the OPPS problem. It should be noted that the problem of current study (Marvin orebody production scheduling) was not solved within 20 days using the exact CPLEX solver, and hence, the authors tried to untangle it by the GA using the commercial scheduler package (SimSched DBS) as an alternative strategy.

The Marvin orebody was characterized as a Three-Dimensional (3D) array of blocks. Likewise, the 3D GA array, as the counterpart of mine 3D block model, was used to represent the OPPS problem solution space. The penalty and normalization methods were employed for handling the capacity and sequencing constraints, respectively. Based on the deterministic OPPS solutions by different solvers, NPV's of SimScehd DBS and GA were obtained to be 3351 M$ and 3491 M$, respectively, for the calculations in the current work. The numerical study was performed on the Intel Core™i7-4470 computer (3.4 GHz) with 16 gigabytes of RAM running under Windows 8.1. The computational time required to solve the problem was about 90-120 minutes in optimum iterations. At the end of the mine life, the total NPV generated from the solution of GA schedule was 139 M$ higher than NPV from SimSched DBS solution, which was about 4%.

After verification of the GA performance, this method was applied to solve the OPPS problem in robust form. It is clear that the OPPS problem solution is sensitive to block economic value perturbation, weights of ore/waste in each period, and operation capacities. Perturbation rate in different states of robust counterpart is represented as follows:

$\hat{V}_{ij,Waste} = 0.05,$

$\hat{V}_{ij,Ore} = 0.2,$

$\hat{r} = 0.06,$

$\hat{d} = 0.03,$

$\hat{C} = 0.15,$

$\hat{A} = 0.10$

The solution of box uncertainty set-based robust counterpart for simultaneous LHS, RHS, and objective function uncertainty is shown in Figure 6. The solution is connected with simultaneous block economic value, block tonnage, and operational capacity (mining and processing) uncertainty.

As a stochastic search method, GA essentially jumped randomly around the solution space, and also different random and approximate solutions were achieved in various runs.

The price of robustness in Marvin orebody production scheduling for box counterpart is represented in Table 5. According to the GA-based solution results, for instance, the NPV's of $\Psi = 0.32$ and $\Psi = 1.2$, considering a simultaneous block economic value, the block tonnage and operational capacity uncertainty were obtained to be 3222×10^6 \$ and 2530×10^6 \$, respectively. It is obvious to note that the optimal value decreased when we increased the protection and conservatism level. Therefore, different production plans according to various conditions can be selected. Indeed, the block sequencing in different scheduling plans changes due to various approach but there is no appropriate visual indicator to show the difference between solutions.

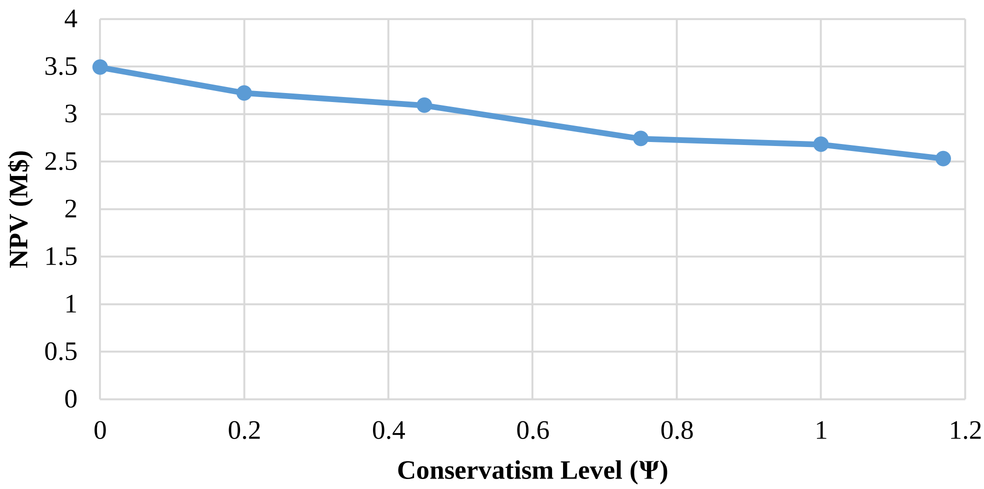

Figure 6. NPV versus different counterpart optimization level, objective function uncertainty, and RHS+LHS uncertainty for all constraints (Marvin 3D orebody).

Table 5. Summary of optimization solutions for box counterpart model (Marvin 3D model).

Conservatism level (Ψ)	GA-based NPV ($10^6$$)	Price of robustness
0, Deterministic model	3491	1
0.2	3222	0.922
0.45	3090	0.885
0.75	2741	0.785
1, Soyster model (interval set)	2680	0.768
1.2	2532	0.725

4. Conclusions

In this paper, the set-induced robust counterpart (box) optimization technique was applied for the OPPS problem. Herein, the uncertainty of the block economic value, block tonnage, and operational capacities (mining and processing) were addressed using the robust box counterpart programming approach. It was concluded that the OPPS solutions were sensitive to the violation of block economic value, block weigh, and operational capacity.

The price of robustness in the OPPS problem was obtained based on the analysis of the impact of robust mathematical framework for quantitatively measuring the sensitivity on the OPPS problem. For the hypothetical 2D open-pit mine, the NPV's of deterministic and the robust box counterpart, also known as "interval set", were obtained to be 41565 and 31421$, respectively. The price of robustness for this condition was calculated to be 0.75.

Furthermore, additional computations were executed on a real-state 3D problem, and GA was used to solve the problem. The investigation revealed that different states of robust planning under uncertainty could be scheduled using the proposed method for the Marvin orebody. It was shown that NPV versus conservatism level (Ψ) had a descending trend.

A possible direction of further studies would be integrating the other set-induced robust counterpart optimization techniques such as polyhedral set, combined box, and polyhedral uncertainty.

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Appendix: box counterpart linear robust optimization with a numerical example [21].

In set-induced robust optimization, the uncertain data is assumed to vary in a given uncertainty set, and the aim is to choose the best solution among those "immunized" against data uncertainty, i.e. candidate solutions that remain feasible for all realizations of the data from the uncertainty set.

Consider the following linear optimization problem:

$$\begin{aligned} &\max \mapsto 8x_1 + 12x_2 \\ &s.t. \\ &\tilde{a}_{11}x_1 + \tilde{a}_{12}x_2 \le 140 \\ &\tilde{a}_{21}x_1 + \tilde{a}_{22}x_2 \le 72 \\ &x_1, x_2 \ge 0 \end{aligned} \tag{A1}$$

Assume that the left-hand side (LHS) constraint coefficients $\tilde{a}_{11}, \tilde{a}_{12}, \tilde{a}_{12}, \tilde{a}_{22}$ are subject to uncertainty, and they are defined as follow:

$$\begin{aligned} &\tilde{a}_{11} = 10 + \xi_{11} \\ &\tilde{a}_{12} = 20 + 2\xi_{12} \\ &\tilde{a}_{21} = 6 + 0.6\xi_{21} \\ &\tilde{a}_{22} = 8 + 0.8\xi_{22} \end{aligned} \tag{A2}$$

where ξ_{11}, ξ_{12}, ξ_{21}, ξ_{22} are independent random variables. The random variables are distributed in the range of $[-1,1]$ (i.e. the constraint coefficients $\tilde{a}_{11}, \tilde{a}_{12}, \tilde{a}_{12}, \tilde{a}_{22}$ have maximum 10% perturbation around their nominal values 10, 20, 6, 8, respectively). Under the set-induced robust optimization framework, finding a robust solution for the above example means to find the best possible candidate solution such that the feasibility of the constraints is maintained no matter what value the random variables realize within a certain set that belongs to the uncertain space defined by $\xi_{ij} \in [-1,1]$.

In general, consider the following linear optimization problem:

$$\begin{aligned} &\max \mapsto cx \\ &s.t. \\ &\sum_j \tilde{a}_{ij} x_j \le \tilde{b}_i, \forall i \end{aligned} \tag{A3}$$

where $\tilde{a}_{ij}$ and $\tilde{b}_j$ represent the true value of the parameters that are subject to uncertainty. Assume that the uncertainty affecting each constraint is independent from each other, and consider the *ith* constraint of the above linear optimization problem, where both the LHS constraint coefficients and RHS parameters are subject to uncertainty. Define the uncertainty as follows:

$$\begin{aligned} &\tilde{a}_{ij} = a_{ij} + \xi_{ij}\hat{a}_{ij}, \forall j \in J_j \\ &\tilde{b}_{ij} = b_{ij} + \xi_{i0}\hat{b}_i \end{aligned} \tag{A4}$$

where a_{ij} and b_j represent the nominal value of the parameters; $\hat{a}_{ij}$ and $\hat{b}_j$ represent constant perturbation/violation (which are positive); J_i represents the index subset that contains the variable indices whose corresponding coefficients are subject to uncertainty; and ξ_{i0} and ξ_{ij} $\forall i, \forall j \in J_i$ are random variables that are subject to uncertainty. With the above definition, the original *ith* constraint can be re-written as:

$$\sum_{j \notin J_i} a_{ij} x_j + \sum_{j \in J_i} \tilde{a}_{ij} x_j \le \tilde{b}_i \qquad \text{(A5)}$$

which can be further reformulated as follows:

$$\sum_j a_{ij} x_j + \left[-\xi_{i0} \hat{b}_i + \sum_{j \in J_i} \xi_{ij} \hat{a}_{ij} x_j \right] \le b_i \qquad \text{(A6)}$$

In the set induced robust optimization method, with a pre-defined uncertainty set *U*, the aim is to find solutions that remain feasible for any ξ in the given uncertainty set *U* so as to immunize against infeasibility, i.e.

$$\sum_j a_{ij} x_j + \left[\max_{\xi \in U} \left\{ -\xi_{i0} \hat{b}_i + \sum_{j \in J_i} \xi_{ij} \hat{a}_{ij} x_j \right\} \right] \le b_i \qquad \text{(A7)}$$

Finally, replacing the original constraint in defined Linear Programing (LP) problem with the corresponding robust counterpart constraints, the robust counterpart of the original LP problem is obtained:

$$\begin{aligned} &\max \mapsto cx \\ &s.t. \\ &\sum_j a_{ij} x_j + \left[\max_{\xi \in U} \left\{ -\xi_{i0} \hat{b}_i + \sum_{j \in J_i} \xi_{ij} \hat{a}_{ij} x_j \right\} \right] \le b_i, \forall i \end{aligned} \qquad \text{(A8)}$$

Example continued. Applying the robust counterpart formulation to the two constraints of the example, their corresponding robust counterpart constraints become:

$$10x_1 + 20x_2 + \max_{(\xi_{11},\xi_{12}) \in U_1} \{\xi_{11} x_1 + 2\xi_{12} x_2\} \le 140 \qquad \text{(A9)}$$

$$6x_1 + 8x_2 + \max_{(\xi_{21},\xi_{22}) \in U_2} \{0.6\xi_{21} x_1 + 0.8\xi_{22} x_2\} \le 72 \qquad \text{(A10)}$$

where U_1 and U_2 are pre-defined uncertainty sets for (ξ_{11}, ξ_{12}) and (ξ_{21}, ξ_{22}), respectively. Example continued. Considering the first constraint of example,

$$(10 + \xi_{11}) x_1 + (20 + 2\xi_{12}) x_2 \le 140 \qquad \text{(A11)}$$

and assuming that the uncertainty set related to (ξ_{11}, ξ_{12}) is defined by box counterpart, the corresponding robust counterpart for this constraint is:

$$10x_1 + 20x_2 + \Psi(|x_1| + 2|x_2|) \le 140 \qquad \text{(A12)}$$

The first robust counterpart constraint with a different value of Ψ is illustrated in Figure A. It can be observed that as the parameter value Ψ increases (i.e. the size of the uncertainty set increases), the feasible set of the resulting robust counterpart optimization problem contracts.

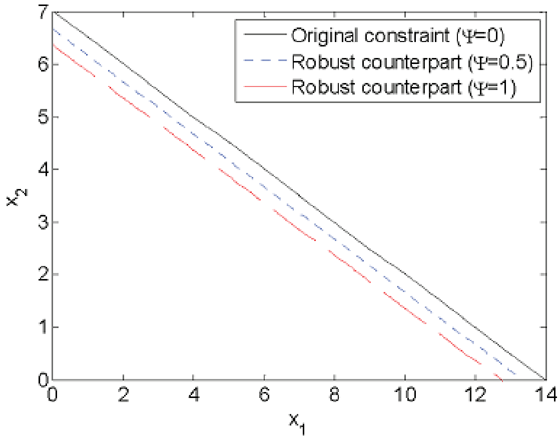

Figure A. Illustration of box counterpart constraint.

Similarly, for the second constraint of the example, the box uncertainty set induced robust counterpart is:

$$6x_1 + 8x_2 + \Psi(0.6|x_1| + 0.8|x_2|) \le 72 \qquad \text{(A13)}$$

Notice that the robust counterpart formulation is constructed constraint by constraint, and different parameter values can be applied for different constraints. The complete box uncertainty set induced robust counterpart formulation of this example with different parameters Ψ_1 and Ψ_2 for the two constraints is:

$$\begin{aligned} &\max \mapsto 8x_1 + 12x_2 \\ &s.t. \\ &10x_1 + 20x_2 + \Psi_1(|x_1| + 2|x_2|) \le 140 \\ &6x_1 + 8x_2 + \Psi_2(0.6|x_1| + 0.8|x_2|) \le 72 \\ &x_1, x_2 \ge 0 \end{aligned} \qquad \text{(A14)}$$

which is equivalent to the following problem since the variables are positive:

$$\begin{aligned} &\max \mapsto 8x_1 + 12x_2 \\ &s.t. \\ &10x_1 + 20x_2 + \Psi_1(x_1 + 2x_2) \le 140 \\ &6x_1 + 8x_2 + \Psi_2(0.6x_1 + 0.8x_2) \le 72 \\ &x_1, x_2 \ge 0 \end{aligned} \qquad \text{(A15)}$$

بهینه‌سازی برنامه‌ریزی زمانی استخراج در معادن روباز تحت شرایط عدم قطعیت- رویکرد استوار با اعمال عدم قطعیت جعبه‌ای

عارف علی‌پور [۱*]، علی‌اصغر خدایاری[۱]، احمد جعفری [۱] و رضا توکلی‌مقدم[۲و۳]

۱- دانشکده فنی و مهندسی، دانشکده معدن، دانشگاه تهران، ایران
۲- دانشکده مهندسی صنایع، پردیس دانشکده‌های فنی، دانشگاه تهران، ایران
3- LCFC, Arts et Métier ParisTech, Centre de Metz

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* نویسنده مسئول مکاتبات: aref.alipour@gmail.com

چکیده:

به طور معمول برنامه‌ریزی زمانی استخراج بلوک‌های معادن روباز با استفاده از مدل‌های برنامه‌ریزی ریاضی صورت می‌گیرد. در پارادایم‌های کلاسیک برنامه‌ریزی ریاضی، داده‌های ورودی مدل (پارامترها) معین (قطعی) و معادل با مقادیر اسمی در نظر گرفته می‌شوند، در صورتی‌که پارامترهای مؤثر در برنامه‌ریزی استخراج، نظیر ارزش اقتصادی بلوک، وزن بلوک و ظرفیت‌های عملیاتی در معرض نوسان هستند؛ از این رو ارائه‌ی مدل برنامه‌ریزی استواری که در مقابل نوسانات و اغتشاشات تاب آورده و در عین بهینه بودن جواب آن شدنی باشد، سطح نگرش به مقوله‌ی برنامه‌ریزی تولید معادن روباز را ارتقا خواهد داد. در تحقیق حاضر مدل برنامه‌ریزی تولید استوار یک کانسار فرضی مس در حالت دوبعدی ارائه و حل شده است. مجموعه‌ی عدم قطعیت محدب جعبه‌ای به منظور معادل‌سازی استوار این مدل به کار رفته و اثر سطوح مختلف محافظه‌کاری مورد توجه قرار گرفته است؛ نتایج حل مدل در دو حالت قطعی و استوار حکایت از تفاوت رویکردها دارد. همچنین مدل استوار پیشنهادی در حالت سه‌بعدی برای کانسار مس ماروین پیاده‌سازی شده و هزینه‌ی استواری در سطوح مختلف محافظه‌کاری برای آن تعیین شده است. نتایج تحقیق با توجه به نوع نگرش و سطح ریسک مورد انتظار، قابل استفاده و پیاده‌سازی در معادن بزرگ است.

کلمات کلیدی: برنامه‌ریزی تولید در معادن روباز، بهینه‌سازی استوار، عدم قطعیت، ارزش اقتصادی بلوک.