JME

Journal of Mining & Environment,
Vol.4, No.1, 2013, 45-55.

Influence of operating parameters on the Apatite flotation kinetics

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Received 2 February 2012; received in revised form 1 January 2013; accepted 14 January 2013
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Abstract
This study aimed to investigate the effect of controllable operating parameters, including pH, solid content, collector, modifier, and depressant dose, and conditioning time, on apatite flotation kinetics. Four first order flotation, kinetic models are tested on batch flotation time-recovery profiles. Model with fast and slow - floating components and classical model gave the best and the worst fit for experimental data respectively. Similarly, rectangular distribution of floatabilities and gamma distribution of floatabilities fitted the experimental data well. In this study, the model with rectangular distribution of floatabilities associated with fractional factorial experimental design was employed to evaluate the effect of operating parameters on kinetic parameters (R_∞ , K). The result indicated that linear effects of depressant dose, conditioning time, and the interaction effects of solid concentration and pH were statistically important on ultimate recovery but the significant parameters for flotation rate constant were linear effects of solids content, depressant dosage and the interaction effect between pH and conditioning time.

Keywords: *phosphate flotation, experimental design, kinetic models.*

1. Introduction

Phosphate rock is a vital nonrenewable resource [1-2]. Phosphates are used in numerous applications ranging from toothpaste, detergents, matches, and fertilizers to food additives [3-5]. The major use of phosphate is as a fertilizer since it is one of the three major plant nutrients - nitrogen, phosphorus, and potassium [4-5]. Phosphate deposits may be divided into three groups according to their origin: (i) deposits from sea sediments; (ii) igneous deposits; (iii) biogenetic deposits [6]. Beneficiation of phosphate ores by flotation has been an important part of the concentration process since the 1920's, when it became possible to recover fine particles of apatite. Today, more than half of the world's marketable phosphate is upgraded by the flotation method [6]. Flotation is one of the most complex mineral processing operations; and it is affected by a very large number of variables. Many of these are beyond the control of the mineral engineer and some cannot be even measured quantitatively with available instruments. The relationship between measured and controlled variables is intricately related. Sometimes simultaneously changing various component settings will reinforce one particular attribute. In addition, various component settings can cancel or counteract each other if changes are not chosen wisely [7].

Flotation of apatite is complicated, owing to its physicochemical similarity to other minerals in the phosphate ores [8]. Many studies have been undertaken to increase understanding of the principles of apatite flotation and its separation from other minerals [9], comparatively few studies have been conducted on the effect of operating parameters on the kinetics of apatite flotation [10-11].

This research was carried out on Esfordi phosphate ore. The Esfordi mine, located in central Iran, is the main phosphate producer in Iran. The Esfordi deposit is of the igneous type and includes three apatite-bearing zones, namely, an apatite-iron zone (12% P_2O_5); a main apatite zone (16.3% P_2O_5); and, a tremolite-actinolite zone (14.4% P_2O_5) [12].

In the present work, firstly the influences of flotation operation parameters were studied on the iron and phosphate recoveries under fixed operating conditions. Secondly, first order flotation kinetic models were investigated on the phosphate recovery. Furthermore, model with rectangular distribution of floatabilities was applied to data from tests and experimental design was employed to relate flotation kinetics and operation parameters.

Kinetic models are often used to analyze batch flotation data and to evaluate various parameters such as flotation chemical and equipment operating conditions for flotation process [13]. On important aspect of the kinetics models is that the model parameters should in some way be characteristic of a flotation process [14]. They can be effectively used to evaluate variables affecting flotation process [15]. Understanding and interpreting changes in the values of ultimate recovery (R_∞) and rate constant (K) are very important and can often be misleading [16]. In many laboratory studies, changing one condition leads to a change in both R_∞ and K values. This can make it difficult to compare flotation rate data between tests or to establish a trend for R_∞ and k values under different conditions [17].

2. Experimental and modeling procedure

2.1. The Design of Experiments (DOE)

More recently, the statistical techniques have been widely used to study the flotation of minerals [18-22]. DOE provides a statistical means for analyzing how numerous variables interact. The tool is a planned approach for determining cause and effect relationships [23]. The statistical design of the experiments has several advantages over the classical method of treating one variable at a time [24].

Fractional factorial designs are experimental designs consisting of a carefully chosen subset (fraction) of the experimental runs of a full factorial design. The subset is chosen so as to exploit the sparsity-of-effects principle to expose information about the most important features of the problem studied, while using a fraction of the effort of a full factorial design in terms of experimental runs and resources [24].The full factorial experiment is a method of design of experiments in which a statistical analysis is performed to evaluate the significance of the main and interaction effects as evaluated from the experimental results. In particular, they are used when several factors have to be studied in order to determine their main effects and interaction effect. The experiments can be conducted in an organized manner and can be analyzed systematically to obtain much needed information. The information can be utilized for optimization purpose [22].

In statistics, fractional designs are expressed using the notation L^{n-p}, where L is the number of levels of each factor investigated, n denotes the number of factors investigated, and p describes the size of the fraction of the full factorial used. Formally, p represents the number of generators, assignments as to which effects or interactions are confounded, i.e., cannot be estimated independently of each other. A design with p such generators is a $(\frac{1}{L^p})$ fraction of the full factorial design [24].

2.2. Flotation experiments

Representative samples from the rougher flotation feed were prepared. These samples contained 16% P_2O_5 and 23% Fe. Flotation experiments were carried out with a Denver D12 laboratory flotation cell using 500 g of ore sample based on a fractional factorial design, while first order flotation kinetic models were used to assess the influence of these parameters on the flotation kinetics. Starch, NaOH, FLO-Y-S20, and Procol3496 were used as the depressant, pH regulator, collector, and modifier respectively. In each experiment, the froth was collected at three time intervals of 0.5, 1 and 1.5 minutes, and the recoveries of P_2O_5 and Fe in the concentrate were calculated [1-2]. The P_2O_5 recovery was calculated by Equation 1.

$$R = \frac{Cc}{(C+T) \times F} \times 100 \tag{1}$$

where, R is % recovery of iron or phosphate, C represents dry weight of concentrate, c denotes % grade of concentrate, T is dry weight of tailing, and f is % grade feed.

The studied operating parameters were: pH, solids content, the collector, modifier, and depressant concentrations, and the conditioning time. Each factor was varied over five levels while the other operational parameters including impeller speed, air flow rate, and temperature were kept constant. The levels of the parameters are shown in Table 1.

A fractional factorial design of experiment ($2^{(6-1)}$) comprising 32 tests were employed to evaluate the effects of the operational parameters on the phosphate ore flotation. In addition, three extra tests were carried out to determine total square error at the central points of parameters. Table 2 shows the results of a fractional factorial design of flotation tests on the phosphate ore samples and responses measured for each experiment. In Table 2, R_1, R_2, R_3, K and R_∞ represent, first stage recovery, second stage cumulative recovery, third stage cumulative recovery, ultimate recovery and flotation rate constant of model with rectangular distribution of floatabilities, respectively.

3. Results and Discussion

3.1. Influence of operating parameters on the iron and phosphate recoveries

An initial review was performed by investigating the effects of pH, solids content, the depressant, collector, and modifier concentrations, and conditioning time on the recovery of iron and phosphate, *i.e.*; all parameters were held constant except one, which was changed at three levels. The results are presented in Figures 1and 2.

According to Figures 1and 2, the following observations can be made:

1. To raise pH, the phosphate recovery increased (Figure 1) and the amount of iron in the concentrate first decreased and so enhanced, rapidly (Figure 2). This affects the quality of the concentrate.

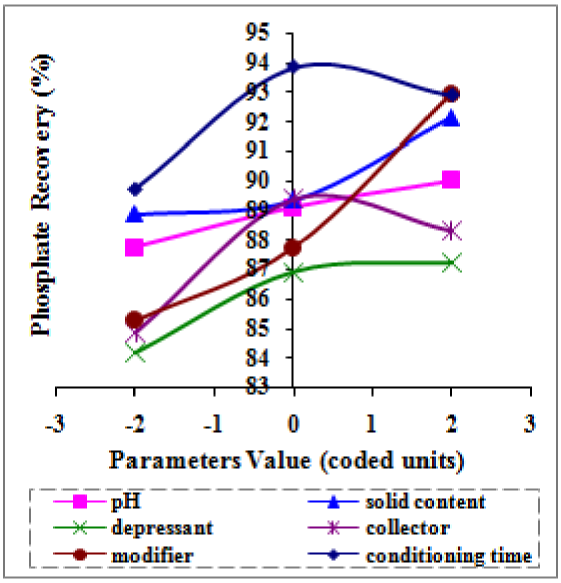

Figure 1. The effect of the operational parameters on phosphate recovery; fixed operating conditions

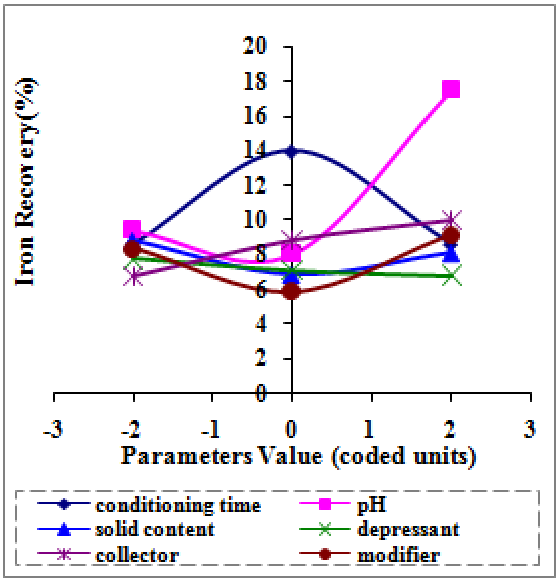

Figure 2. The effect of the operational parameters on iron recovery; fixed operating conditions

Table 1. Selected parameters and their actual and coded levels

Factor	Symbol	High axial level (+2)	High factorial level (+1)	Medium level (0)	Low factorial Level (-1)	Low axial level (-2)
pH	A	10.5	10	9.5	9	8.5
Solid content (%)	B	32.5	30	27.5	25	22.5
Depressant dosage (g/t)	C	480	440	400	360	320
Collector dosage (g/t)	D	680	640	600	560	520
Modifier (g/t)	E	100	90	80	70	60
Conditioning time (min)	F	11	10	9	8	7

Table 2. Fractional factorial design of experiment to evaluate apatite flotation kinetics models

Run	A	B	C	D	E	F	R_1	R_2	R_3	K	R_∞
1	-1	-1	1	1	-1	-1	56.5929	76.3117	88.5261	4.38	93.53
2	1	1	1	-1	-1	1	53.4592	74.5215	85.6345	4.11	91.54
3	-1	-1	1	-1	-1	1	65.7509	79.9178	88.2096	6.84	90.97
4	-1	-1	-1	1	1	-1	62.6656	82.9368	91.9913	5.14	97.13
5	-1	1	-1	-1	1	-1	52.3676	74.2002	90.3086	3.43	97.05
6	1	1	1	-1	1	-1	61.7336	81.3924	89.6332	5.28	94.64
7	-1	-1	1	1	1	1	67.5223	81.3518	87.7214	7.55	90.69
8	1	-1	1	-1	-1	-1	65.0561	80.4837	89.2185	6.35	92.39
9	1	-1	-1	-1	-1	1	58.3284	77.9771	89.9268	4.55	94.79
10	-1	1	1	-1	-1	-1	62.1196	81.5422	91.1769	5.16	95.92
11	-1	1	1	1	1	-1	58.445	79.6707	91.1281	4.4	96.74
12	1	-1	-1	1	-1	-1	55.931	78.0069	89.7948	4.09	95.99
13	1	1	-1	-1	-1	-1	60.736	79.7015	88.9914	5.18	93.63
14	1	-1	1	-1	1	1	58.3639	75.8736	85.2594	5.24	89.37
15	-1	-1	-1	-1	1	1	67.4009	83.8206	90.5797	6.67	94.41
16	1	-1	1	1	-1	1	64.9635	84.1992	91.1143	5.8	96.03
17	-1	-1	1	-1	1	-1	68.5498	84.815	93.0935	6.52	96.59
18	-1	-1	-1	1	-1	1	58.8289	76.5682	88.5539	4.87	92.51
19	-1	-1	-1	-1	-1	-1	54.1748	77.436	87.4382	4.02	94.36
20	1	1	1	1	1	1	56.0117	77.8421	88.7166	4.22	94.8
21	-1	1	-1	1	-1	-1	55.5203	78.6046	91.2865	3.86	98.04
22	-1	1	-1	1	1	1	57.828	78.6609	88.607	4.59	94.15
23	-1	1	-1	-1	-1	1	57.7209	77.5389	89.406	4.49	94.39
24	1	1	1	1	-1	-1	57.5521	77.0741	86.8152	4.79	91.8
25	1	-1	-1	-1	1	-1	63.6635	80.0382	91.6679	5.54	94.85
26	1	1	-1	-1	1	1	57.5335	77.7275	88.4687	4.56	93.69
27	1	-1	1	1	1	-1	58.6185	78.1679	90.1174	4.58	94.91
28	-1	1	1	1	-1	1	51.3233	74.3248	87.7852	3.51	95.02
29	1	1	-1	1	1	-1	57.6878	79.4639	89.8931	4.39	95.85
30	1	1	-1	1	-1	1	57.9708	77.0495	87.8031	4.76	92.49
31	-1	1	1	-1	1	1	62.6167	81.3447	88.7775	5.64	93.48
32	1	-1	-1	1	1	1	60.8882	81.1282	89.259	5.14	94.55
33	0	0	0	0	0	0	58.3416	78.514	89.3801	4.61	94.54
34	0	0	0	0	0	0	58.5163	78.539	88.4582	4.76	93.6
35	0	0	0	0	0	0	54.4769	75.4239	86.6287	4.19	92.4

2. Solids concentration has an important effect on the capacity and the flotation mechanism of the flotation cells. The results indicate that increasing the solids levels have a positive effect on phosphate recovery but it increases the amount of iron in the concentrate as well.

3. The collector dose is a critical flotation parameter in the flotation experiments. Adjusting the collector dose can change the surface

characteristics of the particles and thereby influence recovery efficiency. Collector types, and its concentration, are also important parameters in flotation. Phosphate recovery increases with increasing amount of collector (Figure 1) but at the upper limit iron minerals also begin to float (Figure 2).

4. Modifiers are used to reduce the sensitivity of the process to fines and disturbing ions, and to increase the power for floating the desired minerals. Procol4396 was used as a modifier in the tests. The results show that phosphate recovery increased with increasing Procol4396 concentration. Iron recovery decreased first and then increased. Therefore, recovery of phosphate and iron are very sensitive to co–collector concentration.

5. Starch was used as a depressant in the tests. The influence of starch on the recovery of phosphate and iron indicated that phosphate recovery increased with increasing starch concentration and, iron recovery reduced.

6. In flotation cells or columns, which will carry out separation process, slurry must be properly conditioned with the reagents (depressant, frother, activator, collector, pH regulator) in conditioning tanks agitated by a propeller, during one to several ten or so minutes. This period time is called the conditioning time. By prolonging conditioning time, phosphate recovery reached in maximum peak level and so drop down in gentle gradient, presumably because of the separation of the collector layer from the particle surface.

3.2. Influence of operating parameters on kinetic parameters

3.2. 1. Calculating the kinetics parameters (R_∞ and K)

Based on ($2^{(6-1)}$) fraction factorial design, thirty – two flotation tests associated with three center point tests (to estimate error and standard deviation) were carried out on the phosphate sample of rougher flotation feed to investigate the kinetic models and the effects of operating parameters on the flotation kinetics. Many studies have been carried out in the past to investigate kinetics models in flotation process. These studies indicated that flotation process generally follows a first order kinetics model. Furthermore, studies showed that first order kinetic models could represent the time–recovery curves of flotation tests very well [8 and 17]. Therefore, four first order flotation kinetic models were fitted the tests data. These models are summarized in Table 3. In Table 3, R is recovery, t denotes flotation time, R_∞ represents ultimate recovery, K is referred to flotation rate constant, Z stands for recovery of the slow floating component, R_∞ is rate constant for slow floating component, and K_f is rate constant for fast floating component.

Two parameters, including R_∞, ultimate recovery, and K, first order rate constant, are obtained from the model fit to an experimental recovery–time curve [17].

In this study, the fractional recoveries were fitted to the models after 0.5, 1, 1.5 minutes of flotation time. Ultimate recovery and first order rate constant are obtained from the model fit to experimental data using MATHEMATICA software [25-26] by the following code (For example, the code for test 19 is):

```
<<Statistics`NonlinearFit`
data={{.5,0.541747639},{1.5,0.774360172},{3,0.874382146}};
NonlinearFit[data,r*(1-Exp[-k*t]),{t},{r,k}]
0.854259*(1-e^1.92902)
Best Fit Parameters/.Nonlinear Regress [data,r*(1-Exp[-k*t]),{t},{r,k}, Regression Report□BestFitParameters]
{r=0.854259, k=1.92902}
Nonlinear Regress [data, r*(1-Exp[-k*t]),{t},{r,k}]
```

In this program, r and k are ultimate recovery and rate constant, respectively (Table 4). Table 4 shows kinetic parameters that were derived from first order kinetic flotation models for two tests No.1 and No.19.

3.2.2. Evaluation of first order kinetics models

Four various kinetics models were fitted to the experimental data. Comparison of the results shows that except for classical model, all kinetic models fit well to the experimental data (Tables 5 and 6 and Figures 3and 4).

Tables 5 and 6 introduced recoveries that calculated from tests and kinetic models for the test No. 1 and test No.19. It was observed that the importance weight of models fitting is fast and slow - floating components, rectangular distribution of floatabilities, gamma distribution of floatabilities and classical model to the data, respectively. Figures 3 and 4 approved these results. These figures compare first order kinetic models fitted to the test data. In these figures, vertical axis is recovery in each stage and horizontal axis is time of froth collection.

3.2.3. Effect of operating parameters on ultimate recovery and rate constant

The main purpose of the flotation kinetics is studying the rule of flotation rate constant, and analyzing the effects of various parameters such as the properties of ores, the system of flotation reagent and the Characteristics of flotation machine and so on [27]. As expressed literature, under- standing and interpreting changes in the values of R_∞ and K are very important and can often be misleading [16].

As mentioned earlier, Model with rectangular distribution of floatabilities gave good fit for experimental data; therefore, this model and its kinetic parameters was employed to investigate the influence of flotation operation parameters on kinetic parameters (R_∞, K). Furthermore, in order to find the relationship between operating parameters and kinetic parameters, the regression equations with interactive terms can be written as [28]:

$$Y = (\beta_0 + \varepsilon) + \sum_{i=1}^{n} \beta_i x_i + \sum_{1\langle i \langle j}^{n} \beta_{ij} x_i x_j \qquad (2)$$

Where n, β_0, β_i, x_i, β_{ij} and ε represent, respectively, the number of variables, the constant term, the coefficients of the linear parameters, the variables, the coefficients of the interaction parameters, and residual error associated with the experiments.

The effect of the selected operating parameters on kinetic parameters (R_∞, K) were analyzed using Design Expert statistical software as shown in Tables 7 and 8 and in Figures 5 and 6.

Table 3. First order kinetic models of flotation

Model	Equation
Model 1: Classical model	$R = R_\infty \times (1 - \exp[-K \times t])$
Model 2: Model with rectangular distribution of floatabilities	$R = R_\infty \times (1 - [\frac{1}{K \times t}]) \times (1 - \exp(-K \times t])$
Model 3: Model with fast and slow - floating components	$R = (R_\infty - Z) \times (1 - \exp[-K_f \times t]) +$ $Z \times (1 - \exp[-K_s \times t])$
Model 4: Model with gamma distribution of floatabilities	$R = R_\infty \times (\frac{K + t}{1 + K \times t})$

Table 4. Kinetic parameters obtained from fitting of first order kinetic models

Model	Test number	R_∞	K
1	1	85.0906	2.0682
	19	85.4259	1.92902
2	1	93.526	4.3805
	19	94.3575	4.02042
3	1	50.6298 (R_f)	4.70474 (K_f)
		48.8244 (R_s)	0.498964 (K_s)
	19	30.9701 (R_f)	8.17039 (K_f)
		59.2116 (R_s)	1.02395 (K_s)
4	1	98.2652	0.380817
	19	99.4808	0.420106

Table 5. Recovery obtained from kinetic models fitted for test1

Time	Calculated recovery from test no.1	Obtained recoveries from kinetic models (R_{Model})			
	R	R_{Model1}	R_{Model2}	R_{Model3}	R_{Model4}
0.5	56.5929	54.837	55.6027	56.5929	55.7807
1.5	76.3117	81.2661	79.3123	76.3117	78.369
3	88.5261	84.9187	86.4092	88.5261	87.1965

Table 6. Recovery obtained from kinetic models fitted for test19

Time	Calculated recovery from test no.19	Obtained recoveries from kinetic models (R_{Model})			
	R	R_{Model1}	R_{Model2}	R_{Model3}	R_{Model4}
0.5	54.1748	52.8641	53.7064	54.1746	54.0594
1.5	77.436	80.695	78.7487	77.4359	77.7151
3	87.4382	85.1639	86.5344	87.4381	87.2611

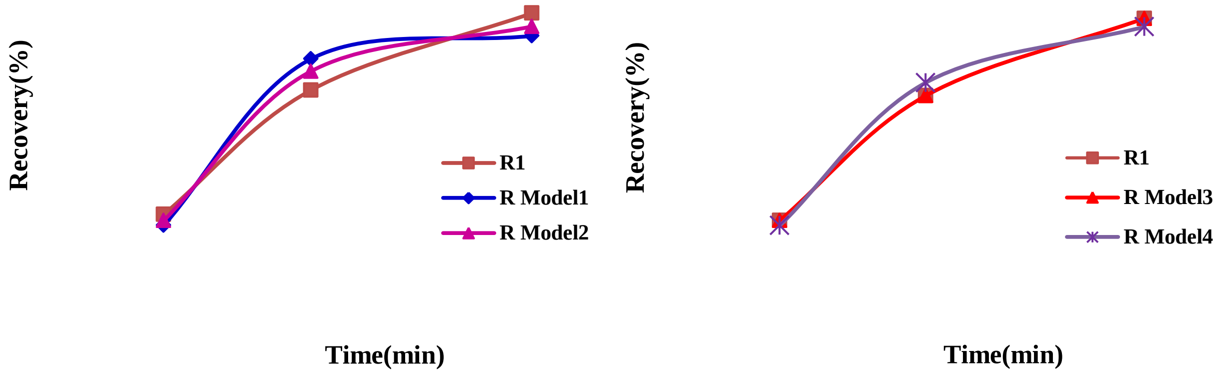

Figure 3. Comparison of different kinetic models fitted to the data for test No.1.

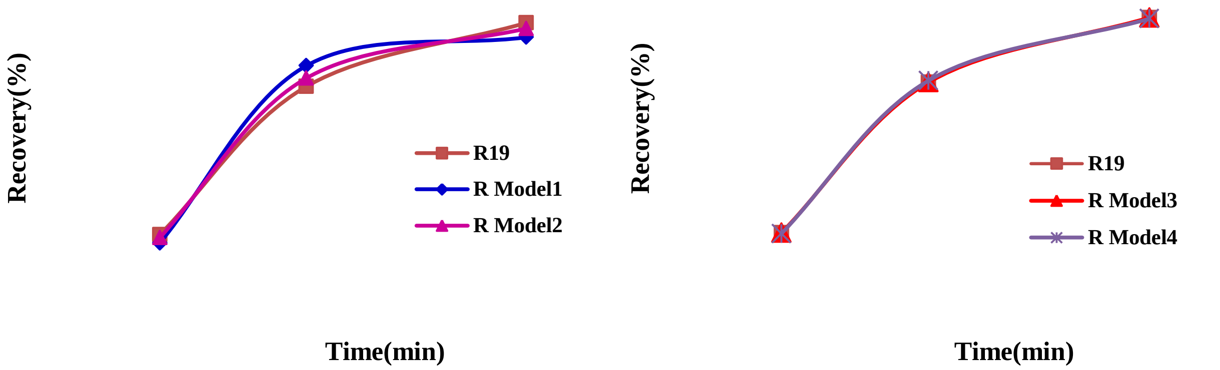

Figure 4. Comparison of different kinetic models fitted to the data for test No.19.

Table 7. Analyses of variance for P_2O_5 ultimate recovery

Source of variation	Sum of Squares	df	Mean Square	F Value	P-value Prob> F
Model	0.0061	5	0.0012	4.986	0.0022
A	0.0006	1	0.0006	2.3652	0.1353
B	0.0003	1	0.0003	1.3067	0.2627
C	0.0012	1	0.0012	4.807	0.0368
F	0.0029	1	0.0029	11.844	0.0018
A×B	0.0011	1	0.0011	4.6069	0.0407
Curvature	0.0002	1	0.0002	0.625	0.4359
Residual	0.0069	28	0.0002		
Lack of Fit	0.0067	26	0.0003	2.2276	0.3568
Pure Error	0.0002	2	0.0001		
Cor Total	0.0132	34			

Table 8. Analyses of variance for phosphate flotation rate constant (K)

Source of variation	Sum of Squares	Df	Mean Square	F Value	P-value Prob> F
Model	13.216	5	2.6432	4.4196	0.0043
A	0.1918	1	0.1918	0.3206	0.5757
B	6.9328	1	6.9328	11.592	0.0020
C	2.5921	1	2.5921	4.3341	0.0466
F	0.9225	1	0.9225	1.5424	0.2246
A×F	2.5769	1	2.5769	4.3088	0.0472
Curvature	0.6079	1	0.6079	1.0164	0.3220
Residual	16.746	28	0.5981		
Lack of Fit	16.569	26	0.6373	7.1901	0.1292
Pure Error	0.1773	2	0.0886		
Cor Total	30.57	34			

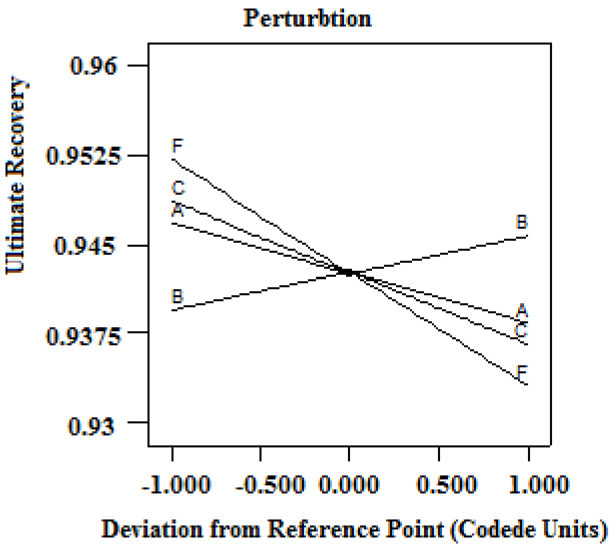

Figure 5. Perturbation plot of the main parameters on P_2O_5 ultimate recovery

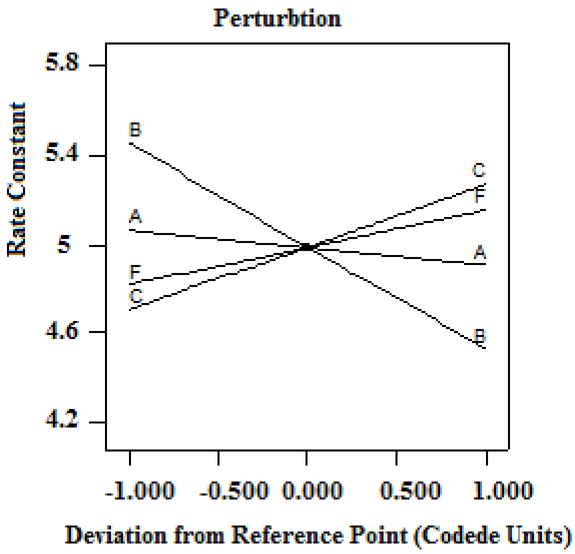

Figure 6. Perturbation plot of the main parameters on flotation rate constant (K)

Table 7 shows an analysis of variance for the ultimate recovery. As it is seen, the model F-value and the corresponding p value is 4.99, indicating that the model is statistically significant, *i.e.*; there is only a 0.22% chance that this "Model F-Value" could occur due to noise. The "Curvature F-value" of 0.62 implies that the curvature, which is obtained by the difference between the average of the center points and the average of the factorial points in the design space, is not statistically significant. There is a 43.59% chance that this "Curvature F-value" could occur due to noise. In addition, the "Lack of Fit F-value" of 2.23 indicates that the Lack of Fit is not significant compared with the pure error. There is a 35.68% chance that this "Lack of Fit F-value" could occur due to noise. It is noteworthy that the "Lack of Fit" value to be non-significant for a proper fitted model.

Furthermore, the results of Table 7 reveal that the linear effects of depressant dosage (C), conditioning time (F), and the interaction effects of solid concentration and pH (A×B) were statistically significant at the 95% confidence level (p-value < 0.05), as well as demonstrate that conditioning time (F) give the most effect to ultimate recovery. Figure 5 approved these result. Hence, the proposed regression model (Equation 3) including these parameters is:

$$R_\infty = 0.94 - 0.0043 \times A + 0.0032 \times B - 0.0061 \times C - 0.0095 \times F - 0.006 \times A \times B \quad (3)$$

The flotation rate constant analysis of variance is shown in Table 8 and Figure 6.

The Model F-value of 4.42 implies the model is significant. Moreover, the "Lack of Fit F-value" of 7.19 indicates that the "Lack of Fit" is not significant compared with the pure error. There is a 12.92% chance that a this "Lack of Fit F-value" could occur due to noise. Thus, the proposed regression model is statistically significant.

$$K = 4.99 - 0.077\times A - 0.47\times B + 0.28\times C + 0.17\times F - 0.28\times A\times F$$

The results show that the linear effects of solids content (B) and depressant (C) as well as the interaction effect between pH and conditioning time (A×F) are all statistically significant at the 95% confidence level. The lack of fit of the proposed model (Equation 4) was not significant indicating that this model can be used to explain the variation in the data. The results also show that solid content was the most important factor over the investigation range. An increase in solid content decreases flotation rate constant. But longer conditioning time and higher depressant dose increase it. Therefore, the proposed regression model is:

$$K = 4.99 - 0.077\times A - 0.47\times B + 0.28\times C + 0.17\times F - 0.28\times A\times F \quad (4)$$

Normal probability plot of residuals is used to check proposed regression models shown in Figure 7.

Figure 7 indicates Normal probability plots of the residuals for both models (Equations 3 and 4). The most important diagnosis of the statistical properties of the model is the normal probability plot of the residuals. The data points should be approximately linear. A non-linear pattern indicates non-normality in the error term, which may be corrected by a transformation. It was observed that Normal probability plots of the residuals of models reveal no unusual data; therefore, regression equations fitted to the data correctl

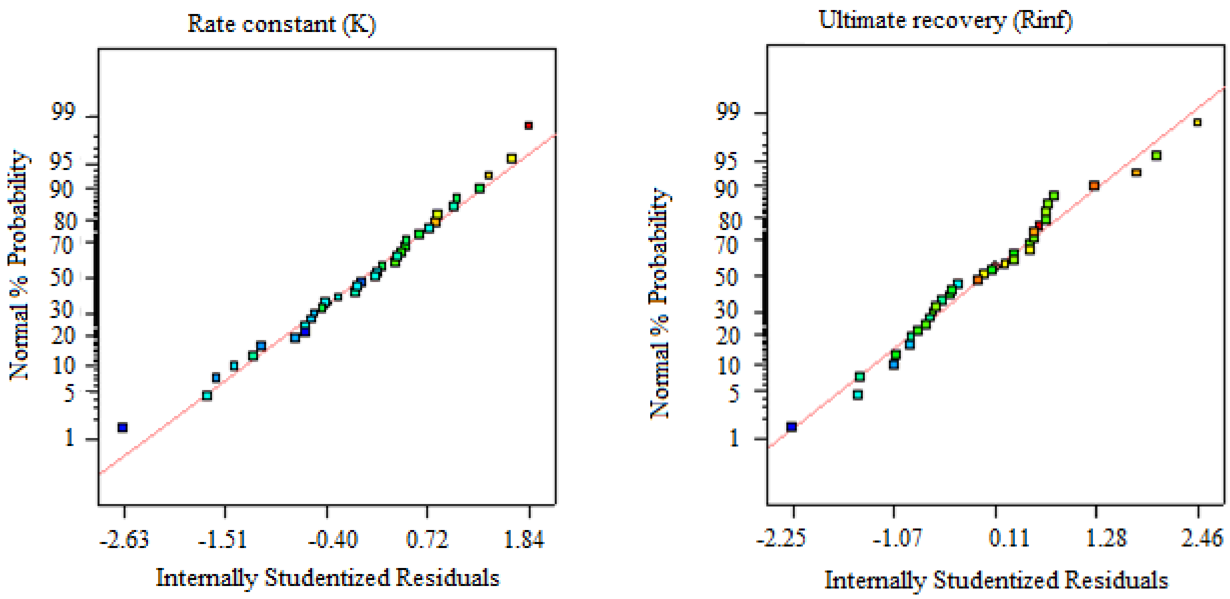

Figure 7. Normal probability plot of model residuals: flotation rate constant (K) (left); P_2O_5 ultimate recovery ($R_{\inf}$) (right)

4.Conclusions

This study was carried out to assess the influence of operating parameters on the flotation kinetics (R_∞, K). The conclusions obtained from the study are as follows:

i) The results indicate that when pH increased from 8.5 to 10.5, the phosphate recovery increased. For a pH range of 8.5 to 9.5, iron recovery decreased gradually but it increased sharply when pH changed from 9.5 to 10.5. A similar behavior can be seen for both phosphate and iron recoveries while modifier varies from 60 to 100 gr/L.

ii) Solid content has a positive effect on phosphate recovery. However, it increased the amount of iron in the concentrate as well. By consuming more collectors, both phosphate and iron minerals flotation recovery increased. Increasing depressant dosage shows a positive effect on phosphate flotation system; and by prolonging conditioning time, phosphate recovery reached the maximum peak level and then drop down with gentle gradient.

iii) Four first order kinetic models of apatite flotation were investigated in batch tests. Evaluation of the kinetic models shows that the significance of these models in this experiments can be ordered as a model with fast and slow - floating components, a model with rectangular distribution of floatabilities, a model with gamma distribution of floatabilities and classical model respectively.

The model with rectangular distribution of floatabilities together with fractional factorial experimental design was employed to evaluate the influence of the flotation parameters on the kinetics parameters. The results indicate that the linear effects of depressant dosage, conditioning time, and the interaction effects between solid content and pH statistically affected the ultimate recovery of apatite. The significant parameters of flotation rate constant were linear effects of solids content and depressant as well as the interaction effect between pH and conditioning time. In addition, conditioning time and solids content in turn are the most dominant parameters affecting the ultimate recovery of apatite. The results further indicate that a nonlinear regression equation presented above can well describe the flotation kinetic parameters.

The results obtained from this investigation provide useful guidance for modification of the Esfordi apatite processing circuit.

Acknowledgements

The authors wish to thank the manger and personnel of the Esfordi Mining Complex for their support during this research.

References

[1] Sis, H and Chander, S. (2003a). Reagents used in the flotation of phosphate ores: A critical review. Miner. Eng. 16: 577 – 585.

[2] Sis, H and Chander, S. (2003b). Improving froth characteristics and flotation recovery of phosphate ores with nonionic surfactants. Miner. Eng. 16: 587 – 595.

[3] McConnell, D. (1973). Apatite – Its crystal Chemistry, Mineralogy, Utilization, and Geologic and Biologic Occurrences. New York: Wein, Springer-Verlag.

[4] Emigh, G. D. (1983). Phosphate rocks, in Lefond, S. J., ed., Industrial minerals and rocks (nonmetallics other than fuels) (5th ed.), vol. 2: New York: AIME, pp. 1017-1047.

[5] Yongqiang, L., Ning, L., Xuming, W. and D.Miller, J. (1999). Improved Phosphate Flotation with Nonionic Polymers. In: Zhang, P., El-Shall H and Wiegel R, Editors, Beneficiation of Phosphates: Advances in Research and Practice, pp. 3–19.

[6] Guimaraes, R. C, Araujo A. C. and Peres, A. E. C. (2005). Reagents in igneous phosphate ores flotation. Miner. Eng. 18: 199-204.

[7] Aplan, F. F. (1999). The historical development of coal flotation in the United States., In: Parekh, B.K., Miller, J.D. (Eds.), Advances in Flotation Technology., SME, pp. 269– 287.

[8] Feng, D. and Aldrich, C. (2004). Influence of operating parameters on the flotation of apatite, Miner. Eng. 17: 453-455.

[9] Lovell, V. M. (1976). Froth characteristics in phosphate flotation. In: Fuerstenau, M.C. (Ed.), Flotation: A.M. Gaudin Memorial Volume, 2. New York, AIME, pp. 597–621.

[10] Singh, R. and Pradip Sankar, T. A. P. (1992). Selective flotation of Maton (India) phosphate ore slimes with particular reference to the effects of particle size. Int. J. Miner. Process. 36: 283–293.

[11] Smar, V. D., Klimpel, R. R. and Aplan, F. F. (1994). Evaluation of chemical and operational variables for the flotation of a copper ore – Part 1: Collector concentration, frother concentration and air flow rate. Int. J. Miner. Process. 42: 225–240.

[12] Jami, M. (2005). Geology, Geochemistry and Evolution of the Esfordi Phosphate-Iron Deposit., PhD thesis, The University of New South Wales, Australia.

[13] Xu, M. (1998). Modified flotation rate constant and selectivity index. Miner. Eng. 11 (3): 271–278.

[14] Woodburn, E. T. (1970). Mathematical modeling of flotation process. Miner. Sci. Eng. 2 (2): 3-17.

[15] Hanumantha Roa, K., Su, F. and Forssberg, K. S. E. (1999). Flotation Kinetics of Apatite from Magnetite. In: Zhang, P., El-Shall, H, Wiegel, R, Editors, Beneficiation of Phosphates: Advances in Research and Practice, pp. 103–125.

[16] Uçurum, M. (2009). Influences of Jameson flotation operation variables on the kinetics and recovery of unburned carbon, Powder Technology, 191: 240–246.

[17] Uçurum, M. and Bayat, O. (2007). Effects of operating variables on modified flotation parameters in the mineral separation, Separation and Purification Technology, 55: 173–181.

[18] Yalsin, T. (1999). Evaluation of Box-Wilson experimental design in flotation research, Transactions of The Institution of Mining and Metallurgy Section C, 108: 109– 112.

[19] Rao, G. V. and Mohanty, S. (2002). Optimization of flotation parameters for enhancement of grade and recovery of phosphate from low grade dolomitic rock

phosphate ore from Jhamarkota, India. Miner. Metall. Process. 19 (3): 154–160.

[20] Cilek, E. C. and Yilmazer, B. Z. (2003). Effect of hydrodynamic parameters on entrainment and flotation performance. Miner. Eng. 16: 745– 756.

[21] Martinez, L. A., Uribe, S. A., Carrillo, P. F. R., Coreno, A. J. and Ortiz, J. C. (2003). Study of celestite flotation efficiency using sodium dodecyl sulfonate collector: factorial experiment and statistical analysis of data. Int. J. Miner. Process.70 (1–4): 83– 97.

[22] Naik, P. K., Reddy, P. S. R. and Misra, V. N. (2005). Interpretation of interaction effects and optimization of reagent dosages for fine coal flotation. Int. J. Miner. Process. 75: 83–90.

[23] Anderson, M. J. and Whitcomb, P. J. (2007). Doe Simplified: Practical Tools for Effective Experimentation, Productivity Press, 2nd Ed. Productivity Press, New York, NY, USA.

[24] Box, G. E.; Hunter, J. S. and Hunter, W. G. (2005). Statistics for Experimenters: Design, Innovation, and Discovery. 2nd Edition. Wiley.

[25] Wolfarm, S. (2003).The Mathematica Book, Fifth Edition, Wolfram Media, Inc. ISBN: 1579550223.

[26] www.wolfram.com/mathematica/

[27] Yi, Di and Li Song-ren. (1993). Mathematical models in mineral processing. ChangSha:ZhongNan university Press, pp.185-208.

[28] Montgomery, D. C. (2001). Design and Analysis of Experiments, John Wiley & Sons, New York.