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Calculation of tunnel behavior in viscoelastic rock mass

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Abstract
Wall displacements and ground pressure acting on the lining of a tunnel increase with time. These time-dependent deformations are both due to face advance effect and to the time-dependent behavior of the rock mass. Viscoelastic materials exhibit both viscous and elastic behaviors. Thorough this study, the effect of different linear viscoelastic models including Maxwell, Kelvin and Kelvin-Voigt bodies on the behavior of tunnel is studied and the interaction of rock mass with elastic lining is analyzed. The surrounding rock mass is assumed to be homogeneous, isotropic and continuous. Hydrostatic stress field is also considered. In this paper, a series of formula for the foregoing models is driven to predict the displacement of lined and unlined circular tunnel and the pressure on the lining. The effect of lining stiffness and delay in installation of lining is analyzed. The results of new analytical relations show good correspondence with existing solutions.

Keywords: *Tunnel, Viscoelastic body, Displacement, Pressure, Kelvin model, Maxwell model, Generalized Kelvin model.*

1. Introduction

Creep, i.e. time-dependenteffectis superimposed to thestrain induced by the front advance stress changes, andin some cases it seems to be very important [1]. The Creep analysis of tunnels excavated in viscoelastic continua under hydrostatic stress fields has been a frequent subject for research.

Ladanyi and Gill (1998) have investigated the effect of long term rock deformation on lining pressure for different types of rock behavior [2]. In rock-support analysis presented by Cristescu et al. (1987), it is assumed that the rock behaves linearly viscoelastically and support has non-linear behavior [3]. An analytical solution is proposed by Fahimifar et al. (2010) for the calculation of stress-displacement field around the circular tunnels in a Burger's body [4]. In closed form solution presented by Sulem et al. (1987) a time-dependent model, taking into account the face advance effect is developed for the prediction of the wall displacements and the ground pressure acting on the lining of a circular tunnel [5]. Sakurai (1978), introducing an equivalent initial stress, proposed a method which takes into account the three-dimensional effects of the tunnel face for analyses of the behaviour of tunnel support structures installed in a viscoelastic medium [6]. An exact closed form solution is derived for the mechanical behaviour of a linear viscoelastic Burgers rockaround an axisymmetric tunnel, supported by a linear elastic ring [7].

In case of driving the tunnel in different viscoelastic material except Burger model, this paper investigates the problem of tunnel wall deformation and pressure on lining. According to Cristescu's procedure, a series of relations for prediction of tunnel wall displacement and pressure on lining is extracted. These relations are compared with the existing Goodman solution [8] and Barla relations [9].

2. Methodology

2.1. Tunnel support analysis incorporating rock creep

Assuming that the rock behaves linear viscoelastically, a rock-support analysis is conducted by Cristescu (1987), [3]. In this research the same method is utilized.

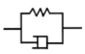
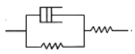
Suppose that circular tunnels are driven in homogeneous isotropic linear viscoelastic incompressible (ν = 0.5) media. The problem is solved for axisymmetric plane strain conditions for sections far from the face. The face effect can be considered by the confinement loss coefficient λ.
The in situ stress field is assumed to be hydrostatic:

$$\sigma_h = \sigma_v = \sigma_0 \tag{1}$$

$$\sigma_0 = \gamma_0 h_0 \tag{2}$$

In this method, the stresses in the constitutive equation of the rock are "relative" stresses, i.e. they are the difference between the actual stress and the hydrostatic "primary" stresses before excavation. In other words, they refer to the reference configurations is the in situ configuration before excavation [3].
Using these assumptions, Massier and Cristcscu [3, 10] showed that the excavation doesn't cause variation in the relative mean stress and relative mean strain. Thus, they proposed the constitutive relations in terms of circumferential components to be used. The structural figures and constitutive relations for three viscoelatic model are presented in Table 1.

Table 1. Viscoelastic models and their parameters [11]

Model	Maxwell	Kelvin	Kelvin-Voigt
Structural figure			
Parameters	G_M η_M	G_K η_K	$G_0 = G_M$ $G_\infty = \frac{G_M G_K}{G_M + G_K}$ $n = \frac{\eta}{G_M + G_K}$
Constitutive relation	$\dot{\varepsilon}_\theta = \frac{\dot{\sigma}_\theta}{2G_M} + \frac{\sigma_\theta}{2\eta_M}$	$\dot{\varepsilon}_\theta = \frac{\sigma_\theta}{2\eta_K} - \frac{G\varepsilon_\theta}{\eta_K}$	$2G_0 n\dot{\varepsilon}_\theta + 2G_\infty\varepsilon = n\dot{\sigma}_\theta + \sigma_\theta$
Number of relation	(3)	(4)	(5)

2.2. Calculation of tunnel wall displacement

This part introduces the derivation procedure of tunnel wall displacement relations.
The displacement is related to the strain component as follow:

$$u = r\varepsilon_\theta \tag{6}$$

the circumferential stress can be expressed as:

$$\sigma_\theta(t) = (\sigma_0 - P_s)(\frac{r_0}{r})^2 \tag{7}$$

where P_s is the possible pressure acting on the tunnel wall.
The lining is an elastic ring of constant thickness, sufficiently strong and rigid to be able to prevent rock mass failure. The lining is set at a time t_0 that corresponds with initial deformation U_0.
For the time before lining installation, the support pressure equals to zero - $P_s = 0$.
After installation of lining $t \geq t_0$, the support pressure is obtained by:

$$P_s = \frac{K_s}{r_0}(U - U_0) \tag{8}$$

By replacing relations (6), (7) and (8) into any of the constitutive relations (3, 4 or 5) in Table 1 a differential equation for the rock -support interface that describes the convergence of the tunnel surface is obtained:

$$\frac{du}{dt} = f(K_s, G, \eta, r_0, \sigma_0) \tag{9}$$

Equation 9 requires an initial condition - U_0 to be solved which is found by considering the deformational properties of the viscoelastic rock at $t = t_0$.
For Maxwell body:

$$U_0 = \frac{r_0\sigma_0}{2G}\left[1 + \frac{t_0}{T}\right] \tag{10}$$

For Kelvin body:

$$U_0 = \frac{r_0\sigma_0}{2G}\left[1 - \exp(-\frac{t_0}{T})\right] \tag{11}$$

and for Kelvin-Voigt body:

$$U_0 = \frac{r_0\sigma_0}{2}\left[\frac{1}{G_\infty} + (\frac{1}{G_0} - \frac{1}{G_\infty})\exp(-\frac{G\infty}{nG_0}t_0)\right] \tag{12}$$

Total displacement consists of elastic and creep components can be obtained as follow:

$$U = U_{el} + U_{cr} \tag{13}$$

In this paper, the resulted relations for total displacement are presented as the multiplication of elastic displacement on the creep functions. The utilized creep functions $Z(t),\ Z^{'}(t)\ and\ Z^{"}(t,t_0)$ show the proportion of time-dependent behavior of rock mass on total displacement for three cases respectively:
when tunnel is unlined,

$$U = U_{el} \times Z(t) \tag{14a}$$

when lining is installed simultaneously with the excavation,

$$U' = U_{el} \times Z'(t) \quad (14b)$$

and when, there is a delay in lining installation,

$$U(t) = U_{el} \times Z''(t, t_0) \quad (14c)$$

Table 2 includes the relations for unlined tunnel for three viscoelastic models.

Table 2. Displacement of unlined tunnel

$U = U_{el} \times Z(t)$, $U_{el} = \frac{r_0 \sigma_0}{2G}$

Number of relation	model	$Z(t)$
15	Maxwell	$\left[1 + \frac{t}{T}\right]$
16	Kelvin	$\left[1 - \exp(-\frac{t}{T})\right]$
17	Kelvin-Voigt	$\left[\frac{G_\infty}{G_0} + (1 - \frac{G_0}{G_\infty}) \times \exp(-\frac{G_\infty}{nG_0} t)\right]$, $G = G_M$

The displacement of lined tunnel for these two cases ($t_0 = 0$ and $t_0 \neq 0$) are given in Tables 3 and 4.

Table 3. Relations for calculation of lined tunnel displacement for case $t_0 = 0$

$U' = U_{el} \times Z'(t)$, $U_{el} = \frac{r_0 \sigma_0}{2G}$

Number of relation	model	$Z'(t)$
18	Maxwell	$\left\{1 + \frac{2G}{K_s}\left[1 - \exp(-\frac{K_s}{TK'} t)\right]\right\}$
19	Kelvin	$\frac{2G}{K'}\left[1 - \exp(-\frac{K'}{2\eta} t)\right]$
20	Kelvin-Voigt	$\left[\frac{K_0'}{K_\infty'} + (1 - \frac{K_0'}{K_\infty'}) \times \exp(-\frac{K_\infty'}{nK_0'} t)\right]$, $G = G_M$

2.3. Pressure on lining

After calculation of displacement, the pressure on lining can be calculated using relation 8.

The achieved relations for calculation of pressure on the lining are expressed in terms of $W(t, t_0)$ function that depends on the selected viscoelastic model (Table 5).

Table 4. Relations for calculation of tunnel displacement with delay t_0 in lining installation Pressure on lining

$U(t) = U_{el} \times Z''(t, t_0)$, $U_{el} = \frac{r_0 \sigma_0}{2G}$

Number of relation	model	$Z''(t, t_0)$
21	Maxwell	$\left\{\left[\frac{t_0}{T} - \frac{2G}{K_s}\right] \times \left[\exp(-\frac{K_s}{TK'}(t - t_0))\right] + \frac{K'}{K_s}\right\}$
22	Kelvin	$\left\{1 - \frac{1}{K'}\left[\begin{array}{l} K_s \times \exp(-\frac{t_0}{T}) + 2G \times \\ \exp(-\frac{2Gt_0 + K'(t - t_0)}{2\eta}) \end{array}\right]\right\}$
23	Kelvin-Voigt	$\left\{\begin{array}{l} \frac{G_0}{G_\infty} + \frac{1}{K_\infty'}\left[K_s + 2G_\infty \times \exp(-\frac{K_\infty'}{K_0' n}(t - t_0))\right] \\ \left[1 - \frac{G_0}{G_\infty}\right]\left[\exp(-\frac{G_\infty}{nG_0} t_0)\right] \end{array}\right\}$, $G = G_M$

Table 5. Relations for calculation of pressure on the lining

$P_s = \frac{K_s \sigma_0}{2G} \times W(t, t_0)$

Number of relation	model	$W(t, t_0)$
24	Maxwell	$\left\{\begin{array}{l} \left[1 + \frac{t_0}{T} - \frac{K'}{K_S}\right] \times \\ \left[\exp(-\frac{K_s}{TK'}(t - t_0))\right] + \frac{2G}{K_S} - \frac{t_0}{T} \end{array}\right\}$
25	Kelvin	$\left\{\exp(-\frac{t_0}{T}) - \frac{1}{K'}\left[\begin{array}{l} K_s \exp(-\frac{t_0}{T}) + 2G \times \\ \exp(-\frac{t_0}{T} - \frac{K'(t - t_0)}{2\eta}) \end{array}\right]\right\}$
26	Kelvin-Voigt	$\left\{(1 - \frac{G_0}{G_\infty}) \times \exp(-\frac{G_\infty}{nG_0} t_0)\left[\begin{array}{l} \frac{1}{K_\infty'}(K_s + 2G_\infty \times \\ \exp(-\frac{K_\infty'}{K_0' n}(t - t_0))) - 1 \end{array}\right]\right\}$, $G = G_M$

3. Results and Discussion

The following sections discuss results of application and validation of developed relations for different models.

3.1. Kelvin body

A circular lined tunnel with a radius of 3 m is driven in rock mass that satisfied kelvin viscoelastic model. Table 6 summarizes the Kelvin parameters of the rock mass surrounding the tunnel.

In Figures 1 and 2, the displacement and scaled presssure on tunnel lining are illustrated (relation 22 and 25). The scaled pressure is equal to the ratio of pressure on lining to overburden weight. According to Figure 2, the pressure increases and finally tends to in situ stresses.

Table 6. Mechanical properties of Kelvin model

σ_0 (MPa)	G (MPa)	Ks (MPa)	η (MPa-Year)	t_0 (Year)
0.34	35	3320	193	0

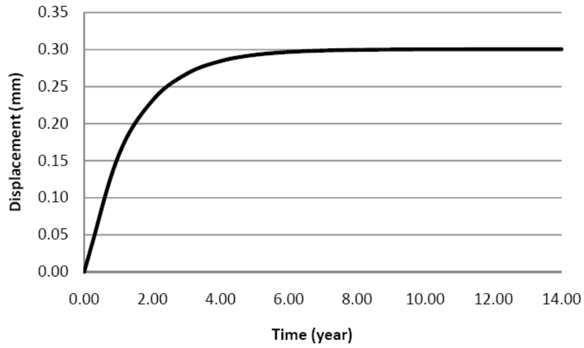


Figure 1. Displacement of tunnel wall

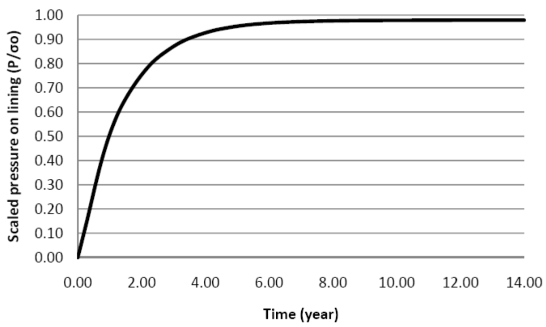


Figure 2. Scaled pressure on lining

Figure 3 shows the effect of rigidity of lining on applied pressure. It is obvious that pressure increases with rigidity of the lining and the applied pressure on an ideal rigid lining equals to overburden weight.

3.2. Maxwell body

Maxwell model consists of an elastic spring and a viscous dashpot that are put in series (Table1). The input data for validation of this model are taken from Barla research work (Tunnelling under squeezing rock conditions) [9]. The relations of this work are presented in appendix A.

In the case of unlined tunnel both Goodman and Cristesco's (Equation 15) methods end into the same result. While, for lined tunnel, Cristesco (relations 18 or 21 for displacement and 24 for pressure) eventuate the same result as Barla (Figure 4). The relations of Goodman solution are presented in appendix B.

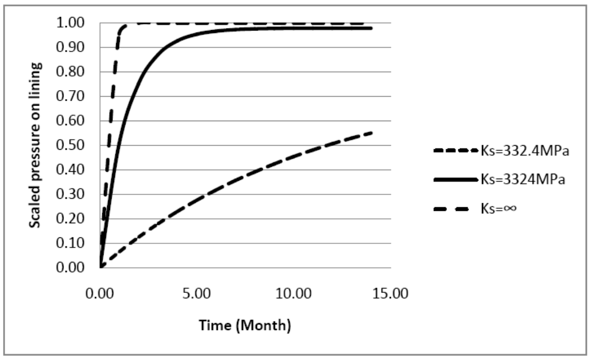


Figure 3. Effect of lining stifness on applied pressure

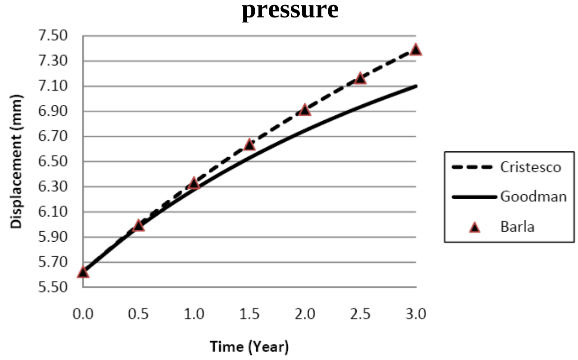


Figure 4. Comparison of wall displacement of lined tunnel

Table 7. Mechanical properties of rock mass and support

G (MPa)	r_0 (m)	η (MPa-Year)	σ_0 (MPa)	E_c (MPa)	ν_c	t_c (m)	t_0 (year)
2000	3	6000	7.50	31000	0.25	0.45	0

Figure 5 represents the scaled pressure on lining for three solutions. According to Figure 5 the ultimate pressure (for Barla and Cristesco methods) on lining is equal to overburden weight.

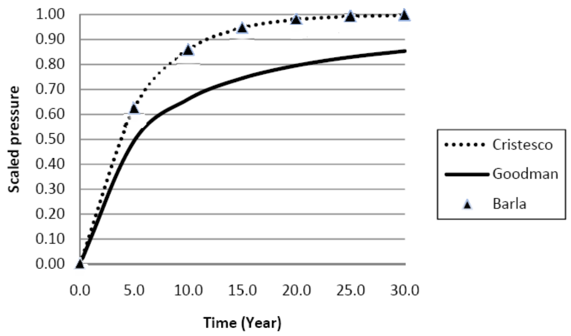


Figure 5. Comparison of scaled pressure on the tunnel lining

Now, we analyses the effect ofthe delay in tunnel lining instalation on the pressure. Figure 6 considers different delays: t0=0, 0.03 and 0.1 year. It can be seen that increase of delay in lining installation causes decrease in the inserted pressure on lining.

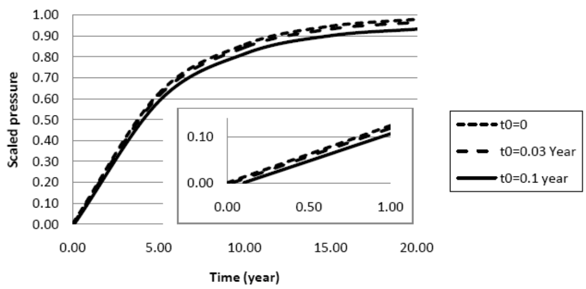


Figure 6. The effect of different delay time on lining pressure

3.3. Kelvin-Voigt model

For comparison of extracted relation with existing ones, the data from Barla research work was used again [9]. The properties of rock mass are given in Table 8.

Table 8. Creep parameters of rock mass

G (MPa)	G_{el} (MPa)	H (MPa-Year)	n
2	2000	6000	3000

The calculated results of unlined tunnel displacemnts by Cristesco (relation 17) would be the same as those of Barla or Gooman solution.

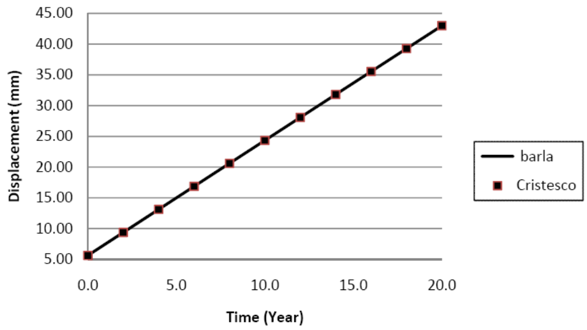


Figure 7. Creep displacement of tunnel wall

At the next step, the same conditions of rock mass with a lining with rigidity equals to1880 MPa was considered.The calculated displacement and pressureare shown in Figure 8 and 9 (relations 20 and 26). From these two curves, it can be concluded that the Kelvin-Voigt model, Goodman solution predict underestimated values than two other methods.

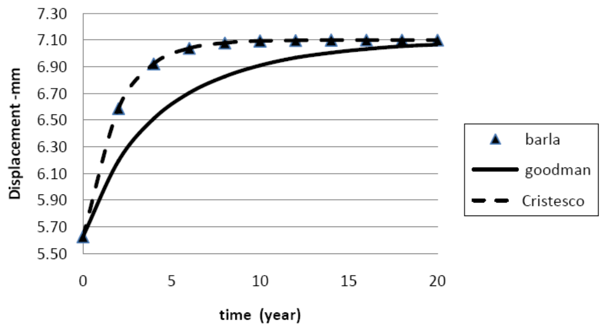


Figure 8. Comparison of Creep displacement

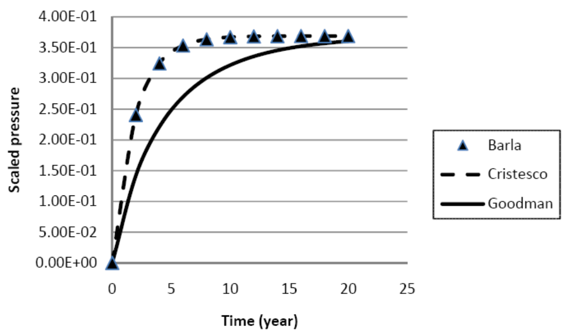


Figure 9. Pressure on tunnel lining

In Kelvin-Viogt model, the final pressure for ideal solid lining is about half of the overburden weight (Figure 10).

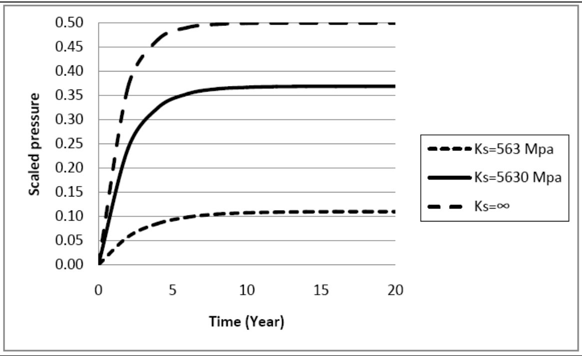


Figure 10. The effect of lining stiffness on the final pressure

4. Conclusions

In this paper a series of analytical relations are provided for the calculation of unlined and lined tunnel wall displacements of three viscoelastic models –Maxwell, Kelvin and Kelvin-Voigt. Furthuremore, new formulae are presented to determine the pressure on lining, considering a delay in lining installation.

The relations based on Cristescue's method are compared with existing ones which were presented by Goodman and Barla solutions. There is full correspondence between results of three solutions for unlined tunnel. In the case of lined tunnel, the Cristesco method has good correspondence with Barla relations for Maxwell and Kelvin-Voigt bodies, but there is a difference in calculated displacement and pressure by Goodman solution. In all cases, Cristesco's model gives higher values of pressure and displacement than Goodman solution for the case of lined tunnel.

The effect of lining stifness and delay in installation of lining is analysed. For Kelvin and maxwell model the ultimate pressure on lining is equal to overburden weight, but it is half of the overburden for Kelvin voigt.

What can be concluded from all of mentioned analyses is that the selection of proper model has an important role in calculation of underground constructions. Each model has its own behaviour in response to tunnel excavation and lining installation.

Acknowledgment

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Appendix A.
Relations of Barla solution

According to Goodman solution, the displacement for Maxwell- u_M and Kelvin-Voigt -u_{KV} substances are respectively expressed as [9]:

$$u_M = \frac{r_0\sigma_0}{2G_M}\left[1-\frac{t}{T_M}\right]+\frac{P_s r_0}{K_s} \quad \text{(A1)}$$

Where the pressure on the lining is obtained by:

$$P_s = \sigma_0\left[1-\exp\left[-\frac{t-t_0}{T(1+\frac{2G_M}{K_s})}\right]\right] \quad \text{(A2)}$$

for Kelvin-Voigt model:

$$u_{KV}=\frac{r_0\sigma_0}{2G_M}\left[1+\left[\frac{G_M}{G_f}-1\right].\left[1-\exp(-\frac{t_0}{T_K})\right]\right]+\frac{P_s r_0}{K_s} \qquad \text{(A3)}$$

and the pressure on lining:

$$P_s=\sigma_0\left[\frac{1-\frac{G_f}{G_0}}{1+2.\frac{G_f}{K_s}}\right]\exp(-\frac{t_0}{T_K}).\left[1-\exp\left[\frac{2+\frac{K_s}{G_f}}{2+\frac{K_s}{G_0}}\right].\left[-\frac{t-t_0}{T_K}\right]\right] \qquad \text{(A4)}$$

where:

$$T_K=\frac{\eta}{G_K}$$

$$G_f=\frac{1}{G_M}+\frac{1}{G_K}$$

Appendix B.
Relations of Goodman solution

According to Goodman solution, the tunnel wall displacement for Kelvin, Maxwell and Kelvin-Voigt substances are respectively expressed as [8, 11]:

$$u_K=\frac{r_0\sigma_0}{2G_K}\left[1-\exp(-\frac{t}{T_K})\right] \qquad \text{(B1)}$$

$$u_M=\frac{r_0\sigma_0}{2G_M}\left[1-\frac{t}{T_M}\right] \qquad \text{(B2)}$$

$$u_{KV}=\frac{r_0\sigma_0}{2}\left[\frac{1}{G_M}+\frac{1}{G_K}\exp(-\frac{t}{T_K})\right] \qquad \text{(B3)}$$

Where:

$$T_K=\frac{\eta}{G_K}$$

$$T_M=\frac{\eta}{G_M}$$

Nomenclature

E_c , t_c and ν_c - Modulus of elasticity, thickness and poisson's ratio

$G=\frac{E}{2(1+\nu)}$ - Shear modulus

G_M, G_K – Shear modulus for Maxwell and Kelvin materials

G_0 – Initial shear modulus

G_∞ - ultimate shear modulus

h_0 -tunnel depth

K_s – Lining stiffness

$K^{'}=K_s+2G$

$K_\infty^{'}=K_s+2G_\infty$ - Current, ultimate and initial

$K_0^{'}=K_s+2G_0$

stiffness of rock mass – support system

r - Radial distance from the tunnel center

r_0 - Radius of tunnel

U_0 - Initial deformation corresponding to time $t=t_0$

$U_{el}=\frac{r_0\sigma_0}{2G}$ - Elastic displacement

U_{cr} - Creep displacement

P_s -Pressure acting on the tunnel surface

t_0 - Time of lining installation

t_c - Thickness of concrete lining

$W(t,t_0)$ - Creep function for calculation of pressure on lining

$Z(t)$, $Z^{'}(t)$, $Z^{''}(t,t_0)$ - Creep function for calculation of displacement

γ_0 -Rock unit weight

σ_0 - Hydrostatic in situ stress state of stress

σ_v and σ_h are respectively vertical and horizontal in situ stress

η - Viscosity of material

η_M and η_K – Viscosity of Maxwell and Kelvin materials

$T=\frac{\eta}{G}$ - Relaxation time

λ - Confinement loss coefficient