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New empirical failure criterion for shale

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Abstract

A new failure criterion was presented to predict the ultimate strength of shale under the triaxial and polyaxial state of stress. A database containing 93 datasets was obtained from the results of the uniaxial, triaxial, polyaxial compressive tests, an indirect tensile test was collected from reliable references, and this test was carried out on the shale samples taken from the southwestern oilfields in Iran. The database was used to evaluate the proposed criterion, and its accuracy was compared against the popular failure criteria in rock mechanics, particularly those used for stability analysis such as the Hoek-Brown, Mohr-Coulomb, Drucker-Prager, and Mogi-Coulomb failure criteria. In order to evaluate the model, seven important statistical indices were selected. Subsequently, curves from various failure criteria were fitted to the triaxial and polyaxial data, and the corresponding coefficients and statistical indices were determined. The results obtained indicated that, in all cases, compared to the other failure criteria, the proposed criterion succeeded to predict the ultimate strength at a higher accuracy. Also the proposed criterion was used calculate the uniaxial compressive and tensile strengths with a minimum error. For a further examination of the proposed criterion, a series of results from the triaxial test including the ductile failure data were utilized for evaluation of the applicability of the proposed criterion to the ductile zone. It showed that the criterion could predict the ultimate strength of shale over a wide range of stresses.

1. Introduction

One of the main aspects of a well stability analysis is the choice of an appropriate failure criterion to predict the ultimate strength of rocks [1]. Rock strength is an important index when designing the rock structure. Thus analysis of rock structure requires an appropriate failure criterion. Any failure criterion has a number of material constants commonly determined via regression analysis of the results of triaxial or polyaxial compressive tests [2]. Researchers have proposed many failure criteria to predict the rock strength for triaxial cases ($\sigma_2=\sigma_3$) neglecting the intermediate stress, such as those proposed by Mohr [3], Fairhurst [4], Hub [5], Murrel [6], Franklin [7], Bieniawski [8], Hoek and Brown [9], Yudhbir et al. [10], Ramamurthy [11], Johnson [12], and sheorey et al. [13], and also for

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the polyaxial failure criteria considered in intermediate stress ($\sigma_2 \neq \sigma_3$) such as those published by Mogi [14], Drucker-Prager [15], Lead and Duncan [16], Zhou [17], You [18], and Mogi-Coulomb [19].

In the present research work, a new behavior model is presented for brittle and ductile failures of an intact rock of shale using the least squares approximation regression method. For this purpose, a curve is empirically fitted to the experimental data, and the results obtained are compared against those of well-known empirical criteria including Hoek-Brown, Mohr-Coulomb, Drucker-Prager, and Mogi-Coulomb. This empirical criterion is obtained via a trial and error approach toward choosing the variables of the best fit to test the results. The proposed failure criteria are evaluated by the results of the triaxial and polyaxial tests compiled from reliable references and multi-stage triaxial compressive tests and the indirect tensile strength test of the shale samples taken from the southwestern oilfields in Iran. The purpose of proposing these empirical failure criteria is to enhance the accuracy of predicting the shale rock ultimate strength, as compared to the existing failure criteria.

2. Strength database

The database used in the present research work contains the results of 93 tensile, uniaxial, and conventional triaxial and polyaxial tests undertaken on an intact rock of shale. It was tried to collect from reliable references the triaxial and indirect tensile tests on deep shale samples obtained from oilfields in Iran.

2.1. Conventional triaxial database

50 datasets of uniaxial and conventional triaxial compressive strength were collected from reliable references (see Table 1).

Table 1. Uniaxial and conventional triaxial experimental data prepared from reliable the published articles [8, 11, 20].

σ_1 (MPa)	σ_3 (MPa)	σ_1 (MPa)	σ_3 (MPa)	σ_1 (MPa)	σ_3 (MPa)	σ_1 (MPa)	σ_3 (MPa)
257.00	6.89	162.00	30	32.40	0	311.90	120
347.00	34.48	179.00	40	35	2.45	426.10	200
440.00	68.98	201.00	50	48.80	4.9	504.00	250
544.00	103.44	62.10	0	63.70	9.80	39.20	0
714.00	172.41	88.10	2.45	78.50	14.70	187.30	50
148.00	6.89	108.30	4.90	63.70	0	395.10	100
203.00	34.48	128.20	9.80	123.00	25	616.70	200
281.00	68.95	156.90	14.70	210.80	50	78.20	0
356.00	103.44	33.90	0	310.20	100	109.40	20.70
493.00	172. 41	35.80	2.45	498.00	200	179.30	48.30
64.00	0	49.30	4.90	98.30	20	188.30	55.20
73.00	10	66.30	9.80	207.90	60	-	-
138.00	20	88.30	14.70	245.80	100	-	-

2.2. Multi-stage triaxial compressive and indirect tensile test on deep shale

The samples were taken from the southwestern oilfields operated by the National Iranian South Oilfields Company (NISOC) (see Figure 1). These samples were about 9 and 6.5 cm in diameter. As such, plugs of 2.45 cm in diameter and 2-3 times the diameter in length were taken from the samples along the normal direction to the bedding and prepared according to the ISRM standard procedure. Due to the deterioration of samples in response to contact with water, we used gasoil for plugging. The sample ends were fully smoothened using an angle grinder. Finally, considering the problems caused by the deterioration of the samples and the presence of joints, we ended up with 6 and 4 samples for the triaxial and indirect tensile (Brazilian) tests, respectively. Figure 2 shows the samples. Also Tables 2 and 3 present the sample properties. Triaxial compressive tests were conducted via the multi-stage loading method according to the ISRM 2007 standard.

This database further contains 14 datasets obtained from the conventional multi-stage triaxial tests under various values of confining pressures as well as four Brazilian tensile tests conducted on the depth samples of shale taken from the southwestern oilfields in Iran. Each dataset includes maximum and minimum stresses at failure. Tables 4 and 5 show the results of the multi-stage triaxial compressive and Brazilian tests. Also, Figures 3 and 4 show the samples before and after the multi-stage triaxial and Brazilian tests, and Figure 5 presents the typical stress strain curves of shale in the triaxial compression tests with various confining pressures.

Figure 1. Samples taken from southern and southwestern oilfields in Iran.

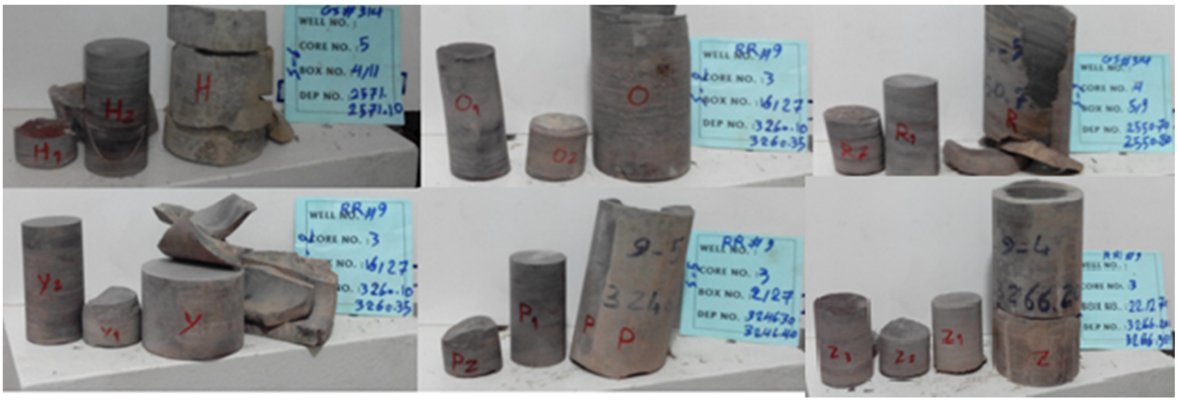

Figure 2. Plugs for triaxial and Brazilian tests.

Table 2. Properties of samples for triaxial test.

Sample	Filed	Well No.	Depth (m)	Length (mm)	Diameter (mm)
H1	Gachsaran	314	2571	60	24.5
O1	Ramshire	9	3260	60	24.5
R1	Gachsaran	314	2550	60	24.5
Y1	Ramshire	9	3266	60	24.5
P1	Ramshire	9	3246	60	24.5

Table 3. Properties of samples for Brazilian test.

Sample	Filed	Well No.	Depth (m)	Length (mm)	Diameter (mm)
H2	Gachsaran	314	2571	20	24.5
O2	Ramshire	9	3260	20	24.5
R2	Gachsaran	314	2550	20	24.5
Y2	Ramshire	9	3266	20	24.5
P2	Ramshire	9	3246	20	24.5

Table 4. Results of triaxial test on deep shale samples.

σ_3(MPa)	σ_1(MPa)	Description	σ_3(MPa)	σ_1(MPa)	Description
5	136	-	27	218.50	-
7	64	Failure on joint	30	244.00	-
10	157	-	33	224.00	-
13	94	Failure on joint	35	241.30	-
15	182	-	38	236.00	-
17	162.20	-	40	256.00	-
20	114	Failure on joint	43	206.80	-
23	199.80	-	47	226.30	-
25	207.70	-	50	183.3	Failure on test

Table 5. Results of Brazilian test on deep shale samples.

Sample	Thickness (mm)	Diameter (mm)	Tensile strength (MPa)
H2	20	24.5	12.82
O2	20	24.5	12.23
P2	20	24.5	14.16
R2	20	24.5	15.39
Average			**13.65**

Figure 3. Samples before and after multi-stage triaxial test.

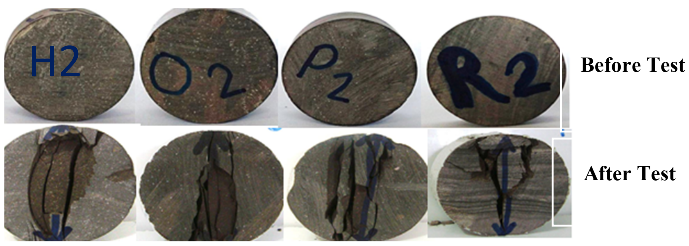

Figure 4. Samples before and after Brazilian test.

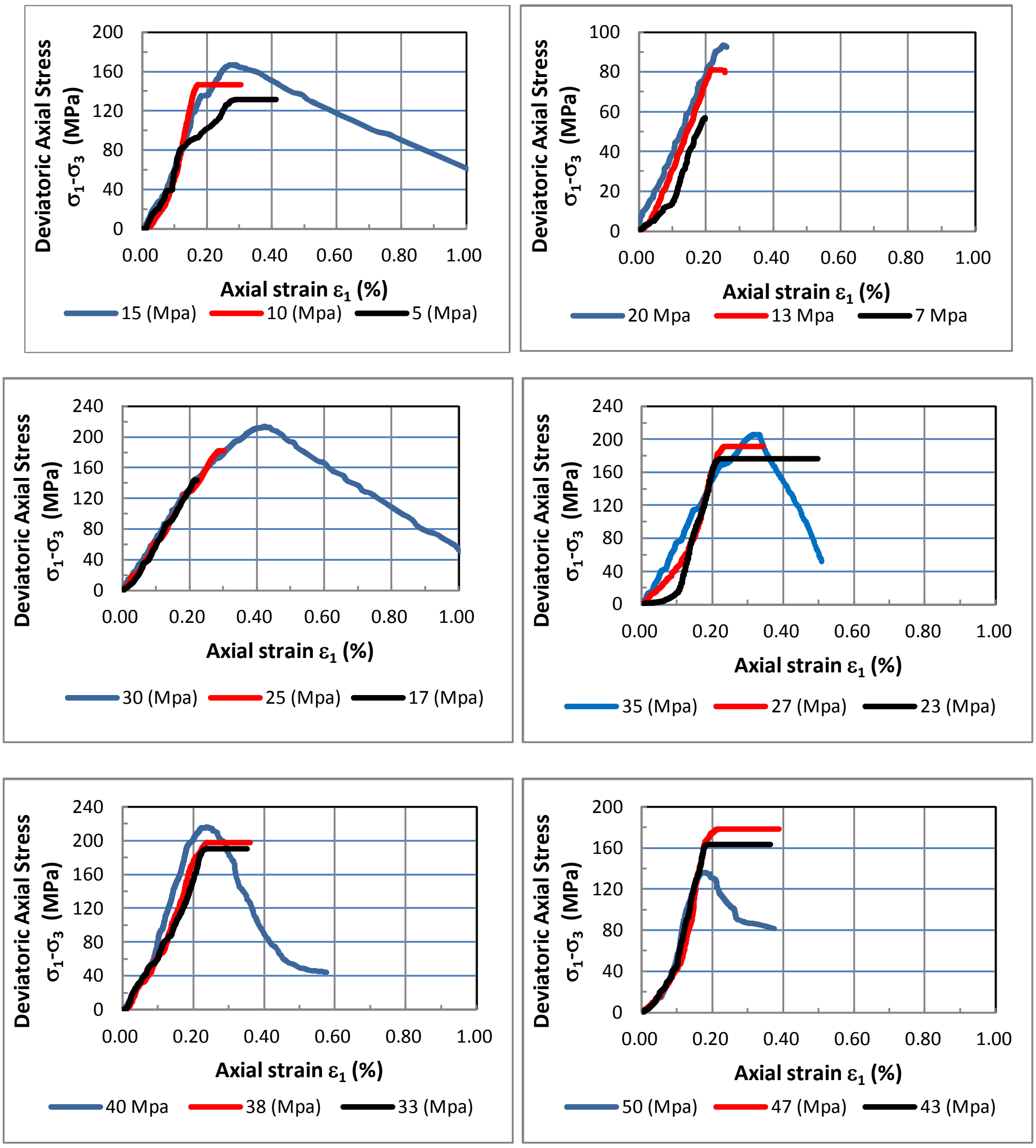

Figure 5. Typical stress strain curves of shales in multi-stage triaxial compression tests with various confining pressures.

2.3. Polyaxial compressive test database

25 datasets of polyaxial compressive strength were collected from reliable references. Table 6 shows the results of the polyaxial tests.

3. Failure criterion

The Hoek-Brown, Mohr-Coulomb, Drucker-Prager, and Mogi-Coulomb failure criteria are among the popular failure criteria in a well-stability analysis [19]. Therefore, these were selected for comparison with the proposed criterion.

3.1. Hoek-Brown criterion

The Hoek-Brown failure criterion was presented in 1980 for prediction of the intact rock and also the rock mass ultimate strength. Many years have passed since modification of the Hoek-Brown failure criterion has started. Among the various proposed criteria, the Hoek-Brown criterion is the most widely applied one in rock mechanics. After studying an extensive range of the laboratory data the relationship between the maximum and minimum stresses at the point of failure was presented as follows:

$$\sigma_1 = \sigma_3 + (m\sigma_c\sigma_3 + s\sigma_c^{\,2})^{\frac{1}{2}} \quad (1)$$

Or

$$(\sigma_1 - \sigma_3)^2 = m\sigma_c\sigma_3 + s\sigma_c^2 \quad (2)$$

where m and s, are the constants that are dependent on the rock properties, so is equal to 1 for intact rock and σ_c denotes the uniaxial compressive strength [21].Tensile strength is obtained by Eq. (3).

$$\sigma_t = \frac{\sigma_c}{2}(\sqrt{m^2 + 4s} - m) \quad (3)$$

Table 6. Polyaxial experimental data collected from reliable published articles [20].

σ_1 (MPa)	σ_2 (MPa)	σ_3 (MPa)	σ_1 (MPa)	σ_2 (MPa)	σ (MPa)
161	26.3	25	199	123.70	25
166.70	26.3	25	186	131.60	25
180.60	36.30	25	230.50	50	50
180.60	36.80	25	238.90	50	50
175	47.40	25	258.30	71	50
175	55.30	25	261	89.50	50
189	65.80	25	266.70	100	50
202	76.30	25	261	110.50	50
191.70	78.90	25	261	121	50
200	86.80	25	288.90	131	50
202	97.40	25	266.70	150	50
191.70	100	27	258.30	157.90	50
186	115.80	25			

3.2. Mohr-Coulomb

This criterion is one of the most important and widely used failure criteria. Failure of a rock under pressure occurs when the shear stress develops in a plane and reaches an amount that overcomes the rock's natural cohesion and the friction force resisting against the failure plane. This criterion is expressed as:

$$\tau = C + \sigma_n Tan\varphi \quad (4)$$

where σ_n is the normal stress acting on the failure plane, C is the cohesion, and ∅ is the angle of the internal friction. The Mohr-Coulomb criterion could be expressed based on the maximum and minimum principal stresses as follows [21]:

$$\sigma_1 = \sigma_c + N\sigma_3 \quad (5)$$

In the above relationship, σ_c is the uniaxial compressive strength and $N = \frac{1+sin\emptyset}{1-sin\emptyset}$.

The uniaxial compressive and tensile strength can be calculated by the following equations:

$$\sigma_c = \frac{2c\cos\phi}{1-\sin\phi} \quad (6)$$

$$\sigma_t = \frac{2c\cos\phi}{1+\sin\phi} \quad (7)$$

3.3. Drucker-Prager criterion

This criterion was first developed for soil mechanics. It is expressed in terms of the principal stresses, as follows:

$$\tau_{oct} = k + m\sigma_{oct} \quad (8)$$

where, τ_{oct} is the octahedral shear stress and σ_{oct} is the octahedral normal stress, which are given by the following expressions:

$$\sigma_{oct} = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \quad (9)$$

$$\tau_{oct} = \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_1 - \sigma_3)^2} \quad (10)$$

m and k are the material constants whose values could be obtained from the drawn failure push in τ_{oct} -σ_{oct} space [19]. Uniaxial compressive and tensile strength are calculated by Eqs. 11 and 12.

$$\sigma_c = \frac{k}{(\frac{\sqrt{2}}{3} - \frac{m}{3})} \quad (11)$$

$$\sigma_t = \frac{k}{(\frac{\sqrt{2}}{3} + \frac{m}{3})} \quad (12)$$

3.4. Mogi-Coulomb criterion

This criterion has been presented by Al-Ajmi, as given below, and has been widely used in the oil wellbore stability analysis. In fact, the Mogi-Coulomb criterion is considered as the extended form of the Mohr-Coulomb criterion in three dimensions [19, 20].

$$\tau_{oct} = a + b\sigma_{m.2} \quad (13)$$

Where:

$$\sigma_{m.2} = \frac{\sigma_1 + \sigma_3}{2} \quad (14)$$

a and b are the material constants whose values could be obtained from the drawn failure push in τ_{oct} -$\sigma_{m.2}$. Uniaxial compressive and tensile strengths are obtained by the following equations.

$$\sigma_c = \frac{a}{(\frac{\sqrt{2}}{3} - \frac{b}{2})} \quad (15)$$

$$\sigma_t = \frac{a}{(\frac{\sqrt{2}}{3} + \frac{b}{2})} \quad (16)$$

4. Least squares approximation method

A good curve fitting not only requires a complete range of data, but also relies on the application of an appropriate mathematical method. Many mathematical methods have been proposed for curve fitting. Among various linear methods used for estimating model parameters (β), the ordinary least squares (OLS) method is known as the most popular and dominant approach due to its desired properties. Attributed to the German mathematician Karl Fredrick Gauss, this method seeks fitting the best regression line to all data points by minimizing the sum of error terms squared.

$$y_i = \beta_1 + \beta_2 x_i + e_i \quad (17)$$

where x_i is the independent variable, y_i is the dependent variable, and e_i is the statistical error that is a measure of deflection of y_i from a straight-line. Therefore, the OLS method tries to minimize Equation (11) (see Figure 6):

$$\min \sum e_i^2 = \sum (y_i - \hat{y}_i)^2 = \sum (y_i - \hat{\beta}_1 - \hat{\beta}_2 X_i^2) \quad (18)$$

By minimizing Eq. (11), the OLS method presents estimations of the parameters that are calculated as follows:

$$\hat{\beta}_1 = \frac{\sum x_i^2 \sum y_i^2 - \sum x_i \sum x_i \sum x_i y_i}{N \sum x_i^2 - (\sum x_i)^2} \quad (19)$$

$$\hat{\beta}_2 = \frac{\sum x_i y_i}{\sum x_i^2 - N\bar{x}^2} \quad (20)$$

where N is the number of samples and $\bar{x}$ represents the average of the independent variables [22].

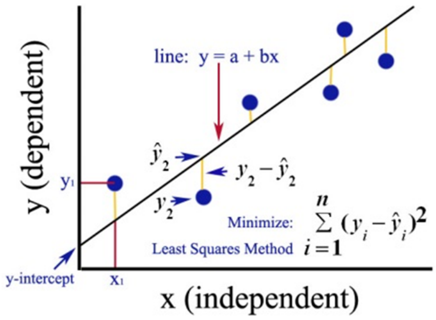

Figure 6. Linear curve fitting to data points.

5. Proposed failure criterion

Differently from other 3D failure criteria that are related to the first stress invariant with the third deviatoric stress invariant, the Lade's failure criterion uses a special relationship between the first and third stress invariants. Therefore, we fitted various functions on the dataset in the first and third stress invariant spaces. The proposed empirical failure criterion is described as a relationship between the first and third stress invariants in a compressive zone and maximum and minimum stresses in the tensile zone:

$$\begin{cases} i_3^{(1/3)} = a + b(i_1 - m\sigma_3), \sigma_3 \geq 0 \\ \sigma_1 = 8\sigma_3 - \frac{a}{b}, \sigma_3 < 0 \end{cases} \quad (21)$$

where i_1 and i_3 are the first and third stress invariants, and can be calculated by Eq. (22), respectively; and b denote material constants which can be obtained by fitting a straight line to the data in the $i_3^{(1/3)} - i_1$ space.

$$\begin{cases} i_1 = \sigma_1 + \sigma_2 + \sigma_3 \\ i_3 = \sigma_1 \sigma_2 \sigma_3 \end{cases} \quad (22)$$

In addition, m is the ductile limit correction factor and can be calculated as follows:

$$m = \frac{\sigma_{3\max} - \sigma_{3\min}}{5\sigma_{3\max}} \tag{23}$$

where σ_{3min} and σ_{3max} are the minimum and maximum confining pressures among the understudied datasets; m changes between $0 \le m \le 0.2$. For the elastic zone ($\sigma_1 > 4.4\sigma_3$), m=0 and ductile zone ($\sigma_1 \le 4.4\sigma_3$), m is calculated by Eq. (23).

The uniaxial compressive (σ_c) and tensile strength (σ_t) can be found by substituting $\sigma_3=0$ and $\sigma_1=0$ in Eq. (21), and thus:

$$\sigma_c = -\frac{a}{b} \tag{24}$$

$$\sigma_t = \frac{a}{8b} \tag{25}$$

6. Performance evaluation indices for prediction model

Various statistical indices have been proposed to investigate the quality of curve fitting and accurate predictions. Since each performance evaluation index considers a certain aspect, seven indices were used to evaluate the proposed criterion. Of these, two indices are of standard average error family: standard mean squared error (MSE) and root-mean-square error (RMSE). Two indices are the mean absolute error (MAE) and root mean squared error (RMSE), and the other two indices are related to the percent errors: prediction percent error (AP_e) and squared average prediction percent error (AVPE). The smaller the above indices, the more accurate the predictions are made by the corresponding criterion [23, 24].

$$MSE = \frac{1}{n}\sum(\sigma_{1i\,\exp} - \sigma_{1ical})^2 \tag{26}$$

$$RMSE = \sqrt{\frac{\sum(\sigma_{1i\,\exp} - \sigma_{1ical})^2}{n}} \tag{27}$$

$$MAE = \frac{\sum\left|(\sigma_{1ical} - \sigma_{1i\,\exp})\right|}{n} \tag{28}$$

$$P_e = \left|\frac{\sigma_{1cal} - \sigma_{1\exp}}{\sigma_{1\exp}} \times 100\right|\% \tag{29}$$

$$AP_e = \frac{\sum p_e}{n} \tag{30}$$

$$AVPE = \sqrt{\frac{1}{n}\sum p_e^2} \tag{31}$$

The remaining two indices are coefficient of determination (R^2) and accommodation coefficient (ψ^2). Coefficient of determination represents the percentage of data that is closer to the fitted line, and can be calculated by Eq. (32). The value for the coefficient of determination ranges within $0 \le R^2 \le 1$. The closer the coefficient approaches to 1, the better the fitting is. ψ^2 is greater than or equal to 0 ($\psi^2 \ge 0$). The smaller the value for ψ^2 is, the better the strength criterion accords with the triaxial test data [25].

$$R^2 = \frac{(\sum\sigma_{1i\,\exp}\sigma_{1ical} - \sum\sigma_{1i\,\exp}\sigma_{1ical}/n)^2}{[\sum\sigma_{1i\,\exp}^2 - (\sum\sigma_{1i\,\exp}^2)^2/n][\sum\sigma_{1ical}^2 - (\sum\sigma_{1ical}^2)^2/n]} \tag{32}$$

$$\psi^2 = \frac{\Sigma(\sigma_{1i\,\exp} - \sigma_{1ical})^2}{\Sigma\sigma_{1i\,\exp} - \bar{\sigma}_{1i\,\exp})^2} \tag{33}$$

In the above relationships, σ_{1iexp} and σ_{3iexp} represent the maximum and minimum observed stresses, respectively, while σ_{1ical} denotes the predicted ultimate stress and n is the number of observations.

7. Comparison between proposed criterion and conventional triaxial criteria

In this section, 50 datasets (results of uniaxial and triaxial tests on shale samples) given in table 1 were used to the undertake regression analysis to obtain constants and statistical indices for model evaluation. Linear regression was used in the statistical analysis phase. Nonlinear indices were also converted into linear models by variable conversion. Eqs. (19) and (20) were utilized to calculate the constants and coefficients.

In the present research work, SPSS 23, Sigma Plot 12.3, MATLAB 2016, and Excel 2013 were utilized to undertake statistical analyses, determine the coefficients and indices, and plot the results.

Figure 7 demonstrates the fitness of different criteria onto the sum of data, while Table 7 presents the coefficients and constants corresponding to different criteria. It can be observed that the proposed criterion demonstrates the best linear fitness value (0.98). Based upon the coefficients obtained and the presented equations in Section 3, the uniaxial compressive strength was calculated for each criterion and then compared against the experimental values. Since more than one uniaxial compressive test results existed within the database, their average value was taken as the experimental uniaxial compressive strength. Table 8 demonstrates the corresponding predicted uniaxial compressive strength and the error associated with each criterion. In this table, it can be observed that, compared to the other criteria, the proposed criterion provides a more realistic prediction of uniaxial compressive strength. This is while other criteria, particularly the Hoek-Brown and Mohr-Coulomb criteria, overestimate the uniaxial compressive strength.

Figure 8 shows the relationship between the ultimate strength predicted by different failure criteria as compared against the ultimate strength obtained from experimentations. Different indices have been proposed to quantitatively compare different criteria. Since each of these indices come with certain advantages and disadvantages, several indices were used to evaluate the failure criteria in the present research work, as explained in the previous section. The results obtained by calculating the values of indices for each criterion are summarized in Table 9. In this table, it is observed that, considering the measured indices, the proposed criterion can predict the ultimate strength at a superior accuracy.

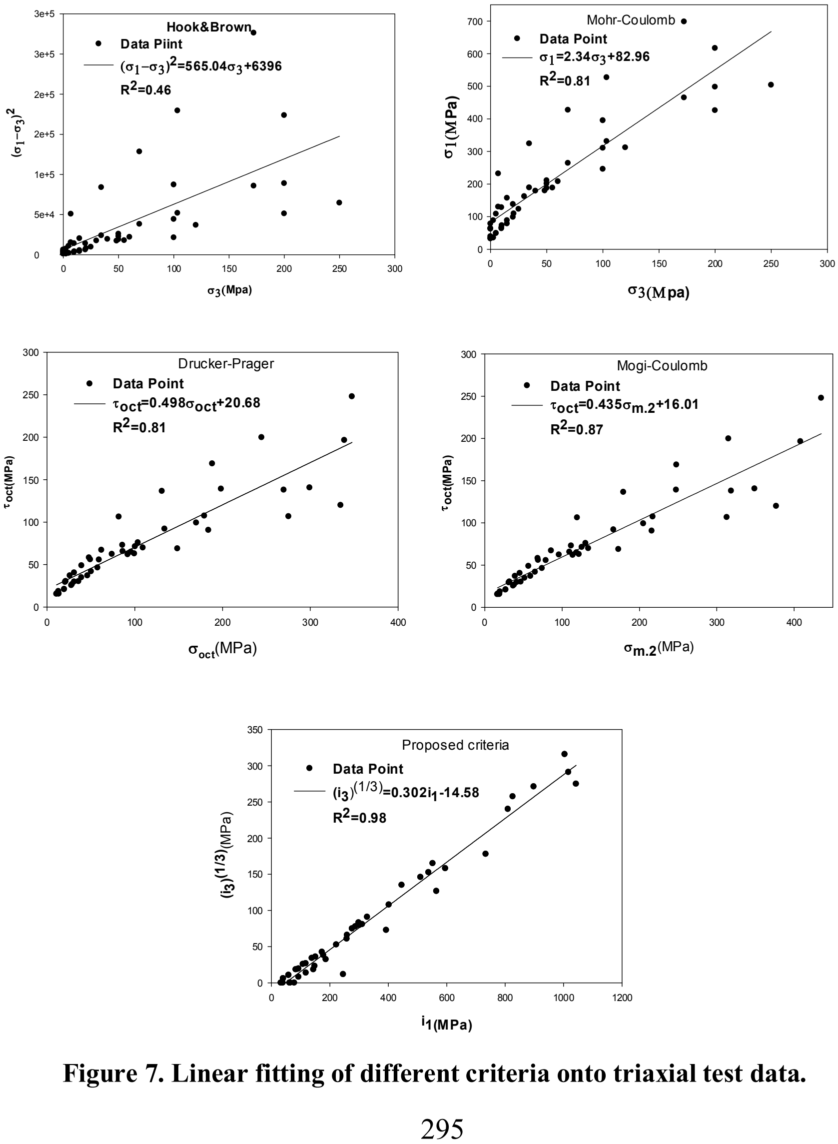

Figure 7. Linear fitting of different criteria onto triaxial test data.

Table 7. Parameters obtained from fitting different failure criteria to available data.

Criteria	R^2	Equation	Parameters	
Hoek&Brown	0.46	$(\sigma_1-\sigma_3)^2 = m\sigma_c\sigma_3 + s\sigma_c^2$	m=7.06	$\sigma_c = 79.97$
Mohr-Coulomb	0.93	$\sigma_1 = \sigma_c + N\,\sigma_3$	N=2.34	$\sigma_c = 82.96$
Drucker-Prager	0.81	$\tau_{oct} = k + m\,\sigma_{oct}$	K=20.68	m=0.498
Mogi-Coulomb	0.87	$\tau_{oct} = a + b\sigma_{m.2}$	a=16.01	b=0.435
Proposed criteria	0.98	$I_3^{\frac{1}{3}} = a + b(I_1 - m\sigma_3)$	a=-14.58	b=0.302

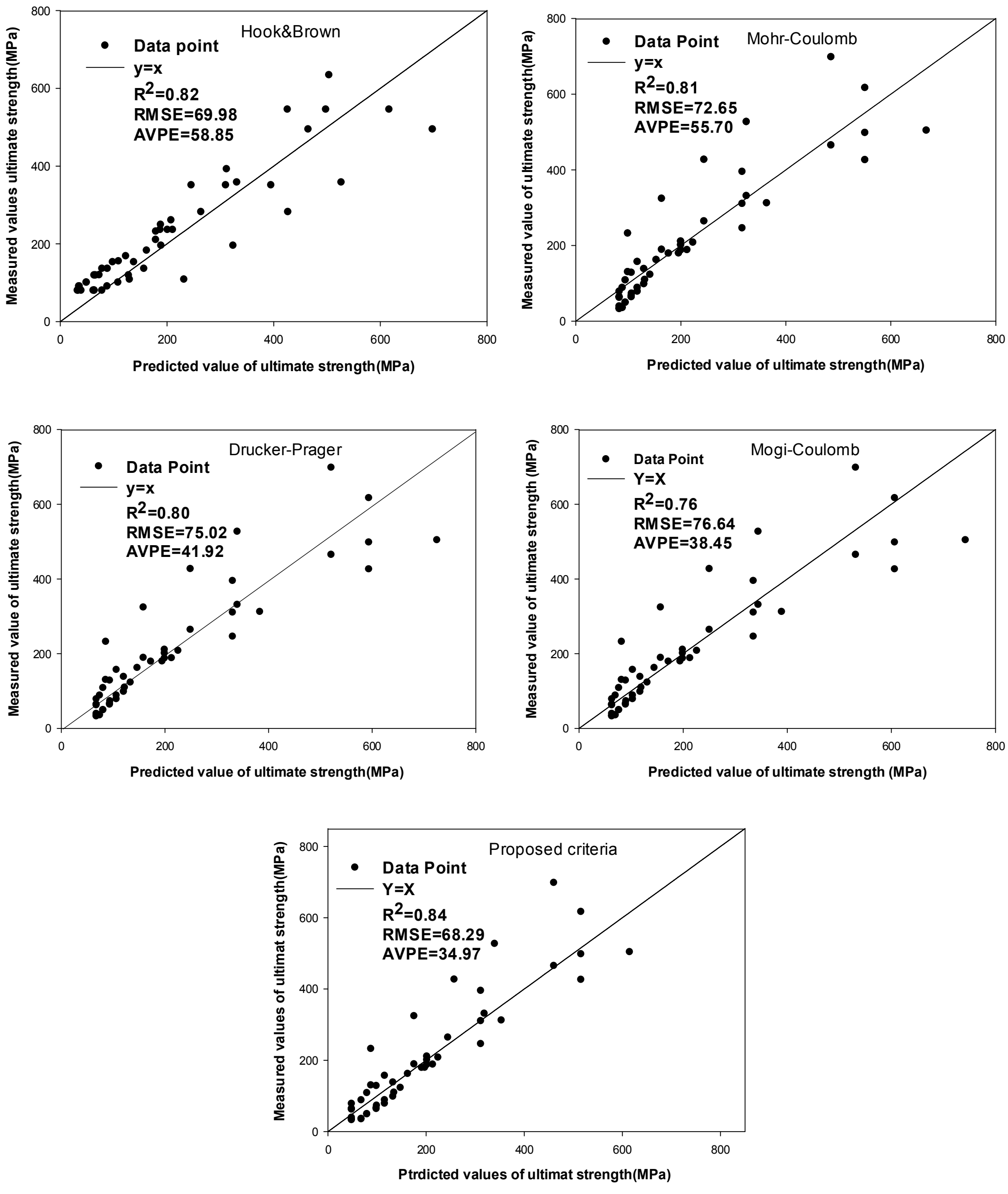

Figure 8. Relationship between, predicted and measured values for ultimate strength for all failure criteria.

Table 8. Predicted and measured uniaxial compressive strengths considering different failure criteria, in MPa.

Type	Experimental	Hoek&Brown		Mohr-Coulomb		Drucker-Prager		Mogi-Coulomb		Proposed criterion	
		Predict	Error	Predict	Error	Predict	Error	Predict	Error	Predict	Error
UCS	50.80	77.38	26.58	80.23	29.43	76.87	26.06	75.65	24.76	49.32	1.57

Table 9. Statistical indices associated with predicting entire database on the basis of different failure criteria.

Criteria	R^2	RMSE	MSE	AVPE	Pe	ψ^2	MAE
Hoek&Brown	0.82	69.98	4897.20	58.85	42.89	0.172	53.02
Mohr-Coulomb	0.81	72.65	5278.02	55.57	37	0.187	48.89
Drucker-Prager	0.80	75.02	5628	41.92	30.44	0.20	48.75
Mogi-Coulomb	0.76	76.64	5873.68	38.45	28.65	0.209	48.91
Proposed criterion	0.84	68.29	4663.57	34.97	27	0.158	43.14

8. Further testing of proposed failure criterion

Since one of the most important objectives followed in the present research work was to use the proposed failure criterion for an oil/gas well-stability analysis, this section is dedicated to the investigation of the accuracy of the proposed model in predicting the ultimate strength at ductile failure and also the performance of the proposed model in terms of strength prediction in deep shale samples.

8.1. Performance of proposed criterion at ductile failure

Deformation behavior most of rocks (shales, in particular) changes from brittle mode to ductile mode. Therefore, the failure envelope reduces near the brittle-to-ductile mode transition pressure. The newly proposed criterion uses coefficient m to predict the ultimate strength under the ductile failure mode. For this propose, the Mogi transition criterion (σ_1=4.4σ_3) was employed to distinguish the ductile data and investigate the model accuracy in predicting the ultimate strength (see Figure 9).

Figure 10 demonstrates the relationship between the predicted and measured values within the ductile zone. Moreover, Table 10 presents the calculated statistical indices for comparing various criteria within the ductile zone. Accordingly, it was observed that the proposed criterion was superior over other criteria in terms of accuracy within the ductile zone as well. In other words, the proposed criterion could be used in a wide range of stress values.

8.2. Performance of proposed criterion in predicting strength of deep shale samples

Using the data obtained from multistage triaxial tests conducted on deep shale samples taken from the southwestern oilfields in Iran, the coefficients corresponding to different criteria were determined by fitting curves to the data points. Figure 11 and Table 11 show the fitness curve and coefficients obtained for each model, respectively. The data is shown in Table 3.

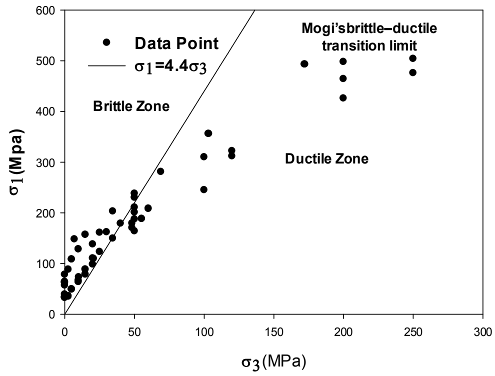

Figure 9. Separation of data points associated with brittle and ductile zones.

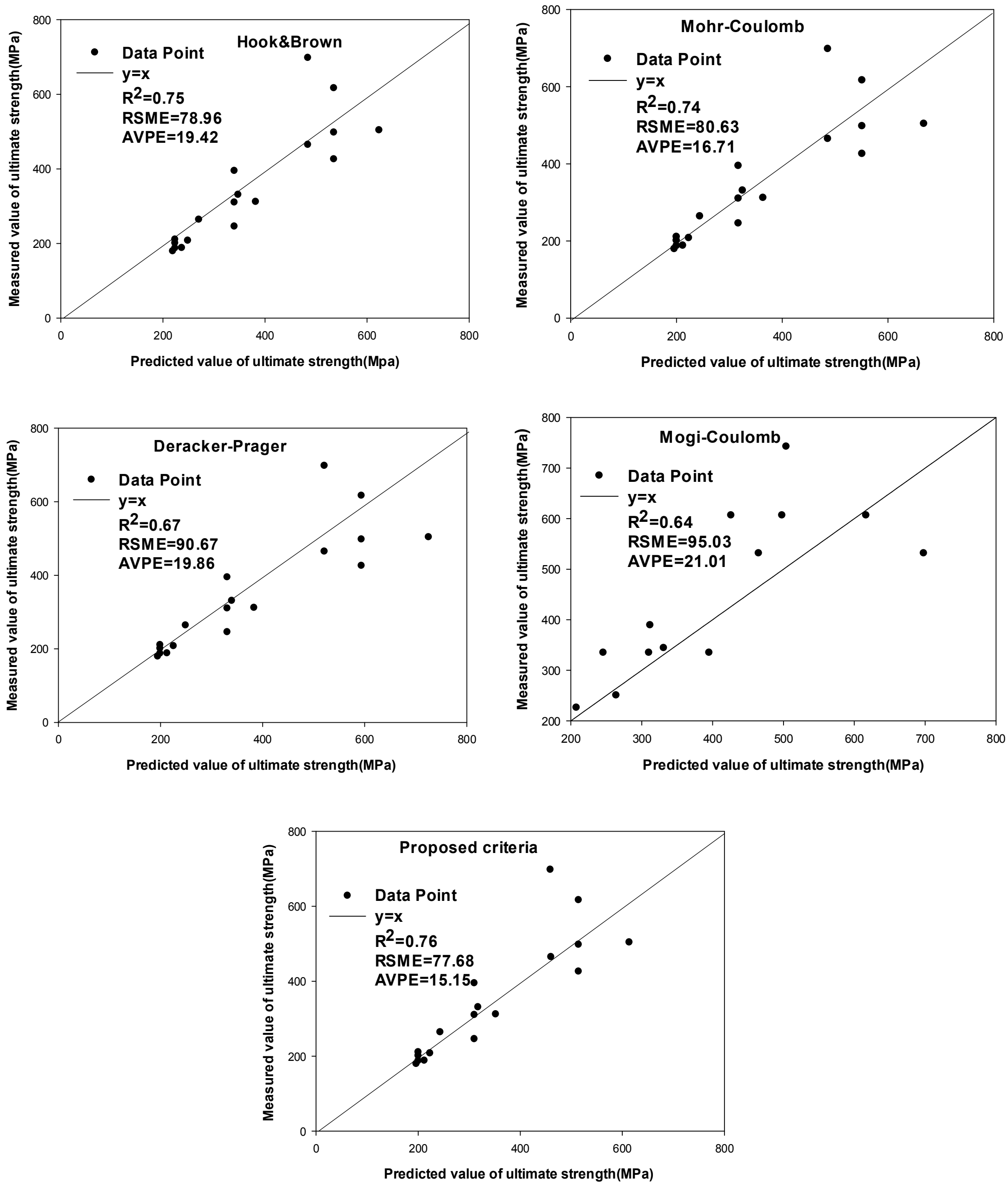

Figure 10. Relationship between measured and predicted values of ductile failure data for different failure criteria.

Table 10. Statistical indices corresponding to prediction of various criteria based on data within ductile zone.

Criteria	R^2	RMSE	MSE	AVPE	Pe	ψ^2	MAE
Hoek&Brown	0.75	78.96	6234.68	19.42	16.74	0.250	58.56
Mohr-Coulomb	0.74	80.63	6501.19	16.71	13.14	0.261	53.04
Drucker-Prager	0.67	90.67	8221.04	19.86	15.28	0.330	60.47
Mogi-Coulomb	0.64	95.03	9030.70	21.01	15.99	0.362	62.93
Proposed criterion	0.76	77.68	6034.18	15.15	11.77	0.242	48.03

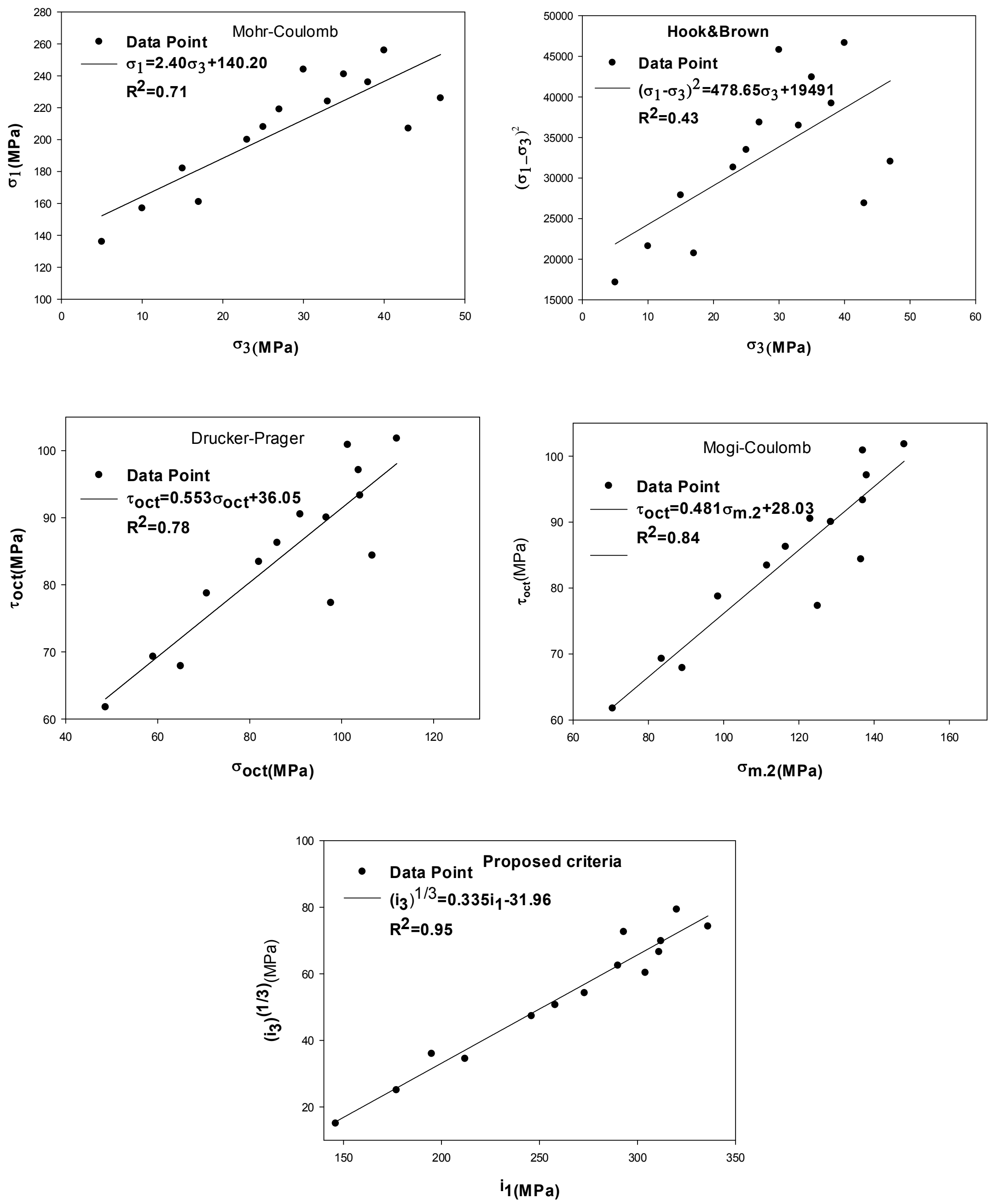

Figure 11. Linearly fitted curves to experimental data obtained from samples taken from Iranian southern oilfields for different failure criteria.

Table 11. Coefficients obtained by fitting curves corresponding to different failure criteria onto experimental data obtained from tests undertaken on samples taken from Iranian southern oilfields.

Criteria	R^2	Equation	Parameters	
Hoek&Brown	0.43	$(\sigma_1-\sigma_3)^2 = m\sigma_c\sigma_3+\sigma_c^2$	m=3.42	$\sigma_c=139.60$
Mohr-Coulomb	0.71	$\sigma_1=\sigma_c+N\sigma_3$	N=2.40	$\sigma_c=140.20$
Drucker-Prager	0.78	$\tau_{oct}=k+m\sigma_{oct}$	K=36.05	m=0.553
Mogi-Coulomb	0.84	$\tau_{oct}=a+b\sigma_{m.2}$	a=28.03	b=0.481
Proposed criterion	0.95	$I_3^{\frac{1}{3}}=a+b(I_1-m\sigma_3)$	a=-31.96	b=0.325

Figure 12 shows the relationship between the predicted values for ultimate strength by different criteria and the one obtained from experimentation. In addition, Table 12 presents the results obtained by calculating different indices for each failure criterion. According to this table, the newly proposed failure and Hoek-Brown criteria ended up with equal MSE and RMSE, but AVPE and MAE of the proposed failure criterion were lower than those of the Hoek-Brown criterion. Undertaking tensile strength tests on the samples, tensile strength was calculated based upon the determined coefficients and the presented equations in Section 3, and then the results were compared against those measured in the tests. The results are summarized in Table 13. These results indicate small errors associated with the proposed failure criterion when it comes to predicting the tensile strength, as compared to the other failure criteria (data shown in Table 5).

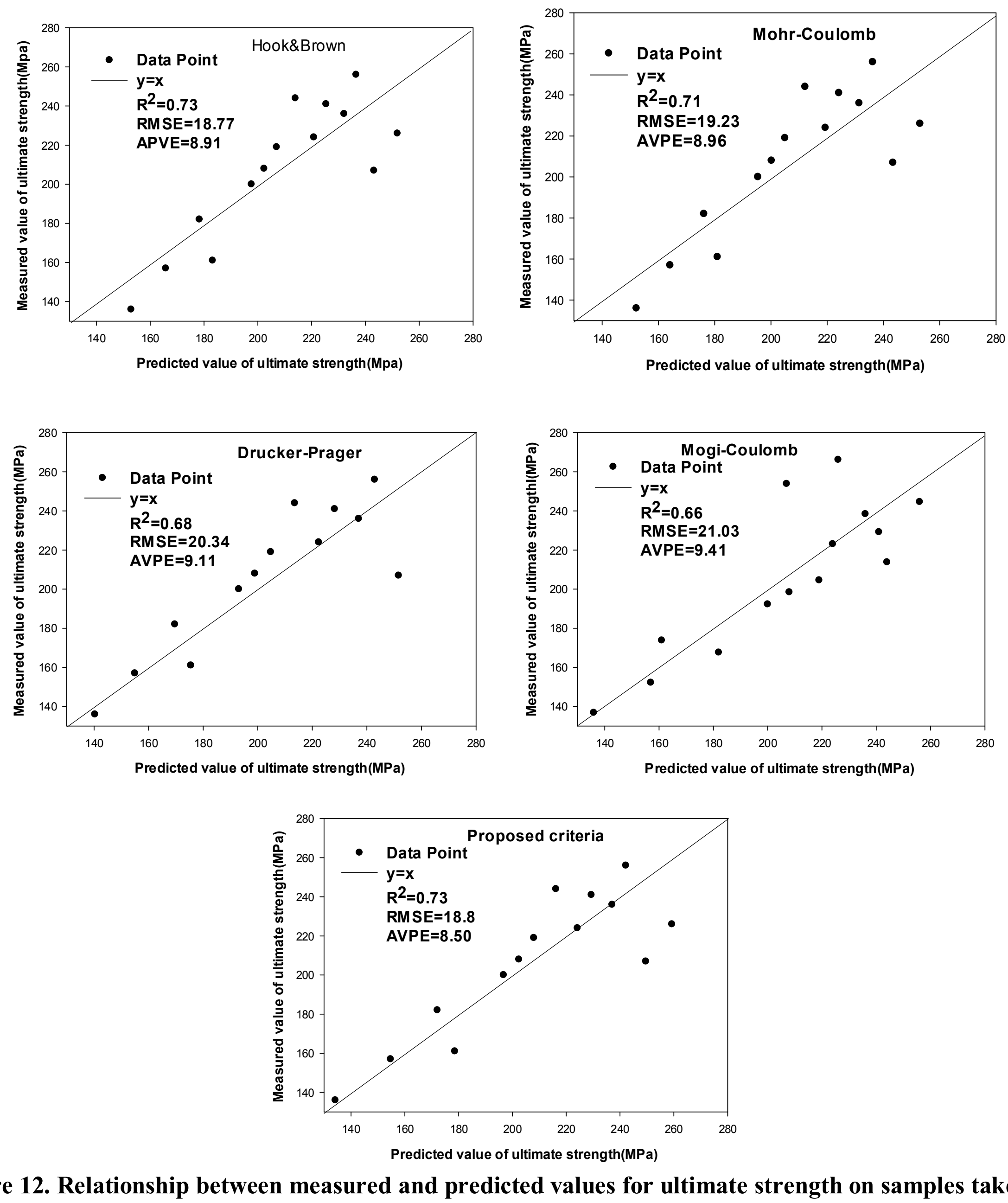

Figure 12. Relationship between measured and predicted values for ultimate strength on samples taken from Iranian southern oilfields for different failure criteria.

Table 12. Statistical indices associated with predicting different criteria based upon experimental data from samples taken from Iranian southern oilfields.

Criteria	R^2	RMSE	MSE	AVPE	Pe	ψ^2	MAE
Hoek&Brown	0.73	18.80	353.44	8.91	7.25	0.271	14.70
Mohr-Coulomb	0.71	19.23	369.80	8.96	7.55	0.285	15.47
Drucker-Prager	0.68	20.34	413.71	9.11	6.91	0.318	14.60
Mogi-Coulomb	0.66	21.03	442.26	9.41	7	0.340	14.88
Proposed criterion	0.73	18.80	353.50	8.50	6.14	0.271	12.99

Table 13. Predicted and measured tensile strength for different failure criteria, in MPa.

Type	Experimental	Hoek&Brown		Mohr-Coulomb		Drucker-Prager		Mogi-Coulomb		Proposed criterion	
		Predict	Error	Predict	Error	Predict	Error	Predict	Error	Predict	Error
BST	13.65	5.84	7.81	58.41	44.76	42.90	29.25	39.37	25.72	12.29	1.36

9. Analysis of polyaxial data

In this section, the proposed failure criterion was compared with the Drucker-Prager and Mogi-Coulomb polyaxial failure criteria using the experimental data obtained from the polyaxial tests on shale samples and compiled from reliable references (Table 6). Figures 13 and 14 and Tables 14 and 15, shows the results of curve fitting onto the data and determination of the coefficients corresponding to different criteria.

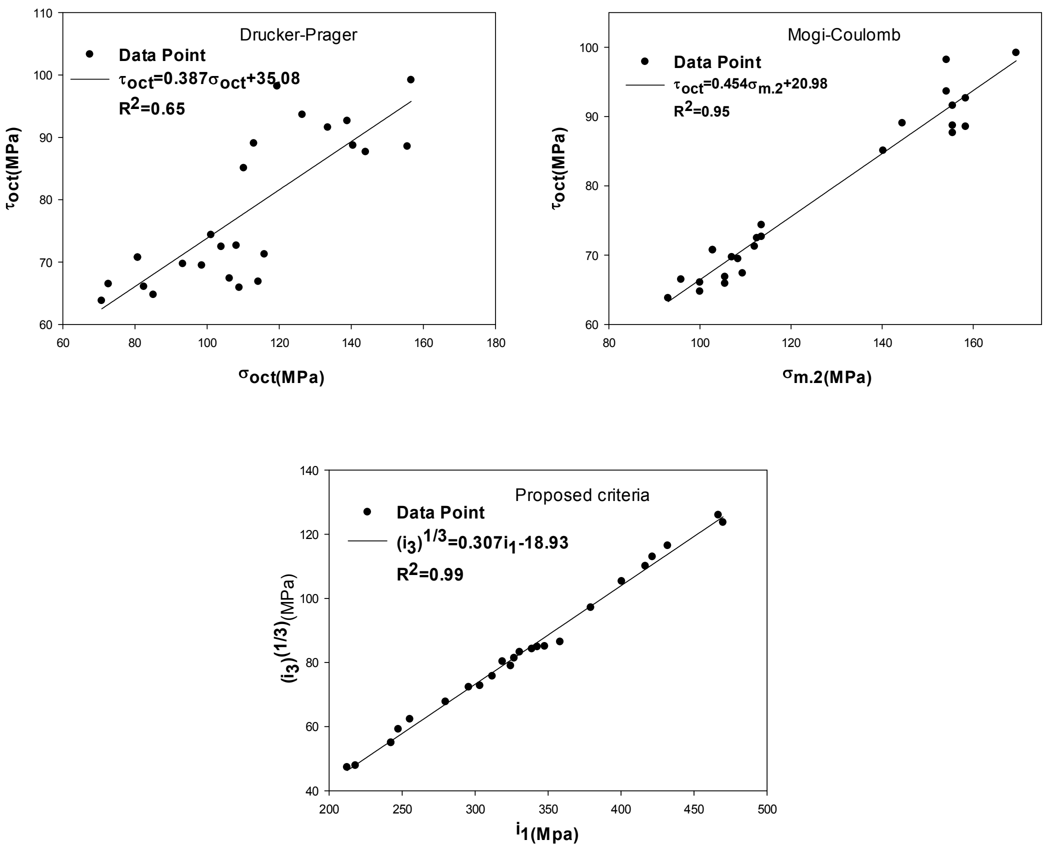

Figure 13. Linearly fitted curve to data obtained from polyaxial compressive strength tests for different failure criteria.

Table 14. Coefficients obtained from curve fitting onto data obtained from polyaxial compressive strength tests for different failure criteria.

Criteria	R^2	Equation	Parameters	
Drucker-Prager	0.65	$\tau_{oct} = k + m\sigma_{oct}$	K=35.08	m=0.387
Mogi-Coulomb	0.95	$\tau_{oct} = a + b\sigma_{m,2}$	a=20.98	b=0.454
Proposed criterion	0.99	$I_3^{\frac{1}{3}} = a + b(I_1 - m\sigma_3)$	a=-18.93	b=0.307

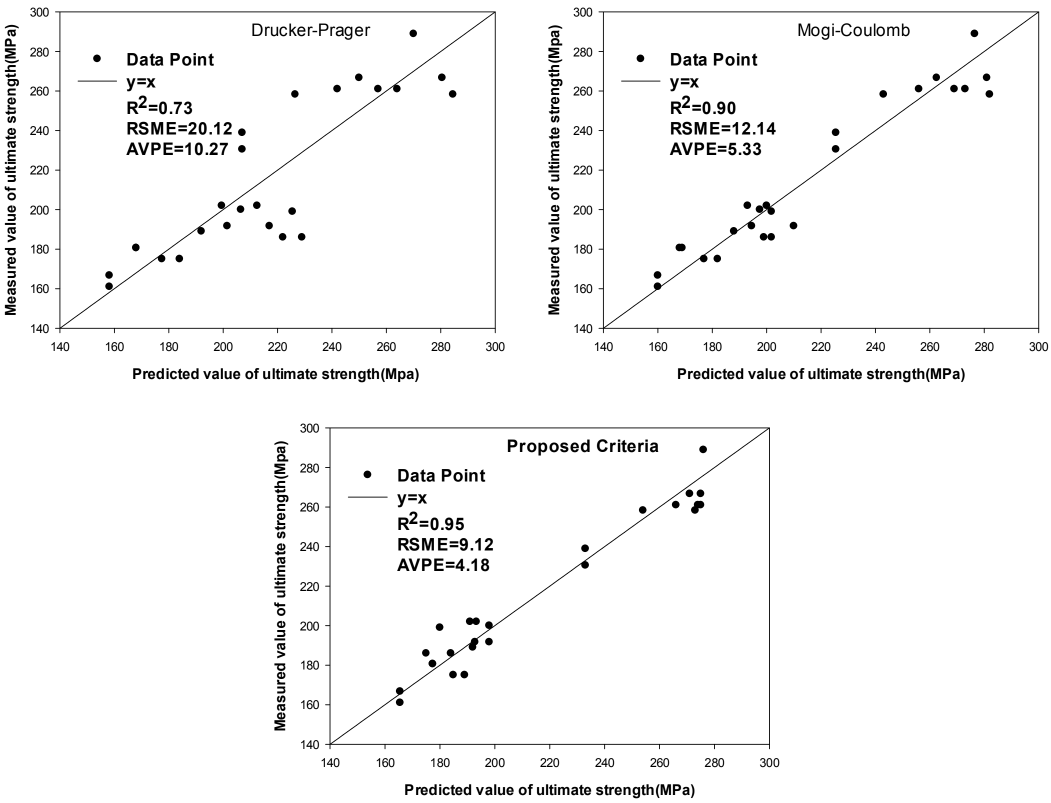

Figure 14. Relationship between predicted and measured data obtained from polyaxial compressive strength tests for different failure criteria.

Table 15. Statistical indices associated with predictions using different criteria based upon data obtained from polyaxial compressive strength tests.

Criteria	R^2	RMSE	MSE	AVPE	Pe	ψ^2	MAE
Drucker-Prager	0.70	21.77	473.93	10.27	8.10	0.309	17.18
Mogi-Coulomb	0.90	12.14	147.37	5.33	4.16	0.096	8.99
Proposed criterion	0.95	9.12	83.17	4.18	3.47	0.054	7.44

10. Conclusions

In the present research work, a new failure criterion has been proposed for intact shale rock under conventional triaxial and polyaxial stress conditions. For this purpose, a database containing the results of uniaxial, triaxial, and polyaxial tests compiled from reliable references as well as the results of triaxial tests on shale samples taken from the southwestern oilfields in Iran were used to evaluate the proposed failure criterion and compared against the most commonly used failure criteria in stability analysis of rock structures.

Curves associated with the Hoek-Brown, Mohr-Coulomb, Drucker-Prager, Mogi-Coulomb criteria and the newly proposed failure criterion was fitted to the data obtained from the triaxial tests. Coefficients corresponding to different criteria were calculated. Afterwards, the compressive and ultimate strength values were predicted and compared. The newly proposed failure criterion was found to produce the lowest RMSE and the best statistical indices.

In order to test the performance of the newly proposed criterion within the ductile zone, the Mogi transition criterion was used to separate the data points within the ductile zone, with the coefficients and statistical indices determined for different failure criteria. The results obtained were indicative of the superior accuracy of the proposed failure criterion in predicting the ultimate strength within a wide range of stresses.

In order to further test the proposed criterion, deep samples taken from the southwestern Iranian oilfields were subjected to multi-stage triaxial tests, and different criteria were fitted to the data obtained, with the corresponding statistical coefficients and indices calculated. The results obtained show the higher accuracy of the proposed failure criterion in predicting the ultimate strength. In addition, the tensile test results on samples were compared with the predicted uniaxial tensile strength values obtained on the basis of different criteria, indicating that the proposed criterion could predict the tensile strength at a minimum error.

In order to test the performance of the proposed failure criterion under the polyaxial stress conditions using curve fitting onto the polyaxial test data on shale samples, the statistical coefficients and indices were determined and the indices corresponding to the proposed failure criterion are compared against the Drucker-Prager and Mogi-Coulomb criteria. The results obtained indicate that the proposed criterion exhibits the best agreement with the experimental data.

In general, it was observed that the proposed failure criterion was well in agreement with the uniaxial, triaxial, and polyaxial compressive/tensile strength tests, so that it could be used within a wide range of stress values.

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معیار شکست تجربی جدید برای شیل

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چکیده:

در این پژوهش یک معیار شکست تجربی جدید برای پیش‌بینی مقاومت نهایی شیل تحت حالت‌های مختلف تنش ارائه شده است. پایگاه داده‌ها شامل ۹۳ داده از نتایج آزمایش‌های تک، سه و چند محوره جمع‌آوری شده از مقاله‌های معتبر و همچنین آزمایش‌های سه محوره و کششی انجام شده بر روی نمونه‌های تهیه شده از میدان‌های نفتی جنوب غرب ایران است. داده‌ها برای ارزیابی و مقایسه، دقت پیش‌بینی مقاومت نهایی شیل بر اساس معیار پیشنهادی با معیارهای شکست پر کاربرد در مکانیک سنگ برای تحلیل پایداری مانند هوک- برون، موهر- کلمب، دراگر- پراگر و موگی- کلمب استفاده شد. برای ارزیابی و مقایسه مدل‌ها از ۷ شاخص آماری مهم استفاده شد. منحنی هر یک از معیارها بر روی داده‌های سه و چند محوره برازش شد و ضریب‌ها و شاخص‌های متناظر هر معیار تعیین شده و با یکدیگر مقایسه شد. نتایج نشان داد در همه حالت‌ها معیار پیشنهادی مقاومت نهایی شیل را با دقت بیشتری نسبت به سایر معیارها پیش‌بینی می‌کند. همچنین معیار پیشنهادی مقاومت فشاری تک محوره و مقاومت کششی را با کمترین خطا تخمین می‌زند. برای ارزیابی بیشتر معیار پیشنهادی عملکرد مدل در پیش‌بینی مقاومت نهایی در شکست‌های خمیری با سایر معیارها با استفاده از داده‌های محدوده خمیری مقایسه شد که نتایج نشان دهنده دقت مدل در محدوده خمیری نسب به سایر معیارها بوده است.

کلمات کلیدی: معیار شکست تجربی، شیل، پایداری چاه، مقاومت نهایی، شکست خمیری، رفتار مکانیکی.