



Predicting Unconfined Compressive Strength of Intact Rock Using New Hybrid Intelligent Models

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Abstract

Bedrock unconfined compressive strength (UCS) is a key parameter in designing the geosciences and building related projects comprising both the underground and surface rock structures. Determination of rock UCS using standard laboratory tests is a complicated, expensive, and time-consuming process, which requires fresh core specimens. However, preparing fresh cores is not always possible, especially during the drilling operation in cracked, fractured, and weak rocks. Therefore, some attempts have recently been made to develop the indirect methods, i.e. intelligent predictive models for rock UCS estimation, which require no core preparation and laboratory equipment. This work focuses on the application of new combinations of intelligent techniques including adaptive neuro-fuzzy inference system (ANFIS), genetic algorithm (GA), and particle swarm optimization (PSO) in order to predict rock UCS. These models were constructed based on the collected laboratory datasets upon 93 core specimens ranging from weak to very strong rock types. The proposed hybrid model results were compared with each other, and the real data and multiple regression (MR) results. These comparisons were made using coefficient of correlation, mean of square error, mean of absolute error, and variance account for indices. The comparison results proved that the ANFIS-GA combination had a relatively higher accuracy than the ANFIS-PSO combination, and both had a higher capability than the MR model. Furthermore, the ANFIS-GA and ANFIS-PSO model results were completely in accordance with the UCS laboratory test, and they were more accurate than the previous single/hybrid intelligent models. Lastly, a parametric study of the suggested models showed that the density and Schmidt hammer rebound had the highest influence, and porosity had the lowest influence on the output (UCS).

1. Introduction

Unconfined compressive strength (UCS) is the main utilized parameter for designing the mining, civil, and geotechnical structures as well as the infrastructure projects. Indeed, the stability analysis of these structures is conducted based on both the intact rock and rock mass geomechanical properties (i.e. UCS), which is crucial to provide the long-term stability for efficiency maintenance. Determining the rock geomechanical properties is also required for floor, roof, pillar, and slope analysis in mining and tunneling projects and also for dam, building,

and road construction in civil operations [1]. Nonetheless, the rock UCS is the most important parameter due to the fact that other rock mechanical characteristics can be determined according to this parameter. Bieniawski [2] has stated that UCS determination is commonly more required for mining engineers than the other rock properties. Therefore, it can be concluded that UCS is the most important rock geomechanical properties whose high significance has been outlined by many investigators.

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Commonly, there are two approaches, i.e. direct (laboratory tests) and indirect (correlated index tests) methods for UCS determination. Standard direct methods have been suggested for UCS determination by ISRM [3] and ASTM [4]. In direct methods, laboratory test is conducted, in which the strain of rock core specimen is determined at the same time as the axial compressive force is incremented. Accordingly, the pressure at which the core is broken is considered as the maximum UCS of the tested rocks [5]. In the second method, index tests such as point load index, Schmidt hammer rebound, impact strength, and wave velocity index as well as the predictive models have been utilized to estimate UCS as an indirect alternative of the direct determination method. The main benefits of the indirect approach utilization are the flexibility and low-cost implementation [6]. Moreover, there are some new indirect methods available to estimate rock mass or intact rock strength including the

Equotip hardness tester [7], block punch index test [8], core strangle test [9], nail penetration test [10], and edge load strength test [11] that require a large number of rock samples with a precise size. Indeed, specific core specimens, i.e. cubical or cylindrical cores, should be prepared with high precision to implement the above-mentioned standard tests. Moreover, some of these indirect methods/relations are concluded from the statistical models to predict UCS, as shown in Table 1. In these statistical relations, only the mean UCS values are estimated, and thus low and high values are often overestimated and underestimated, respectively. On the other hand, the restrictions and strictness of direct methods for UCS determination make them expensive, tedious, and time-consuming. In fact, preparing the standard core specimens is often difficult, particularly during the coring operation in highly broken and weak rocks [12].

Table 1. Some available indirect statistical/empirical relations to estimate UCS.

Equation	Rock type	Reference
$UCS = 0.25EH + 28.14\rho - 0.75n - 15.47GS - 21.55RT$	Granite, limestone, sandstone, granodiorite, and dolomite	[6]
$UCS = 183 - 16.55n$	Granite	[13]
$UCS = 74.4 \exp(-0.04n)$	Sandstone	[14]
$UCS = 0.386EH + 39.268\rho - 1.307n - 246.804$	Granite, limestone, sandstone, granodiorite, and dolomite	[15]
$UCS = 10.1 \exp(-0.821n)$	Shale, claystone, and siltstone	[16]
$UCS = \exp(0.818 + 0.059SH)$	Gypsum	[17]
$UCS = 7.3PLS^{1.71}$	Limestone, sandstone, and marl	[18]
$UCS = 0.0065V_p + 1.468PBI + 4.094PLS + 2.418TS - 225$	Weak, fractured, and thin-bedded rocks	[19]
$UCS = 31.5V_p - 63.7$	Limestone, dolomite, and marble	[20]
$UCS = 1.4459 \exp(0.0706SH)$	Granitic rocks	[21]
$UCS = 0.89SH + 13.1PLS - 1.68V_p - 35.9$	Limestone, dacite, and marble	[22]
$UCS = 0.88\rho^{5.72}SH^{0.22}CI^{0.89}$	Igneous and sedimentary rocks	[23]
$UCS = 0.48SH + 1.863PLS + 0.248WC + 7.972V_p - 23.859$	Gypsum	[24]
$UCS = 0.0028^{2.584SH}$	Travertine, limestone, dolomitic limestone, and schist	[25]
$UCS = 165.05 \exp[-4.452 / V_p]$	Limestone, marble, and sandstone	[26]
$UCS = 29.63SD - 2858$	Travertine, limestone, and dolomitic limestone	[27]
$UCS = 5.734V_p + 10.876TS - 2.408PLS - 10.029$	Sedimentary rocks	[28]
$UCS = 0.458 \exp(1.504V_p)$	Claystone and mudstone	[29]
$UCS = \exp(0.011BPI + 0.065PLS + 0.029SH + 0.000012V_p + 2.157)$	Granite, schist, and sandstone	[30]
$UCS = 12.5PLS$	Pyroclastic rocks	[31]
$UCS = 24.301 + 4.874TS$	Basalt and limestone	[32]
$UCS = -2.56n + 1.384PLS - 127.411v + 18.251\rho - 0.0162V_p - 43.214$	Carbonate rocks	[33]
$UCS = 0.047 \exp(0.065SD)$	Pyroclastic rocks	[34]

UCS Uniaxial compressive strength, n porosity, ρ density, SH Schmidt hardness, PLS point load strength, Vp Primary wave velocity, BPI block punch index, CI cone indenter hardness, WC water content, SD slake durability index, v Poisson's ratio, TS Tensile strength, GS Grain size, RT Rock type, EH Equotip hardness.

In fact, to overcome the above-mentioned difficulties in UCS determination and demand to acquire rock strength properties in cheaper, feasible, and quicker procedure with accurate results lead to develop some intelligent predictive models, i.e. neural network (NN) tool, fuzzy systems (FSs), genetic algorithm (GA), etc. A wide range of applications of these techniques has been reported in the literatures to model and control different problems. Particularly, an increase in the application of these techniques has been reported in the fields of mining and geosciences [35-46]. Regarding the UCS prediction, Madhubabu *et al.* [33] have estimated the carbonate rocks UCS and elastic modulus by means of the ANN technique. Gokceoglu [47] has suggested a triangular fuzzy technique for agglomerate rock UCS estimation considering petrographic properties. Majdi and Rezaei [48] have developed an artificial neural network (ANN) model with a high accuracy to predict UCS of rocks surrounding a roadway. Rezaei *et al.* [49] have proposed a valid Mamdani fuzzy inference system (FIS) for the UCS estimation of the surrounding rocks of longwall access tunnels. Jahed Armaghani *et al.* [50] have suggested an ANFIS model to determine granite rocks UCS and young's modulus. Moreover, Ghasemi *et al.* [51] have proposed a tree-based model for determining the UCS and young's modulus of carbonate rocks. All the above surveyed references proved the capability of the artificial intelligent techniques in UCS estimating. However, these literature surveyings show that few studies have been conducted on the application of hybrid intelligent techniques for UCS estimation, which is the main objective of the current paper.

In this paper, combinations of the intelligent algorithms including the ANFIS, GA, and PSO techniques were utilized for UCS estimation of dissimilar rock types classified as weak to very strong rocks. The two new hybrid algorithms ANFIS-GA and ANFIS-PSO were developed for UCS (output) prediction based on the three easily determinable input variables including Schmidt hammer rebound, density, and porosity. These proposed models were constructed and verified using 93 experimental data determined in the laboratory. Finally, their results were compared with the results of the conventional multiple regression (MR) and available previous intelligent models reported in the current publications.

2. Applied techniques

2.1. ANFIS

The details of the ANFIS algorithm are well-described in the literature [52], and thus it is briefly outlined here. Originally, FIS is able to simulate the inference procedure and linguistic features of the human understanding without application of an accurate quantitative investigation. On the other hand, ANNs are a combination of many interdependent processing components that are comparable to neurons. In conventional ANN, just weight quantity alters throughout the learning phase, whereas in a neuro-fuzzy decision-making system, the learning capability of ANN is coupled with the reasoning process of FIS. In the ANFIS method, the human process of decision-making is intelligently imitated by a combination of ANN and FIS. In this approach, the learning capability of ANN is coupled with the reasoning process of FIS. Basically, ANFIS utilizes a FIS and adjusts it using a back-propagation algorithm and employing a group of input-output information. A combination of ANN and FIS enables FIS to learn. Neural network algorithms in combination with FIS can be applied to calculate the unknown factors, and this decreases the error values, as traditionally described for every parameter of the model, and this optimization process makes the model adaptive [52]. The structure of an adaptive neural network includes several nodes joined via oriented links. A node function with unchangeable or adaptable parameters defines each node. Neural network algorithms, when FIS is loaded, can be applied to calculate the unknown factors, and this decreases the error values, as traditionally described for every parameter of the model, and this optimization process makes the model adaptive. The adaptive neural network and its operationally identical to FIS are presented in Figures. 1a and 1b, respectively. Overall, an ANFIS with two input parameters including x and y and one output z are considered, and its related fuzzy 'if-then' rules based on the Takagi and Sugeno FIS type is presented in the above Figure [52].

2.2. PSO

PSO is a heuristic algorithm-based approach, which was introduced by Kennedy and Eberhart. This technique solves a problem by having a population of possible answers. In this approach, each particle is nominated with a pace that is different from the other soft computing techniques. Particles move around in the search domain with paces that are dynamically regulated based on their previous characteristic. Accordingly, the particles during the

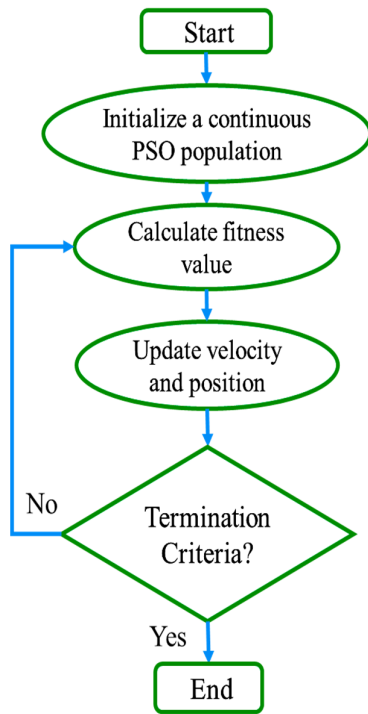


Figure 2. Flow diagram of the PSO algorithm [53].

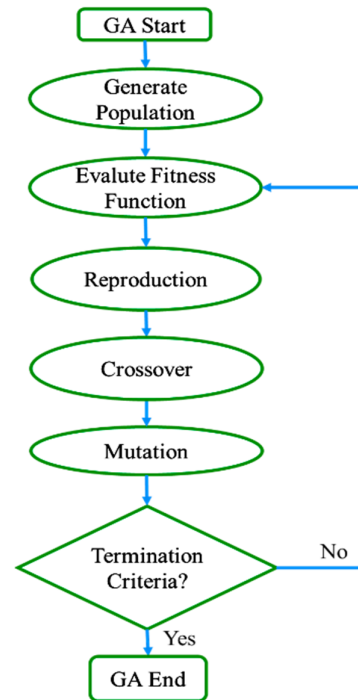


Figure 3. A simple flow sheet of GA modeling processes [54].

2.4. Multiple regression analysis

Multiple regression (MR) analysis is a conventional statistical model available to forecast the value of one or more dependent parameters from a single variable or a group of independent variables [55]. Multiple linear regression models were constructed in the current research work for output prediction. Generally, a typical multiple regression formulation is presented in the following format:

$$y = \beta_0 + \beta_1 x_1 + \dots + \beta_k x_k + e \quad (1)$$

where y is the output parameter whose amount is related to the input parameters (x_1, x_2, \dots, x_k) that is selected by the examiner, $1, \dots, k$ are the parameters of regression, and β_0, \dots, β_k are the constant coefficients.

3. Data collection

Generally, considerable datasets are required for construction/development and evaluation of the artificial intelligence modelling. In fact, collection of data is the main step in developing the intelligent models. For this purpose, about 93 core specimens were prepared upon the different rock blocks classified from weak to very strong types. Core specimens were prepared as a right cylinder shape with 54 mm diameter (NX-size) and 2.5:1 ratio of height-to-diameter according to the suggested

standard method by ISRM (1981). In the next step, UCS, density, porosity, and Schmidt hammer rebound of the rock core specimens were measured in the laboratory. UCS was determined using an automated uniaxial compression testing machine (Figure. 4) according to the proposed standards by ASTM [4]. Also the porosity parameter was determined using the saturation-caliper approach in the lab according to the suggested method by ISRM [56]. Moreover, the density of core specimens was determined by the core weight division to its volume. Finally, a common Schmidt hammer device was applied for the Schmidt hammer rebound determination of the samples in the laboratory. It should be noted that the density and Schmidt hammer rebound tests have been performed according to the procedure proposed by ISRM [56]. During the several recent years, the above-mentioned tests have been performed by the authors in their personal laboratories, and the measured data have been recorded as exhaustive datasets to develop the proposed models in this work. As mentioned earlier, various rock types including weak, medium, strong, and very strong rocks were tested for data preparation. The tested rock types include Gabbro, Amphibolite, Granodiorite, Norite, Quartz diorite, Serpentine, Groana, Dunitite, Peridotite, Olivine, Pyroxenite, Dolerite, Basalt,

Quartzite, Diopside, Diabase, Granite, Eclogite, Syenite, Anorthosite, Slate, Dolomite, Sandstone, Pitchstone, Siltstone, Shale, Limestone, Marl, Gneiss, Gypsum, Anhydride, Chalk, Tuff, Conglomerate, Rhyolite, Schist, and Marble.

As a common process in the intelligent modelling, the prepared datasets were divided into two types including the construction/training (70% of datasets) and evaluation/testing (30% of datasets) data. The construction/training datasets were applied for developing the hybrid intelligent models and MR method, while the evaluation/testing datasets were considered for testing the models capability and their verifications. The statistical characteristics of the prepared datasets and variable symbols are given in Table 2. Moreover, ten samples of the prepared datasets are shown in Table 3.

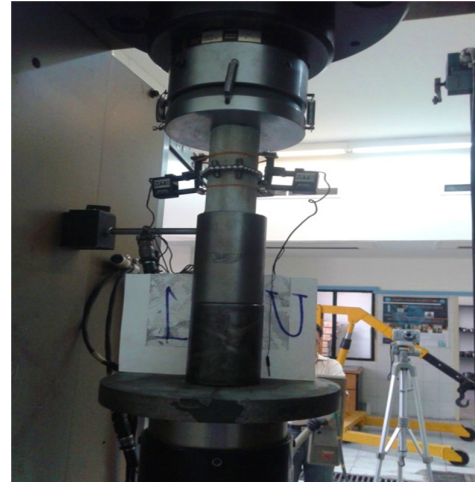


Figure 4. The applied device for UCS determination.

Table 2. Statistical characteristics of prepared datasets used for modeling.

Parameter	Symbol	Min.	Max.	Variance	Std. dev.
Porosity (%)	n	0.1	41.17	60.88	7.8
Density (g/cm ³)	ρ	1.65	3.8	0.19	0.43
Schmidt hammer rebound (-)	R	25.25	71.33	73.65	8.58
Unconfined compressive strength (MPa)	UCS	23	361.37	7072.73	84.09

Table 3. Ten samples of prepared datasets for the current modeling.

n (%)	ρ (g/cm ³)	R (-)	UCS (MPa)
3.56	2.6	61	184
7.7	2.56	54	127
3	3.4	55	260
11.03	1.71	44	23
2.17	2.67	50.5	110
3.7	3.24	55.16	230
9.6	2.33	51	90
0.22	2.77	64.33	282
14.32	2.2	48.33	70
6	3.8	41.66	150

4. UCS modelling

In this paper, two new hybrid intelligent algorithms including the ANFIS-GA and ANFIS-PSO models along with the MR statistical model were developed and proposed to estimate the UCS of rock core specimens. For this purpose, considering a suitable dividing approach is required to divide the prepared input data into the construction/training and evaluation/testing datasets to start the modelling process. As mentioned earlier, 93 data series were provisioned to construct the above three models in this work. In order to divide the prepared datasets, a random approach was utilized. Accordingly, 70% of them was selected for model development (construction/training data) and the remaining 30%

was considered to test the developed models (evaluation/testing data). For modelling the soft computing methods, normalization of such datasets was carried out to the domain of [0, 1] to have an effective training phase. In the present work, a normalization process was conducted by Equation. (2). This helped the better training and developing of the applied models in order to acquire the accurate results.

$$X_{normalized} = \frac{X - X_{min}}{X_{max} - X_{min}} \quad (2)$$

where X is an input variable, X_{min} is the minimum value of a variable, and X_{max} is the maximum value of it. Development of these suggested models will be outlined in the following sub-sections in detail.

4.1. ANFIS-PSO/GA modeling

In the ANFIS-PSO/GA modelling, UCS is basically predicted by ANFIS. However, to train ANFIS, the learning methodology was separately utilized on the basis of the PSO and GA algorithms in order to enhance its prediction capability and accuracy for obtaining the optimum results. The mentioned datasets in Section 3 were utilized for the ANFIS model training using the PSO and GA algorithms separately. The suggested ANFIS structure includes three input variables (n, ρ, R) along with one output

variable (UCS). The PSO/GA-based ANFIS approaches were performed using a MATLAB-based program. Using this program, a specific modelling of UCS was carried out based on the porosity, density, and Schmidt hammer rebound variables. At the first step, the prepared laboratory data was converted to normal values between 0 and 1. After that, the methodology of PSO/GA, which was explicated in the prior section, was functioned to discover the best values/types of ANFIS predictive model parameters. For running these ANFIS-PSO/GA algorithms, 100 generations under the 50 population size were employed. When the last generation was terminated, the best values/types of the ANFIS predictive model parameters to estimate the UCS value were obtained, as provided in Table 4.

4.2 MR modelling

Multivariable regression (MR) is a branch of the statistical regression analysis in which the output variable(s) can be estimated in the form of a predictive equation on the basis of the input (independent) variables. Indeed, determining the inherent relations between the output (dependent) variable(s) and the input (independent) variable(s) is possible by this method. The MR approach was broadly utilized in the geosciences fields, especially in mining, rock engineering, and rock mechanics by

many researchers [57]. In this section, statistical correlation between the output (UCS) and considered input characteristics (n, ρ, R) is surveyed based on the MR analysis. The SPSS23 software package was utilized for multivariate relation generation between the above-mentioned input-output variables based on the same datasets as applied for intelligent modelling (construction data). For MR analysis of UCS, all of the possible combinations of input parameters along with the independently status of single inputs were tested in order to predict UCS. The MR models summary including the correlation coefficient (R), determination coefficient (R²), adjusted value of R², and estimations standard error are demonstrated in Table 5. According to this Table, the MR model, considering all of the three input parameters (model 7), have high correlation values (R, R², and adjusted R²) and the minimum value of estimation standard error. Therefore, it is considered as the best MR model to predict UCS based on the current input variables. Accordingly, the main results of this optimum statistical analysis and its obtained coefficients are given in Table 6. Finally, the acquired relation from this optimum MR analysis is given in Equation. (3).

$$UCS = -350.784 - 1.825n + 82.749\rho + 5.708R \quad (3)$$

Table 4. The best achieved values/types for the applied models.

ANFIS values/types	Description/value
Fuzzy inference engine	Sugeno-type
Input membership function type	Gaussian (“gaussmf”)
Output membership function type	Linear
Cluster influence center	0.7
Number of inputs	3
Number of Outputs	1
Approach of optimization	PSO/GA
Number of iterations	1000
Training data number	65
Testing data number	28
Size of the initial step	0.3
Decrease rate step size	0.9
Increase rate step size	1.10
Fuzzy rules number	6
GA values/types	Description/value
Size of population	50
Rate of mutation	0.05
Cross-over	0.7
PSO values/types	Description/value
Size of population	50
W	0.5
C1	2
C2	2

Table 5. MR models summary for UCS prediction.

Model output	Model No.	Input(s)	R value	R ² value	Adjusted value of R square	Estimations Std. error
UCS	1	n	0.585	0.342	0.335	68.58464
	2	ρ	0.655	0.429	0.422	63.91807
	3	R	0.794	0.630	0.626	51.42595
	4	n, ρ	0.771	0.594	0.585	54.14955
	5	n, R	0.825	0.680	0.673	48.0898
	6	ρ, R	0.907	0.822	0.818	35.83491
	7	n, ρ, R	0.918	0.843	0.838	33.83951

Table 6. The main results of optimum MR statistical analysis for UCS prediction.

Predictor	Coefficient	Standardized coefficients (Beta)	t-value	Sig.
Constant	-350.784	-	-10.532	0
n	-1.825	-0.169	-3.454	0.001
ρ	82.794	0.431	9.631	0
R	5.708	0.583	11.893	0

5. Results and Discussion

5.1. Verification using laboratory data

In order to verify the proposed ANFIS-GA, ANFIS-PSO, and MR models, the results achieved for them were compared with the real data attained from the laboratory experiments. The evaluation/testing data (19 data series), which was not utilized in the model construction, was applied for this verification. Indeed, the prediction performance and ability of the proposed models were controlled based on the evaluation/testing datasets. For this aim, four statistical indices (SIs), i.e. the coefficient of correlation (R), mean of square error (MSE), mean of absolute error (MAE), and variance account for (VAF) were computed for all the developed models. The R index stands for the correlation of the model results with the real datasets. On the other hand, the MSE and MAE indices demonstrate the model errors compared to the real values. Finally, different amounts of the real dataset variance with the model output(s) were calculated using the VAF index. In general, the higher values for R and VAF (nearer to 100%) and the lower amounts of MSE and MAE (near to zero) proved the better performance and capability of a model. The following equations were utilized to calculate the above-mentioned indices:

$$R = 100 \left[\frac{\sum_{i=1}^n (A_{ipred} - \bar{A}_{pred})(A_{imeas} - \bar{A}_{meas})}{\sqrt{\sum_{i=1}^n (A_{ipred} - \bar{A}_{pred})^2 \sum_{i=1}^n (A_{imeas} - \bar{A}_{meas})^2}} \right] \quad (4)$$

$$MSE = \frac{1}{n} \sum_{i=1}^n (A_{imeas} - A_{ipred})^2 \quad (5)$$

$$MAE = \frac{\sum_{i=1}^n | (A_{imeas} - A_{ipred}) |}{n} \quad (6)$$

$$VAF = 100 \left(1 - \frac{\text{var}(A_{imes} - A_{ipred})}{\text{var}(A_{imeas})} \right) \quad (7)$$

where n is the datasets number, \bar{A}_{imeas} is the average of actual datasets, \bar{A}_{ipred} is the average of forecasted datasets, and A_{imeas} and A_{ipred} are the ith actual and forecasted elements, correspondingly. Based on the 19 numbers of the evaluation/testing dataset, the prior mentioned statistical indices were calculated for all the suggested models (Table 7). As revealed in the this table, the anticipation capability of the proposed intelligent models (ANFIS-GA and ANFIS-PSO models) in terms of R, MSE, MAE, and VAF was much higher than the statistical model. However, the correctness of the ANFIS-GA approach was somewhat better than the ANFIS-PSO procedure. For more evaluation, correlation between the real data and the predicted ones from the ANFIS-GA, ANFIS-PSO, and MR models are illustrated in Figures. 5-7, respectively. The last comparison was also proved that the results of the suggested hybrid intelligent models were more correlated with the real data in comparison with the statistical approach, and their outputs were completely nearer to the real ones. Finally, comparing the results of the suggested models with the real evaluation datasets is depicted in Figure. 8, which verify the prediction capability of the proposed hybrid intelligent models.

Table 7. Comparing the proposed model performances using the computed statistical indices.

Index	ANFIS-GA model	ANFIS-PSO model	MR model
R	97.57	97.41	89.99
MSE	192.20	154.23	612.52
MAE	9.60	9.98	21.87
VAF	97.29	97.32	89.95

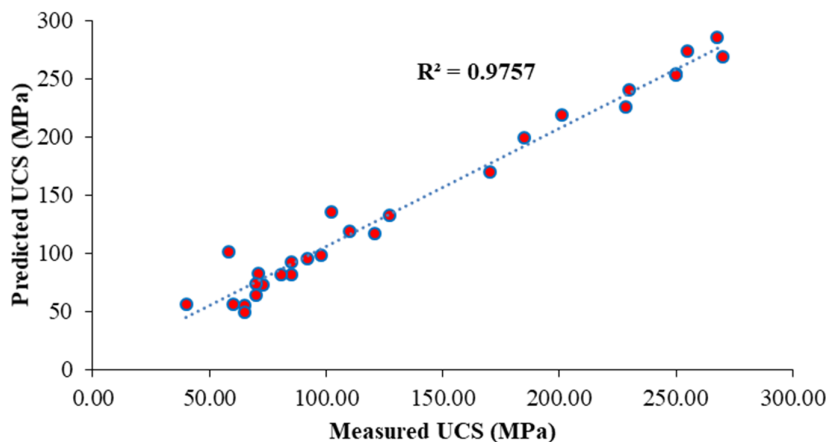


Figure 5. Correlation between the ANFIS-GA model results and the real data.

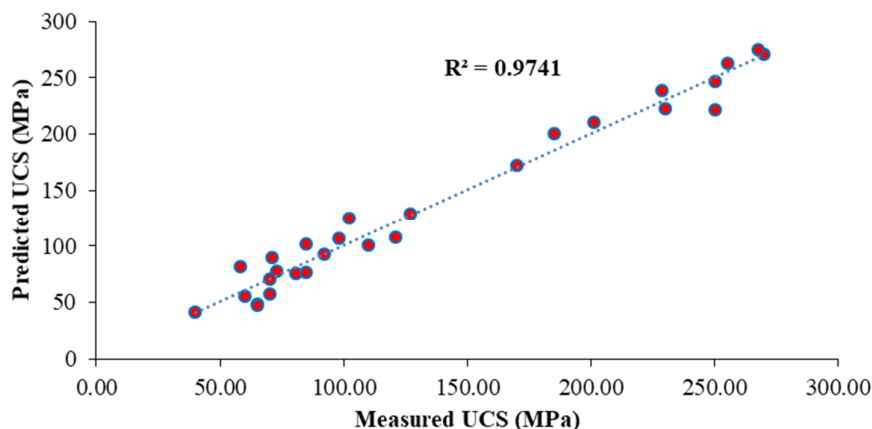


Figure 6. Correlation between the ANFIS-PSO model results and the real data.

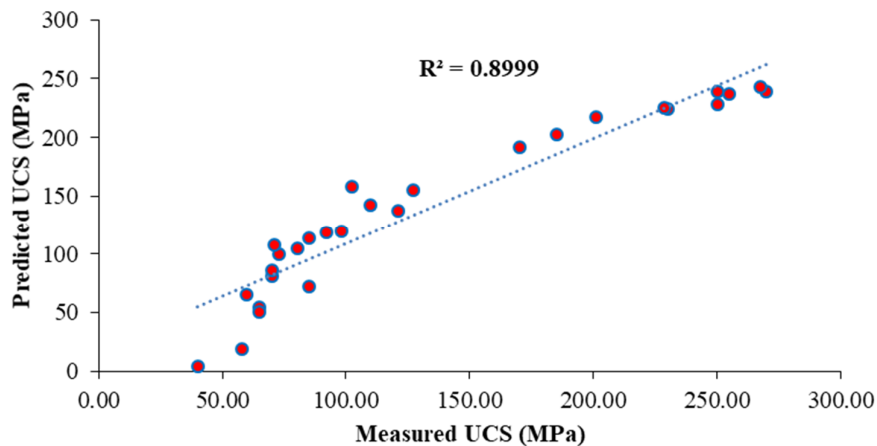


Figure 7. Correlation between the LR model results and the real data.

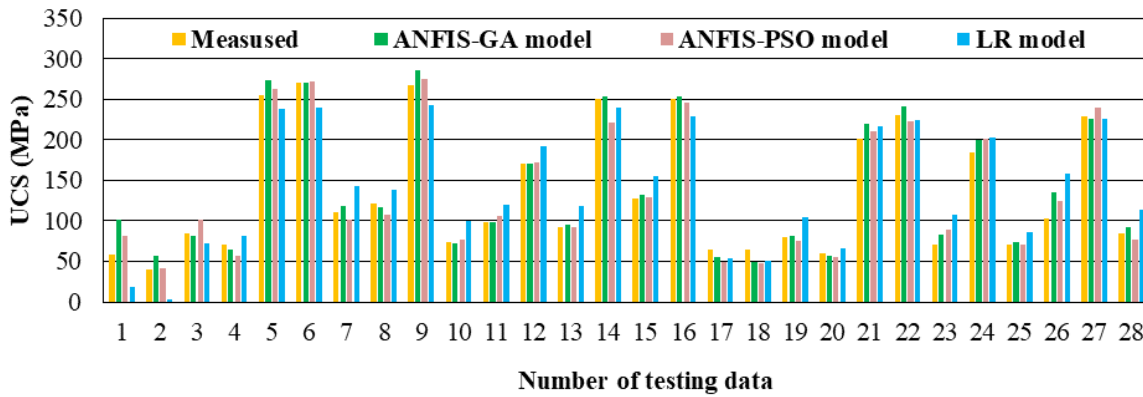


Figure 8. Comparing the suggested model results with the real evaluation datasets.

5.2. Comparison with available intelligent models

For further validation and evaluation of the suggested hybrid intelligent models, they were compared with the other single/hybrid intelligent models proposed by some researchers in the recent years, which were reported in the literature. The determination coefficient (R^2) index resulting from these models is used as a comparison basis for comparison object in order to confirm the suggested models. The values of R^2 index resulted from the available/previous intelligent single/hybrid models, and the proposed hybrid models in the current research work for UCS estimation are given in Table 8. As it can be concluded from this table, the prediction capacity and ability of the suggested hybrid intelligent models in this paper is considerably higher than the majority of available/previous single/hybrid intelligent models. Nevertheless, only the proposed ANFIS model by Jahed Armaghani *et al.* [50] is relatively as accurate as the suggested hybrid intelligent models in the current work. In addition to their accuracies, the most important benefit of the suggested hybrid intelligent models compared to the previous single/hybrid intelligent models is that only the rock easily definable properties (n , ρ , R) are regarded as their input variables. On the other hand, expensively and hardly definable variables were considered as the input parameters of the previous single/hybrid intelligent models. This comparison proved that the suggested hybrid intelligent models in this paper were quite precise and reasonably priced compared to the previous single/hybrid intelligent models. The above two mentioned points are the main superiorities and novelties of the current research work compared to the similar available works.

Table 8. Comparing the proposed models in this work with the available intelligent single/hybrid models for UCS prediction.

Model Type	Acquired R^2	Reference
ANN	0.94	[6]
FIS	0.67	[19]
FIS	0.97	[22]
ANN	0.4	[23]
ANFIS	0.94	[24]
FIS	0.98	[30]
ANN	0.97	[33]
GP	0.83	[37]
FIS	0.92	[47]
ANN	0.97	[48]
FIS	0.94	[49]
ANFIS	0.99	[50]
Un-pruned type of TA	0.89	[51]
Pruned type of TA	0.80	
FIS	0.64	[58]
GP	0.86	[59]
ANN	0.67	[60]
ANN	0.93	[61]
FIS	0.88	[62]
GP	0.88	[63]
ANN	0.86	[64]
ANN	0.97	[65]
ANN	0.50	[66]
GA-ANN	0.96	[67]
GP	0.63	[68]
ANFIS	0.83	[69]
SVR	0.77	[70]
PSO-ANN	0.97	[71]
PSO-ANN	0.97	[72]
ICA-ANN	0.94	[73]
ICA-ANN	0.92	[74]
ANFIS-GA	0.98	This research
ANFIS-PSO	0.97	This research

Here, ANN states the artificial neural network, FIS expresses the fuzzy inference system, GP imparts the genetic programming, ANFIS intends the adaptive neuro-fuzzy inference system, SVR demonstrates the support vector regression, PSO remarks the particle swarm optimization, ICA declares the imperialist competitive algorithm, and TA pronounces the tree algorithm.

5.3. Parametric study

A parametric study was performed in this section for all the three input variables to realize the relative influence of each input on the UCS resulting from the three suggested models. The results of the parametric study for the ANFIS-GA, ANFIS-PSO, and MR models are depicted in Figures. 9a, 9b, and 9c, respectively. This parametric study was carried out by keeping constant the two variables inputs and changing the third variable. Accordingly, the influence of the third input on UCS can be discovered. For instance, the porosity was removed from the input variables, and then the ANFIS-GA/ANFIS-PSO/MR model

was run by the other two input variables. Consequently, the influence of porosity on UCS could be realized through comparison of the new simulation results with the previous results. This process was also performed for the other input variables. According to this parametric study procedure, it was proved that density was the most effective variable on UCS in ANFIS-GA and LR models, whereas Schmidt hammer rebound was the most influence input on UCS in the ANFIS-PSO model. On the other hand, porosity was the least effectual variable on UCS in all of the above proposed models.

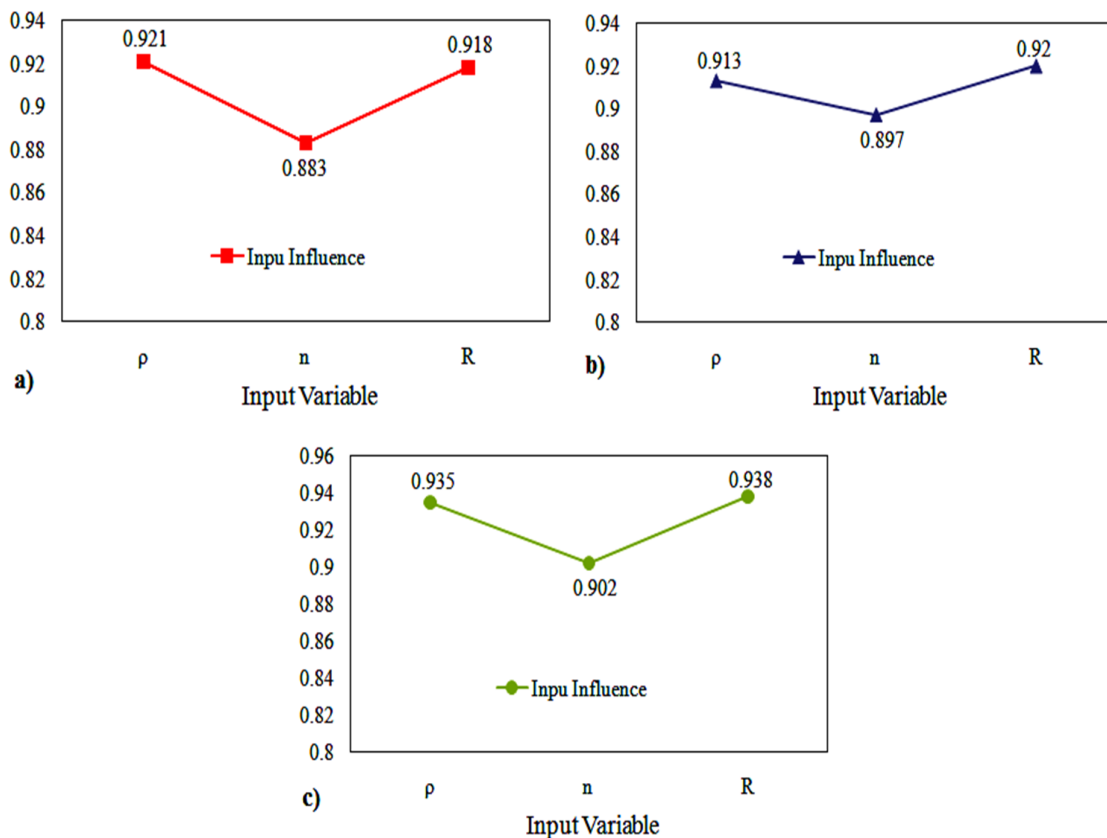


Figure. 9. Sensitivity analysis of UCS with related inputs: a) ANFIS-GA model; b) ANFIS-PSO model; c) MR model.

6. Conclusions

Two new hybrid intelligent models (ANFIS-GA and ANFIS-PSO) along with a conventional statistical approach (MR model) were developed in this work for predicting UCS of core rock specimens. For UCS (output) prediction in the above models, three easily definable variables including porosity, density, and Schmidt hardness were regarded as inputs. The construction and evaluation of the aforementioned models were made based on the 93 datasets that were determined on the different rock

core specimens in the laboratory. In order to verify the new hybrid intelligent models, their achieved results were compared with the statistical model results and the real evaluation datasets using the indices R, MSE, MAE, and VAF. Finally, the proposed hybrid intelligent models were compared with the available/previous intelligent single/hybrid models for UCS prediction based on the concluded R^2 values. According to the above tasks, the main achieved conclusions are outlined below.

- Comparative analysis proved that the performance of the ANFIS-GA model was rather better than those of the ANFIS-PSO model.
- Both hybrid intelligent models are considerably superior compared to the MR model.
- The simulation results of the new hybrid intelligent models are in extremely close agreement with the determined values of UCS in laboratory.
- It was revealed that the suggested hybrid intelligent models were relatively more accurate than the previous/available intelligent single/hybrid models proposed by other researchers.
- By a parametric study, it was discovered that density and Schmidt hammer rebound were the most influential variables, and porosity was the least input in UCS.
- The major benefit of the recommended models is that the easily definable variables are considered as their inputs, unlike the previous intelligent models.
- According to the above findings, it can be concluded that the proposed hybrid intelligent models have a good capability in UCS prediction, and they are more economical than the other available similar approaches. Thus they can be used for UCS determination in practice successfully.

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پیش‌بینی مقاومت فشاری تک محوری سنگ بکر با استفاده از مدل‌های هوشمند ترکیبی

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چکیده:

مقاومت فشاری تک محوری (UCS) سنگ بستر یک پارامتر کلیدی در طراحی پروژه‌های مرتبط با علوم زمین و ساختمان‌سازی است که هر دو نوع ساختارهای سنگی سطحی و زیرزمینی را شامل می‌شود. تعیین مقاومت فشاری تک محوری سنگ با استفاده از تست‌های آزمایشگاهی استاندارد یک فرآیند پیچیده، پرهزینه و زمان‌بر است که نیازمند نمونه‌های مغزه‌ای سالم است. با این حال، تهیه مغزه‌های سالم به‌ویژه هنگام عملیات حفاری در سنگ‌های درزه‌دار، شکسته و ضعیف معمولاً امکان‌پذیر نخواهد بود. بنابراین، اخیراً تلاش‌هایی در زمینه توسعه روش‌های غیر مستقیم مانند مدل‌های پیشگویانه هوشمند برای تخمین مقاومت فشاری تک محوری سنگ صورت گرفته است که نیازی به آماده‌سازی نمونه و تجهیزات آزمایشگاهی ندارد. تحقیق حاضر بر کاربرد ترکیب‌های جدیدی از تکنیک‌های هوشمند شامل سیستم استنتاج عصبی-فازی تطبیقی (ANFIS)، الگوریتم ژنتیک (GA) و بهینه‌سازی ازدحام ذرات (PSO) به‌منظور پیش‌بینی مقاومت فشاری تک محوری سنگ تمرکز دارد. این مدل‌ها بر اساس داده‌های آزمایشگاهی جمع‌آوری شده بر روی 93 نمونه مغزه‌ای از انواع سنگ‌های ضعیف تا بسیار قوی ساخته شده‌اند. نتایج مدل‌های ترکیبی پیشنهادی با همدیگر و با داده‌های واقعی و نتایج روش رگرسیون چند متغیره (MR) مقایسه شده است. این مقایسه‌ها با استفاده از شاخص‌های ضریب همبستگی، میانگین مربعات خطا، میانگین خطای مطلق و حساب واریانس انجام شده است. نتایج مقایسه فوق‌الذات که ترکیب ANFIS-الگوریتم ژنتیک تا حدودی دقیق‌تر از ترکیب ANFIS-بهینه‌سازی ازدحام ذرات است و هر دو مدل فوق دارای عملکرد بهتری نسبت به مدل رگرسیون چند متغیره هستند. بعلاوه، نتایج مدل‌های ANFIS-الگوریتم ژنتیک و ANFIS-بهینه‌سازی ازدحام ذرات کاملاً در تطابق با نتایج آزمایش مقاومت فشاری تک محوری در آزمایشگاه بوده و بسیار دقیق‌تر از مدل‌های هوشمند ترکیبی قبلی می‌باشند. در نهایت، مطالعه پارامتری مدل‌های پیشنهادی نشان داد که متغیرهای چگالی و قرائت چکش اشمیت دارای بیشترین تأثیر و متغیر تخلخل دارای کمترین تأثیر بر خروجی (مقاومت فشاری تک محوری) می‌باشند.

کلمات کلیدی: سنگ بکر، مقاومت فشاری تک محوری، سیستم استنتاج عصبی-فازی تطبیقی، الگوریتم ژنتیک، بهینه‌سازی ازدحام ذرات.