

## Application of stochastic programming for iron ore quality control

J. Gholamnejad<sup>1\*</sup>, A. Azimi<sup>1</sup> and M.R. Teymouri<sup>2</sup>

1. Department of Mining and Metallurgical Engineering, Yazd University, Yazd, Iran

2. Ore Processing Supervisory and Management, Asphalt-Tous Company, Yazd, Iran

Received 30 June 2017; received in revised form 4 February 2018; accepted 26 February 2018

\*Corresponding author: j.gholamnejad@yazd.ac.ir (J. Gholamnejad).

### Abstract

Stockpiling and blending play a major role in maintaining the quantity and quality of the raw materials fed into processing plants, especially the cement, iron ore and steel making, and coal-fired power generation industries that usually require a much uniformed feed. Due to the variable nature of such materials, they even come from the same source and the produced ores or concentrates are seldom homogeneous enough to be directly fed to the processing plant ore furnaces. Processing plants in iron ore mines need uniform feed properties in terms of each variable (in this work, iron phosphorous ratio and Fe content in magnetite phase) grade of ore, and therefore, homogenization of iron ore from different benches of an open pit or ore dumps has become an essential part of modern mine scheduling. When ore dumps are considered as an ore source, the final grade of the material leaving the dump to the blending bed cannot be easily determined. This difficulty contributes to mixing the materials of different grades in a dump. In this work, the ore dump elements were treated as normally distributed random variables. Then a stochastic programming model was formulated in an iron ore mine in order to determine the optimum amount of ore dispatched from different bench levels in open pit and also four ore dumps to a windrow-type blending bed in order to provide a mixed material of homogenous composition. The chance-constrained programming technique was used to obtain the equivalent deterministic non-linear programming problem of the primary model. The resulting non-linear model was then solved using LINGO. The results obtained showed a better feed grade for the processing plant with a higher probability of grade blending constraint satisfaction.

**Keywords:** *Stochastic Programming, Iron Ore Mine, Homogenization, Processing Plant.*

### 1. Introduction

In mining industries, raw material stockpiling reduces the amount of fluctuation of feed quality characteristics sent to the processing plant and convert heterogeneous materials into a stable homogenous product. The raw materials going from multiple production faces and ore dumps must be blended to provide the required minimum quality of ore. It is rarely possible to have an even quality of mined ore with direct feeding from mine. Robinson also explained the buffering function of stockpiles. Stockpiles can operate as buffers so that previous processes and the subsequent processes can operate independently [Robinson]. Therefore, many open-pit or underground mines require a level of raw material

homogenization as a part of their process. Commonly, stockpiles are constructed by stackers and are depleted by reclaimers by slicing across the pile perpendicular to the direction of layering. Therefore, multiple layers are simultaneously reclaimed from the base of a stockpile to its surface. Achieving the required level of blending and the efficiency of a stockpile system depend on [1, 2]:

- Stacking method: the blending process starts with stacking. Four basic stacking methods are cone shell, chevron, strata, and windrow.
- Stockpiling parameters such as the stockpile length, stockpile width, number of

stockpile layers, size and shape of the slice, and layer and position of the reclaimer in relation to the stockpile.

- The equipment properties of the stackers and reclaimers.
- Raw material characteristics such as bulk density, particle size, and oxidation.
- Variability of stockpile inputs.

As it is clear, both the quality and quantity of input materials from different sources and also their nature of variation affect the quality of homogenized ore. Usually there are several sources of raw materials such as open-pit mines, underground mines, and ore dumps. The mined materials are either sent to the stockpiles for homogenization or handled as waste or sent to the ore dumps for a later re-handling and processing according to their composition. The blasted block grade is the weighted average of the blast holes within the block. Incorporating ore dump in this model causes difficulty in estimating the final grade of material leaving the ore dumps to stockpile. In reality, the final grade of material leaving the dump becomes a complex function of the material inside it. Many researchers like Ramazan and Dimitrakopoulos [3], Senecal and Dimitrakopoulos [4], and Goodfellow and Dimitrakopoulos [5] assumed that the materials within ore dump were mixed homogeneously before later removal for processing. Bley et al. supposed that at the beginning of each time period of scheduling horizon, the ore-metal ratio of the material leaving the ore dump equaled the ore-metal ratio in the dump itself [6]. This causes the non-linearity of some of the equations in the model. Moreno et al. presented linear models for the upper and lower approximations of the non-linear formulation of the long-term production scheduling problem considering ore dumps [7].

In this work, we assumed that the final grade of material leaving the dump was a random variable with a normal distribution. Accordingly, a stochastic optimization model was developed in an iron ore mine to solve the ore homogenization problem.

## 2. Definition of problem

The selected case study was a large open-pit iron ore mine at the center of I.R. of Iran, which was exploited using the open-pit mining method. The mine was supposed to produce 12 million tons of annual ore feed to beneficiation plant and around 1 million tons of annual lump ore (high Fe content and low phosphorous). In this mine, the orebody

was divided into nine categories based on the rock type and its grade. The high-grade iron ore had a Fe content of more than 45% and the low-grade iron ore had an Fe content of 20-45%. The low phosphorous iron ore type had lower than 0.2% of phosphorous, and the high phosphorous iron ore type had more than 0.2% phosphorous. The sulfur cut-off for separating the high and low sulfur contents was 0.3%. The mine reserves are presented at Table 1.

As the ore materials were extracted, they were either sent to an ore dump for later re-handling and processing or sent to a 2300 t/h gyratory crusher for crushing and homogenization on the blending bed. This dispatching was based upon their Fe%, P%, and ore type. There were four separate ore dumps named CF3H (high grade-high phosphorous magnetite ore), CF3L (high grade-low phosphorous magnetite ore), CF2 (low grade magnetite ore), and SOD (low grade-high grade hematite ore). The blasted ore coming from the open pit and that dispatched from ore dumps supplied the crusher feed. The primary crusher reduced the material size to 300 mm. After crushing the material, it was transported via a 1256 m conveyer belt to the blending bed. The four variables Fe (total Fe), P, ratio (Fe/FeO), and Fem (Fe content in magnetite phase) were considered as critical variables due to the quality requirement of the processing plant. The operating conditions of the processing plant in terms of critical variables can be seen in Table 2.

In order to determine the ore dump characteristics, the samples were collected from each dump and assayed for Fe, P, ratio, and Fem. Table 3 shows the mean, variance, minimum, maximum, and median of ore variables in each dump.

There are two stockpile lines for homogenization. One is stacking, while the other is reclaiming. Once stockpile reaches its nominated tons, it is closed-off and is then available to be reclaimed until empty, while a new similar stockpile begins. The stacking method of piles is the windrow method (Figure 1). This stockpile is stacked by means of many lines of a small volume, and the main direction of movement of the stacker boom is parallel to the base area of the stockpile. Stacking starts in the lowest position of the hoisting gear at the edge of the stockpile opposite to the stacker (Figure 2). For stacking a line, the travel gear travels between two specified final positions. After the first line has been completely stacked, the slewing angle is changed by a specified value and the next line is stacked in the opposite direction. The whole base area of the

stockpile is stacked in this way. After the first level has been completed, the boom is lifted to the second level and the next lines are stacked in the

spaces of the first ones [8]. This is repeated accordingly until the desired height of the stockpile has been completed.

**Table 1. Rock types within orebody according to its characteristics.**

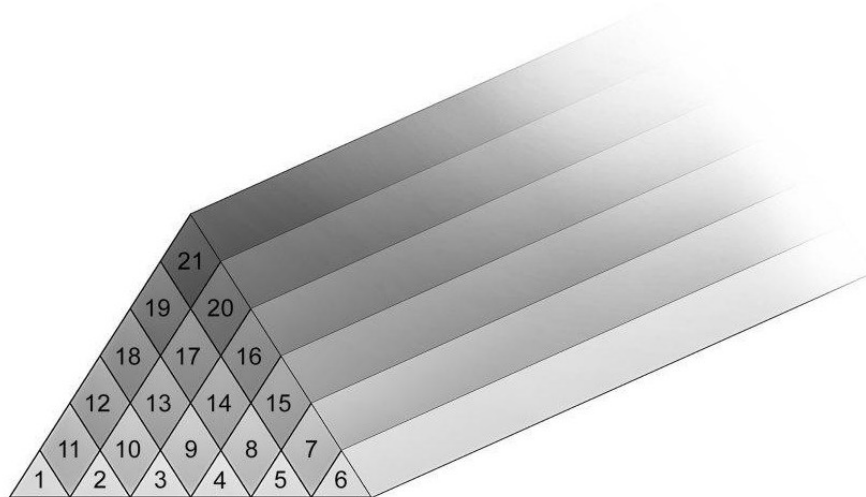
Rock type	Reserve (million tons)
Low phosphorus - low sulfur - non oxidized - high grade iron ore	27.9
Low phosphorus - low sulfur - oxidized - high grade iron ore	6.3
High phosphorus - low sulfur - non oxidized - high grade iron ore	179.2
High phosphorus - low sulfur - oxidized - high grade iron ore	80.1
Low phosphorus - high sulfur - non oxidized - high grade iron ore	3.8
High phosphorus - High sulfur - non oxidized - high grade iron ore	52.1
High phosphorus - High sulfur - oxidized - high grade iron ore	2.1
Non oxidized - Low grade iron ore	37.4
Oxidized - Low grade iron ore	10
<b>Total</b>	<b>398.9</b>

**Table 2. Operating conditions of processing plant.**

	Fe (%)	P (%)	Ratio	Fem (%)
<b>Lower bound</b>	52	-	3.5	40
<b>Upper bound</b>	54	1	5	50

**Table 3. Results of data analysis from sampling process in four dumps.**

Dump	Variable	Mean	Variance	Minimum	Median	Maximum
<b>CF3H</b>	Fe	50.89	67.14	34.97	53.96	61.63
	Fe/FeO	3.903	1.215	2.402	4.019	7.704
	P	0.9839	0.059	0.56	0.953	1.584
	Fem	42.63	200.67	23.29	41.37	73.13
<b>CF3L</b>	Fe	58.58	25.865	41.04	60.575	64.72
	Fe/FeO	3.321	0.943	2.588	3.237	8.904
	P	0.9698	0.0489	0.524	0.99	1.689
	Fem	58.4	135.67	18.3	59.14	76.02
<b>CF2</b>	Fe	49.501	71.316	19.54	49.21	67.66
	Fe/FeO	4.382	5.498	2.241	3.435	15.593
	P	0.9115	0.0766	0.383	0.899	2.093
	Fem	42.11	289.67	7.51	41.67	84.41
<b>SOD</b>	Fe	56.175	17.448	46.38	56.18	65.51
	Fe/FeO	8.324	21.478	2.598	7.782	21.117
	P	1.0548	0.0759	0.501	1.008	2.13
	Fem	29.29	360.51	6.41	20.79	78.52



**Figure 1. Windrow method of stacking.**

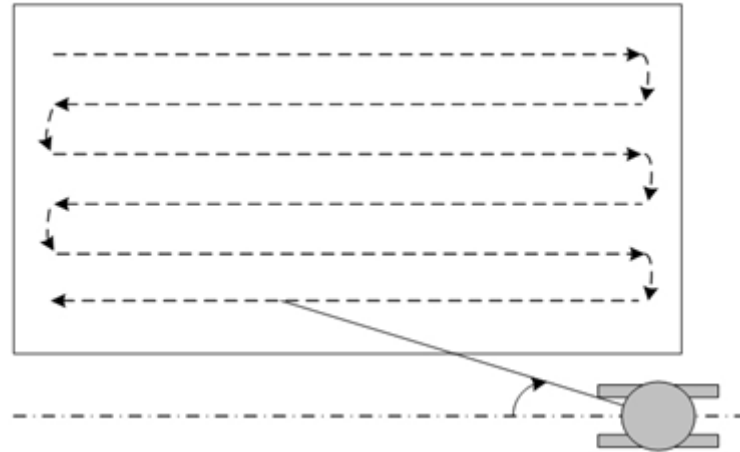


Figure 2. Boom movement for first level of stacking in windrow method [8].

As is clear in Figure 1, each stockpile has 21 blocks; the blocks of the first level (numbers 1 to 6) are called the base blocks, whilst the others are called the main blocks. If all blocks have the same triangle cross-section, each pile contains 36 blocks (Figure 3). The cross-sectional area of the base blocks is only half of the main blocks. Each main block is composed of 7 rows (each sweep of loading boom makes a row), and each row is composed of 17 trucks with a carrying capacity of about 120 tons; as a result, each row must be about 2000 tons, each base block is about 7500 tons, and each main block is about 15000 tons. With these values, each pile has a weight of about 270000 tons.

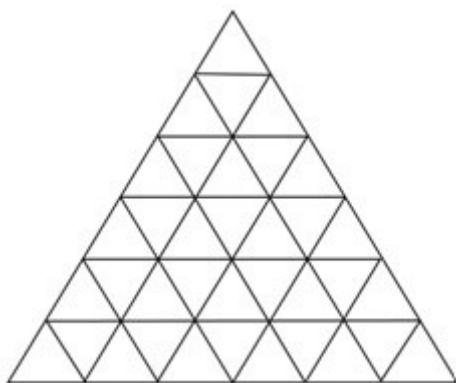


Figure 3. Section view of stockpile with 36 blocks of equal cross-section area.

As mentioned earlier, each block of these 36 triangular blocks of the piles comes from different sources including open-pit levels or ore dumps. The number of blocks from different sources should be determined in such a way as:

- The blending specifications regarding both the quality and the tonnage requirement to be met.

- Sending the blasted high grade-low phosphorous material ( $Fe \geq 60\%$  and  $P \leq 0.1\%$ ) to the blending bed should be minimized. These materials are only required to be crushed and sieved, and no more beneficiation is required.
- Extraction of low-grade and high-phosphorus dumps should be prioritized.
- Extraction from the nearest dump should be prioritized.

In the next section, a mathematical optimization model will be developed to solve the homogenization problem in this open-pit mine.

### 3. Mathematical modeling of problem

In this section, at first, the basic linear programming model for the homogenization problem is presented. Symbols are introduced as follow:

- $i$  Ore dump identification number,  $i = 1, 2, \dots, n$
- $j$  Blasted block identification,  $j = 1, 2, \dots, m$
- $n$  Number of ore dumps
- $m$  Number of accessible blasted blocks.
- $Fed_i$  Average grade of iron at the  $i^{\text{th}}$  dump, which is a random variable with the expected values for  $E(Fed_i)$  and variance of  $Var(Fed_i)$
- $Pd_i$  Average grade of phosphorous at the  $i^{\text{th}}$  dump, which is a random variable with the expected value of  $E(Pd_i)$  and variance of  $Var(Pd_i)$
- $Rd_i$  Average ratio at the  $i^{\text{th}}$  dump, which is a random variable with the expected value of  $E(Rd_i)$  and variance of  $Var(Rd_i)$
- $Magd_i$  Average grade of  $Fem$  at the  $i^{\text{th}}$  dump, which is a random variable with the expected value of  $E(Magd_i)$  and variance of  $Var(Magd_i)$
- $Feb_j$  Average grade of iron at the  $j^{\text{th}}$  blasted block

$Pb_j$  Average grade of phosphorous at the  $j^{th}$  blasted block  
 $Rb_j$  Average ratio at the  $j^{th}$  blasted block  
 $Magb_j$  Average grade of Fem at the  $j^{th}$  blasted block  
 $MinFe$  Minimum acceptable grade of iron ore  
 $MaxFe$  Maximum acceptable grade of iron ore  
 $MinR$  Minimum acceptable grade of iron ore ratio  
 $MaxR$  Maximum acceptable grade of iron ore ratio  
 $MinMag$  Minimum acceptable grade of Fem  
 $MaxMag$  Maximum acceptable grade of Fem  
 $MaxP$  Maximum acceptable grade of phosphorus  
 $L$  Total number of blocks at each pile (in this study, 36)  
 $TBW_j$  Total weight of the  $j^{th}$  blasted block  
 $X_i$  Number of blocks dispatching from the  $i^{th}$  dump  
 $Y_j$  Number of blocks dispatching from the  $j^{th}$  blasted block within open pit.

*Objective function:* Here, we want to maximize the total weight attributed to the piled blocks in the stockpile:

$$Max Z = \sum_{i=1}^n W_i X_i + \sum_{j=1}^m W_j Y_j \quad (1)$$

$X_1, X_2, X_3,$  and  $X_4$  are decision variables related to the SOD, CF2, CF3H, and CF3L ore dumps, respectively.

*Constraint:* The average grade (in terms of Fe, Fem, P, ratio) of the material sent to the stockpile has to be more than a lower bound and less than an upper bound:

$$\frac{\sum_{i=1}^n Fed_i X_i + \sum_{j=1}^m Feb_j Y_j}{L} \leq MaxFe \quad (2)$$

$$\frac{\sum_{i=1}^n Fed_i X_i + \sum_{j=1}^m Feb_j Y_j}{L} \geq MinFe \quad (3)$$

$$\frac{\sum_{i=1}^n Magd_i X_i + \sum_{j=1}^m Magb_j Y_j}{L} \geq MinMag \quad (4)$$

$$\frac{\sum_{i=1}^n Magd_i X_i + \sum_{j=1}^m Magb_j Y_j}{L} \leq MaxMag \quad (5)$$

Phosphorous content of feed must be as low as possible, and therefore, the lower bound for phosphorous is not considered:

$$\frac{\sum_{i=1}^n Pd_i X_i + \sum_{j=1}^m Pb_j Y_j}{L} \leq MaxP \quad (6)$$

$$\frac{\sum_{i=1}^n Rd_i X_i + \sum_{j=1}^m Rb_j Y_j}{L} \geq MinR \quad (7)$$

$$\frac{\sum_{i=1}^n Rd_i X_i + \sum_{j=1}^m Rb_j Y_j}{L} \leq MaxR \quad (8)$$

The total weigh of the dispatched material from the  $j^{th}$  blasted block within the open pit should be equal or less than its weight:

$$7500 \times Y_j \leq TBW_j \text{ for } j = 1, 2, \dots, m \quad (9)$$

The total number of stacked blocks within the stockpile should be equal to L:

$$\sum X_i + \sum Y_j = L \quad (10)$$

All the variables should be non-negative and integer:

$$X_i, Y_j \geq 0 \text{ and Integer} \quad (11)$$

#### 4. Stochastic formulation of iron ore homogenization

As mentioned earlier, ore dump grades are not known with certainty. Only the statistical information of the random grades is available; therefore, constraints 2 to 8 contain random parameters. The main difficulty of such models is due to the optimal decisions that have to be taken prior to the observation of random parameters.

There are several methods available to handle the uncertainty in this problem. The chance-constrained programming method was used in this work. This approach ensures that the probability of meeting a certain constraint is above a certain level ( $\beta$ ). Chance-constrained programming was originally proposed by Charnes, Cooper, and Symonds [9] and Charnes and Cooper [10], and then applied by Charnes and Cooper [11]. This approach was previously used in mining industries by many researchers like Gholamnejad et al. [12], Gangwar [13], and Kumral [14].

In the following, the chance constrained programming approach was applied to handle

dump grade uncertainty to the proposed integer model. We began with constraint (2). The generic way to express such constraints is:

$$\Pr \left[ \frac{\sum_{i=1}^n Fed_i X_i + \sum_{j=1}^m Feb_j Y_j}{L} \leq MaxFe \right] \geq \beta \quad (12)$$

The value for  $\beta$  is called the probability level, and it is chosen by the decision-maker in order to model the safety requirements. Equation (12) indicates that constraint (2) has to be satisfied with the probability of at least  $\beta$ . Let's define:

$$H = \frac{\sum_{i=1}^n Fed_i X_i + \sum_{j=1}^m Feb_j Y_j}{L} \quad (13)$$

As a result, Equation (12) can be re-written as:

$$\Pr[H \leq MaxFe] \geq \beta \quad (14)$$

As it is clear, H is the average grade of material sent to the stockpile, which is a random variable. It is assumed that the distribution of H can be approximated by a normal distribution function with the following mean and variance:

$$E(H) = \frac{\sum_{i=1}^n E(Fed_i) X_i + \sum_{j=1}^m Feb_j Y_j}{L} \quad (15)$$

$$\sum_{i=1}^n E(Fed_i) X_i + \sum_{j=1}^m Feb_j Y_j + \eta_\beta \sqrt{\sum_{i=1}^n Var(Fed_i) X_i^2 + \sum_{i=1}^n \sum_{k=1}^n (X_i X_k) Cov(Fed_i, Fed_k)} \leq L.MaxFe \quad (20)$$

Similarly, the equivalent form of Equation (3) is:

$$\sum_{i=1}^n E(Fed_i) X_i + \sum_{j=1}^m Feb_j Y_j + \eta'_\beta \sqrt{\sum_{i=1}^n Var(Fed_i) X_i^2 + \sum_{i=1}^n \sum_{k=1}^n (X_i X_k) Cov(Fed_i, Fed_k)} \geq L.MinFe \quad (21)$$

where:

$$\Pr(\bar{H} \leq \eta'_\beta) = \int_{-\infty}^{\eta'_\beta} \frac{1}{2\pi} \exp\left(-\frac{x^2}{2}\right) dx = 1 - \beta \quad (22)$$

The value for  $1-\beta$  is the acceptable risk level for not satisfying the grade constraint.

$$\sum_{i=1}^n E(Pd_i) X_i + \sum_{j=1}^m Pb_j Y_j + \eta_\beta \sqrt{\sum_{i=1}^n Var(Pd_i) X_i^2 + \sum_{i=1}^n \sum_{k=1}^n (X_i X_k) Cov(Pd_i, Pd_k)} \leq L.MaxP \quad (23)$$

$$\sum_{i=1}^n E(Rd_i) X_i + \sum_{j=1}^m Rb_j Y_j + \eta_\beta \sqrt{\sum_{i=1}^n Var(Rd_i) X_i^2 + \sum_{i=1}^n \sum_{k=1}^n (X_i X_k) Cov(Rd_i, Rd_k)} \leq L.MaxR \quad (24)$$

$$\sum_{i=1}^n E(Rd_i) X_i + \sum_{j=1}^m Rb_j Y_j + \eta'_\beta \sqrt{\sum_{i=1}^n Var(Rd_i) X_i^2 + \sum_{i=1}^n \sum_{k=1}^n (X_i X_k) Cov(Rd_i, Rd_k)} \geq L.MinR \quad (25)$$

$$Var(H) = \frac{\sum_{i=1}^n Var(Fed_i) X_i^2 + \sum_{i=1}^n \sum_{k=1}^n (X_i X_k) Cov(Fed_i, Fed_k)}{L^2} \quad i \neq k \quad (16)$$

Equation (14) can be re-written by subtracting  $E(H)$  from both sides of Equation (14), and dividing by  $\sqrt{Var(H)}$ , as follows:

$$\Pr \left[ \frac{H - E(H)}{\sqrt{Var(H)}} \leq \frac{MaxFe - E(H)}{\sqrt{Var(H)}} \right] \geq \beta \quad (17)$$

Let's define  $\bar{H} = \frac{H - E(H)}{\sqrt{Var(H)}}$ . Therefore,  $\bar{H}$  is a

standard normal distribution function with a zero mean and unit standard deviation. There will be a value of  $\eta_\beta$  that can then be determined from the area under normal curve such that:

$$\Pr(\bar{H} \leq \eta_\beta) = \int_{-\infty}^{\eta_\beta} \frac{1}{2\pi} \exp\left(-\frac{x^2}{2}\right) dx = \beta \quad (18)$$

Thus combining Equations (17) and (18) results in:

$$\frac{MaxFe - E(H)}{\sqrt{Var(H)}} \geq \eta_\beta \Rightarrow E(H) + \eta_\beta \sqrt{Var(H)} \leq MaxFe \quad (19)$$

The deterministic equivalent form of constraint (12) can be achieved by combination of Equations (15), (16), and (19), as follows:

The deterministic equivalents of Equations (4) to (8) can be achieved similarly as follow:

$$\sum_{i=1}^n E(\text{Magd}_i).X_i + \sum_{j=1}^m \text{Magb}_j.Y_j + \eta_\beta \sqrt{\sum_{i=1}^n \text{Var}(\text{Magd}_i).X_i^2 + \sum_{i=1}^n \sum_{k=1}^n (X_i.X_k).Cov(\text{Magd}_i, \text{Magd}_k)} \leq L.\text{MaxMag} \tag{26}$$

$$\sum_{i=1}^n E(\text{Magd}_i).X_i + \sum_{j=1}^m \text{Magb}_j.Y_j + \eta'_\beta \sqrt{\sum_{i=1}^n \text{Var}(\text{Magd}_i).X_i^2 + \sum_{i=1}^n \sum_{k=1}^n (X_i.X_k).Cov(\text{Magd}_i, \text{Magd}_k)} \geq L.\text{MinMag} \tag{27}$$

**5. Solving non-linear model for a homogenization problem**

Suppose that in the iron ore mine we have nine blasted blocks within the open pit. The characteristics of these nine blocks and also four ore dumps can be seen in Table 4.

Using the existing data, the non-linear programming model is developed and then solved using the LINGO software for different values of  $\beta$ . Execution of the program yields the following solutions:

$$X_1=5, X_2=4, X_3=1, X_4=3, Y_1=0, Y_2=3, Y_3=4, Y_4=3, Y_5=4, Y_6=0, Y_7=1, Y_8=3, Y_9=5.$$

Also the corresponding maximum probabilities for satisfaction of Fe, P, Ratio, and Fem constraints were 72.5%, 84.1%, 93.3%, and 90.3%, respectively. In order to the compare this

solution with the traditional one, we also resolved the deterministic model assuming  $\eta_\beta=0$ . The corresponding solutions are:

$$X_1=13, X_2=2, X_3=0, X_4=21, Y_1=0, Y_2=0, Y_3=0, Y_4=0, Y_5=0, Y_6=0, Y_7=0, Y_8=0, Y_9=0.$$

In this case, the probability of constraint satisfaction is 50% for all grade blending constraints. As a result, the proposed model has increased the probability of constraint satisfaction from 50% to at least 72.5%. Also due to considering the uncertainty associated with the ore dump characteristics, the solution forced the model to decrease the dump re-handling. The average grade of each variable in the stockpile, which is obtained using the deterministic and stochastic methods, is shown in Table 5.

**Table 4. Means of four variables in production sources.**

Source	Fe	Ratio	P	Fem	Decision variable
<b>B4273</b>	55.27	3.97	1.05	53.5	Y <sub>1</sub>
<b>B4291</b>	54.63	3.60	1.05	49.15	Y <sub>2</sub>
<b>B4284</b>	51.05	3.81	0.88	42.92	Y <sub>3</sub>
<b>B4302</b>	49.07	3.28	0.93	51.02	Y <sub>4</sub>
<b>B4308</b>	53.97	3.05	0.99	56.84	Y <sub>5</sub>
<b>B4310</b>	58.54	3.37	0.87	54.91	Y <sub>6</sub>
<b>B496</b>	52.73	8.79	1	29	Y <sub>7</sub>
<b>B4319</b>	55.82	5.53	0.98	49.69	Y <sub>8</sub>
<b>B4295</b>	55.43	3.48	0.88	50.68	Y <sub>9</sub>
<b>SOD</b>	54	7.65	0.93	24.55	X <sub>1</sub>
<b>CF2</b>	50.48	3.76	0.85	45.10	X <sub>2</sub>
<b>CF3L</b>	50.54	3.08	0.85	58.74	X <sub>3</sub>
<b>CF3H</b>	52.54	3.24	0.91	51.03	X <sub>4</sub>

**Table 5. Average grade of each variable in stockpile.**

	Fe (%)	P (%)	Ratio	Fem (%)
<b>Deterministic model</b>	52.95	0.9139	4.86	41.14
<b>Stochastic model</b>	53.01	0.9275	4.36	45.72

**6. Conclusions**

Iron ore quality control between mine and processing plant is a complex issue, especially when ore dumps are one of the suppliers of the feed mill. This is due to the difficulty of correctly evaluating the grade of material leaving the ore dumps. In this work, the average grade of ore dump materials fed into the stockpile was treated

as a normally distributed random variable. A stochastic programming model was then presented to solve a homogenization and blending problem in the case of multiple feed resources containing ore dumps in an iron ore mine. The stochastic model was then converted to its equivalent non-linear model using the chance-constrained programming approach. By solving the non-linear

programming model, the amount of ore sent from each source to the stockpile was determined. In this model, the probability of satisfying each constraint was also calculated. Comparison of the original deterministic model with the stochastic model shows that the proposed model reduces the risk of non-satisfaction of grade blending constraints from 50% to at most 27.5%. As a result, the stochastic programming model provides a useful decision tool for homogenizing problems.

## References

- [1]. Pavloudakis, F. and Agioutantis, Z. (2001). Development of a software tool for the prediction of the coal blending efficiency, Proceedings, 17<sup>th</sup> International Mining Congress and Exhibition of Turkey (IMCET 2001), June 19-22, 2001, Ankara, Turkey. pp. 675-681.
- [2]. KUMRAL, M. (2006). Bed blending design incorporating multiple regression modeling and genetic algorithms. *Journal of the South African Institute of Mining and Metallurgy*. 106 (3): 229-236.
- [3]. Ramazan, S. and Dimitrakopoulos, R. (2013). Production scheduling with uncertain supply: a new solution to the open pit mining problem. *Optimization and Engineering*. 14 (2): 361-380.
- [4]. Senecal, R. and Dimitrakopoulos, R. (2014). Parallel implementation of a tabu search procedure for stochastic mine scheduling". Proceedings orebody modeling and strategic mine planning symposium 2014. The Australian Institute of mining and Metallurgy: Melbourne. pp. 405-414.
- [5]. Goodfellow, R.C. and Dimitrakopoulos, R. (2016). Global optimization of open pit mining complexes with uncertainty. *Applied Soft Computing*. 40: 292-304.
- [6]. Bley, A., Boland, N., Froyland, G. and Zuckerberg, M. (2012). Solving mixed integer nonlinear programming problems for mine production planning with stockpiling. *Optimization Online*. ([http://www.optimization-online.org/DB\\_HTML/2012/11/3674.html](http://www.optimization-online.org/DB_HTML/2012/11/3674.html)).
- [7]. Moreno, E., Rezakhah, M., Newman, A. and Ferreira, F. (2017). Linear models for stockpiling in open-pit mine production scheduling problems. *European Journal of Operational Research*. 260 (1): 212-221.
- [8]. S/SR Reclaiming/stacking methods. <http://www.abb.com>.
- [9]. Charnes, A., Cooper, W.W. and Symonds, G.H. (1958). Cost horizons and uncertainty equivalents: An approach to stochastic programming of heating oil. *Management Science*. 4 (3): 235-263.
- [10]. Charnes, A. and Cooper, W.W. (1959). Chance-constrained programming. *Management Science*. 6: 73-79.
- [11]. Charnes, A. and Cooper, W.W. (1963). Deterministic equivalents for optimizing and satisficing under chance-constraints. *Operation Research*. 11: 18-39.
- [12]. Gholamnejad, J., Osanloo, M. and Karimi, B. (2006). A chance-constrained programming approach for open pit long-term production scheduling in stochastic environments. *The Journal of the South African Institute of Mining and Metallurgy*. 106: 105-114.
- [13]. Gangwar, A. (1982). Using Geostatistical Ore Block Variance in Production Planning by Integer Programming. 17<sup>th</sup> APCOM Symposium. pp. 443-460.
- [14]. Kumral, M. (2004). Genetic algorithms for optimization of a mine system under uncertainty. *Production Planning & Control*. 15 (1): 34-41.



## استفاده از برنامه ریزی تصادفی در کنترل عیار سنگ آهن

جواد غلام نژاد<sup>\*</sup>، علی عظیمی<sup>۱</sup> و محمدرضا تیموری<sup>۲</sup>

۱- دانشکده مهندسی معدن و متالورژی، دانشگاه یزد، ایران

۲- واحد نظارت کارخانه فرآوری، شرکت آسفالت طوس، یزد، ایران

ارسال ۲۰۱۷/۶/۳۰، پذیرش ۲۰۱۸/۲/۲۶

\* نویسنده مسئول مکاتبات: j.gholamnejad@yazd.ac.ir

### چکیده:

دپوسازی و همگن سازی ماده معدنی نقش مهمی را در کنترل کمیت و کیفیت خوراک ورودی به کارخانه های فرآوری، به خصوص در صنایعی نظیر سیمان، فولاد و نیروگاه های برق با سوخت زغال سنگ که نیازمند خوراک یکنواخت می باشند، بازی می کند. به خاطر طبیعت متغیر مواد معدنی و لزوم تأمین خوراک از منابع مختلف، خوراک تولیدی به ندرت همگن بوده به طوری که نمی تواند مستقیماً در کارخانه فرآوری مورد استفاده قرار گیرد. کارخانه های فرآوری در معادن سنگ آهن نیازمند خوراکی هستند که عیار کانسنگ (برحسب متغیرهایی نظیر عیار آهن، فسفر، درجه اکسیدگی و درصد آهن در فاز مگنتیت) یکنواخت باشد، بنابراین همگن سازی ماده خام ارسالی از پله های مختلف معدن و دپوهای کانسنگ یک بخش ضروری در برنامه ریزی تولید معدن است. زمانی که دپوهای کانسنگ به عنوان منبع تأمین خوراک کانسنگ هستند نمی توان عیار کانسنگ ارسالی از دپوها به کارخانه فرآوری را به آسانی مشخص کرد. این مسئله ناشی از اختلاط کانسنگ با عیارهای مختلف در دپوها است. در این پژوهش عیار کانسنگ ارسالی از دپوها متغیر تصادفی با توزیع نرمال در نظر گرفته شد. سپس در یک معدن روباز سنگ آهن یک مدل برنامه ریزی تصادفی توسعه داده شد که هدف آن تعیین میزان بهینه خوراک ارسالی از پله های مختلف معدن و همچنین چهار دپوی کانسنگ به سایت اختلاط کارخانه فرآوری به منظور تأمین خوراک همگن برای کارخانه فرآوری است. سپس با استفاده از برنامه ریزی توأم با شانس مدل معادل قطعی و غیرخطی مدل اولیه به دست آورده شد. سپس مدل خطی حاصل، توسط نرم افزار LINGO حل شد. نتایج حاصل نشان داد که با این مدل می توان کنترل عیار بهتری را برای کارخانه فرآوری انجام داد به طوری که محدودیت های اختلاط عیار با احتمال بالاتری برآورده شود.

**کلمات کلیدی:** برنامه ریزی تصادفی، معدن سنگ آهن، همگن سازی، کارخانه فرآوری.