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Fuzzy tonnage-average grade model based on extension principle

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Abstract

Due to the nature of the geological and mining activities, different input parameters in the grade estimation and mineral resource evaluation are always tainted with uncertainties. It is possible to investigate the uncertainties related to the measurements and parameters of the variogram model using the fuzzy kriging method instead of the kriging method. The fuzzy kriging theory has already been the subject of relatively various research studies but the main weak point in such studies is that the results of the fuzzy estimations are not used in decision-making and planning. A very common, but key, tool of decision-making for mining engineers is the tonnage-average grade models. Under conditions where measurements or/and variogram model parameters are tainted with uncertainties, the tonnage-average grade model will be uncertain as well. Therefore, it is necessary to use the fuzzy tonnage-grade model instead of the crisp ones, and the next analysis steps and decision-makings are done accordingly. In this paper, the computational principles of the fuzzy tonnage-average grade curve and a case study regarding its usage are presented.

1. Introduction

Nowadays, kriging is known to be the most common estimation method in the evaluation of mineral deposits, and the very important inputs of the kriging estimator are 1) assay data gathered from drillholes and 2) variogram model parameters, both assumed to be certain but in reality, something else because 1) preparation and analysis of the assay data are not certain due to the sampling errors [1], and 2) the fitted variogram model is tainted with epistemic uncertainty [2] due to insufficient data during the structural analysis [3-5]; therefore, the assumptions are invalid, and hence, the results obtained by kriging are uncertain as well [1, 3-6]. Geostatisticians have dealt with such uncertainties in 3 different ways: 1) ignoring them, 2) defining a unique prior distribution function for every parameter with uncertainty and using the Bayesian kriging tool [7-9], and 3) defining the uncertain parameters in the form of interval-valued or fuzzy-interval parameters and using the fuzzy kriging tool [1, 3-5]. In most geostatistical studies, use has been made of the first method because of the insufficient uncertainty of the variogram model parameters and variables. The epistemic uncertainty in the Bayesian kriging methods has been investigated through attaching prior subjective probabilities to each potential model [9]. Using a single subjective probability to describe epistemic uncertainty represents much more information than what is actually available, and therefore, application of Bayesian kriging could be debatable [2]. Also a Bayesian kriging approach requires extensive calculations [10]. There are 2 general objections to this method: 1) a subjective probability presents much more information than what really exists, and 2) since the subjective and objective probabilities present information related to 2 very different natures, their product (like what happens in the Bayes' Law) is incompatible [11]. Fuzzy kriging can be done based on 2 different algorithms: 1) extending

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the principles of the random functions to the triangular fuzzy random function [5, 12], and 2) applying the extension principle [13] on some selected specific operators [1, 3, 4, 14]. Different applications of both algorithms have been presented before [4, 15-19] but they have all sufficed only on the algorithm of fuzzy kriging; they have not used the results of fuzzy kriging in decision-making or planning. A very common, but key, tool of decision-making for mining engineers is the tonnage-average grade models. Under conditions where measurements or/and variogram model parameters are tainted with uncertainties, the tonnage-average grade model will be uncertain as well. Therefore, it is necessary to use the fuzzy tonnage-grade model instead of the crisp ones, and the next decision-making steps take place accordingly. Effort has been made in this work to calculate the tonnage-average grade model based on the fuzzy extension principle and make use of it in decision-making. To check the efficiency of the proposed algorithm, the results of a case study in Jajarm Zu2 bauxite deposit, North Khorasan Province, Iran, have also been presented.

2. Problem formulation

Consider grade as a realization of a second order stationary random function $Z(x), x \in D \subset R^d$, $d = 3$, where x is a location in mineral deposit *D*. Divide the deposit into v_i blocks $i = 1, \ldots, N$ having an equal size and a similar shape. The average grade of each block $v \in D$ can be estimated by the ordinary kriging using the surrounding information $Z(x_i)$ $i = 1, \ldots$ n, as follows:

$$z^*(v) = f(z(x_1), \ldots, z(x_n), a, v) = \sum_{i=1}^{n} \lambda_i(x_1, \ldots x_n, a, v) Z(x_i), \tag{1}$$

$$\sigma_K^2(v) = g(x_1, \ldots, x_1, a, v) = \sum_{j-1}^{n} \lambda_j(x_1, \ldots, x_1, a, v)\ \bar{\gamma}(x_j, v) + \mu - \bar{\gamma}(v, x_i) \tag{2}$$

where $z^*(v)$ is the kriged block value; $\sigma_K^2(v)$ is the block kriging variance; $\gamma(x_i, x_j)$ is the variogram between locations x_i and x_j; $\bar{\gamma}(v, x_i)$ is the average variogram value between block v and location x_i; μ is the Lagrange multiplier; $a = \{a_j, j = 1, \ldots, p\}$ are the variogram model parameters (i.e. nugget effect, sill, and range); p is the number of variogram model parameters (commonly p = 3); and λ_i, $i = 1, \ldots, n$, are weights that can be obtained by solving the kriging system [20]. According to the above equations, the results obtained from kriging depend, in addition to the variables at the sampling points, on the variogram model parameters and the arrangement of the samples with respect to the block [21]. Since it is generally impossible, due to insufficient data or behavior of the experimental semi-variogram (modeling difficulties), to exactly fit a variogram model with no uncertainties, then it is necessaryto show the effects of uncertainties in the results using the fuzzy kriging method. Bardossy *et al*. modeled the uncertain parameters of variogram with fuzzy numbers $\hat{a} = \{\hat{a}_j, j = 1, \ldots, p\}$ and calculated the membership functions $\sigma_K^2(v)$ and $z^*(v)$ as follow:

$$\mu_{\hat{\sigma}^0}\left(\sigma_K^2(v)\right) = \sup_{x,v:\sigma^2 = g_0(x,a,v)} \left(\mu_{\hat{a}_i}(a_i)\right) \tag{3}$$

$$\mu_{\hat{z}^0}\left(z^*(v)\right) = \sup_{z,v:z^*(x_0) = f_0(z,a,v)} \left(\mu_{\hat{a}_i}(a_i)\right) \tag{4}$$

Now, if for a specified cut-off grade, g, the average grade C_g and tonnage T_g (of those parts of the deposit with grades greater than g) are defined as follow:

$$T_g = \sum_{i \in s} \rho V(v_i) z^*(v_i), s = \{1, \ldots, n | z^*(v_s) > g\} \tag{5}$$

$$C_g = \frac{\sum_{i \in s} V(v_i)\, z^*(v_i)}{\sum_{i \in s} V(v_i)}, s = \{1, \ldots, n | z^*(v_s) > g\} \tag{6}$$

where ρ is the deposit average density and $V(v_i)$ is the volume of the i^{th} block, and then, according to the extension principle (Appendix A), the membership functions of the fuzzy average grade and tonnage of those parts of the deposit that have grades greater than g, are defined as follow:

$$\mu_{\widehat{T_g}^0}\left(T_g\right) = \sup_{T_g = K(z)} \left(\mu_{\hat{z}_i}(z_i)\right) \tag{7}$$

$$\mu_{\widehat{C_g}^0}\left(C_g\right) = \sup_{C_g = L(z)} \left(\mu_{\hat{z}_i}(z_i)\right) \tag{8}$$

Therefore, based on the fuzzy grade value estimated for each block, it is possible to extract the tonnage and average grade according to the cut-off grades at different degrees of membership, and finally, draw the tonnage-average grade fuzzy curve. A useful property of such curves is that it is possible to extract from them, at any desirable cut-off grade, triangular fuzzy numbers for both the tonnage and the average grade, and use them in risk evaluations.

3. Case study

3.1. Jajarm Zu2 deposit

The Jajarm bauxite deposit complex is located at 56.25° to 56.45° east longitude and 37.2 to 37.3 north latitude, 19 km NE part of the city of Jajarm, North Khorasan Province, Iran (Figure 1). Aluminum anomalies in the Jajarm bauxite deposit are divided into the 4 separate zones of lower Kaolin, shale bauxite (SB), hard bauxite (HB), and upper Kaolin (KB) with HB being economically the most important zone (9). The complex has been divided, due to different faults, to 4 blocks, one of which is called “Zu”; this too, has been divided, due to the same reason, to 4 different parts named Zu1, Zu2, Zu3, and Zu4. In Zu2, by boring 72 drillholes, a total of 4439 m of exploratory drilling has been carried out for part of which (approximately 574 m) the geological and assay data is available. The exploratory drilling pattern is presented in Figure 2. Statistical studies done on the regional variable of AL_2O_3 and SiO_2 in HB zone of Zu2 deposit show that these variables are normal, and there is no trend in the data.

Figure 1. Geographic location of the Zu2 deposit in the Jarajm Bauxite Complex.

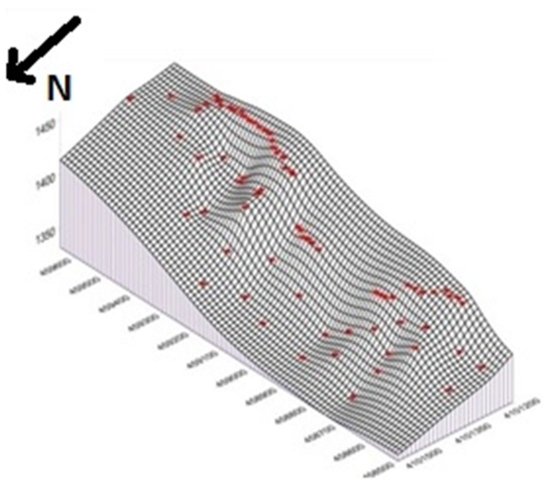
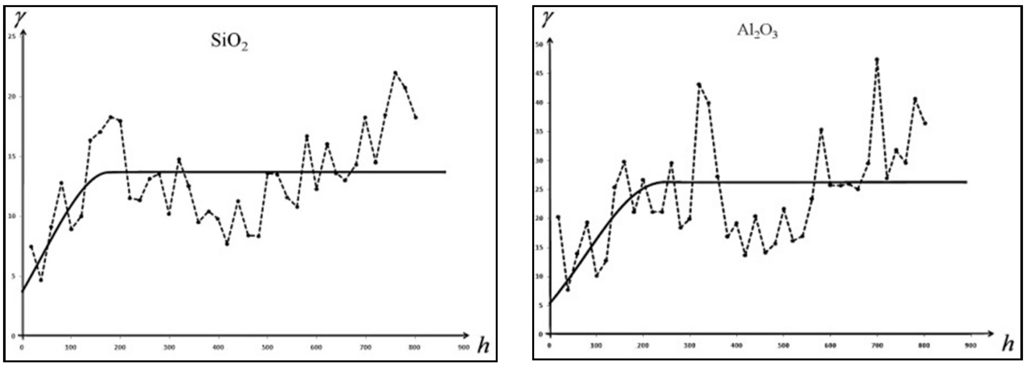


Figure 2. The exploratory drilling pattern in Zu2 deposit.

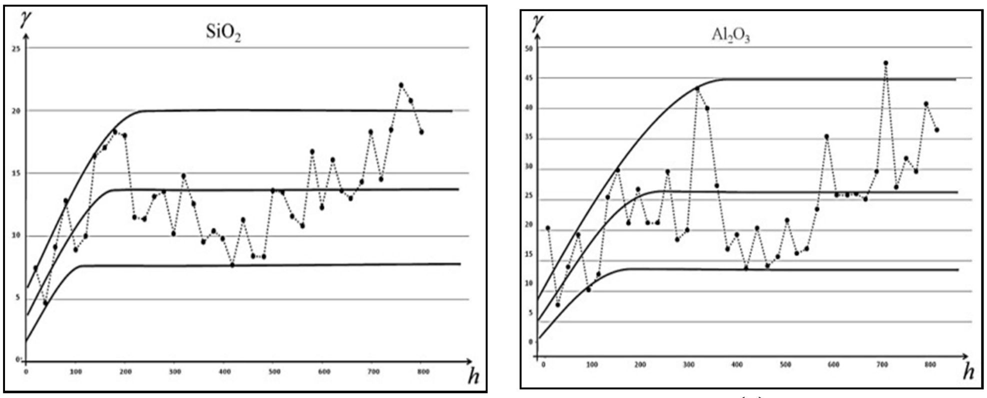
To structurally analyze the AL_2O_3 and SiO_2 variables, the experimental semi-variograms were drawn directionally and directionless. The latter not only studies and identifies the structural properties of the regional variable and show its trend of variations but also summarizes the data. Before using the experimental semi-variograms in estimation processes, it is necessary that the most appropriate theoretical model be fitted to them. In the present work, fitting model to directional experimental semi-variograms was not possible due to insufficient data; therefore, the deposit was assumed to be isotropic and the model was fitted only to the directionless ones. The spherical models fitted to the non-directional experimental semi-variograms of AL_2O_3 and SiO_2 are shown in Figure 3.

As shown, the superposition of the variogram model is facing many uncertainties, especially regarding the parameters of the variogram model; therefore, a fuzzy variogram model can be quite helpful. Then it is necessary to use three models for fitting the lower, middle, and upper bound models of an experimental variogram instead of just using one crisp model (Figure 4). Then for each variable, three models with parameters such as the nugget effect, sill, and range would be defined (Table 1). To avoid more complexities, the upper and lower bound models have also been fitted spherically.



(b) (a)

Figure 3. Model fitted to the experimental semi-variograms in zone HB a) Al_2O_3 b) SiO_2.



(b) (a)

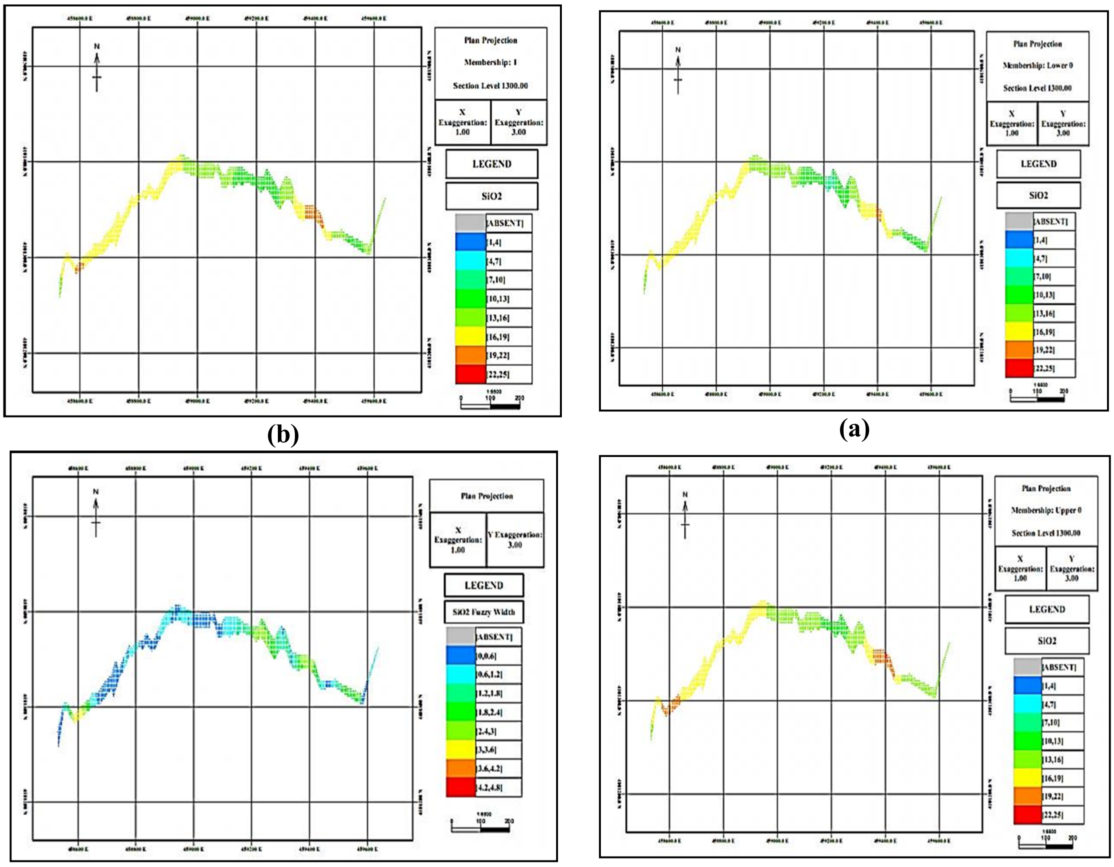
Figure 4. Fuzzy variogram model a) Al_2O_3 b) SiO_2.

3.2. Fuzzy kriging estimation

To implement the fuzzy kriging, use was made of the "FuzzyKrig" program (prepared in the University of Kashan, Iran) [18] in MATLAB R2012a. The inputs to the program included the data gathered from drillholes, block model, fuzzy variogram model, and search ellipsoid parameters. Since this program is capable of including uncertainties related to data and variogram model parameters separately or combined, it is necessary to specify the fuzzy kriging method before performing the program. In this case study, only the variogram model parameters were tainted with uncertainties, and then we used the Bardossy method [3] (presented for crisp data and variogram model). Figure 5 shows the SiO_2 fuzzy grade model in membership degrees of the lower zero and upper zero and the width of fuzzy kriged values at the 1300 m level.

Table 1. Parameters of the fuzzy variogram model.

	SiO_2			Al_2O_3		
	Sill	**Range**	**Nugget effect**	**Sill**	**Range**	**Nugget effect**
Lower limit	6.197	113.54	1.945	11.287	186.995	2.281
Crisp value	10.003	175.189	3.712	20.882	244.709	5.494
Upper limit	14.053	242.086	5.947	36.12	378.5	8.489



(b) (a)

(d) (c)

Figure 5. Fuzzy estimated block model for SiO_2 at 1300 m level a) Lower zero membership grade b) upper one membership grade c) upper zero membership grade d) width of the fuzzy number.

3.3. Fuzzy tonnage–average grade model preparation

The fuzzy tonnage and average grade for Zu2 deposit at different cut-off grades were calculated according to relations 7 and 8. A necessary parameter for calculation of the fuzzy tonnage is the specific gravity, which has been taken equal to 3 for the HB zone of Zu2 deposit. Table 2 shows the fuzzy tonnage and average grade for the zero membership grades at different cut-off grades, and Figure 6 shows the Al_2O_3 tonnage–average grade curve.

As mentioned earlier, a useful property of the fuzzy tonnage–average grade curve is that it makes it possible to extract a triangular fuzzy number both for the tonnage and average grade at the desirable cut-off grades, e.g. at 40% cut-off grade for Al_2O_3, the fuzzy tonnage and average grade will be, respectively, in the ranges of [4263274, 4654240] and [42, 27, 42, 92]; and, at 15% cut-off grade for SiO_2, the ranges will be, respectively, [675897, 949674] and [16, 17, 16, 56]. As an example, Figure 7 shows the triangular fuzzy number related to Al_2O_3 tonnage and average grade at 40% cut-off grade.

Table 2. Lower and upper tonnage and fuzzy average grade at different cut-off grades for zero membership grade.

	Cut-off grade (%)	Average grade (%) Membership value Upper zero	Average grade (%) Membership value Lowe zero	Tonnage (ton) Membership value Upper zero	Tonnage (ton) Membership value Lower zero
Al_2O_3	35	42.64	41.82	4959040	4831433
	40	42.92	42.27	4654240	4263274
	45	46.03	45.79	654170	292854
SiO_2	12	15.29	14.68	1756741	1554703
	15	16.56	16.17	949674	675897
	18	19.28	18.37	152686	34117
Module	1.67	3.02	2.77	350658	322353
	2.67	3.07	2.91	324371	249033
	3.67	3.92	3.67	46370	3810

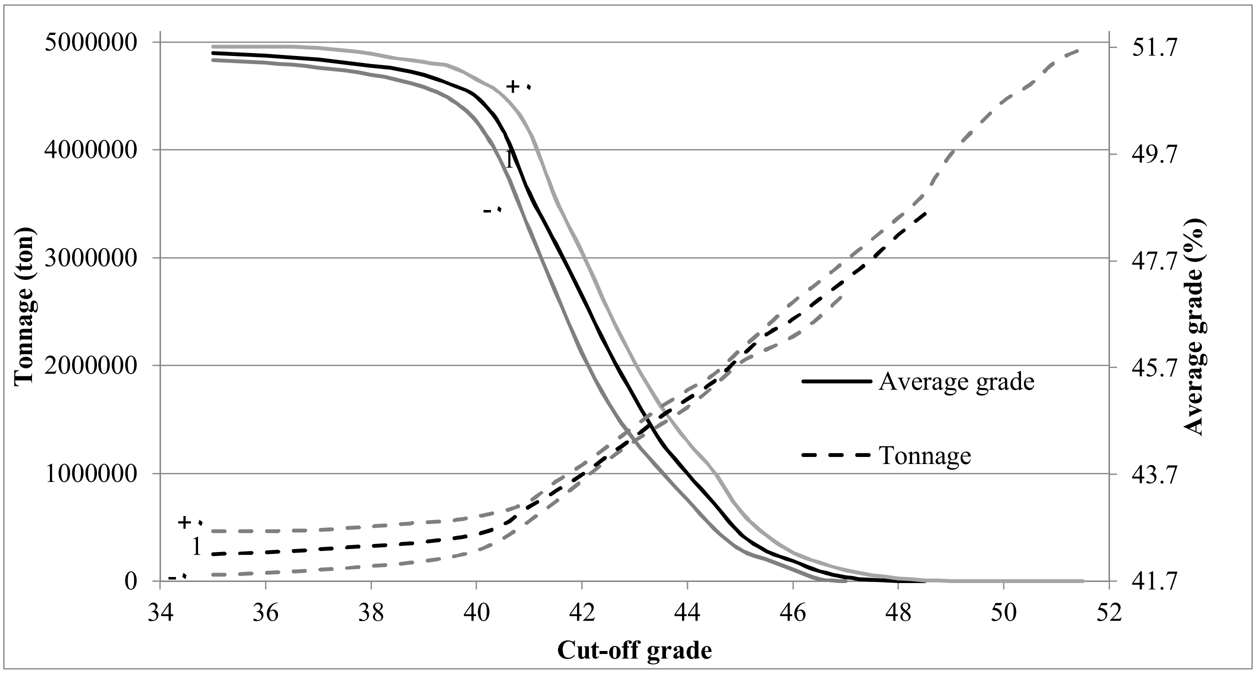
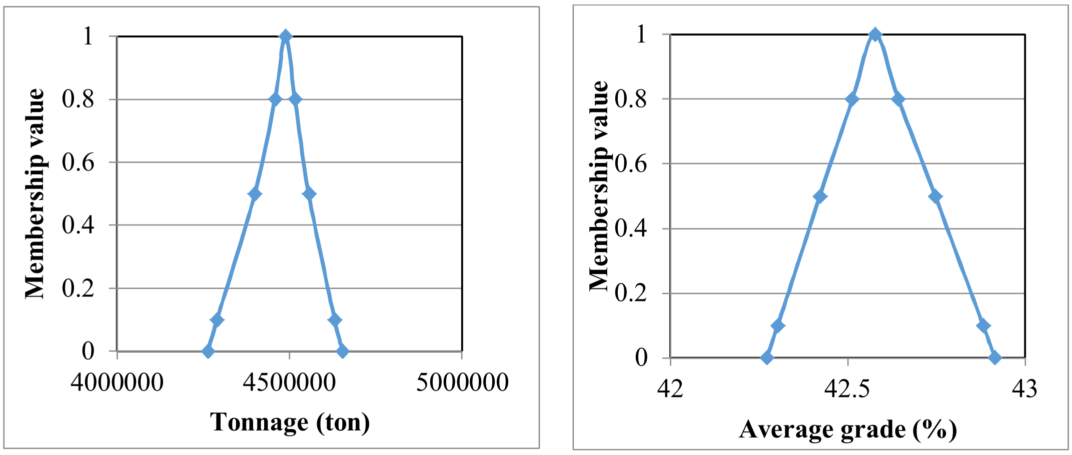


Figure 6. Fuzzy tonnage–average grade curve for Al_2O_3.



(b) (a)

Figure 7. Triangular fuzzy numbers extracted from fuzzy Al_2O_3 tonnage–average grade curve at 40% cut-off grade a) average grade b) tonnage.

4. Conclusions

Due to the nature of the geological and mining activities, different steps in the process of grade estimation and mineral deposit evaluation are always tainted with uncertainties. It is possible to consider and evaluate some of the uncertainties using new methods such as the fuzzy logic. For example, we may investigate uncertainties related to the measurements and variogram model parameters through using the fuzzy kriging instead of the kriging method. There have already been various research studies regarding the fuzzy kriging theory but their main drawback is that they have not used the results in decision-makings and planning. A very common, but key, decision-making tool for mining engineers is using the tonnage–average grade models. Under conditions where measurements lack uncertainties, and the model fitted to the experimental variogram is assumed to be crisp, the tonnage–average grade model will be crisp as well. However, in reality, the data is tainted with uncertainties and the fitted model is not crisp; therefore, it is necessary that the tonnage–average grade curves be used instead of the crisp ones, and the analyses and decision-makings be done accordingly. In this paper, the principles of calculating the fuzzy tonnage–average grade curves, and the results of a case study wherein they have been used, are presented. The proposed fuzzy kriging algorithm was implemented on the Jajarm Zu2 deposit data, the fuzzy grade of every block was estimated, and, based on the fuzzy results obtained, the tonnage–average grade curve was drawn so that the fuzzy tonnages and average grades could be attributed to different cut-off grades. For example, for a 40% cut-off grade for Al_2O_3, 15% for SiO_2 and 2.67 for the module, the tonnages will be, respectively, in the ranges of [4263274, 4654240], [675897, 949674], and [249033, 324371] tons.

In order to determine the optimum mining method, the tonnage, metal content, and average grade must be estimated accurately. When the estimation parameters are tainted with uncertainty, estimated block models and average grade-tonnage models will be tainted with uncertainty, and therefore, it is necessary to determine the magnitude of this uncertainty. Compared to the classical average grade-tonnage models, the proposed approach provides the uncertainty-based format of modeling the relationships between cut-off grade with average grade and tonnage parameters. The uncertainty of variogram parameters in kriging could also be accounted by Bayesian kriging. The problems with a Bayesian approach are that 1) a prior distribution has to be selected and 2) its procedure is time-consuming. The effect of variogram model uncertainty on the average grade-tonnage model could be accounted by results of the Bayesian kriging in the future studies.

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Appendix A: Extension Principle

The "Extension Principle" is one of the main concepts of the theory of fuzzy sets that can be used to extend the crisp mathematical concepts to fuzzy sets. Let us consider a function $f: X \rightarrow Y$ and let F(X) and F(Y) be, respectively, the fuzzy power sets *X* and *Y*. Then for every set $A \in \mathrm{F}(X)$,

$$f: F(X) \rightarrow F(Y)$$
$$A \rightarrow f(A) = \{y: y = f(x) \wedge x \in A\},$$

and the degree of belonging of each value $y \in Y$ to $f(A)$ is given by:

$$\mu_{f(A)}(y) = \begin{cases} \sup\limits_{x, y=f(x)} [\mu_A(x)] & if \quad \exists x: y = f(x) \\ o & if \quad \nexists x: y = f(x) \end{cases}$$

مدل تناژ- عیار متوسط فازی بر اساس اصل گسترش

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چکیده:

به علت ماهیت فعالیت‌های زمین‌شناسی و معدنی، مراحل مختلف فرآیند تخمین عیار و ارزیابی ذخایر معدنی همیشه توأم با انواع مختلف عدم قطعیت می‌باشند. با استفاده از علوم نوینی همچون منطق فازی و ترکیب آن با زمینه‌های متنوع علوم زمین، می‌توان برخی از این عدم قطعیت‌ها را لحاظ کرد. به عنوان نمونه می‌توان عدم قطعیت توأم با عیارسنجی و پارامترهای مدل نیمه‌تغییرنمای برازش یافته را با استفاده از روش فازی کریگینگ به جای کریگینگ مورد بررسی قرار داد. پیش از این، مطالعات نسبتاً متنوعی در زمینه تئوری فازی کریگینگ صورت گرفته، ولی عمده ضعف موجود در مطالعات پیشین را می‌توان به عدم استفاده آن‌ها از نتایج فازی تخمین در تصمیم‌گیری و طراحی‌های متعاقب تخمین نسبت داد. یکی از معمول‌ترین و کلیدی‌ترین ابزارهای تصمیم‌گیری برای مهندسین معدن، استفاده از منحنی‌های تناژ- عیار متوسط می‌باشد. در شرایطی که اندازه‌گیری‌ها فاقد عدم قطعیت باشند و مدل برازش یافته به واریوگرام تجربی قطعی فرض شوند، مدل تناژ- عیار متوسط نیز قطعی خواهد بود. در حالی که در واقعیت هم داده‌ها، توأم با عدم قطعیت هستند و هم مدل برازش یافته قطعی نیست، درنتیجه لازم است تا به جای منحنی تناژ- عیار متوسط قطعی از منحنی‌های تناژ- عیار متوسط فازی استفاده شود و تحلیل‌ها و تصمیم‌گیری‌ها بر اساس آن صورت گیرد. در این پژوهش ضمن ارائه مبانی محاسبات منحنی تناژ- عیار متوسط فازی، یک مطالعه موردی نیز در خصوص استفاده از آن ارائه شده است.

کلمات کلیدی: زمین‌آمار، مدل واریوگرام فازی، تصمیم‌گیری، عدم قطعیت.