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Determination of the largest pit with the non-negative net profit in the open pit mines

J. Gholamnejad[1]*, A.R. Mojahedfar[2]

1. Assistance professor, Department of Mining and Metallurgical engineering, Yazd University, Yazd, Iran
2. M.Sc. student, Department of Mining and Metallurgical engineering, Yazd University, Yazd, Iran

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*Corresponding author: j.gholamnejad@yazduni.ac.ir (J. Gholamnejad).

Abstract
The determination of the Ultimate Pit Limit (UPL) is the first step in the open pit mine planning process. In this stage that parts of the mineral deposit that are economic to mine are determined. There are several mathematical, heuristic and meta-heuristic algorithms to determine UPL. The optimization criterion in these algorithms is maximization of the total profit whilst satisfying the operational requirement of safe wall slopes. In this paper the concept of largest pit with non- negative value is suggested. A mathematical model based on integer programming is then developed to deal with this objective. This model was applied on an iron ore deposit. Results showed that obtained pit with this objective is larger than that of obtained by using net profit maximization and contains more ore, whilst the total net profit of ultimate pit is not negative. This strategy can also increase the life of mine which is in accordance to the sustainable development principals.

Keywords: *Ultimate pit limit, mathematical modeling, integer programming, largest pit*.

1. Introduction
The size, location and final shape of an open pit are important in the planning of the location of waste dumps, stockpiles, processing plant, access roads and other surface facilities and for development of a production program. The pit design also defines minable reserves and associated amount of waste to be removed during the life of the operation. Over the past 45 years the determination of optimum open pits has been on the areas of operational research in the mining industry and many algorithms have been published. Heuristic techniques, Dynamic programming, linear programming, Graph theory and Network flow theory are mathematical methods that have been applied to determine the UPL. The most common optimizing criterion in these algorithms is maximization of the total undiscounted net profit within the designed pit limits subject to mining (access) constraints. The UPL problem does not impose any limitations on the amount of ore tons to be mined during life of mine. This objective function may lead to generate a pit shell excluding a huge amount of ore because of economic considerations. In spite of the importance of economic issues during mining process, especially for strategic deposits, the environmental and social concerns cannot also be neglected. Environmental issues may contains Land use, management and rehabilitation, Solid waste, Water use, Acid mine drainage, Product toxicity, Nuisance, etc. Also social issues may contain Creation of employment, Employee education and skills development, Health and safety, Wealth distribution, Relationship with local communities etc. Some of these issues like job creation and improvement of life quality of people may force the government managers to extend the life of mine in light of economic consideration. Therefore, the ore tonnage that one would like to mine from a deposit can be maximized such that the total net profit is greater than or equal to zero. This goal can guarantee the maximum resource exploitation in light of economic consideration. In this paper a mathematical

model is developed to incorporate this objective.

2. Pit limit optimization

Open pit design is a computer-bused implementation of an algorithm that is applied to a three dimensional block model of an ore body, i.e. a three-dimensional array of identically sized blocks that covers the entered ore-body and sufficient surrounding waste to allow access to the deepest ore blocks. The dimensions of the block vary depending on the operational restrictions, mineralization and initial drill hole spacing. In the block model, each block can be identified by (x, y, z) triplet. Each number indicates corresponding axis value in (x, y, z) triplet. Figure 1 shows an example of a 3D block model for UPL determination.

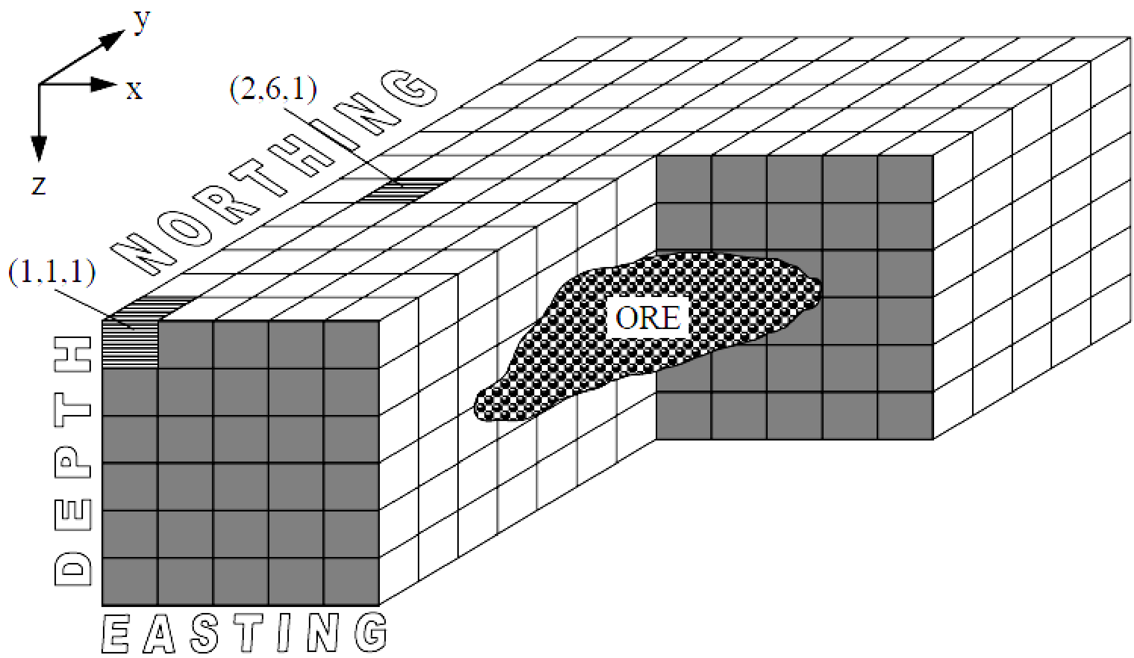

Figure 1. An example of a 3D block model for ultimate pit limits determination [1]

Each block is assigned a value in the context of its use with optimization algorithms. Then a value is estimated for each block. This is done by assuming production and process costs and commodity prices at current economic conditions (i.e. current costs and prices). This value is the net (undiscounted) revenue that would be obtained by mining and treating the block and selling its contents. A block is considered as a possible ore block if its net profit value obtained from mining is positive. Otherwise, the block is considered as a waste block. According to the surface topography many blocks are air blocks and a value of zero is assigned to them.

Optimum pit limit determination is one of the most important steps in mine planning design process. Feasibility analysis, long term production scheduling and the assessment of the capital exposure and corporate risk can be significantly impacted by the results of the optimum pit limit determination. The UPL problem can be defined as determining the final mining limits of a mineral deposit in such a way that some standard of maximum value or profitability is obtained from it's extraction. The standard of profitability is defined as maximizing the difference between the profit obtained from extracted ores and the costs incurred in removing associated waste materials. By using ultimate pit design techniques, economic depth and limit of an open pit can be easily determined. The optimization techniques in designing UPL can be classified into two categories:

- Heuristic techniques (Floating cone, Korobov algorithm, etc.)

- Rigorous techniques (Dynamic programming, Linear programming, Graph theory, Network flow theory, etc.)

Heuristic methods have been one of the most widely used methods for UPL analysis; however, these methods mostly fail to generate true optimal designs. Moving cone method is the most common heuristic technique [2]. An upward cone is made up of slopes corresponding to the restriction on the pit's slopes on each ore block. If the material inside the cone contains a profitable amount of ore, then the material inside the cone is removed. The process is repeated until no more profitable cones of material exist. In spite of its rapid execution speed and easy conceptualization, this method may fail to generate true ultimate pit limits due to mutual support between overlapping cones. Another heuristic method is Korobov algorithm [3]. Korobov algorithm operates by basing a cone on every positive block in the pit and allocating the positive values within the cone against the negative values within the cone until no negative values remain, so that the positive blocks pay for the negative blocks.

There is still no heuristic based method which has been proven to converge to an optimal answer.

Dynamic programming approach [4] as originally defined by Lerchsand Grossmann is able to generate the optimal pit contours in given 2D cross sections , however, three dimensional extensions produce erratic results. Several attempts have been made to extend the original 2D algorithm concepts to the design of truly optimum 3D ultimate pit limits. The algorithms suggested by Johnson & Mickle [5], Johnson and Sharp [6], Konigsberg [7] used the original 2D concept to produce a 3D pit contour, but these methods may yield final pit limits that are far from optimal.

The problem of determining the optimum pit limit with the objective of profit maximization in the form of Linear Programming (LP) can be formulated as:

$$Max \quad \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{O} c_{ijk} X_{ijk} \tag{1}$$

Subject to:

$$X_{ijk} \in \Gamma_{X_{ijk}} \quad \forall \text{ijk} \quad \text{Slope contraints}$$

$$0 \le X_{ijk} \le 1 \quad \text{and integer}$$

where, *ijk* are indices correspond to the row, column and level of blocks in the block model. *N, M and O* are the number of blocksin different direction. c_{ijk} is the net value of block *ijk*, $\Gamma_{X_{ijk}}$ is the set of blocks which must be removed in order to mine block *ijk*. X_{ijk} is a binary variable that is 1 if the block *ijk*is mined, and zero otherwise.

Slope constraints insure that all overlaying blocks to be mined be for mining a given block. There are a two block configurations to define mine slopes, 1:5 block configuration (five overlaying blocks must be removed to mine one) and a 1:9 (nine overlaying blocks must be removed to mine one). If 1:5 pattern is carried up over several levels, an undesirable wall slope will be obtained [8]. Therefore, in this paper a nine-above relationship is used. Figure 2 shows a 1:9 block precedence relationship described. There are two ways of implementing these constraints [9]:

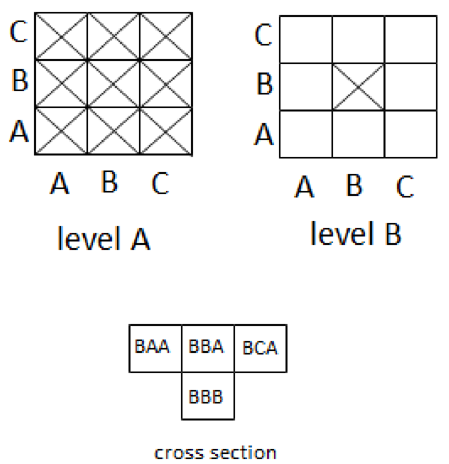

Figure 2. Simple precedence relationships [9]

- ***Using 9 constraints for each block:***

In this method nine separate linear constraints should be written to insure that all nine blocks must be mined first, these equations are:

$$\begin{aligned} &X_{AAA} - X_{BBB} \geq 0 \\ &X_{ABA} - X_{BBB} \geq 0 \\ &X_{ACA} - X_{BBB} \geq 0 \\ &X_{BAA} - X_{BBB} \geq 0 \\ &X_{BBA} - X_{BBB} \geq 0 \\ &X_{BCA} - X_{BBB} \geq 0 \\ &X_{CAA} - X_{BBB} \geq 0 \\ &X_{CBA} - X_{BBB} \geq 0 \\ &X_{CCA} - X_{BBB} \geq 0 \end{aligned} \tag{3}$$

Generally above inequalities can be re-written as:

$$\begin{aligned} &X_{l,m.k-1} - X_{ijk} \geq O \quad l = i-1, i, i+1 \\ &X_{ijk} = 0 \text{ or } 1 \qquad m = j-1, j, j+1 \end{aligned} \tag{4}$$

An advantageous of this formulation is that constraints matrix is totally unimodular, which insures that all X_{ijk} will take on integer values in the optimal solutions; therefore, the integer requirements, $X_{ijk} = o$ or 1, can be eliminated from the formulation [9]. Unfortunately, if other kinds of constraints ore added to this model this property is lost and should be solved by integer programming solution methods.

- ***Using one constraint for each block***

$$\sum_{l=j-1}^{i+1} \sum_{m=j-1}^{j+1} X_{l,m,k-1} - 9X_{ijk} \geq 0 \quad X_{ijk} = 0 \text{ or } 1 \tag{5}$$

The advantage of this formulation is that only one constraint per block instead of nine is incorporated in the modeling procedure, which results in decreasing the model size. In this paper this strategy is applied for UPL modeling.

While Integer programming based models produce optimal solution for ultimate pit limit problems, it cannot be readily implemented for realistically size mines.

Network flow model based on the network theory on maximum flow and minimum cut [10] and also Graph theory of Lerchs & Grossmann (LG) [4] are alternate solution techniques for the linear programming problem. In the network flow analysis, the block model of the deposit is represented by a network. A detailed Analysis of the algorithm can be found in [11] and [12].

The Lerchs & Grossmann 3D Graph Theory algorithm make use of the property of a block model of an open –cut mine that it can be modeled as a weighted directed graph in which the vertices represented blocks and the arcs represented mining restrictions on other blocks. The ultimate pit problem can be cast as one of finding the maximal valued closure of a directed graph. This algorithm will converge to an optimal solution. Zhao and Kim [13] also presented a 3D graph theory oriented algorithm for optimum ultimate pit limit design. They claimed that their algorithm performs much faster than the LG algorithms and produce a true optimal solution to the ultimate pit limit problem.

As mentioned before, all the above algorithms try to find the UPL with the objective of net profit maximization. In the next section a new mathematical algorithm is proposed which produce the largest pit with non-negative value.

3. The largest pit with a non-negative net value

The objective of this model is finding a pit with a non-negative net profit such that the total ore tonnage to be mined from a deposit is maximized. This problem can be formulated as:

Objective function: objective function is set to maximization of the total amount of ore tons extracted. This objective forces the model to extract the whole ore blocks. Therefore, it can be written in the following form:

$$Max \quad \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{O} p_{ijk} X_{ijk} \tag{6}$$

where, p_{ijk} is the tonnage of ore per each block.

Non-negative profit constraints: this constraint insures that the total net profit obtained from extracting the volume of material (ore and waste blocks) is non-negative:

$$\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{O} c_{ijk} X_{ijk} \geq 0 \tag{7}$$

Slope constraints:

$$\begin{aligned}
&X_{i-1,j-1,k-1} - X_{ijk} \leq 0\\
&X_{i-1,j,k-1} - X_{ijk} \leq 0\\
&X_{i,j+1,k-1} - X_{ijk} \leq 0\\
&X_{i,j-1,k-1} - X_{ijk} \leq 0\\
&X_{i-1,j,k-1} - X_{ijk} \leq 0\\
&X_{i-1,j+1,k-1} - X_{ijk} \leq 0\\
&X_{i+1,j,k-1} - X_{ijk} \leq 0\\
&X_{i+1,j+1,k-1} - X_{ijk} \leq 0\\
&X_{i+1,j-1,k-1} - X_{ijk} \leq 0
\end{aligned} \tag{8}$$

Integer constraints:

$$0 \leq X_{ijk} \leq 1 \quad \text{and integer} \tag{9}$$

This problem can be solved by popular integer programming algorithms like Branch and Bound or cutting plane techniques [14].

This problem can also be transformed so that it looks like the ultimate pit limit problem and therefore, it can be solved by one of the ultimate pit limit problem. This can be done by using Lagrangian relaxation method [15]. Lagrangian relaxation is a relaxation technique which works by moving hard constraints into the objective so as to exact a penalty on the objective if they are not satisfied. Transformation is done by multiplying the profit constraint by a Lagrange multiplier, λ, and subtract it from the objective function. Therefore, the Lagrangian form is:

$$\begin{aligned}
&Max \quad \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{O} p_{ijk} X_{ijk}\\
&-\lambda\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{O} c_{ijk} X_{ijk}\\
&=\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{O} (p_{ijk} - \lambda c_{ijk}) X_{ijk}\\
&=\sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{O} d_{ijk} X_{ijk}
\end{aligned} \tag{10}$$

Thus, the final form of objective function is:

$$Max \quad \sum_{i=1}^{N}\sum_{j=1}^{M}\sum_{k=1}^{O} d_{ijk} X_{ijk} \tag{11}$$

Subject to:

- Slope constraints (equations 8)
  $0 \leq X_{ijk} \leq 1 \quad \text{and integer}$

where, $d_{ijk} = p_{ijk} - \lambda c_{ijk}$.

The above formulation is exactly the same as conventional ultimate pit limit problem. Now this problem can be solved using one of the existing ultimate pit limit algorithms like LG. The only difference between these two problems is that parameter λ is incorporated in the objective function. Note that setting $\lambda = 0$, the profit constrain is completely relaxed and the largest pit which contains the whole ore blocks. By increasing the value of λ $(\lambda > 0)$ the total profit is increased until the profit constraint is satisfied. There are a number of ways to modify this multiplier. The simplest way is to change multiplier incrementally by some fixed amount at each iteration until the convergence is reached. Another and more efficient way is subgradient method which was first suggested by Held and Karp [16]. Some applications of this method can be seen in [17] and [18].

4. Application of suggested algorithm in an iron ore deposit

In order to compare the suggested formulation for ultimate pit limit determination with the traditional one, a small iron ore mine is selected as a case study. The block model of this mine contains 33263 blocks with the dimensions of 25m*25m*15m.

In the first step the ultimate pit limit is obtained via traditional objective function (profit maximization) by using SURPAC 6.1.2 software [19] and applying LG algorithm. Figure 3 shows a cross section through the ultimate pit limit. The pit limit is then designed using the proposed model via Surpac software and subgradient method. By changing the λ values in each step, a new economic block model is obtained and then the traditional ultimate pit limit algorithms (using LG algorithm) are usedon each economical block models. This process is repeated until the right λ value, which results in satisfying constraint (7), is obtained. Figure 4 shows a cross section through the ultimate pit limit obtained using suggested method. The overall results of pit optimization using these two objectives are shown in Table 1.

As can be seen in the above case study, using the suggested method results in producing the largest pit whilst its total net profit is positive. This strategy can extend the life of mine and increase the minable ore reserve.

5. Conclusions

In this paper a new criterion was proposed in order to determination the ultimate pit limit. This criterion was set to maximization of ore extraction in such a way that the total net profit obtained from removal of ore and waste material is non-negative and slope constraints are satisfied. Then the proposed mathematical model was transformed so that it looked like the traditional ultimate pit limit problem by using Lagrangian multipliers. By changing this multiplier the largest pit with non-negative value can be obtained. Applying this model on an iron ore deposit showed that resultant pit using suggested algorithm contains moreore whilst the total net profit is still positive. This objective is well suited for strategic natural resource like scarce natural resources deposits (deposits containing uranium, diamonds, especially pure quartz, and the yttria rare earth group), deposits located in territory used by the country's defense sector and large deposits. This strategy can increase the life of open pit and minable ore reserve. Also this objective is more consistent with the social issues of sustainable development principals.

Table 1. The results of pit optimization using traditional and suggested methods.

Pit characteristics	Total No. of blocks	No. of ore blocks	No. of waste blocks	Total net profit (*10^6)
Traditional model	809	532	277	4.3
Suggested model	1630	1146	484	1.6

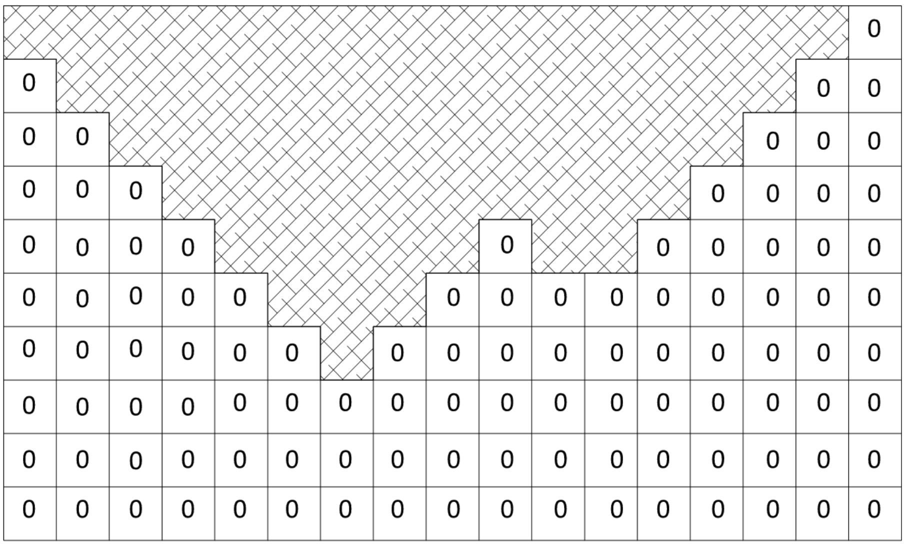

Figure 3. A cross section of final pit using the traditional objective function

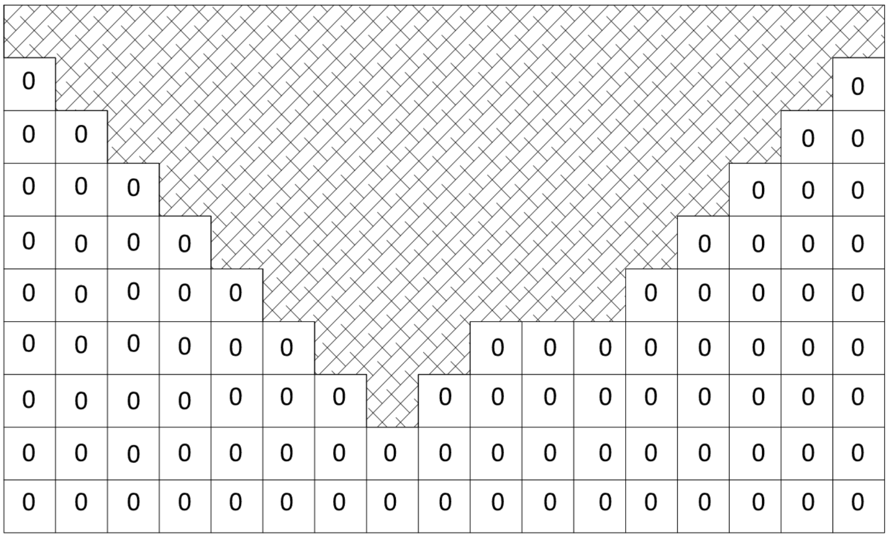

Figure 4. A cross section of final pit using the suggested objective function

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