



## A Hybrid Fuzzy MCDM Approach to Determine an Optimal Block Size in Open-Pit Mine Modeling: a Case Study

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### Abstract

The computer-based 3D modeling of ore bodies is one of the most important steps in the resource estimation, grade determination, and production scheduling of open-pit mines. In the modeling phase, the volume of the orebody model is required to be filled by the blocks and sub-blocks. The determination of Block Size (BS) is important due to the dependence of the geostatistical issues and calculations related to mining capabilities on it. There are some factors effective in the determination of an optimal BS including the metal content, estimation error, recovery percentage, mining ability, safety, and dilution. In this work, an optimal BS is determined using a two-stage approach. In the proposed approach, the Fuzzy Delphi Analytic Hierarchy Process (FDAHP) and Fuzzy Multi-Objective Optimization by Ratio Analysis (FMOORA) methods are used. In the first phase, the weight of each criterion is calculated based on the opinions of the experts using the FDAHP method. In the second phase, the FMOORA method is applied in order to determine a suitable BS for the design and operation of mining considering the extracted weights in the previous phase. The block model of the Sungun copper mine is studied as a case study to evaluate the capability of the proposed approach. The results of implementation of this approach are desirable because of converting the opinions of the experts to fuzzy values, weighing the experts according to the experience and technical knowledge, weighting the criteria by FDAHP, and choosing the optimal option with FMOORA. Furthermore, the 12.5×12.5×12.5 m<sup>3</sup> block (A5) is chosen as an appropriate BS, which is compatible with the real conditions of the studied mine.

### 1. Introduction

The preparation of a block model of mineral reserves is one of the most important stages in the implementation of modeling due to the impact of block size (BS) on various exploitation and exploration parameters. In order to perform various phases of mine design and planning, a block model of the ore and waste around it should be created. In fact, the mineral resource estimation and mine production scheduling are the main purposes of a mine design process. The utilization of 3D computer-based models is inevitable to achieve these goals [1]. In the recent years, the usage of computers in economic calculations and production planning of mines has caused the

creation of computerized models of orebodies. The role of this process is significant in the mining-related design fields [2]. It is possible to plan the production, mining and blending of ores extracted from different parts of a deposit with a sufficient accuracy by relying on the precise 3D models of mineral resources. Several steps are taken to carry out modeling of mineral deposits. This process begins with the preparation of a geological model and continues with the extraction engineering phases. A main step of the orebody modeling is to make a block model that is divided into small cubic blocks. The size of the blocks and their shape have a great impact on the

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engineering calculations and they ensure the accuracy of a modeling [3]. The design and production scheduling of open-pit mines, which is a multi-variable dynamic process, could be summarized into the following steps: a) creating a 3D block model of the ore body, b) optimizing the pit boundaries (determination of the ultimate pit limit), c) designing the push-backs, d) scheduling the mine production.

Generally, determination of a suitable BS is one of the most effective factors involved in the production scheduling and an optimum mining operation in open-pit mines [4]. Many geostatistical, technical, environmental and economic components are impressive in determining BS. The block models are the main inputs of the pit optimization algorithms. These algorithms try to make a list of blocks whose extraction has a maximum revenue and an adequate safety. BS depends on the distance between the exploration boreholes and Selective Mining Unit (SMU). As a general and preliminary method, BS in a horizontal direction of the orebody should be more than 1/4 of the average distance between the exploration boreholes, and this distance should not be shorter than 1/3 of the distance between two exploration boreholes [5].

Also the height of the blocks is limited by the height of the working benches of the open-pit mine.

In the modeling process, the overall cut-off grade is used to make a distinction between the boundary of the ore and the waste [6]. In the first stage of a block modeling, a mine designer makes a great rectangular block model entitled “Main Block” or “Mother Block” (MB). The volume of MB for each deposit is greater than the volume of the orebody and all the deposit volume is surrounded by MB. Also MB is divided into the blocks and sub-blocks. There are several types of block models but the conventional model is a 3D block model with a constant size in three dimensions of blocks. In this method, each block in the modeling space is distinguished by the (x,y,z) coordinates. Also according to the results obtained from the geostatistical studies and variography analyses of the 3D models, which are obtained from the “assay” file used in modeling, the cubic form of blocks with a constant size in three dimensions is a suitable form of blocks (Figure 1). The utilization of cubic blocks culminating in the estimation variance in three dimensions is constant and the variogram studies have been accomplished exactly [7].

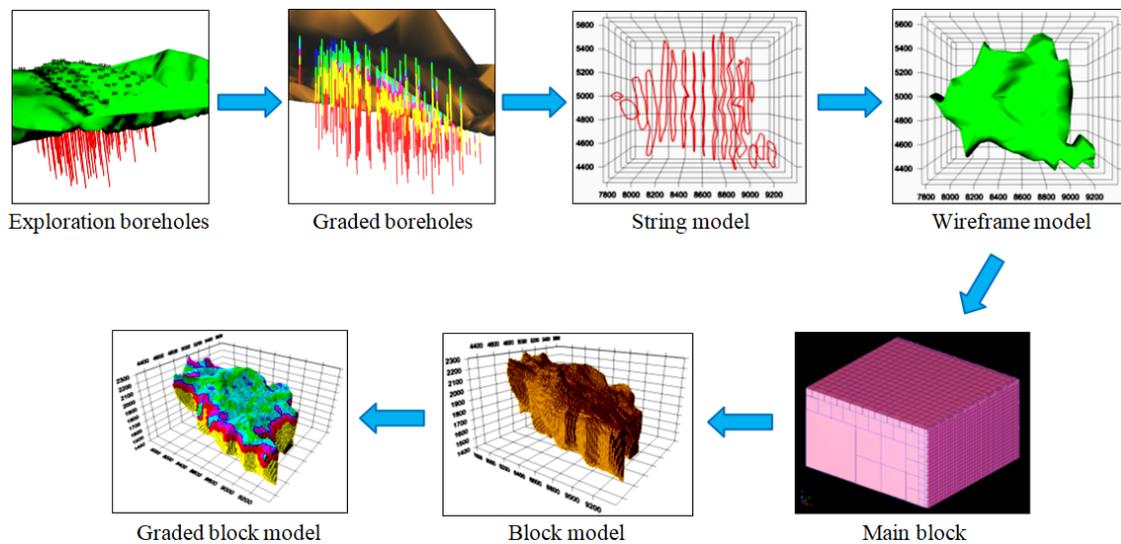


Figure 1. Steps of 3D modeling of an orebody.

BS in the block model depends on the mining equipment, topography of open-pit mine and shape of the mineral resource. Also the volume of the available data for block grade estimation by geostatistical methods affects BS. If BS is very large, the calculation time is but the ore/waste boundary and the grade change ability of the deposit is not performed exactly. On the other

hand, a very small BS leads to an increase in the computer storage for saving data and the calculation time. Also if BS is much smaller than the exploration network, the estimation errors increased and the variance of the estimated data is decreased artificially [8]. The grade estimation of the ore in smaller blocks is far more difficult than the larger blocks since bases with a larger size

have a lower variability [4]. More over, the high grade distribution in a deposit leads to a less accuracy of grade estimation [9]. In addition to the above studies, one of the main issues in solving the block sequencing problem of open-pit mines is BS [10]. Several studies have been done to propose a methodology in this field. Also in the past years, selection of BS has been implemented for the computer-based design methods of open-pit mines by using the experience of mining engineers and managers.

In the current work, a novel hybrid approach is proposed due to the importance of BS in the open-cast mine design process. This approach has been presented based on the Multi-Criteria Decision-Making (MCDM) methods and the fuzzy multi-factorial technique for selection of BS. In fact, the determination of an optimal BS has been performed using a combination of the MCDM methods and the fuzzy Delphi theories. This approach is a two-stage one based on the Fuzzy Delphi Analytical Hierarchy Process (FDAHP) and Fuzzy Multi-Objective Optimization by Ratio Analysis (FMOORA) methods. In the first phase, the weight of each criterion is calculated with the opinions of the experts using the FDAHP method. The reason for using FDAHP is its favorable and consistent results in engineering analyses [11-14]. Also this method has a high capability in solving the problems related to the technical-managerial analyses and engineering choices. Also it should be noted that the implementation steps of this method are very simple and do not require complex calculations. In this sense, in addition to getting the opinions of the experts, the outputs of this method have a significant adaptation to real conditions by converting the definite inputs into the fuzzy data and performing a computational process. In the second phase, the FMOORA method is used to prioritize and select the optimal dimensions. According to the engineering studies conducted on MOORA and FMOORA, it is a useful method for solving the MCDM problems in the engineering science [15, 16]. The ability of this method is significant in engineering rankings and selection of an optimal option [17]. In the proposed approach, all the benefits of Analytical Hierarchy Process (AHP), Delphi, and MOORA methods, and the fuzzy theory are combined and the disadvantages of any of these methods including the lack of compatibility of the results with the actual conditions, application of qualitative comments by experts to prioritize, weighting each expert based on the amount of expertise and experience, etc. are solved.

This work is organized as what follows. In Section 2, an overview of the previous studies relative to the BS selection in open-pit mining is presented. Section 3 gives a general overview of the parameters affecting BS. Section 4 reviews the methodology of fuzzy sets, FDAHP and FMOORA. The proposed approach of study is presented in Section 5. The case study is introduced in Section 6. The results and discussions of the work are presented in Section 7. Finally, Section 8 concludes the paper.

## 2. Literature review

Determination of BS in the 3D models of mineral deposits is the ultimate goal of the mining engineering studies. In this section, the most important studies on the selection of an optimal BS in open-pit mines are reviewed. Leuangthong et al. [18] have used BS in selecting the optimal selective mining unit size in open-pit mines. This research work shows that BS is an effective factor in the production scheduling. Also Jara et al. [10] have studied the impact of BS on the open-pit mine design and production planning. The results of this study indicated that when BS increased, the ore tonnage increased and the average grade decreased. Also the discounted cash flow was lower as BS increased. On the other hand, the loss of selectivity is more important when passing from a model with blocks of  $2.5 \times 2.5 \times 2 \text{ m}^3$  to those of  $5 \times 5 \times 4 \text{ m}^3$ . In another study, Huang et al. [19] have created a 3D model of the Cangshang gold mine based on the Surpac software. In the modeling process, first, the wireframe model of the orebody was made, and then the block model was created based on the wireframe model; in this step of the modeling process, BS had an important role. Birch [20] has presented a new system for geological modelling based on the block modelling and the studied impact of BS in a 3D model of the orebody. Also Moharaj and Wangmo [21] have used computerized 3D modeling in comparison with different reserve estimation techniques and analyzed BS as an important parameter. In another study, Hayati et al. [4] have determined the optimal BS of the Angouran Pb-Zn mine in Iran. They selected the 10-m blocks as the best option for this mine using the VIKOR method. More recently, Sirelda & Resmi [7] have studied the 3D modeling of mineral reserves and introduced a new effective solution for optimization of the geological and mining works. Chanderman [22] has presented a geological block model and estimated the mineral resource for the FE2 gold deposit in Mali. In addition, the

3D modeling of orebodies is a great part of the engineering studies. For example, David [23,24] has studied BS in the block models using the advanced geostatistical approaches.

Some of the most important applications of the MCDM methods in solving the important mining engineering problems have been mentioned below. In a study, Mikaeil et al. [25] have predicted vibration during rock sawing using the fuzzy AHP process. Also the usage of fuzzy AHP approach in ranking the sawability of carbonate rocks has been presented by Ataei et al. [26]. In another study, Mikaeil et al. [27] have ranked the sawability of carbonate rocks by a combined Fuzzy AHP (FAHP)-TOPSIS approach. Naghadehi et al. [28] have presented an application of the FAHP approach in selecting the suitable and optimal underground mining method for a bauxite mine in Iran. The FAHP method as a fuzzy MCDM method has been used to define an optimum post-mining land around a pit area by Bangian et al. [29] to clarify the reclamation costs. In another study, Mikaeil et al. [30] have selected the optimal underground mining method using an integrated FAHP-TOPSIS approach by providing a decision support system. Bangian et al. [31] have chosen the optimal area for the reclamation of the open-pit mines using the fuzzy MADM modeling.

On the other hand, the applied MCDM methods in the solution of technical, economic, and statistical problems of mining engineering problems are other branch of the studies. Using the methods in selection of the mining method [32, 33], location problems [34], open-cast mining equipment [35], tunnel supporting system [36], etc. show that these methods have the most favorable results in the selection and determination problems of mining. Also using a combination of the MCDM methods have had suitable results [37-41]. In these studies, the problems of block extraction sequence and open-cast mining planning have been considered, and regarding the choice of the optimal BS in the open-pit research mines with a multi-stage approach it has not been done. Also according to the previous studies, it is obvious that the use of fuzzy theory in the subject matter has not been taken into consideration by the researchers. The refore, considering the desired results of the fuzzy multi-stage approaches and the accuracy of the results of studies in other engineering fields, a fuzzy two-stage approach can be used in the selection of the optimal BS in the modeling of open-pit mines.

### **3. Effective parameters on block size**

There are many parameters that affect BS in the orebody's 3D block model. BS is affected by a variety of factors whose exact identification and how they affect the improvement of the blocking process will be of great assistance to an engineering team. BS in the metal reserves is controlled by two groups of statistical and exploitation parameters. Based on the geostatistical parameters, a suitable BS is equal to half the distance between the dimensions of the exploration network. On the other hand, BS is heavily influenced by the extractive features and designs and must be selected in such a way that it can respond to mining plans. Given the size of benches in open-pit mining, BS must be in accordance with the height of the working benches of the mine [42]. At first, various parameters are collected based on a questionnaire. Then using the opinions of the active experts in mining engineering (mine managers, mining industry experts, and mine colleges masters), the most influential factors in determining BS are identified. The criteria affecting BS in open-pit mine modeling include the metal content, estimation error, recovery percentage, mining-ability, safety, and dilution.

#### **3.1. Metal content**

One of the most applicable factors in the orebody's 3D modeling is the metal content of each block. Determination of the mass of metal in each block is done based on calculation of the mining economics. The by-products in each block are very important in the modeling of mineral deposits (e.g. silver in the gold deposits or gold in the copper deposits). Also the by-product extraction in mines can cover a main part of the mining operation costs of the main mineral/metal [43]. The value of metal in each block can be calculated more precisely by reducing BS.

#### **3.2. Estimation error**

The estimation error in the geostatistical analysis of the orebody's 3D modeling is used as one of the main indices [44] that has an effective role in the determination of BS. In other words, the estimation error is calculated in the geostatistical studies of the modeling process by the mine design soft wares such as Micromine, Datamine, and Surpac [45]. The estimation error is used when one of the kriging methods (e.g. point, block, ordinary, simple, linear, and universal kriging) is used to estimate the engineering parameters (such as grade in the mineral resource

modeling). Kriging is the best linear unbiased estimator that can also calculate the best linear weighted average of a piece of ground (or a block of mine). Also Kriging is an estimator that can calculate the minimum error estimation of each point separately, which is very important in the 3D modeling of mineral reserves and can be used to determine the statistical confidence interval [46].

### 3.3. Recovery percentage

The value of the metal absorbed and extracted during mineral processing operations is called “recovery”. The recovery value indicates that more metal is extracted during the processing process with increase in the amount of this factor [47]. This factor is directly related to BS. Also the recovery can be controlled more and better with smaller block sizes. In order to put it differently, a more accurate evaluation of the process recovery is possible with a smaller size of the mine blocks.

### 3.4. Mining-Ability

The mining-ability parameters including the capacity of loader equipment, geomechanical problems, mining operation risks, and technical-economic factors play a major part in the determination of BS. Thus this criterion is the most effective parameter in the selection of BS. On the other hand, according to the qualitative form of the mining-ability, this criterion is ranked with a 1-9 scale of Saaty [48] for different dimensions of BS. The ranking is based on the geomechanical conditions, economical problems, annual production rate and mining equipment [49].

### 3.5. Safety

By increasing BS, management and control of the safety are reduced. On the opposite side, in the case of the abnormal reduction in BS, the safety is decreased sharply due to the increase in the number of blocks. The reason for this is that the effect of discontinuities (joints, faults, etc.) on smaller blocks is more than the blocks with a large size [50].

### 3.6. Dilution

Dilution is the mixing waste with the ore. As a result of dilution, the quality of the mineral (grade) is decreased and its quantity (tonnage) is increased [51]. Since the profitability of mining is sensitive to grade changes, dilution is a critical variable in the evaluation of deposits, and also dilution is one of the most important factors

involved in the mining project economics. This factor is closely related to the recovery rate so that the recovery is decreased with increase in the amount of dilution. Dilution increases the processing plant operating costs because the tonnage of ore entering the crushing unit is increased with this factor [52]. This factor causes significant changes in the factors that reduce the overall project value over a long term and it has a direct impact on the short-term incomes of a mine. For example, it increases the life of the mine, which is also due to the reduction in the effective capacity of the crushing operation. It also reduces the grade of the feed sent to the processing plant. In most cases, a low feed grade means a low crushing recovery. Also dilution reduces the cut-off grade, which reduces the optimum use of the ore [53].

## 4. Methods and concepts used

In this work, the FDAHP and FMOORA methods were used in the proposed two-stage approach, respectively. In accordance with this approach, the weight of criteria was calculated using FDAHP and used as a part of the inputs of the FMOORA method. In the following, the theoretical discussion of these methods is described briefly.

### 4.1. Fuzzy sets

The theory of “fuzzy sets”, which has been proposed by Zadeh [54] in 1965, has been introduced in order to enable one to solve decision-making problems in the uncertainty environment and the ease of carrying out complex real-world calculations. The uncertainty in decision-making is due to the necessity of applying the opinions of the experts as the initial data on engineering decisions. The linguistic propositions and variables that are considered by the experts (usually expressed qualitatively) should be quantified and must be applied in a decision-making process. Therefore, replacement of the definite values (numbers) with the fuzzy values (fuzzy numbers) has provided different fuzzy MCDM methods, which will make the results of any decision-making method more consistent with the actual circumstances and increase the efficiency of the method.

If the range of  $\{0, 1\}$  is converted to interval  $[0, 1]$ , then the crisp set is converted to a fuzzy set. In other words, assuming the universal set  $U$ , the fuzzy set  $A$  in  $U$  is defined as:

$$A:U \rightarrow [0,1] \quad ; \quad A(u) \in [0,1] \quad (1)$$

Also the fuzzy set  $A$  can be represented as follows:

$$A = \left\{ \frac{\mu_A(x_1)}{x_1}, \frac{\mu_A(x_2)}{x_2}, \dots, \frac{\mu_A(x_n)}{x_n} \right\} \quad (2)$$

where  $\mu_A(x_i)$  is the membership degree of  $X_i$  in the set  $A$ , which is varied in the interval of  $[0, 1]$  [55].

According to the fuzzy theory, the definite sets are replaced by the membership degree sets. Also the use of fuzzy numbers in fuzzy calculations has led to the study of different types of fuzzy numbers by the researchers. The fuzzy numbers may be expressed in the triangular, trapezoidal or other types. The utilization of triangular numbers is common in solving various engineering problems with the MCDM methods because the decision-makers can easily express the linguistic variables with a greater consistency with the existing facts when they use these types of numbers.

#### 4.2. Fuzzy delphi analytical hierarchy process

The Delphi technique creates a group communication process in a way that the process involves independent components when the researchers try to solve complex problems [56]. Due to the multiple interactions between the experts, the Delphi technique has a high level of richness than the scrolling methods. Some researchers use this method for cases where the judgment and vote information are important, which is typically done using a series of questionnaires with feedback controls [57]. The purpose of these questionnaires and the aggregation of their feedback provide a more limited dispersion of the experts' opinions. On the other hand, this technique has executive disabilities because of the high execution cost of the Delphi as well as the low convergence of the experts' opinions in some cases [58]. In order to improve the traditional Delphi technique, the fuzzy logic can be considered. Accordingly, the Fuzzy Delphi (FD) technique was developed by Kaufman and Gupta in 1980s [59]. In the Delphi technique, the predictions and opinions are provided by the experts in the form of definite numbers, while in the long run, these predictions lose their value. On the other hand, the experts and analysts, who are in favor of the Delphi technique, use some predictions based on their mental assumptions and their perceived abilities.

Thus the uncertainty in these predictions exists and this leads to the usage of fuzzy sets in the Delphi technique. Additionally, FDAHP is a combination of a hierarchical analysis processes with FD.

AHP is an approach that has been developed to deal with complex systems and leads to the decisions to choose among multiple options and compare them together [48]. This method simplifies the complicated and faulty structures by arranging indicators and decision options in a hierarchical structure and with the aid of a series of pairwise comparisons. Analysis of the conventional hierarchy is problematic due to the use of definitive amounts to reflect the decision-makers' comparison of alternatives [60]. In addition, the AHP method is often criticized for using an unbalanced scale in judgments and the inability to manage the uncertainty and inherent inaccuracy in the paired comparison process [61]. In order to overcome all these shortcomings, FDAHP has been created to solve the hierarchical issues. Decision-makers usually find that they can achieve more certainty by providing a range of judgments rather than their constant values. Therefore, an FDAHP is a combination of a hierarchical analysis process with FD. Although FDAHP is a developed method for decision-making, it can also be used in the determination of the weights of criteria. The FD technique is based on the experiences and opinions of the experts in a specific field. Therefore, the results obtained from this method can be suitable for evaluating the importance of the parameters affecting a phenomenon and a concept.

The process of implementation of FDAHP consists of several main steps [62]. After the preliminary step including a survey of the experts in the form of qualitative or quantitative questionnaires, the fuzzy number calculation is based on the results of the survey. In this work, the triangular fuzzy numbers are defined as follow:

$$\tilde{a}_{ij} = (\alpha_{ij}, \delta_{ij}, \gamma_{ij}) \quad (3)$$

$$\alpha_{ij} = \min(\beta_{ijk}) \quad , \quad k = 1, 2, \dots, n \quad (4)$$

$$\delta_{ij} = \left( \prod_{k=1}^n \beta_{ijk} \right)^{\frac{1}{n}} \quad , \quad k = 1, 2, \dots, n \quad (5)$$

$$\gamma_{ij} = \max(\beta_{ijk}) \quad , \quad k = 1, 2, \dots, n \quad (6)$$

where  $\alpha_{ij} \leq \delta_{ij} \leq \gamma_{ij}$  and  $\beta_{ijk}$  indicate the relative importance of  $i$  on  $j$  from the viewpoint

of the  $k^{th}$  expert. Also  $\gamma_{ij}$  and  $\alpha_{ij}$  demonstrate the upper and lower bounds of fuzzy number ( $\tilde{\alpha}_{ij}$ ), respectively (Figure2).

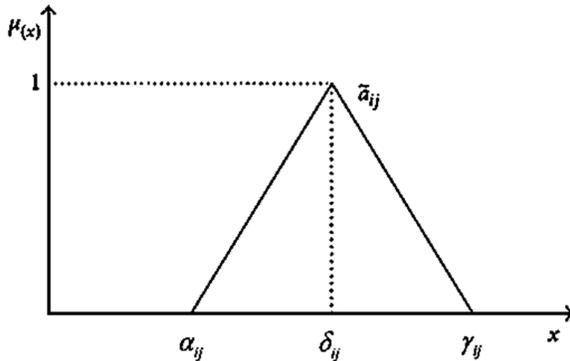


Figure2. A triangular fuzzy membership function [13].

After formation of the above fuzzy numbers, the matrix of the fuzzy pairwise comparison is composed of the following components:

$$\tilde{A} = [\tilde{\alpha}_{ij}] \quad , \quad \tilde{\alpha}_{ij} \times \tilde{\alpha}_{ji} \approx 1 \quad , \quad \forall i, j = 1, 2, \dots, n \quad (7)$$

Another representation of this matrix is as follows:

$$\tilde{A} = \begin{pmatrix} (1,1,1) & (\alpha_{12}, \delta_{12}, \gamma_{12}) & (\alpha_{13}, \delta_{13}, \gamma_{13}) \\ \left(\frac{1}{\alpha_{21}}, \frac{1}{\delta_{21}}, \frac{1}{\gamma_{21}}\right) & (1,1,1) & (\alpha_{23}, \delta_{23}, \gamma_{23}) \\ \left(\frac{1}{\alpha_{31}}, \frac{1}{\delta_{31}}, \frac{1}{\gamma_{31}}\right) & \left(\frac{1}{\alpha_{32}}, \frac{1}{\delta_{32}}, \frac{1}{\gamma_{32}}\right) & (1,1,1) \end{pmatrix} \quad (8)$$

The relative fuzzy weight of the parameters is also calculated using Equations 9 and 10:

$$\tilde{Z}_i = [\tilde{\alpha}_{ij} \otimes \dots \otimes \tilde{\alpha}_{in}]^{\frac{1}{n}} \quad (9)$$

$$\tilde{W}_i = \tilde{Z}_i \otimes [\tilde{Z}_i \oplus \dots \oplus \tilde{Z}_n]^{-1} \quad (10)$$

where  $\oplus$  and  $\otimes$  denote addition and multiplication of the numbers in the fuzzy environment, respectively; it means that  $\tilde{\alpha}_1 \otimes \tilde{\alpha}_2 = (\alpha_1 \times \alpha_2, \delta_1 \times \delta_2, \gamma_1 \times \gamma_2)$ .  $\tilde{W}_i$  is a row vector that shows the fuzzy weight of the  $i^{th}$  parameter. Finally, the geometric mean of the parameter weight ( $\tilde{W}_i$ ) is obtained in the form of a definite number in order to defuzzify the weight of parameters:

$$\tilde{w}_i = \left( \prod_{j=1}^3 W_{ij} \right)^{\frac{1}{3}} \quad (11)$$

### 4.3. Fuzzy multi-objective optimization by ratio Analysis

The Multi-Objective Optimization by Ratio Analysis (MOORA), presented by Brauers & Zavadskas [62], is an MCDM method that has a high level of comprehensive evaluation of alternatives that are faced with a wide variety of factors. Due to the diversity of criteria in engineering decisions, a method should be used that matches the rationality with the decision-makers with an engineering vision. In order to optimize the problem, the satisfaction of the decision should be considered. The MOORA method, like other Multi-Purpose Optimization (MPO) methods, has many applications in solving various complex decision-making problems in the engineering field [16]. The MOORA method as well as the other MPO methods is used to effectively address a variety of scientific issues. Due to the greater compatibility of the Fuzzy MOORA (FMOORA) with real conditions, this method has also been considered by the researchers. In the following, the process steps of the FMOORA method are described:

**a. Identifying the selected alternatives and criteria related to the problem and formulate a decision matrix using fuzzy numbers:** In this matrix, all scores for each alternative are shown based on each criterion [63].

$$\tilde{X} = \begin{pmatrix} (x_{11}^l, x_{11}^m, x_{11}^u) & (x_{12}^l, x_{12}^m, x_{12}^u) & \dots & (x_{1n}^l, x_{1n}^m, x_{1n}^u) \\ \vdots & \vdots & \vdots & \vdots \\ (x_{m1}^l, x_{m1}^m, x_{m1}^u) & (x_{m2}^l, x_{m2}^m, x_{m2}^u) & \dots & (x_{mn}^l, x_{mn}^m, x_{mn}^u) \end{pmatrix} \quad (12)$$

where  $m$  is the number of alternatives,  $n$  is the number of criteria, and  $x_{mn}$  is the value of the  $m^{th}$  option of the  $n^{th}$  criterion (function size).

**b. Normalizing the decision matrix:** The decision matrix formed must be normalized so that all its layers are dimensionless and comparable. The normalization process causes the matrix to have the correct form and helps to form a more comparable structure [64].

$$\tilde{X}_{ij}^* = (x_{ij}^{l*}, x_{ij}^{m*}, x_{ij}^{u*}) \quad \text{and} \quad \forall i, j : \quad (13)$$

$$x_{ij}^{l*} = \frac{x_{ij}^l}{\sqrt{\sum_{i=1}^m [(x_{ij}^l)^2 + (x_{ij}^m)^2 + (x_{ij}^u)^2]}} \quad (14)$$

$$x_{ij}^{m*} = \frac{x_{ij}^m}{\sqrt{\sum_{i=1}^m [(x_{ij}^l)^2 + (x_{ij}^m)^2 + (x_{ij}^u)^2]}} \quad (15)$$

$$x_{ij}^{u*} = \frac{x_{ij}^u}{\sqrt{\sum_{i=1}^m [(x_{ij}^l)^2 + (x_{ij}^m)^2 + (x_{ij}^u)^2]}} \quad (16)$$

**c. Applying the normalized weights of each criterion in the normalized matrix:** The fuzzy weight of each criterion ( $\tilde{W}$ ), which is derived from each one of the fuzzy weight calculation methods (FAHP, FDAHP, etc.), is normalized in the normalized fuzzy decision matrix after normalization [63]:

$$\tilde{v}_{ij} = (v_{ij}^l, v_{ij}^m, v_{ij}^u); \quad (17)$$

$$v_{ij}^l = w_j \times x_{ij}^{l*} \quad (18)$$

$$v_{ij}^m = w_j \times x_{ij}^{m*} \quad (19)$$

$$v_{ij}^u = w_j \times x_{ij}^{u*} \quad (20)$$

**d. Calculating  $\tilde{y}_i$ :** The normalized performance values are calculated by reducing the undesirable criteria of the total desirable criteria (that are determined by the type of problem) [64]:

$$\tilde{y}_i = \sum_{j=1}^g \tilde{v}_{ij} - \sum_{j=g+1}^n \tilde{v}_{ij} \quad (21)$$

where  $\sum_{j=1}^g \tilde{v}_{ij}$  is the total performance of the desired criteria,  $\sum_{j=g+1}^n \tilde{v}_{ij}$  is the total performance of the undesirable criteria,  $g$  is the maximum number of criteria, and  $(n-g)$  is the minimum number of criteria.

**e. Defuzzifying the values of functions:** Since the normalized performance values are fuzzy numbers, these values should be transformed to the performance values that are not fuzzy (Best Non-fuzzy Performance/BNP). In this work, Equation 22 is used to calculate the BNP values of fuzzy triangular numbers, which are  $\tilde{y}_i = (y_i^l, y_i^m, y_i^u)$  [64]:

$$BNP_i(y_i) = \frac{(y_i^u - y_i^l) + (y_i^m + y_i^l)}{3} + y_i^l \quad (22)$$

**f. Calculating  $y_i$ , sorting alternatives based on  $y_i$  values and selecting the best alternative:** The calculated  $y_i$  values are ranked from the highest to the smallest, respectively and an assessment is made between the alternatives. Depending on the total values (desirable criteria) and the minimum

values (undesirable criteria) in the decision matrix,  $y_i$  may be positive or negative. When  $y_i$  is arranged in a descending order, the final ranking is determined. Thus the best alternative has the highest  $y_i$ , while the worst alternative has the lowest  $y_i$ .

### 5. Proposed approach

In this work, a hybrid decision-making approach based on the FDAHP and FMOORA methods is proposed in order to solve the studied problem. In addition, fuzzy numbers are used to provide a greater compatibility with the proposed algorithm's performance. At first, the parameters that affect the selection of the optimal BS are identified by the experts. Subsequently, the weighting process of these criteria is done using the FDAHP method. The experts in this work comprise mining engineers, masters of mining engineering colleges, and open-pit mines managers. Their comments were in response to the questionnaires the researchers gave them. Regarding the questionnaire, it should be noted that the parameters were first extracted from the library studies. After filtering the parameters, the effective factors were selected. Then the questionnaires of effective selection criteria were prepared and provided to 15 academic masters from the Urmia University, Urmia University of Technology, University of Tehran, and Iran's open-pit mine experts. In the following, the results of the completed questionnaires were evaluated. To In order to validate the results of the questionnaires, the Cronbach's alpha test was used because it specified the reliability of the questionnaires [65]. The Cronbach's alpha value in this work was 0.82, which was confirmed.

The fuzzy ranking method was used to give the weights given to the realistic conditions. To this end, the experts' comments were used as fuzzy numbers. Then the results were calculated using the FDAHP method. The fuzzy weight of each criterion was entered into the second phase of the calculation (input of the FMOORA method). According to this method, after forming the fuzzy decision matrix and normalizing it, the weight of the criteria is applied in the normalized decision matrix. Then the  $y_i$  values are calculated and the ranking of options is performed after defuzzifying this score. The work flow of the proposed approach used in the current work is shown in Figure 3.

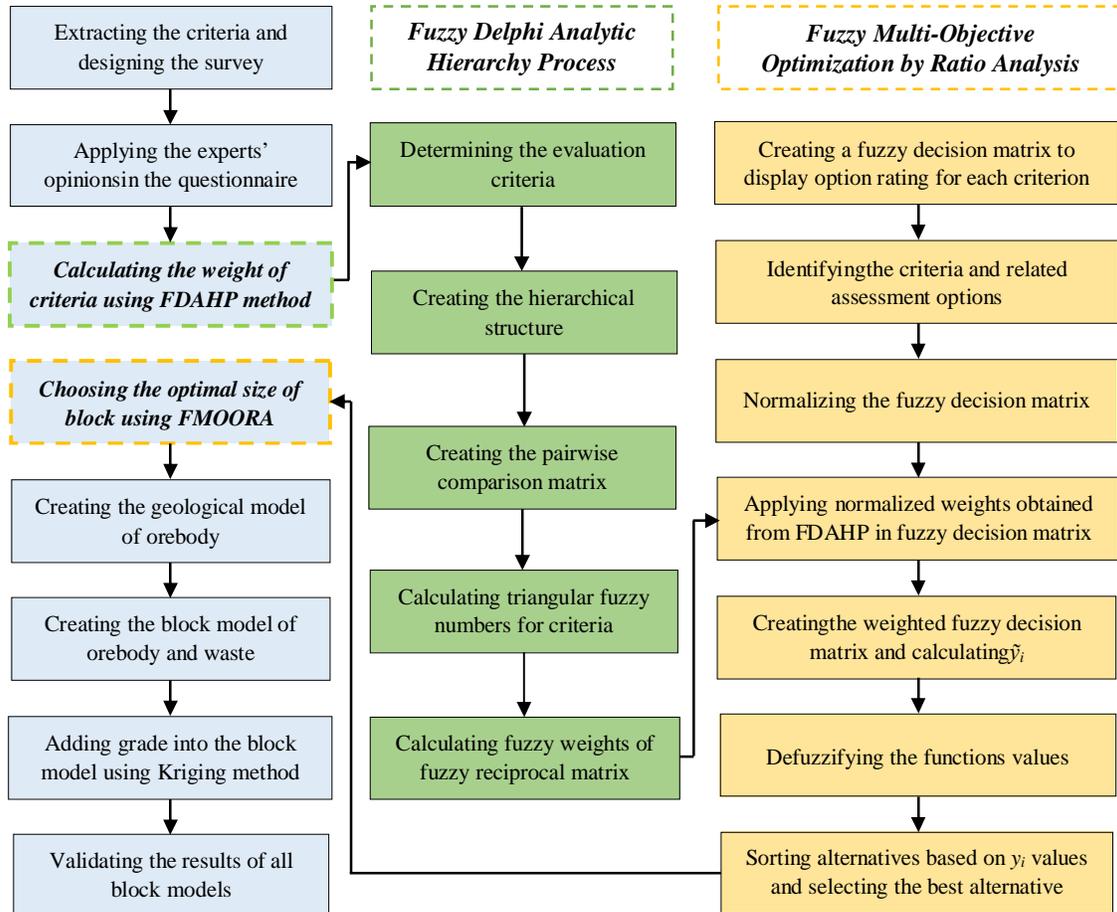


Figure 3. FDAHP-FMOORA approach used in this work.

## 6. Case study

The Sungun porphyry copper mine is located in the Varzeqan City, about 75 km NW of the Ahar City, and 100 km NE of the Tabriz City, Eastern Azerbaijan Province, NW of Iran (Figure 4). The Sungun mine is the most important geological and

industrial feature in the area, containing more than 500 million tons of sulfide copper ore comprising 0.76% Cu and ~0.01% Mo. Therefore, the Sungun copper mine is one of the largest open-pit copper mines in Iran [66-68].

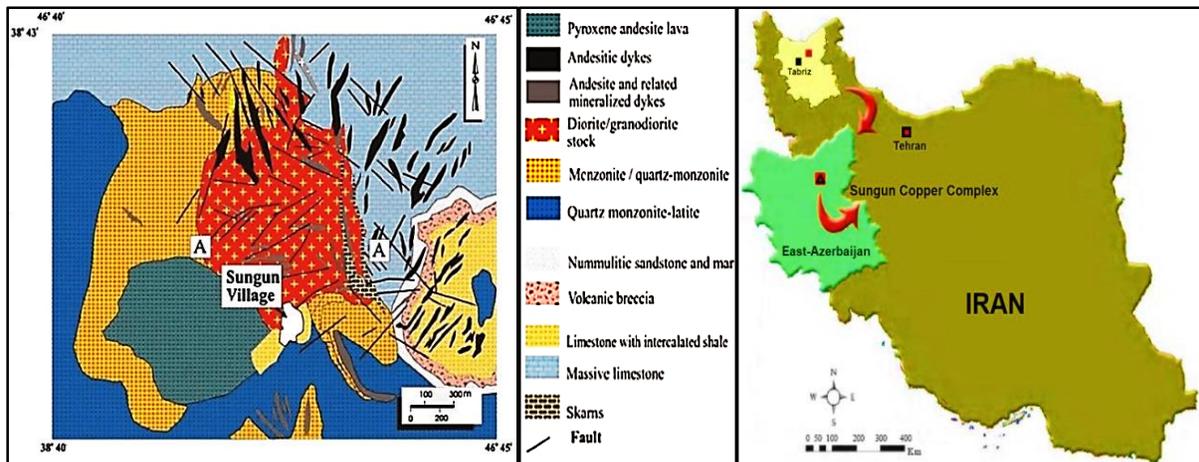


Figure 4. Location and geological map of the Sungun copper complex [66].

For perception of the situation of the Sungun porphyry deposit and its geometrical condition, the data of 230 drilled boreholes saved in four files (Collar, Assay, Survey and Geology) was used as the main inputs of the 3D modeling. After connecting the strings of orebody in 2D sections, a wireframe model was made, also named as

“solid model”. Then the wireframe file was converted to a block model of the deposit. In this work, several block models with different BSs were created. After wards, the geostatistical parameters (estimation error and dilution) were calculated for each one of the dimensions (Table 1).

**Table 1. Statistical outputs of different BSs.**

	<b>BS (m<sup>3</sup>)</b>	<b>Estimation error (%)</b>	<b>Dilution (%)</b>
<b>A<sub>1</sub></b>	2.5 × 2.5 × 2.5	10.27	13.76
<b>A<sub>2</sub></b>	5 × 5 × 5	17.81	14.52
<b>A<sub>3</sub></b>	7.5 × 7.5 × 7.5	29.10	17.89
<b>A<sub>4</sub></b>	10 × 10 × 10	26.94	19.27
<b>A<sub>5</sub></b>	12.5 × 12.5 × 12.5	21.28	19.65
<b>A<sub>6</sub></b>	15 × 15 × 15	24.05	25.36

**7. Implementation of proposed approach**

In the first step, with preliminary studies, the selection of BS was considered, which was considered as the initial option in the proposed algorithm included in the 6 inputs as alternative in the structure of the algorithm. In the second step, a list of several engineering, technical, and statistical parameters related to the determination of optimal BS was prepared. The parameters in the form of questionnaires were available to the experts to determine their most important factors and parameters in terms of engineering and the interconnection between criteria and weight percentages of each criterion based on their experiences available to the researchers. After completing the completed questionnaires, the following parameters were considered as the most important ones in selection of the optimal BS: metal content, estimation error, recovery value, mining-ability, safety, and dilution.

questionnaires were prepared and sent to the open-pit mining experts in Iran. The experts comprising the mining engineers, masters of mining engineering colleges, and mine managers and experts were used in this work. In these questionnaires, it was asked from the experts to mark the importance of each parameter in a very simple way. In order to use the data derived from the questionnaires in the FDAHP method, for each important level, an intensity number from 1 to 9 was assigned based on the Saaty’s scale [48]. Overall, 10 completed questionnaires were incorporated to determine the weights of each criterion in the FDAHP process. A sample of the questionnaire form completed by one of the experts is shown in Table 2. A summary of the experts’ opinion rates are mentioned in Table 3. As it can be seen in this table, the standard deviation and the mining-ability had the highest frequency of rate 9. It shows that they are the most important parameters for optimal BS selection from the experts’ viewpoint.

**7.1. Identification of effective criteria**

After a literature review and recognition of the effective parameters, some technical

**Table 2. A questionnaire sample completed by D<sub>1</sub>.**

<b>Selected parameters</b>	<b>Degree of importance</b>				
	<b>VW</b> (1)	<b>W</b> (3)	<b>M</b> (5)	<b>S</b> (7)	<b>VS</b> (9)
<i>C<sub>1</sub></i> Metal content					•
<i>C<sub>2</sub></i> Estimation error				•	
<i>C<sub>3</sub></i> Value of recovery					•
<i>C<sub>4</sub></i> Mining-ability					•
<i>C<sub>5</sub></i> Safety	•				
<i>C<sub>6</sub></i> Dilution			•		

VW: Very Weak importance, W: Weak importance, M: Moderate importance, S: Strong importance, VS: Very Strong importance.

**Table 3. Summary of the experts' opinions.**

	D <sub>1</sub>	D <sub>2</sub>	D <sub>3</sub>	D <sub>4</sub>	D <sub>5</sub>	D <sub>6</sub>	D <sub>7</sub>	D <sub>8</sub>	D <sub>9</sub>	D <sub>10</sub>
C <sub>1</sub>	9	7	5	7	7	3	7	9	7	7
C <sub>2</sub>	7	7	3	9	7	1	7	7	7	9
C <sub>3</sub>	9	5	5	5	5	3	7	7	9	5
C <sub>4</sub>	9	9	9	9	9	9	9	9	5	9
C <sub>5</sub>	1	7	9	5	9	3	9	3	7	7
C <sub>6</sub>	5	9	9	7	7	3	3	1	3	7

**7.2. Determination of criteria weights**

FDAHP was proposed to take the decision-makers' subjective judgments in to consideration and to reduce the uncertainty and vagueness in the decision-making process. Decision-makers from different backgrounds may define different weight vectors, which usually lead to imprecise evaluations. Therefore, this study proposes a group decision-making based on FDAHP to improve pairwise comparison. Firstly, each decision-maker (*D<sub>i</sub>*) will individually carry out pairwise comparison using the Saaty's 1–9 scale [48]. An example of these pairwise comparisons is shown as Equation 23. *C<sub>1</sub>*, ... , *C<sub>6</sub>* are the criteria describing the metal content, estimation error, value of recovery, mining-ability, safety, and dilution, respectively.

$$D_1 = \begin{bmatrix} & C_1 & C_2 & C_3 & C_4 & C_5 & C_6 \\ C_1 & 1.00 & 1.29 & 1.00 & 1.00 & 9.00 & 1.80 \\ C_2 & 0.78 & 1.00 & 0.78 & 0.78 & 7.00 & 1.40 \\ C_3 & 1.00 & 1.29 & 1.00 & 1.00 & 9.00 & 1.80 \\ C_4 & 1.00 & 1.29 & 1.00 & 1.00 & 9.00 & 1.80 \\ C_5 & 0.11 & 0.14 & 0.11 & 0.11 & 1.00 & 0.20 \\ C_6 & 0.56 & 0.71 & 0.56 & 0.56 & 5.00 & 1.00 \end{bmatrix} \quad (23)$$

Weighting the factors for each criterion (*Z<sub>i</sub>*) is presented in the following steps:

- Computing the triangular fuzzy numbers (using 11-point scale) according to Equations 3-6;
- Creating a fuzzy pairwise comparison matrix *A*; decision-makers' pair wise comparison values were transformed in to triangular fuzzy numbers, as in Table 4.
- Calculating the relative fuzzy weights of the evaluation factors (*Z<sub>i</sub>*):

**Table 4. Fuzzy pairwise comparison matrix.**

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>
C <sub>1</sub>	(1,1,1)	(0.78,1.17,3)	(0.78,1.14,1.4)	(0.33,0.77,1.4)	(0.56,1.29,9)	(0.56,1.37,9)
C <sub>2</sub>	(0.33,0.85,1.29)	(1,1,1)	(0.33,0.97,1.8)	(0.11,0.66,1.4)	(0.33,1.1,7)	(0.33,1.16,7)
C <sub>3</sub>	(0.71,0.87,1.29)	(0.56,1.03,3)	(1,1,1)	(0.33,0.67,1.8)	(0.56,1.13,9)	(0.56,1.2,7)
C <sub>4</sub>	(0.71,1.3,3)	(0.71,1.52,9)	(0.56,1.48,3)	(1,1,1)	(0.71,1.67,9)	(1,1.06,9)
C <sub>5</sub>	(0.11,0.78,1.8)	(0.14,0.91,3)	(0.11,0.89,1.8)	(0.11,0.6,1.4)	(1,1,1)	(0.2,1.06,3)
C <sub>6</sub>	(0.11,0.73,1.8)	(0.14,0.86,1.4)	(0.14,0.84,1.8)	(0.11,0.56,1)	(0.33,0.94,5)	(1,1,1)

$$\tilde{Z}_1 = [\tilde{a}_{11} \otimes \tilde{a}_{12} \otimes \dots \otimes \tilde{a}_{16}]^{\frac{1}{6}} = [0.6595, 1.1055, 2.7945]$$

$$\tilde{Z}_2 = [\tilde{a}_{21} \otimes \tilde{a}_{22} \otimes \dots \otimes \tilde{a}_{26}]^{\frac{1}{6}} = [0.3333, 0.9412, 2.3269]$$

$$\tilde{Z}_3 = [\tilde{a}_{31} \otimes \tilde{a}_{32} \otimes \dots \otimes \tilde{a}_{36}]^{\frac{1}{6}} = [0.5868, 0.9663, 2.7551]$$

$$\tilde{Z}_4 = [\tilde{a}_{41} \otimes \tilde{a}_{42} \otimes \dots \otimes \tilde{a}_{46}]^{\frac{1}{6}} = [0.7662, 1.4347, 4.3267]$$

$$\tilde{Z}_5 = [\tilde{a}_{51} \otimes \tilde{a}_{52} \otimes \dots \otimes \tilde{a}_{56}]^{\frac{1}{6}} = [0.1843, 0.8574, 1.8556]$$

$$\tilde{Z}_6 = [\tilde{a}_{61} \otimes \tilde{a}_{62} \otimes \dots \otimes \tilde{a}_{66}]^{\frac{1}{6}} = [0.2092, 0.8084, 1.9103]$$

$$\sum \tilde{Z}_i = [2.7095, 6.1136, 15.9693]$$

$$\tilde{W}_1 = \tilde{Z}_1 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3)^{-1} = [0.0394, 0.1808, 1.0313]$$

$$\tilde{W}_2 = \tilde{Z}_2 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3)^{-1} = [0.0208, 0.1539, 0.8588]$$

$$\tilde{W}_3 = \tilde{Z}_3 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3)^{-1} = [0.0367, 0.1580, 1.0168]$$

$$\tilde{W}_4 = \tilde{Z}_4 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3)^{-1} = [0.0479, 0.2346, 1.5968]$$

$$\tilde{W}_5 = \tilde{Z}_5 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3)^{-1} = [0.0115, 0.1402, 0.6848]$$

$$\tilde{W}_6 = \tilde{Z}_6 \otimes (\tilde{Z}_1 \oplus \tilde{Z}_2 \oplus \tilde{Z}_3)^{-1} = [0.0131, 0.1322, 0.7050]$$

The final fuzzy weights of each criterion are indicated in Table 5.

**Table 5. Fuzzy weights for the criteria.**

Criteria	Symbol	Fuzzy weights
Metal content	C <sub>1</sub>	(0.0394,0.1808,1.0313)
Estimation error	C <sub>2</sub>	(0.0208,0.1539,0.8588)
Recovery value	C <sub>3</sub>	(0.0367,0.1580,1.0168)
Mining-ability	C <sub>4</sub>	(0.0479,0.2346,1.5968)
Safety	C <sub>5</sub>	(0.0115,0.1402,0.6848)
Dilution	C <sub>6</sub>	(0.0131,0.1322,0.7050)

According to the results presented in Table 5, it can be concluded that the C<sub>4</sub> criterion, which is related to the mining capability, is most effective on the choice of an optimal BS. Given the fact that this criterion has a direct relationship with the type of machinery and the amount of mine production, this effect is also operationally justifiable. Also the statistical criteria (estimation error and dilution) for determining the optimal BS are the most important criteria after mining-ability. The safety and environmental impact are less important than the other criteria with the values presented in Table 5. The significant difference between the C<sub>4</sub> and other criteria indicates the tremendous importance of this criterion in calculating the optimal size of the blocks.

**7.3. Selection of optimal BS by FMOORA**

In the previous step, according to the completed questionnaires, the relationships between the

selected criteria were obtained using the FDAHP method. Now, in order to implement the FMOORA method based on the above data, the steps are executed in order to select the optimal size. According to the experts' opinions, the fuzzy decision matrix is created for the problem of BS using six criteria and six alternatives (Table 6).

After forming the fuzzy decision matrix, this matrix must be normalized to allow the decision process to continue. Equations 14-16 were used to achieve this purpose (Table 7).

The weights of different criteria should be applied in the normalized fuzzy decision matrix that has been obtained in the previous step. To do so, the fuzzy weight of the criteria is applied in normalized fuzzy matrix according to Equations 18-20 (Table 8). The weighted normalized decision matrix is shown in Table 9.

**Table6. Fuzzy matrix of decision.**

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>
A <sub>1</sub>	(0.7,0.8,0.9)	(0.4,0.5,0.6)	(0.6,0.7,0.8)	(0.4,0.5,0.6)	(0.7,0.8,0.9)	(0.8,0.9,1)
A <sub>2</sub>	(0.7,0.8,0.9)	(0.7,0.8,0.9)	(0.6,0.7,0.8)	(0.6,0.7,0.8)	(0.7,0.8,0.9)	(0.8,0.9,1)
A <sub>3</sub>	(0.7,0.8,0.9)	(0.6,0.7,0.8)	(0.6,0.7,0.9)	(0.6,0.7,0.9)	(0.5,0.6,0.7)	(0.7,0.8,0.9)
A <sub>4</sub>	(0.6,0.7,0.8)	(0.8,0.9,1)	(0.7,0.8,0.9)	(0.7,0.8,0.9)	(0.5,0.6,0.7)	(0.7,0.8,0.9)
A <sub>5</sub>	(0.6,0.7,0.8)	(0.8,0.9,1)	(0.5,0.6,0.7)	(0.8,0.9,1)	(0.6,0.7,0.8)	(0.6,0.7,0.8)
A <sub>6</sub>	(0.4,0.5,0.6)	(0.7,0.8,0.9)	(0.5,0.6,0.7)	(0.6,0.7,0.8)	(0.4,0.5,0.6)	(0.4,0.5,0.6)

**Table 7. Normalized fuzzy decision matrix.**

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>
A <sub>1</sub>	(0.22,0.25,0.29)	(0.12,0.15,0.18)	(0.20,0.23,0.27)	(0.12,0.16,0.19)	(0.24,0.27,0.31)	(0.24,0.27,0.30)
A <sub>2</sub>	(0.22,0.25,0.29)	(0.21,0.24,0.27)	(0.20,0.23,0.27)	(0.19,0.22,0.25)	(0.24,0.27,0.31)	(0.24,0.27,0.30)
A <sub>3</sub>	(0.22,0.25,0.29)	(0.18,0.21,0.24)	(0.20,0.23,0.27)	(0.19,0.22,0.25)	(0.17,0.20,0.24)	(0.21,0.24,0.27)
A <sub>4</sub>	(0.19,0.22,0.25)	(0.24,0.27,0.30)	(0.23,0.27,0.30)	(0.22,0.25,0.30)	(0.17,0.20,0.24)	(0.21,0.24,0.27)
A <sub>5</sub>	(0.19,0.22,0.25)	(0.24,0.27,0.30)	(0.17,0.20,0.23)	(0.25,0.29,0.32)	(0.20,0.24,0.27)	(0.18,0.21,0.24)
A <sub>6</sub>	(0.12,0.16,0.19)	(0.21,0.24,0.27)	(0.17,0.20,0.23)	(0.19,0.22,0.25)	(0.13,0.17,0.20)	(0.12,0.15,0.18)

**Table 8. Weight obtained from the normalized FDAHP method for each criterion.**

Criteria	Symbol	Fuzzy weight of criteria
Metal content	C <sub>1</sub>	(0.0394,0.1808,1.0313)
Estimation error	C <sub>2</sub>	(0.0208,0.1539,0.8588)
Value of recovery	C <sub>3</sub>	(0.0367,0.1580,1.0168)
Mining-ability	C <sub>4</sub>	(0.0479,0.2346,1.5968)
Safety	C <sub>5</sub>	(0.0115,0.1402,0.6848)
Dilution	C <sub>6</sub>	(0.0131,0.1322,0.7050)

**Table 9. Weighted normalized decision matrix.**

	C <sub>1</sub>	C <sub>2</sub>	C <sub>3</sub>	C <sub>4</sub>	C <sub>5</sub>	C <sub>6</sub>
A <sub>1</sub>	(0.0089,0.0468,0.3000)	(0.0025,0.0232,0.1551)	(0.0075,0.0377,0.2772)	(0.0062,0.0378,0.3087)	(0.0028,0.0388,0.2134)	(0.0032,0.0358,0.2122)
A <sub>2</sub>	(0.0089,0.0468,0.3000)	(0.0044,0.0371,0.2326)	(0.0075,0.0377,0.2772)	(0.0093,0.0529,0.4116)	(0.0028,0.0388,0.2134)	(0.0032,0.0358,0.2122)
A <sub>3</sub>	(0.0089,0.0468,0.3000)	(0.0038,0.0324,0.2068)	(0.0075,0.0377,0.2772)	(0.0093,0.0529,0.4616)	(0.0020,0.0291,0.1660)	(0.0028,0.0318,0.1910)
A <sub>4</sub>	(0.0076,0.0409,0.2667)	(0.0050,0.0417,0.2585)	(0.0088,0.0431,0.3119)	(0.0108,0.0605,0.4631)	(0.0020,0.0291,0.1660)	(0.0028,0.0318,0.1910)
A <sub>5</sub>	(0.0076,0.0409,0.2667)	(0.0050,0.0417,0.2585)	(0.0063,0.0323,0.2426)	(0.0123,0.0680,0.5146)	(0.0024,0.0340,0.1897)	(0.0024,0.02779,0.1697)
A <sub>6</sub>	(0.0089,0.0468,0.3000)	(0.0040,0.0343,0.2190)	(0.0078,0.0386,0.2792)	(0.0069,0.0422,0.3450)	(0.0014,0.0223,0.1364)	(0.0016,0.00199,0.1273)

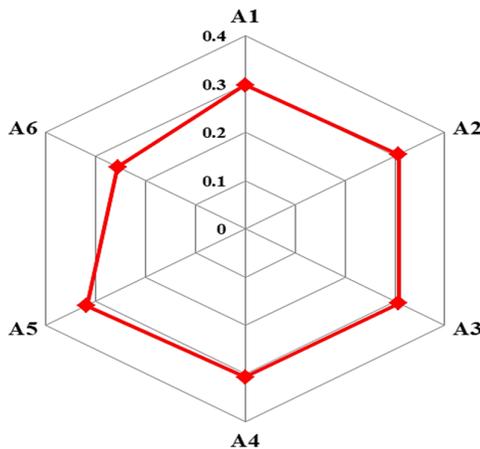
In the following step, the normalized performance values are calculated by subtracting the non-standard criteria from all the desired criteria (determined by the type of problem). Table 10 reflects the fact that all the criteria used in this work are all positive. Therefore, the value of the function is equal to the sum of the fuzzy values of the criteria. After calculating the  $y_i$  score, the final prioritization of the options is done (see Table 11 and Figure 5).

**Table 10. Type of criteria used in the current study.**

Criteria	Symbol	Criteria type
Metal content	$C_1$	Positive criterion
Estimation error	$C_2$	Negative criterion
Value of recovery	$C_3$	Positive criterion
Mining-ability	$C_4$	Positive criterion
Safety	$C_5$	Positive criterion
Dilution	$C_6$	Negative criterion

**Table 11. Ranked alternatives by FDAHP-FMOORA.**

Alternatives	Performance value			$y_i$	Rank
	$y_i^l$	$y_i^m$	$y_i^u$		
A <sub>1</sub>	0.0197	0.1021	0.7321	0.2978	5
A <sub>2</sub>	0.0209	0.1033	0.7575	0.3079	2
A <sub>3</sub>	0.0212	0.1022	0.7522	0.3076	3
A <sub>4</sub>	0.0214	0.1001	0.7582	0.3075	4
A <sub>5</sub>	0.0213	0.1057	0.7853	0.3183	1
A <sub>6</sub>	0.0162	0.0818	0.6366	0.2557	6



**Figure 5. Prioritization of alternatives based on the  $y_i$  score.**

According to Table 11 and Figure 5, the alternative A<sub>5</sub> (with 12.5×12.5×12.5 m<sup>3</sup> for BS) is selected as the best-rated option and the most desirable BS. By examining the results of Table 11 with Table 5, it can be concluded that, given the high importance of mining-ability in choosing the optimal BS, the alternative A<sub>5</sub> is selected for

the purpose of quantifying mining-ability and high value of recovery rate due to its utility. On the other hand, according to the experts' opinions given in Table 5, the impacts of other criteria in the process of selecting the optimal sizes of open-pit blocks are seen. The relatively similar values ( $y_i$ ) for the alternatives A<sub>2</sub>, A<sub>3</sub>, and A<sub>4</sub> indicate that the different studied factors have different effects on BS but the cumulative effect of the factors has made these options a second option. Of course, the final selection and application of these BSs should be done based on the technical and specialized considerations by the mining engineers and designers.

**8. Conclusions**

In this work, we tried to introduce a novel hybrid decision-making approach based on the FDAHP and FMOORA methods in order to determine BS of the open-pit mines in an uncertain environment. The results of implementation of this approach indicate that 12.5×12.5×12.5 m<sup>3</sup> can be calculated for an optimal BS of the mine. According to the suitable results obtained from the current work, it is obvious that the mining-ability has a great influence on the BS selection process because this factor has the most important role in the open-pit mine operations. It was observed that the second selected alternatives were blocks A<sub>2</sub>, A<sub>3</sub> and A<sub>4</sub>, which could interestingly be due to the operational limitation of the mining. The dimensions of 15×15×15 m<sup>3</sup> for a block (A<sub>6</sub>) were also in the last priority because the alternative A<sub>6</sub> was not compatible with mine equipment. Due to the technical and operational conditions of mining in the Sungun copper mine, the results of the work were compatible with the real mining operation in the studied mine. In the future studies, the proposed approach can be used to solve similar decision-making problems in other engineering sciences, especially the mining engineering problems as well as the non-engineering problems. Furthermore, by extension of the proposed approach based on the gray logic, one can study the problem of optimal BS selection in another project. In addition, the Z-number theory can be used in the MCDM method as a hybrid decision-making approach in order to simultaneously take the reliability and uncertainty into account.

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## ارائه رویکرد ترکیبی چندمعیاره فازی جهت تعیین ابعاد بهینه بلوک در مدل‌سازی معادن روباز: مطالعه موردی

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### چکیده:

یکی از مهم‌ترین مراحل اجرایی در ارزیابی ذخیره، تخمین عیار و برنامه‌ریزی تولید معادن روباز، مدل‌سازی سه‌بعدی کنسارها مبتنی بر نرم‌افزارهای رایانه‌ای است. در فرایند مدل‌سازی، به منظور انجام محاسبات آنی ضروری است که پیکره کنسار و باطله توسط بلوک‌ها و ریزبلوک‌هایی به قطعات کوچک قابل استخراج و برنامه‌ریزی تقسیم شود. تعیین ابعاد بلوک به دلیل وابستگی به مباحث زمین‌آماري و مسائل فنی مربوط به تجهیزات معدن‌کاری حائز اهمیت است. محتوای فلز، خطای تخمین، درصد بازیابی، قابلیت استخراج، ایمنی و ترقیق، عوامل مؤثر در تعیین ابعاد بهینه بلوک محسوب می‌شوند. در این پژوهش، ابعاد بلوک با استفاده از یک رویکرد ترکیبی دو مرحله‌ای تعیین شد. در رویکرد پیشنهادی، از روش‌های تحلیل سلسله‌مراتبی دلفی فازی (FDAHP) و بهینه‌سازی چندمنظوره بر اساس تحلیل نسبت فازی (FMOORA) استفاده شد. در گام نخست، وزن هر معیار بر اساس نظرات خبرگان با استفاده از روش FDAHP محاسبه شده و در گام دوم، با بکارگیری وزن‌های استخراج‌شده، از روش FMOORA به منظور تعیین ابعاد بهینه بلوک برای طراحی و بهره‌برداری معدن استفاده شد. مدل بلوکی معدن مس سونگون در قالب یک مطالعه موردی برای ارزیابی قابلیت رویکرد پیشنهادی مورد بررسی قرار گرفت. نتایج اجرای این رویکرد به دلیل تبدیل نظرات خبرگان به مقادیر فازی، وزن‌گیری از متخصصین با توجه به تجربه و دانش فنی، وزن‌دهی به معیارها به روش FDAHP و انتخاب گزینه بهینه با روش FMOORA مطلوب است. در نهایت، بلوک دارای ابعاد  $12/5 \times 12/5 \times 12/5$  مترمکعب به‌عنوان گزینه مناسب و مطلوب انتخاب شد که با شرایط واقعی معدن مورد مطالعه کاملاً سازگار است.

کلمات کلیدی: تعیین ابعاد بلوک، معادن روباز، FDAHP، FMOORA