Shahrood University of Technology

Iranian Society of Mining Engineering (IRSME)

Journal of Mining and Environment (JME)

journal homepage: www.jme.shahroodut.ac.ir

THOMSON REUTERS

Semi-Analytical Study of Settlement of Two Interfering Foundations Placed on a Slope

Hadi Haghgouei[1], Ali Reza Kargar[1*], Mohammad Hossein Khosravi[1] and Mehdi Amini[2]

1. School of Mining Engineering, College of Engineering, University of Tehran, Tehran. Iran
2. Rock Mechanics Specialist, Thurber Engineering Ltd., Vancouver, British Columbia, Vancouver, Canada

Article Info

Received 28 January 2021

Received in Revised form 13 February 2021

Accepted 20 February 2021

Published online 20 February 2021

DOI:10.22044/jme.2021.10509.2001

Keywords

Foundations
Settlement
Interfering footing
Slope stability
Elasticity

Abstract

In many engineering constructions, the foundations should be built adjacent to each other. Therefore, the effect of interfering of close foundations should be considered in the design stage. In this research work, the effect of interference of closely separated foundations resting on a slope on the elastic settlement is investigated by considering a semi-analytical solution. The distribution of stress due to the footing pressure in the slope is computed by a proposed Airy stress function, and then by employing the finite difference scheme, the displacement of the footings is calculated. The results obtained show that by increasing the distance between the foundations, the interference influence on the ratio of settlement will be diminished. However, this behavior is highly linked to the slope characteristics. For a slope with a height of 10 times of footing width, beyond an S/B ratio larger than 10, the effect of interference is not tangible, and the footings behave like an isolated foundation. By decreasing the slope height, this behavior will occur at a lower S/B.

1. Introduction

The foundations built adjacent to the slope are common in the engineering practice [1], and in many practical construction projects, the foundations should be built near each other. In this case, the interference between two adjacent footings may lead to a damage to the structures. When it comes to the safety of the foundations, two different views exist: investigating the bearing capacity and studying the settlement of the footings. Many published research works exist in the literature that have focused on the effect of interfering of footings resting on the horizontal ground surface. However, the investigation on the effect of interfering of close foundations resting on a slope has not drawn much attention, especially in the case of studying the settlement.

Stuart [2] has studied the interference of foundations based on the limit equilibrium method for the first time. He illustrated that by increasing the spacing between the adjacent footings, the interference effect would diminish, and the footings behaved like an isolated foundation. He also introduced the factors of efficiency for the bearing capacity. The influence of interference of closely separated foundations on the bearing capacity has been studied experimentally by Das and Larbi-Cherif [3]. Their outcomes manifested that by increasing the spacing between the adjacent foundations, at first, the value of bearing capacity would rise and then drop. Once the spacing of the foundations reaches 5 to 6 times of the foundation width, the footings behave like an isolated foundation. Graham *et al*. [4] have studied the interference effect of three nearby footings. They showed that by decreasing the distance between the foundations, the failure load would increase.

Corresponding author: ar.kargar@ut.ac.ir (A.R. Kargar).

Kumar and Ghosh [5, 6] have investigated the impact of interference of two footings on the ultimate bearing capacity by considering the upper bound theory. Their results showed that the bearing capacity was linked to the distance between the foundations as well as the soil friction. Mabrouki *et al*. [7] have investigated the bearing capacity of the foundations that were placed close to each other numerically. They proposed the efficiency factors related to the soil cohesion, footing load, and unit weight of the material. Ghosh and Sharma [8], by employing the theory of elasticity, have studied the interference effects of two nearby foundations on the settlement by considering the different factors such as the elastic moduli, layer depth, and surcharge load. Alimardani Lavasan and Ghazavi [9] have studied two adjacent circular and square foundations on the reinforced and unreinforced sand experimentally. They reported that reinforcing the sand led to an increase in the ultimate bearing capacity (25-40%); however, the settlement of the foundations also increased to about 60–100%. Javid *et al*. [10] have numerically investigated the effect of interfering of two foundations built on the rock mass. They investigated the effect of the Hoek-brown failure criterion factors on the bearing capacity. Shamloo and Imani [11] have used the upper bound theory in order to study the influence of numerous factors, for instance, the spacing between the foundations, Hoek-Brown failure criterion parameters, rock mass unit weight, and surcharge loading on the bearing capacity.

The above-mentioned investigation is related to the development of the bearing capacity and settlement of close foundations resting on a half-space. However, the study of the interference effects of foundations resting on a slope has not been found to be explored much in the literature. Many engineering structures are required to be built close to a slope [12]. Also, sometimes, in urban areas, the footings are built adjacent to an excavation for the basement construction of high-rise buildings [13]. Therefore, the study of the safety of the footings resting on a slope is amply clear. Consequently, this work aims to present an analytic method for evaluating the settlement of the foundations resting on a slope. Indeed, such analytical solutions could give an insight of how ultimate results are affected by different factors, and may be used as a fast solver with high accuracy compared to the other methods [16, 17].

In order to investigate the elastic settlement of a foundation in the presence of other foundations, at first, the stress distribution within the slope due to two nearby footing loads will be analyzed based on the theory of elasticity and the integral transformation method, and then using the finite difference scheme, the settlement of the foundations will be investigated. In the present work, the footings were considered as shallow foundations with no embedment, the boundary of slope and foundations was presumed as a rough interface, the elastic and homogeneous material was considered for the slope, and the applied pressure by footings were considered to be such that the slope's material did not trespass the elastic regime as defined by Maheshwari and Viladkar [14] and Zhu *et al*. [15].

2. Details of analysis

Applying the Airy stress function, φ, is a well-known method in the theory of elasticity in order to obtain the stress and displacement distribution in 2D engineering problems. Equation (1) represents a bi-harmonic equation, and it satisfies the compatibility and equilibrium equations.

$$\left(\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}\right)^2 \varphi = 0 \qquad (1)$$

The schematic representation of the problem is illustrated in Figure 1.

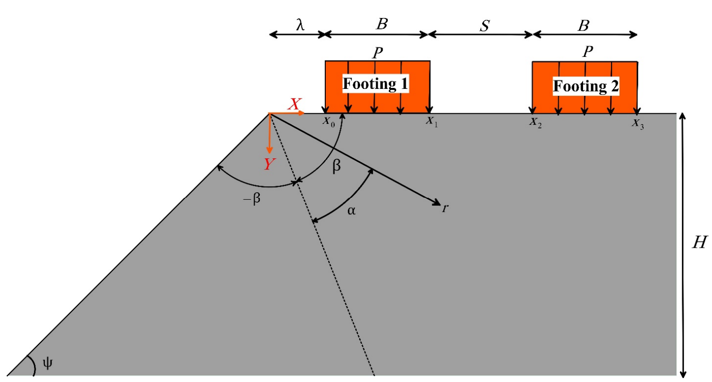


Figure 1. A schematic representation of two foundations resting on a slope.

By finding the proper Airy stress function that satisfies Equation (1) and the problem boundary conditions, as presented in Equation (2), the stress state can be defined by the derivative of the Airy stress function, as illustrated in Equation (3).

$$\sigma_\theta = f\left(r\right) = P, \qquad \theta = \alpha, \qquad x_0 \le r \le x_1$$

$$\sigma_\theta = f\left(r\right) = P, \qquad \theta = \alpha, \qquad x_2 \le r \le x_3$$

$$\sigma_\theta = 0, \ \theta = -\alpha$$

$$\tau_{r\theta} = 0, \ \theta = \pm\alpha \tag{2}$$

$$\sigma_\theta = \frac{\partial^2 \varphi}{\partial r^2}$$

$$\sigma_r = \frac{1}{r}\frac{\partial \varphi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \varphi}{\partial \theta^2} \tag{3}$$

$$\tau_{r\theta} = -\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial \varphi}{\partial \theta}\right)$$

Based on the complex integral transformation rule, namely the Mellin transformation, the authors proposed an Airy stress function in order to study the toppling-slumping failure of the rock slope [18] and finding the bearing capacity of the shallow footing resting on the slope [19]. This proposed transformed Airy stress function is introduced in Equation (4), and will be used in this work in order to find the stress state within the slope due to two nearby foundation loads.

In Equation (4), F(z) is the loading of foundation. In this work, in order to consider the two foundation loads, a step function, *t*, has been used, as shown in Equation (5).

By considering the inverse Mellin transformation rule [20], as presented in Equation (6), the stress components in the real space can be defined by Equation (7).

$$\Phi(z,\theta) = \frac{F(z)}{2z(z+1)}\left[\frac{z\sin(z\alpha)\cos(z+2)\theta - (z+2)\sin(z+2)\alpha\cos(z\theta)}{(z+1)\sin(2\alpha) + \sin 2(z+1)\alpha} + \frac{(z+2)\cos(z+2)\alpha\sin(z\theta) - z\cos(z\alpha)\sin(z+2)\theta}{(z+1)\sin(2\alpha) - \sin 2(z+1)\alpha}\right] \tag{4}$$

$$F(z) = \int_0^{x_3}\left[\sum_{i=0}^{3}(-1)^i\, t\left(x - x_i\right)\right]x^{z+1}dx = \frac{1}{z+2}\sum_{i=0}^{3}(-1)^i\, x_i^{z+2} \tag{5}$$

$$\sigma_\theta = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\Phi(z,\theta)z\left(z+1\right)r^{-z-2}dz$$

$$\sigma_r = \frac{-1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\left[z\,\Phi(z,\theta) - \frac{d^2\Phi(z,\theta)}{d\theta^2}\right]r^{-z-2}dz$$

$$\tau_{r\theta} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\frac{d\,\Phi(z,\theta)}{d\theta}\left(z+1\right)r^{-z-2}dz \tag{6}$$

$$\sigma_\theta = \frac{-1}{4\pi i}\int_{c-i\infty}^{c+i\infty}F(z)\left[\frac{-\sin(z\alpha)\cos((z+2)\beta) + \frac{z[\cos(z\beta)\sin((z+2)\alpha)]}{z+2}}{-\sin(2(z+1)\beta) + (z+1)\sin(2\beta)} + \frac{\sin((z+2)\beta)\cos(z\alpha) - \frac{z[\sin(z\beta)\cos((z+2)\alpha)]}{z+2}}{\sin(2(z+1)\beta) + (z+1)\sin(2\beta)}\right]\mathrm{dz}$$

$$\sigma_r = \frac{-1}{4\pi i}\int_{c-i\infty}^{c+i\infty}F(z)\left[\frac{\cos((z+2)\beta)\sin(z\alpha) - \frac{(z+4)[\cos(z\beta)\sin((z+2)\alpha)]}{z+2}}{-\sin(2(z+1)\beta) + (z+1)\sin(2\beta)} + \frac{\frac{(z+4)[\sin(z\beta)\cos((z+2)\alpha)]}{z+2} - \sin((z+2)\beta)\cos(z\alpha)}{\sin(2(z+1)\beta) + (z+1)\sin(2\beta)}\right]\mathrm{dz} \tag{7}$$

$$\tau_{r\theta} = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}F(z)\left[\frac{\cos(z\beta)\cos((z+2)\alpha) - \cos((z+2)\beta)\cos(z\alpha)}{-\sin(2(z+1)\beta) + (z+1)\sin(2\beta)} + \frac{\sin(z\beta)\sin((z+2)\alpha) - \sin((z+2)\beta)\sin(z\alpha)}{\sin(2(z+1)\beta) + (z+1)\sin(2\beta)}\right]\mathrm{dz}$$

Equation (7) is a meromorfic function, and has a single pole, and therefore, a line integration method along the z = -1 path is considered here to obtain the stress state. Equation (8) is the outcome of the line integration method.

$$
\begin{aligned}
\left(\sigma_{\theta}-\sigma_{r}\right) &= \frac{x_0}{r\pi}\left[\int_0^{\infty}\left(f_1+f_2\right)\left[Sin\left(yLog\left(\frac{x_0}{r}\right)\right)\right]dy\right]-\frac{r_1}{r\pi}\left[\int_0^{\infty}\left(f_1+f_2\right)\left[Sin\left(yLog\left(\frac{x_1}{r}\right)\right)\right]dy\right]+ \\
&\frac{x_2}{r\pi}\left[\int_0^{\infty}\left(f_1+f_2\right)\left[Sin\left(yLog\left(\frac{x_2}{r}\right)\right)\right]dy\right]-\frac{x_3}{r\pi}\left[\int_0^{\infty}\left(f_1+f_2\right)\left[Sin\left(yLog\left(\frac{x_3}{r}\right)\right)\right]dy\right]+[(Residu)\frac{x_1+x_3-(x_0+x_2)}{r\pi}]
\end{aligned}
$$

$$
\begin{aligned}
\left(\sigma_{\theta}+\sigma_{r}\right) &= \frac{x_0}{r\pi}\left[\int_0^{\infty}\frac{(f_5y+f_3)+(f_6y+f_4)}{1+y^2}Cos\left(yLog\left(\frac{x_0}{r}\right)\right)dy+\int_0^{\infty}\frac{(f_3y-f_5)+(f_4y-f_6)}{1+y^2}Sin\left(yLog\left(\frac{x_0}{r}\right)\right)dy\right] \\
&-\frac{x_1}{r\pi}\left[\int_0^{\infty}\frac{(f_5y+f_3)+(f_6y+f_4)}{1+y^2}Cos\left(yLog\left(\frac{x_1}{r}\right)\right)dy\right]+\int_0^{\infty}\frac{(f_3y-f_5)+(f_4y-f_6)}{1+y^2}Sin\left(yLog\left(\frac{x_1}{r}\right)\right)dy+ \\
&\frac{x_2}{r\pi}\left[\int_0^{\infty}\frac{(f_5y+f_3)+(f_6y+f_4)}{1+y^2}Cos\left(yLog\left(\frac{x_2}{r}\right)\right)dy+\int_0^{\infty}\frac{(f_3y-f_5)+(f_4y-f_6)}{1+y^2}Sin\left(yLog\left(\frac{x_2}{r}\right)\right)dy\right]- \\
&\frac{x_3}{r\pi}\left[\int_0^{\infty}\frac{(f_5y+f_3)+(f_6y+f_4)}{1+y^2}Cos\left(yLog\left(\frac{x_3}{r}\right)\right)dy\right]+\int_0^{\infty}\frac{(f_3y-f_5)+(f_4y-f_6)}{1+y^2}Sin\left(yLog\left(\frac{x_3}{r}\right)\right)dy-[(Residu)\frac{x_1+x_3-(x_0+x_2)}{r\pi}]
\end{aligned}
$$

$$
\begin{aligned}
\tau_{r_{\theta}} &= \frac{x_0}{2r\pi}\int_0^{\infty}\left(f_7-f_8\right)Cos\left(yLog\left(\frac{x_0}{r}\right)\right)dy-\frac{x_1}{2r\pi}\int_0^{\infty}\left(f_7-f_8\right)Cos\left(yLog\left(\frac{x_1}{r}\right)\right)dy+ \\
&\frac{x_2}{2r\pi}\int_0^{\infty}\left(f_7-f_8\right)Cos\left(yLog\left(\frac{x_2}{r}\right)\right)dy-\frac{x_3}{2r\pi}\int_0^{\infty}\left(f_7-f_8\right)Cos\left(yLog\left(\frac{x_3}{r}\right)\right)dy
\end{aligned}
\tag{8}
$$

The functions f_1 to f_8 and the Residue value are presented in Appendix A. The integrals of Equation (8) could not be solved analytically, and therefore, we used the Filon numerical integration technique in order to obtain the stress components [21]. In order to calculate the Airy stress function and more details about the procedure of finding the stress components, the interested researchers are referred to the authors' recently published works [19, 18]. Also in the proposed Airy stress function, the role of gravity was neglected. Hence, to take the effect of gravity into account, the gravitational stress components within the slope proposed by Goodman and Brown [22] were added up to the stress components computed from the proposed solution by considering the superposition scheme. Figure 2 illustrates a comparison between the outcome of the analytical solution and the stress components predicted using the Abaqus FEM software. 3260 triangular elements were considered as mesh generation in FEM. Also the Poisson's ratio, Young's modulus, and material unit weight were assumed as 0.3, 1 GPa, and 20 KN/m^3, respectively.

Two cases were considered. In the first case, first, the footing was rested on a 45^o slope with a zero distance from the crest (i.e. λ = 0), and secondly, footing was modeled with a 0.5 m spacing from the first one (i.e. S = 0.5 m). In the second case, first, the footing was rested on a 60^o slope with a zero distance from the crest, and secondly, the footing was modeled with a 2 m spacing from the first one. As it can be seen, there is a very good agreement between the outcome of proposed method and those predicted using the Abaqus software.

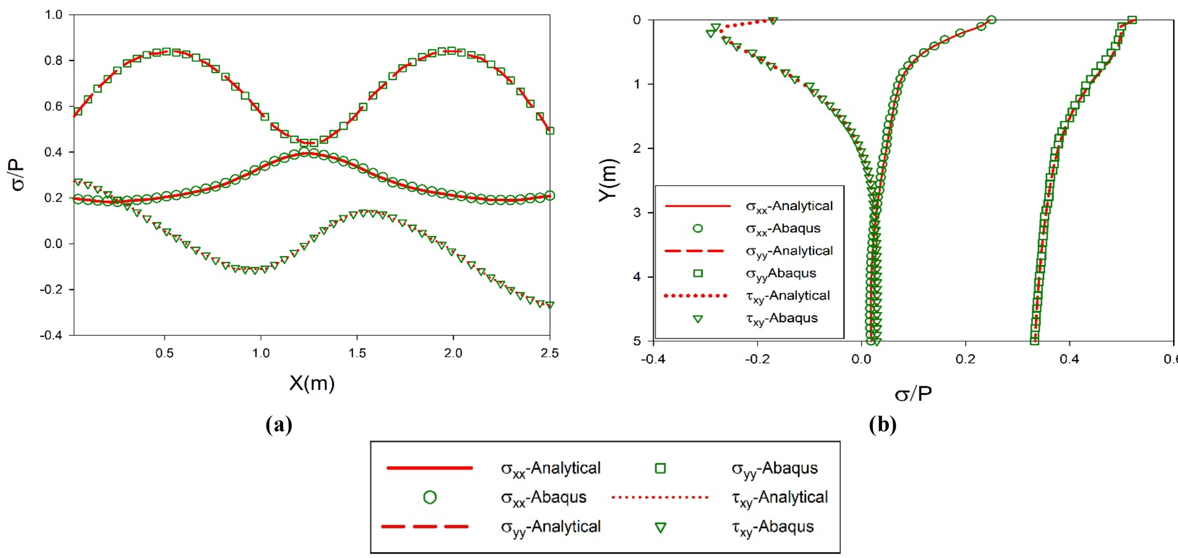


Figure 2. Comparison of stress distribution between the proposed solution and the Abaqus software (a) 45^o slope with λ = 0 and S = 0.5 m at Y = 0.5 m (b) 60^o degree slope with λ = 0 and S = 2 m at X = 1 m.

By finding the stress state, the strain was calculated through Equation (9) by considering the plane strain condition.

$$e_{ij} = \frac{1+\nu}{E}\left(\sigma_{ij} - \nu\sigma_{kk}\delta_{ij}\right) \tag{9}$$

In order to find the settlement of the foundations, the finite difference method was used in order to obtain the displacement based on the distributed strain in the slope. The slope was meshed, and at each grid point, the vertical displacement was computed using Equation (10). In this equation, *h, i,* and *u* represent the mesh size, number of steps, and vertical displacement, respectively.

$$e = \frac{\partial u}{\partial y} = \frac{u\left(y + \boldsymbol{i}h\right) - u\left(y_0\right)}{\boldsymbol{i}h} \tag{10}$$

Based on the method proposed by Ghosh and Sharma [8], a sensitive analysis was performed in order to obtain the displacement by considering the various sizes of the mesh, and the results obtained manifested that beneath the mesh size 0.05 m, no significant change in the outcomes could be noticed. Therefore, in this work, a mesh size of 0.05 m was considered.

3. Results and discussion

In this work, the settlement of the first footing under the interference effect was normalized by that of an isolated footing without the second effect, introduced as ζ.

The height of the slope was considered as 10, 5, and 1 times of the footing width and the slope angles of 90^o, 80^o, 70^o, 60^o, and 45^o were assumed. The values of 0, 0.5, 1, and 2 times of the footing width were taken into account as a space of the first footing from the slope crest. The spacing between two nearby footings was considered as 0.5, 1, 2, 4, 6, 8, and 10 times of the foundation width. It should be noted that in this work, the footings width as well as the footing loads were assumed to be equal. Figure 3 represents the variation in the settlement ratio with the normalized spacing between the footings for different spacings of the first foundation from the slope crest and the different normalized heights of the 90^o slope.

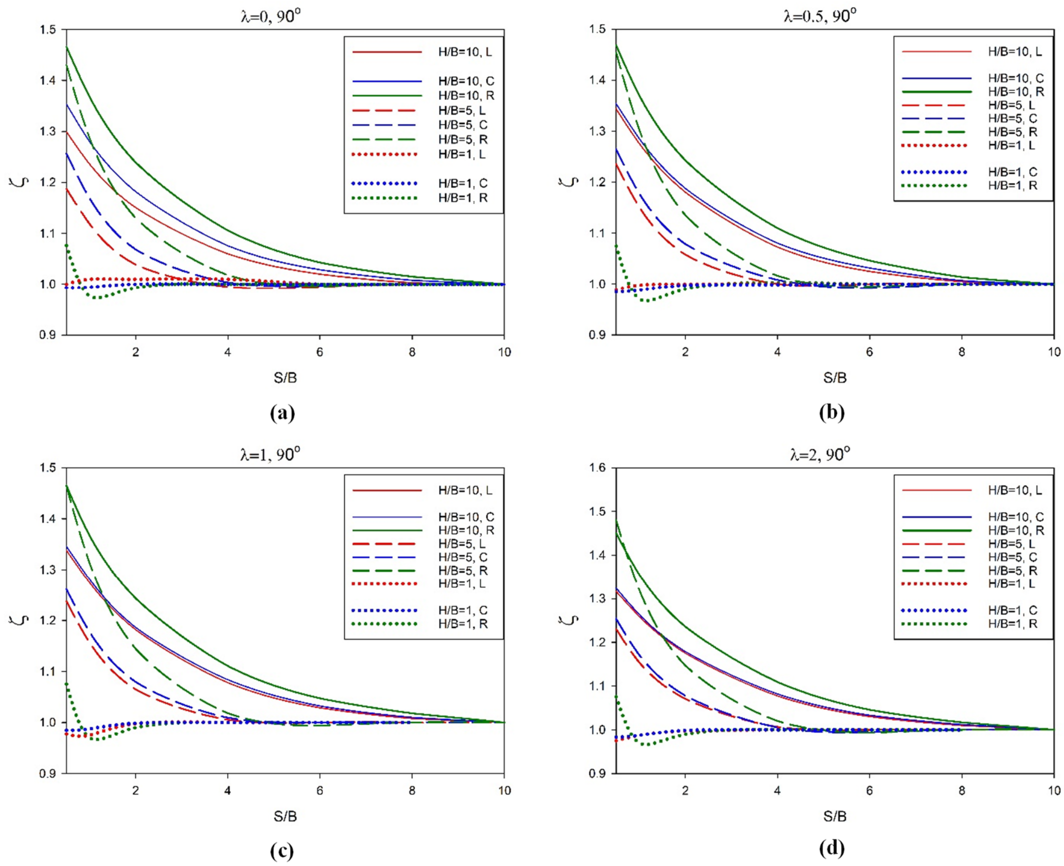


Figure 3. Variation in the settlement ratio with the normalized spacing between the footings for 90° slope at (a) λ = 0, (b) λ = 0.5, (c) λ = 1, and (d) λ = 2.

As it can be seen, the largest ζ is related to the right side of the foundation. By increasing the spacing of the footings, the curves will approach to a unit value. This is due to the fact that by increasing the spacing of the foundations, the interference effect will diminish, and at a large spacing, both footings behave as an isolated foundation placed on a slope. However, this spacing (i.e. the spacing after which the behavior like an isolated footing will be observed) is completely related to the height of the slope. As it is manifested in Figure 3, in the slope with a height of 10 times the footing width, beyond the S/B ratio greater than 10, the ratio of settlement reaches the unit value. However, for the cases of the slopes with normalized heights of 5 and 1 times the footing width, this behavior occurred at the S/B ratio equal to 6 and 4, respectively. Also by increasing the spacing of the first foundation from the crest of the slope, the settlement ratio of the left and center of the footing will approach the same value. An interesting point was observed when the normalized height of the slope was equal to 1. In this case, the ratio settlement of the right side of the footing at first increase then decreased to bellow the unit value, and finally, increased and reached the value of 1. This indicates that at S/B = 1, for the normalized slope height equal to 1, the settlement at the right side of the foundation, when the second foundation presents, is lower than the isolated footing. In the case of S/B = 0.5, the settlement ratios of the center and left sides of the footing is below the unit, and by increasing S/B, the settlement ratio will increase and reach the value of 1. Also when S/B = 0.5, the settlement ratios of the right side of the footing for H/B = 10 and 5 are almost equal. However, by increasing S/B, the two curves representing the settlement ratio of the right side of footings for H/B = 10 and 5 will diverge.

The variations in the settlement ratio with the normalized spacing between the footings for different distances of the first footing from the crest and the different normalized heights for 80°, 70°, 60°, and 45° slope are illustrated in Figures 4 to 7.

The same trend as discussed for the 90° slope is also evident here. In order to elaborate on the effect of the slope angle on the settlement ratio, two extreme slope cases (i.e. 90° and 45°) were compared in Figure 8 for the slope with a normalized height of 10.

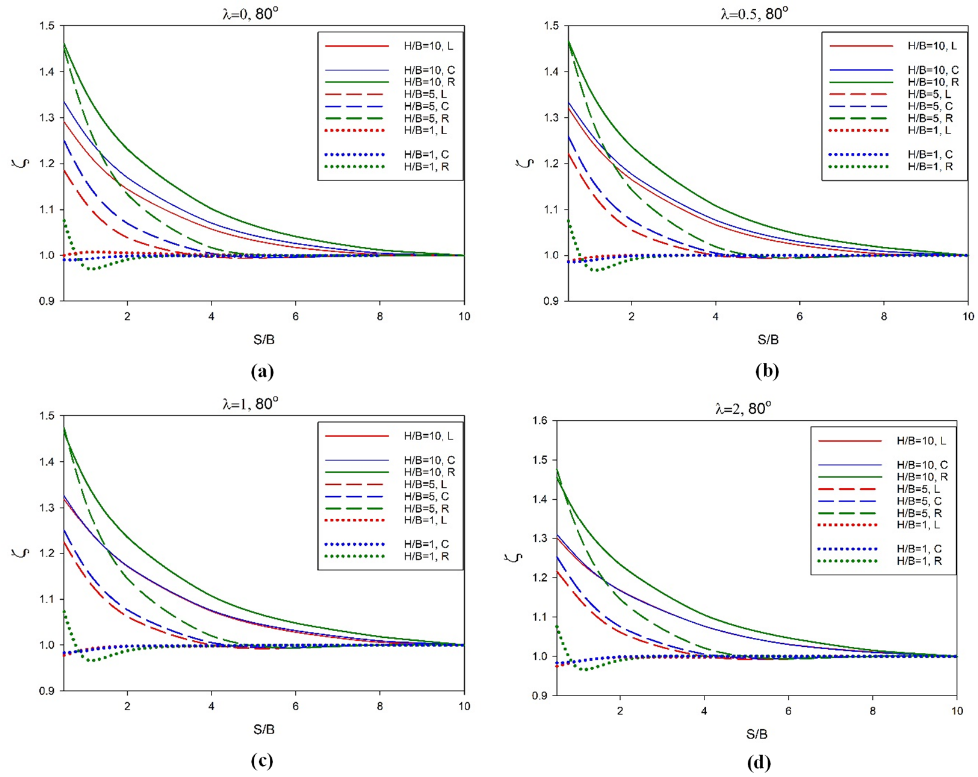


Figure 4. Variations in the settlement ratio with the normalized spacing between the footings for 80º slope for (a) λ = 0 (b) λ = 0.5 (c) λ = 1, and (d) λ = 2.

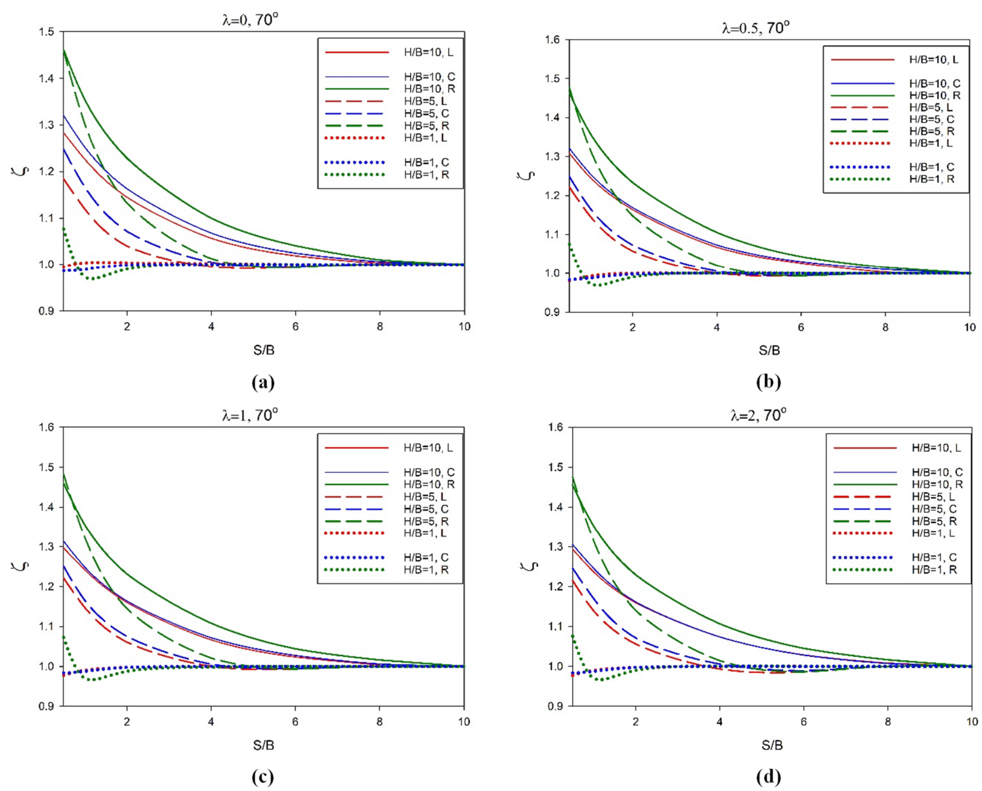


Figure 5. Variations in the settlement ratio with the normalized spacing between the footings for 70º slope for (a) λ = 0 (b) λ = 0.5 (c) λ = 1, and (d) λ = 2.

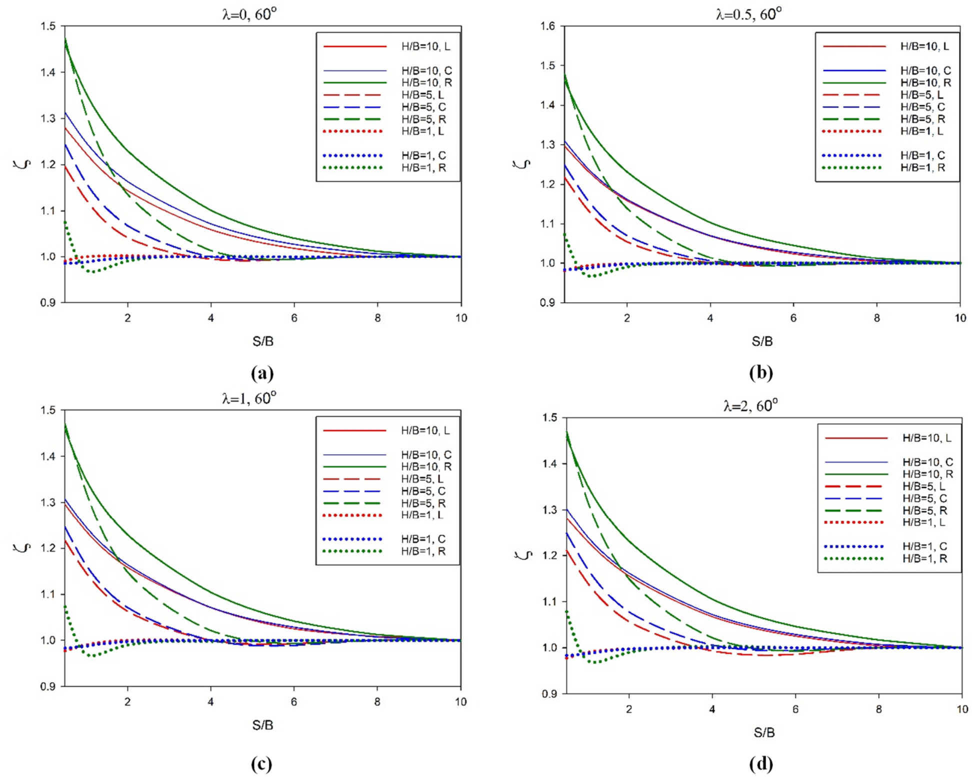


Figure 6. Variations in the settlement ratio with the normalized spacing between the footings for 60º slope for (a) λ = 0 (b) λ = 0.5 (c) λ = 1, and (d) λ = 2.

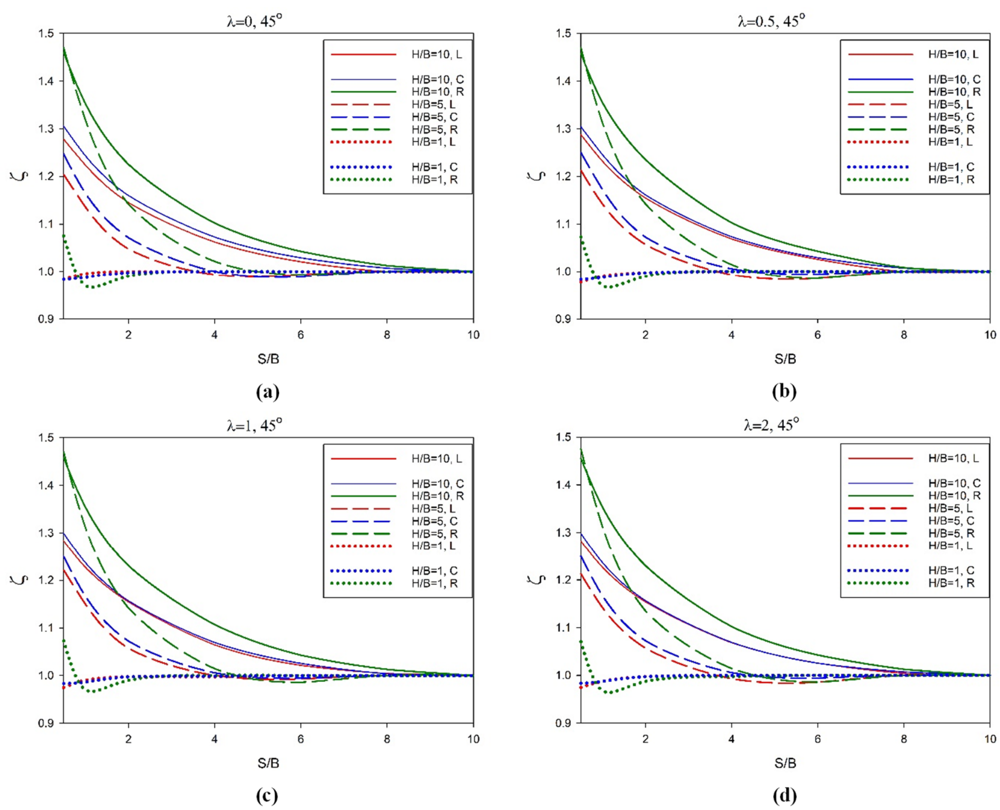


Figure 7. Variations in the settlement ratio with the normalized spacing between the footings for 45º slope for (a) λ = 0 (b) λ = 0.5 (c) λ = 1 (d) λ = 2.

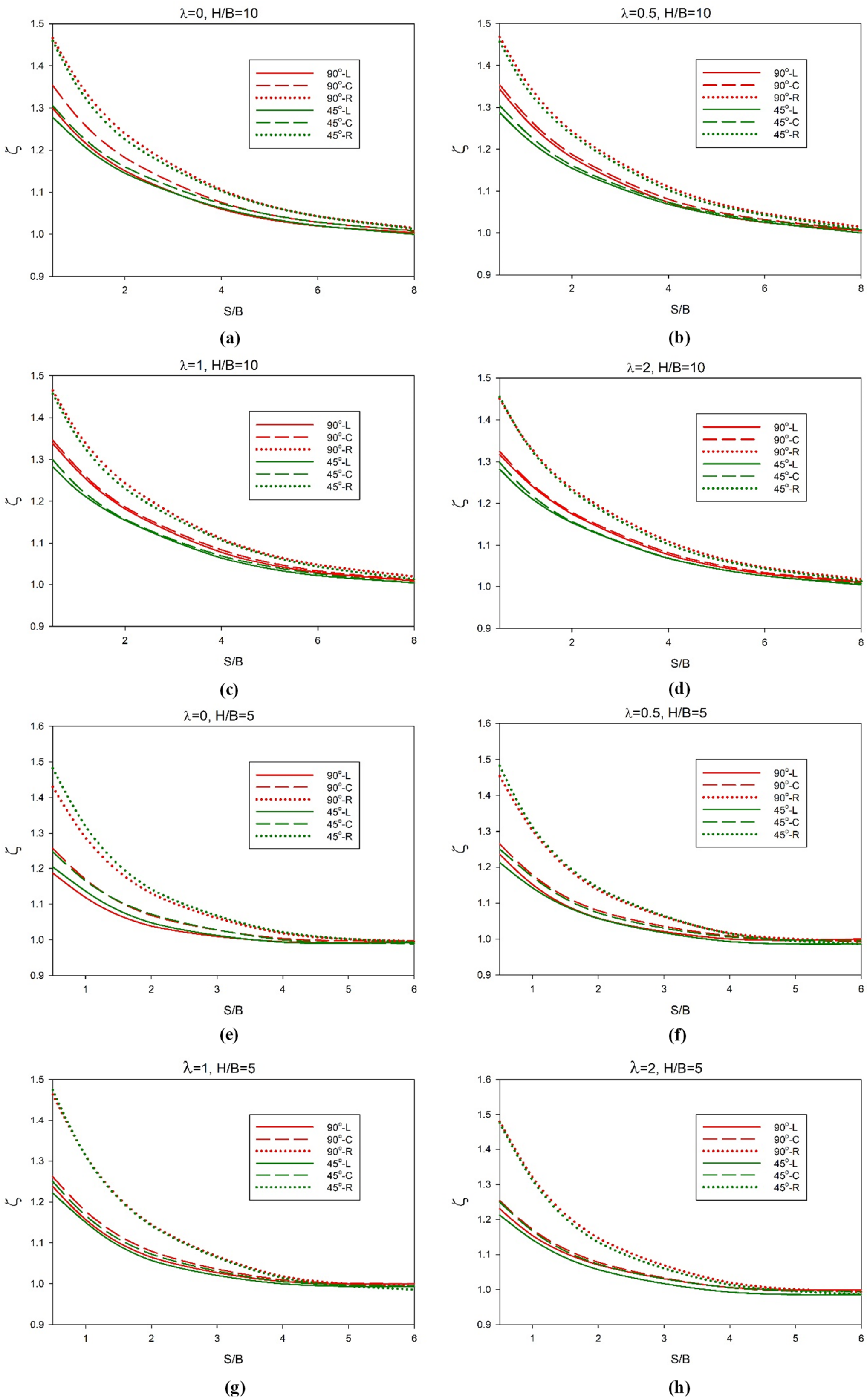


Figure 8. Variations in the settlement ratio with the normalized spacing between the footings for the 90º and 45º degree slope with (a) H/B = 10, λ = 0, (b) H/B = 10, λ = 0.5 (c), H/B = 10, λ = 1, (d) H/B = 10, λ = 2, (e) H/B = 5, λ = 0, (f) H/B = 5, λ = 0.5 (g) H/B = 5, λ = 1, (h) H/B = 5, and λ = 2.

As it can be seen, the settlement ratio of the right side of the footing is not affected by the slope angle, and the curves representing the settlement ratios for the 45º and 90º slopes are almost the same. However, the settlement ratio for the left and center of the footing is affected by the slope angle. Also by increasing the value of λ, the curves representing the settlement ratio of the center of footing tend to cover each other. This behavior can also be observed for the left side of the footing. Also at λ = 2, the settlement ratios of the center and left sides of the footing are approximately equal, regardless of the slope angle.

The variation in the settlement ratio of the center of footing for different slope angles with λ = 0 and H/B = 10 is presented in Figure 9. As it can be seen, there is a smooth transition from the curve representing the 90º degree slope to the curve demonstrating the 45º slope.

Figure 10 represents the change in the ratio of settlement with normalized spacing between the footings for different slope angles and for two different unit weights of the slope's material. Although by increasing the unit weight the absolute value of settlement of the foundation increases, the settlement ratio will decrease, as it can be seen. However, by increasing S/B, the effect of unit weight on ζ is negligible.

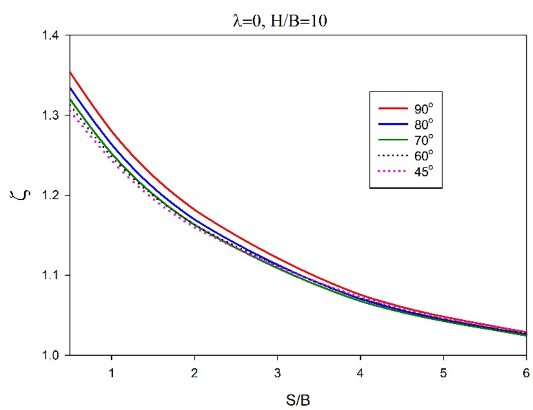
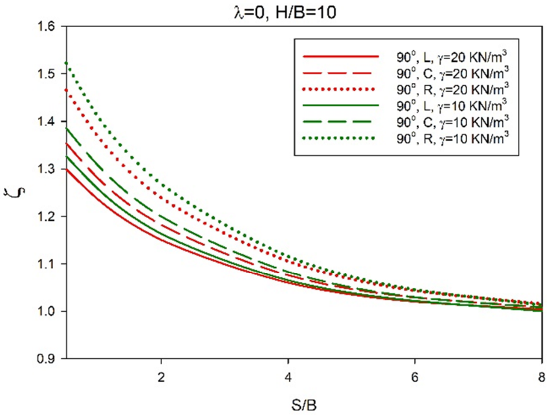


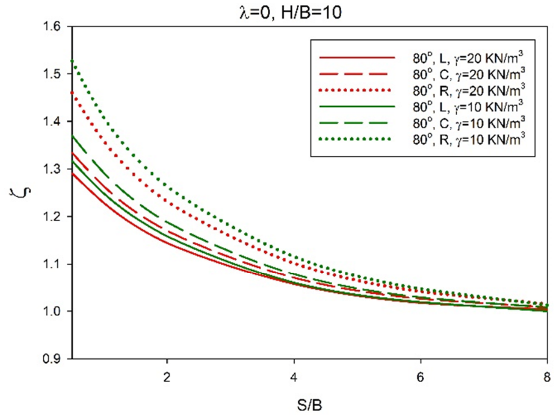
Figure 9. Variation in the settlement ratio of the center of footing with the normalized spacing between the footings for different slope angles with λ = 0 and H/B = 10.

4. Conclusions

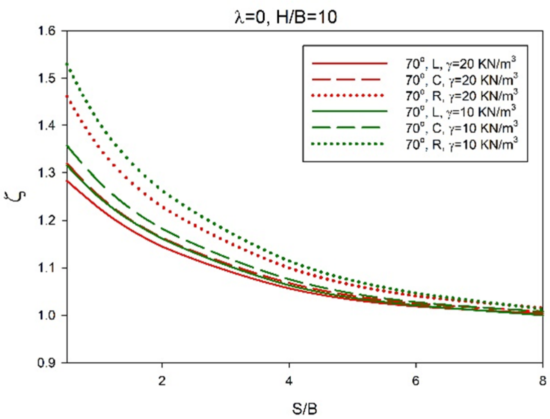
In many cases, the structures require the footing systems to be built near a slope [13], while the footings are not usually isolated [5], and therefore, the interference effects between the foundations should be taken into account. In this research work, the interference effect of the nearby footings on the elastic settlement was examined. For this aim, based on the theory of elasticity and using a proposed transformed Airy stress function, the stress distribution due to the footing pressure in the slope was computed, and subsequently, based on the finite difference scheme, the settlement of the foundations was calculated. The outcomes manifested that by increasing the distance between the foundations, the settlement ratio tended to reach the unit value. This means that by increasing the spacing between two adjacent foundations, the footings will behave as an isolated foundation resting on a slope. This behavior is highly dependent on the slope height. For the case of a slope normalized height equal to 10, the results obtained showed that beyond an S/B larger than 10, the settlement ratio would be equal to the unit value. However, for the cases of H/B = 5 and 1, the S/B values were equal to 6 and 4, respectively. Also the results obtained illustrated that by increasing the λ value, the curves representing the settlement ratio of the left and center sides of the footing would tend to converge to one. Furthermore, the outcome of the proposed solution manifested that when H/B = 1, the settlement ratio of the right side of the footing at first increased, then decreased to quantities bellow the unit value, and finally, increased to reach the value of 1. The settlement ratio of the right side of the footing is not affected by the slope angle, and the curves representing the settlement ratio for different slope angles are almost the same. However, the settlement ratio of the center and left sides of the footing is dependent on the slope angle, and by increasing the value of λ, this dependency will diminish.



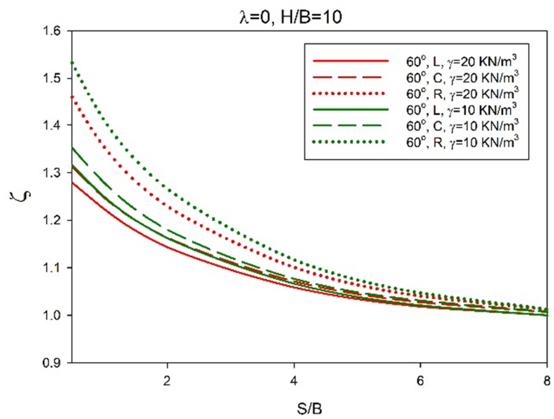
(a)



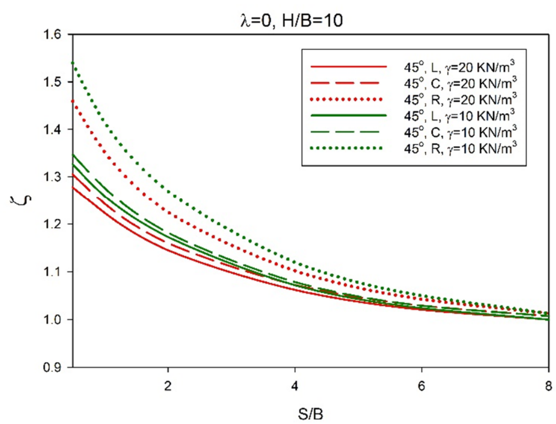
(b)



(c)



(d)



(e)

Figure 10. Variation in the settlement ratio with the normalized spacing between the footings for different slope angles with $\lambda = 0$, H/B = 10, and (a) $\psi = 90^o$, (b) $\psi = 80^o$, (c) $\psi = 70^o$, (d) $\psi = 60^o$, and (e) $\psi = 45^o$.

Appendix A

$$f_1 = \frac{-\sin(\beta-\alpha)\cosh(\beta+\alpha)y + \sin(\beta+\alpha)\cosh(\beta-\alpha)y}{(y\sin 2\beta - \sinh(2\beta y))}$$

$$f_2 = \frac{\sin(\beta+\alpha)\cosh(\beta-\alpha)y + \sin(\beta-\alpha)\cosh(\beta+\alpha)y}{(y\sin 2\beta + \sinh(2\beta y))}$$

$$f_3 = \frac{-\cos(\beta-\alpha)\sinh(\beta+\alpha)y + \cos(\beta+\alpha)\sinh(\beta-\alpha)y}{(y\sin 2\beta - \sinh(2\beta y))}$$

$$f_4 = \frac{\cos(\beta+\alpha)\sinh(\beta-\alpha)y + \cos(\beta-\alpha)\sinh(\beta+\alpha)y}{(y\sin 2\beta + \sinh(2\beta y))}$$

$$f_5 = \frac{-\sin(\beta-\alpha)\cosh(\beta+\alpha)y + \sin(\beta+\alpha)\cosh(\beta-\alpha)y}{(y\sin 2\beta - \sinh(2\beta y))}$$

$$f_6 = \frac{\sin(\beta+\alpha)\cosh(\beta-\alpha)y + \sin(\beta-\alpha)\cosh(\beta+\alpha)y}{(y\sin 2\beta + \sinh(2\beta y))}$$

$$f_7 = \frac{\sin(\beta-\alpha)\sinh(\beta+\alpha)y + \sin(\beta+\alpha)\sinh(\beta-\alpha)y}{y\sin(2\beta) - \sinh(2\beta y)}$$

$$f_8 = \frac{\sin(\beta-\alpha)\sinh(\beta+\alpha)y - \sin(\beta+\alpha)\sinh(\beta-\alpha)y}{y\sin(2\beta) + \sinh(2\beta y)}$$

$$Residue = [\frac{\pi \sin\beta \cos\alpha}{\sin 2\beta - 2\beta} + \frac{\pi \sin\beta \cos\alpha}{\sin 2\beta + 2\beta}]$$

References

1. Zhou, H., Zheng, G., Yin, X., Jia, R., and Yang, X.: The bearing capacity and failure mechanism of a vertically loaded strip footing placed on the top of slopes. Computers and Geotechnics **94**, 12-21 (2018).

2. Stuart, J.: Interference between foundations, with special reference to surface footings in sand. Geotechnique **12**(1), 15-22 (1962).

3. Das, B.M. and Larbi-Cherif, S.: Bearing capacity of two closely-spaced shallow foundations on sand. Soils and foundations **23**(1), 1-7 (1983).

4. Graham, J., Raymond and G., Suppiah, A.: Bearing capacity of three closely-spaced footings on sand. Geotechnique **34**(2), 173-181 (1984).

5. Kumar, J. and Ghosh, P.: Ultimate bearing capacity of two interfering rough strip footings. International Journal of Geomechanics **7**(1), 53-62 (2007).

6. Kumar, J. and Ghosh, P.: Upper bound limit analysis for finding interference effect of two nearby strip footings on sand. Geotechnical and Geological Engineering **25**(5), 499 (2007).

7. Mabrouki, A., Benmeddour, D., Frank, R., and Mellas, M.: Numerical study of the bearing capacity for two interfering strip footings on sands. Computers and Geotechnics **37**(4), 431-439 (2010).

8. Ghosh, P. and Sharma, A.: Interference effect of two nearby strip footings on layered soil: theory of elasticity approach. Acta. Geotech. **5**(3), 189-198 (2010).

9. Lavasan, A.A. and Ghazavi, M.: Behavior of closely spaced square and circular footings on reinforced sand. Soils and Foundations **52**(1), 160-167 (2012).

10. Javid, A.H., Fahimifar, A., and Imani, M.: Numerical investigation on the bearing capacity of two interfering strip footings resting on a rock mass. Computers and Geotechnics **69**, 514-528 (2015).

11. Shamloo, S. and Imani, M.: Upper bound solution for the bearing capacity of two adjacent footings on rock masses. Computers and Geotechnics **129**, 103855.

12. Shields, D., Chandler, N., and Garnier, J.: Bearing capacity of foundations in slopes. Journal of geotechnical engineering **116**(3), 528-537 (1990).

13. Shiau, J., Merifield, R., Lyamin, A., and Sloan, S.: Undrained stability of footings on slopes. International Journal of Geomechanics **11**(5), 381-390 (2011).

14. Zhu, H.-H., Liu, L.-C., Pei, H.-F., and Shi, B.: Settlement analysis of viscoelastic foundation under vertical line load using a fractional Kelvin-Voigt model. Geomechanics and Engineering **4**(1), 67-78 (2012).

15. Maheshwari, P. and Viladkar, M.: Strip footings on a three layer soil system: theory of elasticity approach. International Journal of Geotechnical Engineering **1**(1), 47-59 (2007).

16. Kargar, A.R. and Haghgouei, H.: An analytical solution for time-dependent stress field of lined circular tunnels using complex potential functions in a stepwise procedure. Applied Mathematical Modelling **77**, 1625-1642 (2020).

17. Kargar, A.R., Haghgouei, H., and Babanouri, N.: Time-dependent analysis of stress components around lined tunnels with circular configuration considering tunnel advancing rate effects. Int. J. Rock. Mech. Min. Sci. **133**, 104422 (2020).

18. Haghgouei, H., Kargar, A.R., Amini, M., and Esmaeili, K.: An analytical solution for analysis of toppling-slumping failure in rock slopes. Eng. Geol. **265**, 105396 (2020).

19. Haghgouei, H., Kargar, A.R., Amini, M., and Khosravi, M.H.: Semianalytical Solution for Evaluating Bearing Capacity of a Footing Adjacent to a Slope. International Journal of Geomechanics **21**(2), 06020041 (2021).

20. Tranter, C.J.: Integral transforms in mathematical physics. (1951)

21. Filon, L.N.G.: III.—On a Quadrature Formula for Trigonometric Integrals. Proceedings of the Royal Society of Edinburgh **49**, 38-47 (1930).

22. Goodman, L. and Brown, C.: Dead load stresses and the instability of slopes. Journal of the Soil Mechanics and Foundations Division **89**(3), 103-136 (1963).

بررسی اثر تداخل دو پی در مجاورت شیروانی بر نشست آن ها به روش نیمه تحلیلی

هادی حقگویی[۱]، علیرضا کارگر[۱*]، محمدحسین خسروی[۱] و مهدی امینی[۲]

۱- دانشکده مهندسی معدن، پردیس دانشکده های فنی، دانشگاه تهران، تهران، ایران
۲- شرکت مهندسین مشاور تربر، ونکوور، بریتیش کلمبیا، ونکوور، کانادا

ارسال ۲۰۲۱/۰۱/۲۸، پذیرش ۲۰۲۱/۰۲/۲۰

* نویسنده مسئول مکاتبات: ar.kargar@ut.ac.ir

چکیده:

در بسیاری از پروژه‌های مهندسی، پی‌ها در مجاورت هم ساخته می شوند. بنابراین، اثر تداخل فونداسیون‌های ساخته شده در مجاورت هم باید موردبررسی قرار گیرد. در این تحقیق، با استفاده از یک روش نیمه تحلیلی به بررسی نشست الاستیک ناشی از تداخل پی‌های ساخته‌شده در مجاورت شیروانی پرداخته‌شده است. با استفاده از یک تابع تنش ایری پیشنهادی، در ابتدا توزیع تنش در شیروانی براثر فشار ناشی از پی محاسبه گردید، و سپس با استفاده از روش تفاضل محدود، جابجایی پی‌ها موردبررسی قرار گرفت. نتایج حاصله نشان داد که با افزایش فاصله‌داری پی‌ها، تأثیر تداخل پی‌ها بر روی نشست کم خواهد شد. این رفتار بشدت به مشخصات شیروانی مرتبط است به‌نحوی‌که برای یک شیروانی با ارتفاع ۱۰ برابر عرض پی، پس از نسبت S/B بزرگ‌تر از ۱۰، اثر تداخل محسوس نیست و پی‌ها همانند یک فونداسیون جدا عمل خواهند کرد. با کاهش ارتفاع شیروانی، این رفتار در S/B های کوچک‌تری رخ خواهد داد.

کلمات کلیدی: پی، نشست، پی های متداخل، پایداری شیروانی، الاستیسیته