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A Stochastic Model Upgrading Gold Content in Cyanide Leaching using Monte Carlo Simulation

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Abstract

This paper elucidates a new idea and concept in order to estimate the gold content in cyanide leaching method. The cyanidation method is traditionally used for gold extraction. However, this method is laborious, time-consuming, costly, and depends upon the availability of the processing units. In this work, an attempt is made to upgrade the gold ore content by Monte Carlo based simulation. An excellent approach always requires a high quality of the datasets for a good model. A total of 48 incomplete datasets are collected from the Shogore district, Chitral area of Khyber Pakhtunkhwa, Pakistan. The cyanidation leaching test is carried out in order to measure the percentage of the gold ore content. In this work, the mean, median, mode, and successive iteration substitute methods are employed in such a way that they can compute the datasets with missing attributes. The multiple regression analysis is used to find a correlation between the potential of hydrogen ion concentration (pH), solid content (in %), NaCN concentration (in ppm), leaching time (in Hr), particle size (in μm), and measured percentage of gold recovery (in %). Moreover, the normal and exponential distributions are employed to forecast the uncertainty in the measured gold ores. The performance of the model reveals that the Monte Carlo approach is more authentic for the probability estimation of gold ore recovery. The sensitivity analysis reveals that pH is the most influential parameter in the estimation of the gold ore deposits. This stochastic approach can be considered as a foundation to foretell the probabilistic exploration of the new gold deposits.

1. Introduction

The gold demand has reached an all-time high requirement over the last few decades, and has now gained a 4 Mt/year market [1, 2]. Due to its extraordinary physical and chemical properties, gold has a lot of applications in different enterprises including the medical, electronics, electrical, and chemical industries [3, 4]. However, from its first recommendation, it has been almost 15 years that there is a serious decline in the exploration of new gold deposits, and there is a high demand about the time-saving, practical, profitable, and laborsaving exploration for new gold deposits [5 -6]. The chemistry and morphology of the sludge gold atoms and their

appropriations are considered as the most important key performance attributes for the exposure of gold reserve mineralization, mineralization style, and latent of surrounding rocks [7-12]. The placer gold mining techniques have been used in order to assess the gold grain texture [13]. Many studies have been effectuated in the northern area of Pakistan, aiming to explore new gold ore deposits [14-20]. An integrated geophysical approach with fusion representation for the geological model has been proposed in order to improve the Makran subduction belt by employing a magnetic and gravity database [21]. The seismic reflection data has been practiced,



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lying out a distinguished structures model and their ability to unfold the hydrocarbon reservoirs [22].

Uncertainty has been commonly used in rock engineering during the analysis and design of various structures. The input attributes are greatly influenced by uncertainty as a random characteristic rather than a single value [23]. The Monte Carlo simulation uses a random number as a representative from a probability distribution, and a large enough number of trials are produced and employed in the computation so that the final value calculation will be massive [24]. Considerable copper populations have been established on the outcome of U-V and C-V 3D modeling and the most ubiquitous model of various types of rock [25]. A study has been implemented in order to compare the results of the minimum/maximum auto-correlation factor approach with some

traditional approach for multivariate simulation in ore body evaluation [26].

The original contribution of this research work is to develop a methodology that utilizes the multiple regression analysis and Monte Carlo simulations on gold extraction from the rock deposits. First, the incomplete data was collected from different experiments on cyanide leaching, the required representative sample were compiled from Shogore, in the district of Chitral, Pakistan. The missing values were replaced by various replacement methods. Then the multiple regression analysis and the Monte Carlo-based simulation were used in order to predict the model. Later on, the prognostic execution of every model was thoroughly examined and evaluated. A flowchart of the proposed research works is revealed in Figure 1.

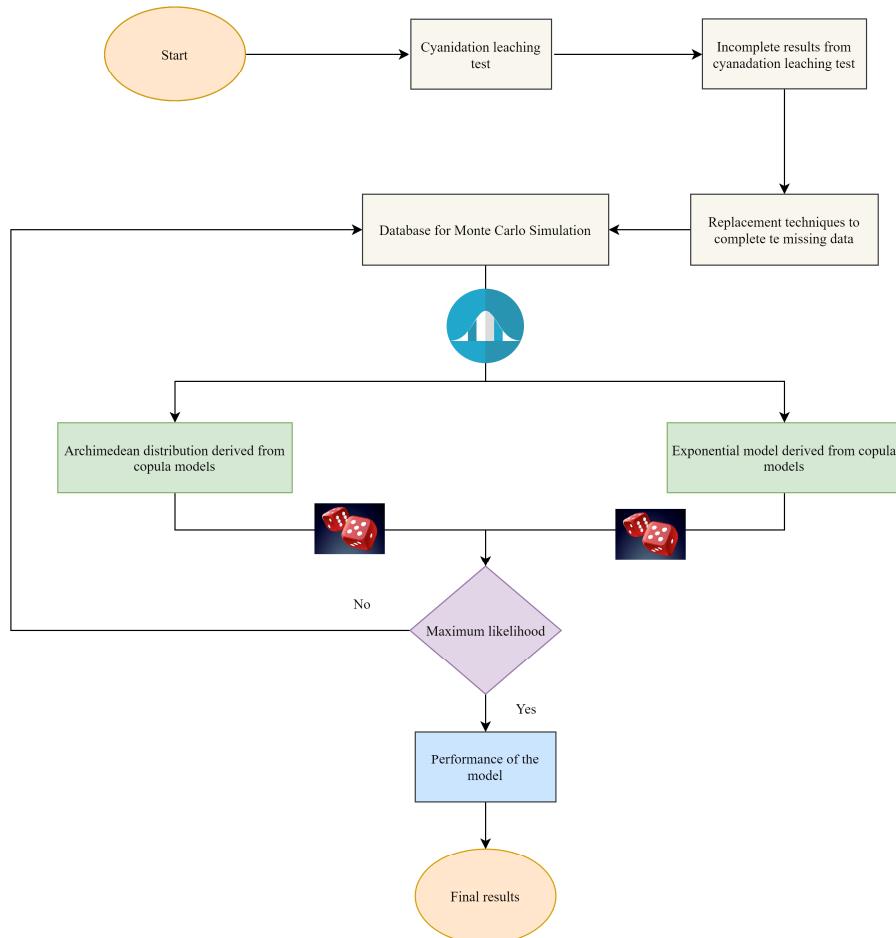


Figure 1. A flow chart of the work.

2. Case study

The studied area is located in Khyber Pakhtunkhwa, Pakistan, and is accessed through

the Karakoram highway from Islamabad, the capital city of Pakistan. Shogore is an administrative authority recognized as Union Council of the Chitral district, Pakistan as, shown

in Figure 2. Its geographical coordinates are 35.8461° North, 71.7858° East, while having an altitude of 1,100 m (3,600 ft). The samples are arranged succeding two phases, namely crushing and milling. During this work, as a leaching agent,

sodium cyanide was used. Furthermore, in order to sustain the potential of hydrogen ion concentration (pH) at the targetted attribute during leaching, $\text{Ca}(\text{OH})_2$ was employed as a pH modifier.

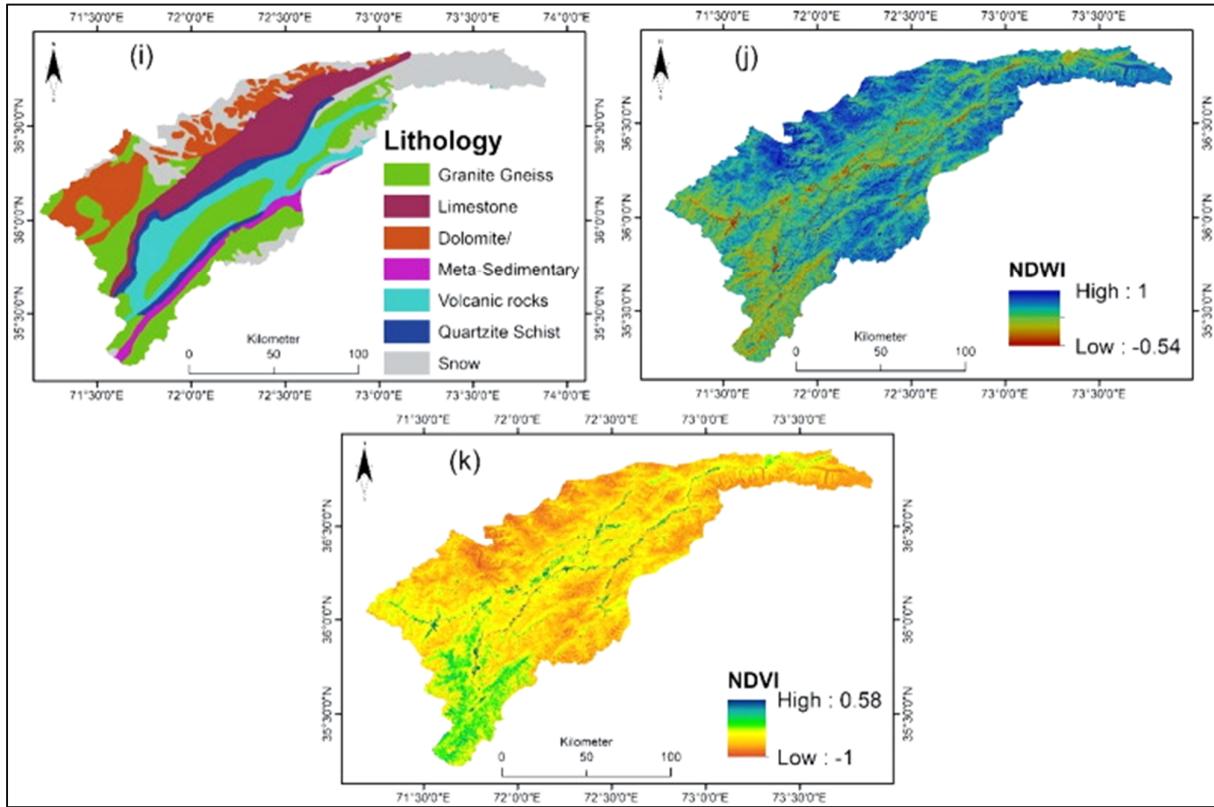


Figure 2. Geological map of the case study area [27].

3. Cyanide leaching

The experiments were conducted in a beaker at normal room temperature under various range states of pH of 10.22–12.3, concentration of sodium cyanide of 1000–10000 ppm, solid content of 25–50%, size of the particle of 63–106 μm , and leaching time of 0–39 h. For every trial 15 ppm, an illustrative sample was chosen. Firstly, the pulp was formed in the beaker, and the pH was modified using $\text{Ca}(\text{OH})_2$ for the targeted attributes; later on, the sodium cyanide was combined with the pulp and was stirred. The pulp was filtered following each trial, the pulp was separated, and the liquid stage was examined for the extraction. Various parameters in the literature were used for cyanide leaching [28–31]. Based on the literature, five parameters were used in this work, i.e. potential of hydrogen (pH), solid content (in %), NaCN concentration (in ppm), leaching time (in Hr), and particle size (in μm) in order to predict the gold ore deposit.

4. Methodology

Data pre-processing is one of the major steps in the analysis of incomplete datasets preceding further data analysis [32]. Various methods are utilized to treat the incomplete data [33–38]. The mean, median, and mode methods [39] and the successive iteration techniques [38] are utilized in order to complete the data.

4.1. Replacement of missing values

Succeed the missing values by any one of the missing value replacement methods (mean, median, mode or successive iteration).

i) Mean method

First, calculate the mean (average) value for the feature. (This feature contains the missing values.) Secondly, replace all the missing values of this feature with this mean value.

ii) Median method

First, calculate the median (middle attribute) for the feature. (This feature contains the missing values.) Secondly, replace all the missing values of this feature with this median value.

iii) Mode method

First, calculate the highest value for the feature. (This feature contains the missing values.) Secondly, replace all the missing values of this feature with this highest value.

iv) Successive iteration method

First, find the mean value for the feature. (This feature contain the missing values.) Secondly, succeed this mean for the initial missing value. Repeatedly, compute the mean for the entire feature. If the new mean and old mean are matching, then replace this mean value for all the missing values and stop the iteration; otherwise, repeat the process. The measured average and range of mean, median, mode, and successive iteration are given in Table 1.

Table1. Measured average and range of substitute methods.

	Mean method measured Au (%)	Median method measured Au (%)	Mode method measured Au (%)	Successive iteration method measured Au (%)
Average	3.7181	3.7342	3.7336	3.7181
Range	0.1397-4.58585	0.1397-4.58585	0.1397-4.58585	0.1397 - 4.58585

4.2. Prediction models performance indicators

The key role in attaining an ore reserve genetic illustration is the primary geometrical model of a complete deposit attribute database. Consequently, significant time and effort are required in order to establish a large amount of database that incorporates the data characterized by all mines from the whole world. The petrological and geological data is employed in order to explain the model construction of reserve genetic representation [40]. The prediction models are implemented to build a genuine correlation between the various input attributes for the estimation of gold traps. The various prediction models performance is computed using the index i.e the squared coefficient of correlation between the measured and predicted attributes [41-43]. The formulas for calculating the R^2 is given by Eq. (1).

$$R^2 = 1 - \frac{\sum_{i=1}^n (Z - Z')^2}{\sum_{i=1}^n (Z - Z'')^2} \quad (1)$$

where Z is the measured value, Z' is the predicted value, and Z'' is the mean value; N represents the total number of the dataset. For $R^2 = 100\%$, the predictive model is defined as a perfect model.

Concerning the determination of capability of the various Monte Carlo simulation models, the computed performance attribute of the generated database was enumerated.

4.3. Multiple regression analysis (MRA)

MRA has been conventionally utilized in order to foretell the real-world problems in numerous engineering fields [44]. In regression, a correlation is established between one or more input and output datasets [45]. In this contribution, Excel 2007 was employed in order to compute a multiple regression analysis between the input and output datasets. The output values were implemented in their logarithmic form. After computing the logarithm of the feature, it does not alter the relationship of the datasets but it only flattens the dimension of the variable and lowers the absolute desirability of the dataset [46]. Table 2, 3, 4 and 5 represents the statistical characteristics from MRA mean, median, mode and successive iteration respectively. As can be seen from the results, the coefficient of correlation is relatively low for all the models because the dataset is unbalanced and is relatively small. Hence a MC simulation has been adopted in this research to establish a stochastic model for the gold reserve estimation.

Table 2. Statistical characteristics obtained from MRA mean method.

	Coefficients	Standard error	t Stat	P-value	Statistical parameters
Constant	-4.3789	10.4993	0.4170	0.6787	Multiple R
Potential of hydrogen (pH)	0.5866	0.9611	0.6103	0.5448	R^2
Solid content (%)	0.0077	0.0132	0.5807	0.5644	Adjusted R square
NaCN concentration (ppm)	1.52E-05	6.55E-05	0.2313	0.8181	Standard error
Retention time (Hr)	0.02937	0.01379	2.1288	0.0391	Total trials
Particle size (μm)	0.01125	0.0077	1.4607	0.1515	

Table 3. Statistical characteristics obtained from MRA median method.

	Coefficients	Standard error	t Stat	P-value	Statistical parameters
Constant	-7.2755	11.2554	-0.6464	0.5215	Multiple R
Potential of hydrogen (pH)	0.8701	1.0307	0.8441	0.4033	R^2
Solid content (%)	-0.00026	0.0155	-0.0164	0.9869	Adjusted R Square
NaCN concentration (ppm)	1.43E-05	6.63E-05	0.2151	0.8307	Standard error
Leaching time (Hr)	0.0309	0.0148	2.0894	0.0427	Total trials
Particle size (μm)	0.0143	0.0077	1.8623	0.0695	48

Table 4. Statistical characteristics obtained from MRA mode method.

	Coefficients	Standard error	t Stat	P-value	Statistical parameters
Constant	-4.1688	10.1891	-0.40915	0.6845	Multiple R
Potential of hydrogen (pH)	0.5760	0.9306	0.6189	0.5392	R^2
Solid content (%)	-0.00208	0.0154	-0.1349	0.8933	Adjusted R Square
NaCN concentration (ppm)	4.59E-05	5.87E-05	0.7819	0.4386	Standard error
Leaching time (Hr)	0.0281	0.0133	2.1055	0.0412	Total trials
Particle size (μm)	0.0146	0.0076	1.9052	0.0636	48

Table 5. Statistical characteristics obtained from MRA successive replacement method.

	Coefficients	Standard error	t Stat	P-value	Statistical parameters
Constant	-3.5395	10.4578	-0.3384	0.736702	Multiple R
Potential of hydrogen (pH)	0.5264	0.9594	0.5486	0.586132	R^2
Solid content (%)	0.000603	0.0154	0.0391	0.968973	Adjusted R Square
NaCN concentration (ppm)	6.14E-06	6.67E-05	0.0919	0.927139	Standard error
Leaching time (Hr)	0.0288	0.0138	2.0874	0.042951	Total trials
Particle size (μm)	0.01298	0.0075	1.7216	0.092483	48

4.4. Monte Carlo (MC) simulation

The MC simulation is employed as a uncertainty simulation in a variety of real-world issues [47-48]. This simulation is utilized as a computerized calculation for recognition of the abundant complications including the impact of uncertainty and the risk of different models including assessment, prediction, appraisal, and evalution in different engineering sectors including finance, project management, decision making, etc. In order to attain the probabilistic estimation of a mathematical equation, the MC simulation is employed on monotonous indiscriminate sampling [49-50]. Each model that forecasts the consequences requires an investigative set of hypotheses correlated with the real-world problems and evaluation of the expected values proceeding the data [49].

MC simulation has two foremost intentions. First is the numerical testing of dissimilarities and uncertainties, and the second is the attribute investigation affecting uncertainty, variation, and their proportion [51]. In contrast to the conventional established methods, the MC simulation utilizes an array of evaluated attributes

as an input, and afterward, the simulations give an extent of the output values for a whole data. Hence, in the MC simulation, a more practical picture of a simulation model can be generated. In this approach, a random attribute is taken for each input attribute on the basis of the range of output value, and then the output datasets are evaluated depending on these random values. Subsequently, the consequences of the MC simulation are noted, and this method is continuously replicated using various random numbers. The simulation technique is reciprocated over 1000 times. After evaluation and prediction of the simulation technique, various outcomes are obtained as an output, which can be utilized in order to rademonstrate the product [52-56].

The prediction probabilistic model of gold reserve estimation was constructed based on five parameters: potential of hydrogen (pH), solid content (in %), NaCN concentration (in ppm), leaching time (in Hr), and particle size (in μm). RISK SOLVER APP was used as the MC simulator in this approach. The calculation model for the prediction of gold estimation is given as:

$$G = t(A) \quad (2)$$

where G is the gold estimation prediction value and $t(A)$ is the calculation model of gold estimation prediction.

Considering that A is a random set of attribute, B is also a random attribute. Hence, the estimation probability P_f is given as:

$$P_f = P(B \leq B_{cri}) \quad (3)$$

$$\int_{-\infty}^{+\infty} I(t(A) \leq B_{cri}) f_a(A) dx$$

where:

$$I(t(A) \leq B_{cri}) = \begin{cases} 1 & t(A) \leq B_{cri} \\ 0 & t(A) > B_{cri} \end{cases} \quad (4)$$

where B_{cri} is the critical prediction value of gold ore deposit estimation.

On the basis of Eq. (2), it is very laborious to execute a direct combination. Hence, the probability of prediction P_f is used in order to predict the gold ore deposit estimation of Eq. (2). Eq. 5 can also be generated by the MC simulation:

$$P_f = \frac{1}{N} \sum_{i=1}^N I(t(A) \leq B_{i,cri}) \quad (5)$$

Table 6. Spareman's correlation coefficient for various input attributes.

	pH	Solid content (%)	NaCN concentration (ppm)	Leaching time (Hr)	Particle size (μm)
Potential of hydrogen (pH)	1				
Solid content (%)	-0.04472	1			
NaCN concentration (ppm)	0.168338	0.155135	1		
Leaching time (Hr)	-0.66512	-0.01357	-0.067457	1	
Particle size (μm)	-0.27857	0.316228	-0.235922	-0.00429	1

4.5. Univariate probability distribution in estimating gold reserve

By utilizing five attributes including potential of hydrogen (pH), solid content (in %), NaCN concentration (in ppm), leaching time (in Hr), and particle size (in μm) of gold ore reserve estimation, different normal probability distributions were obtained for the mean, median, mode, and successive iteration methods, as shown in Figure 3. Firstly, the distinguished univariate normal distribution was determined. Secondly, the joint normal probability distribution function was

where N is the total number of representation of the MC simulation, A_i is the i-th attribute set of A, and $B_{i,cri}$ is B_{cri} corresponding to the A_i . Consequently, as long as Eq. (5) is acquired, the probability P_f of gold reserve estimation can be obtained. The calculation steps are as follow:

1. The attribute values of the certain number of times were executed using five attributes.
2. The acquired attribute values are replaced into Eq. (2) in order to compute the analogous gold deposit estimation value.
3. Finally, the prediction values of gold reserve estimation are computed by the stochastic approach, and the uncertainty and distribution function of gold deposit estimation are acquired. Later on, the prediction grade probability is calculated using Eq. (4).

It is also practicable to compute the prediction number less than the critical value gold reserve estimation, and Eq. (5) could be used to compute the occurrence probability of the gold deposit estimation. The spareman's correlation between various input attributes is shown in Table 6.

executed by the MC simulation technique on the mean, median, mode, and successive iteration. Various functions were employed to perform the MC simulation, and the $1,000 \times 5$ -dimensional random attributes generated were stored in the attributes datasets. The generally employed normal distribution model of various attributes is always not suitable to get the optimal probability distribution model that specifies the respective anomalies encompassed for different attributes [57]. The range and average obtained by the normal probability distribution are given in Table 7.

Table 7. Predicted normal distribution average, range of mean, median, mode, and successive iteration.

	Mean method measured Au (%)	Median method measured Au (%)	Mode method measured Au (%)	Successive iteration method measured Au (%)
Average	3.7176	3.7345	3.7336	3.7183
Range	0.4232-6.6605	0.1915-7.4665	0.5204-6.8914	0.6258-6.8515

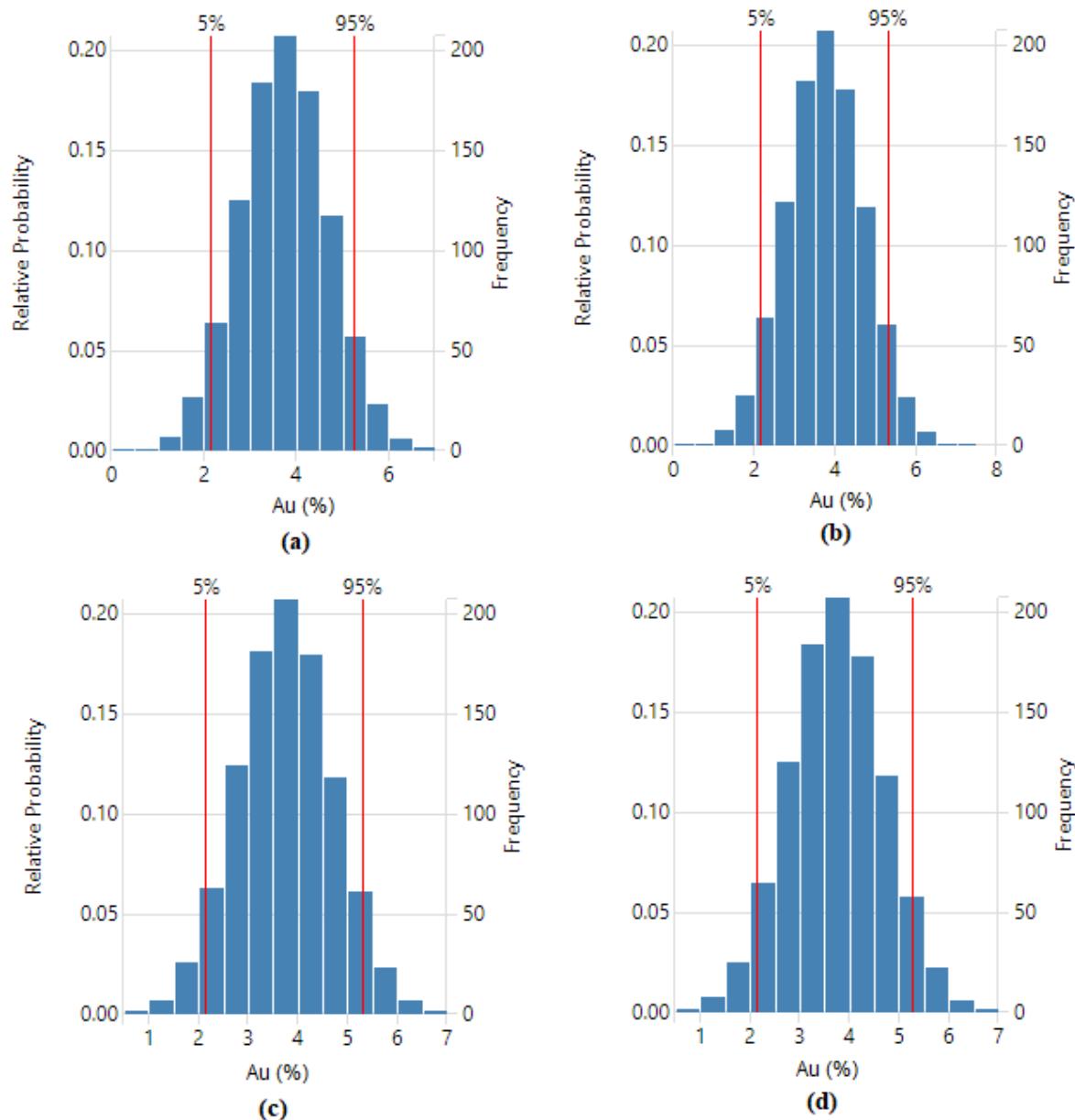


Figure 3. Probability density function of Au (%) obtained by MC simulation; (a) Mean replacement; (b) Median replacement; (c) Mode replacement; (d) Successive iteration replacement Au (%).

For different functions, there are different correlations between the attributes, and have various different structures of correlation. The normal univariate probability distribution can only explain the positive relationship between the attributes. Therefore, the exponential probability distribution function distribution were further

acquired to narrate the correlation between the five attributes influencing gold deposit estimation of mean, median, mode and successive mean iteration replacement (Figure 4). Table 8 shows the predicted exponential distribution of all the replacement methods.

Table 8. Predicted exponential distribution average, range of mean, median, mode, and successive iteration.

	Mean method measured Au (%)	Median method measured Au (%)	Mode method measured Au (%)	Successive iteration method measured A (%)
Average	3.7162	3.7329	3.7408	3.7217
Range	0.0017-28.3231	0.0022-26.9374	0.0002-32.2795	0.0032-31.8126

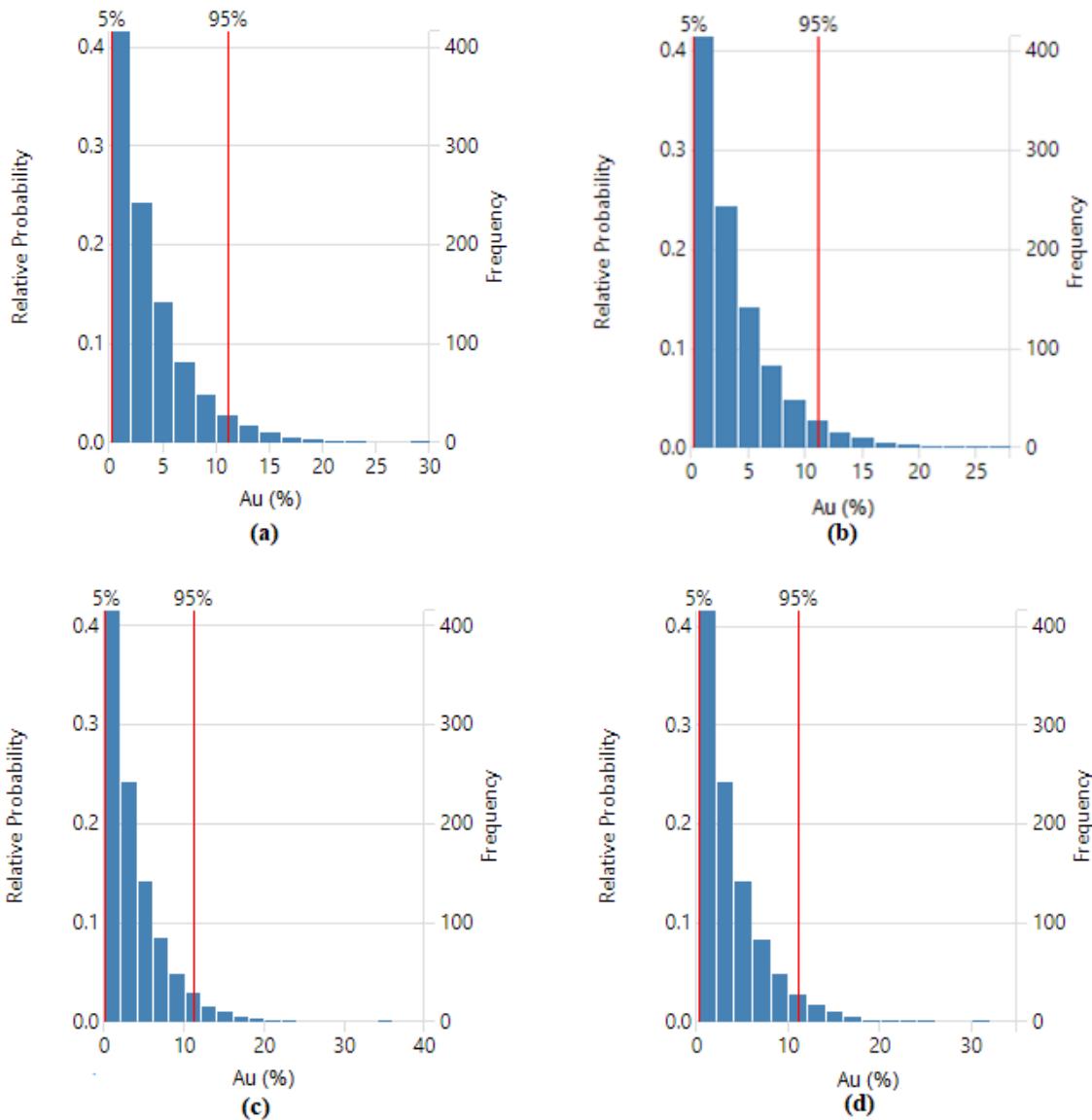


Figure 4. Exponential distribution of probability of Au (%): (a) Mean replacement Au; (b) Median replacement Au (%); (c) mode replacement; (d) successive iteration replacement Au (%).

With the aim to affirm, all the possible correlation is presented in a stochastic aspect. A 1000 iterations were investigated during this work, meaning that 1000×5 -dimensional simulations between the input parameters were employed during the MC simulation. The normal and exponential simulations were implemented in order to achieve a statistical illustration of the gold ore data. The maximum likelihood method for both the normal and exponential MC Simulation was employed in order to compute the parameter of the model. The values in Tables 1, 7, and 8 were compared, showing that the predicted exponential mode Au percentage with 3.7408% was higher

than all the predicted models with the range of 0.000233–35.27975%. Hence, all the possible situations were investigated by the MC simulation. A comparison of the normal and exponential distributions of the MC simulation is shown in Figure 5. Hence, this stochastic approach could be applied to probabilistic analysis of Au estimation in gold deposit estimation.

5. Sensitivity analysis

In order to analyze the sensitivity of gold reserve estimation to the input parameters, a sensitivity analysis was carried out using the most accurate

model. In order to study the sensitivity of gold reserve prediction, a sensitivity analysis was realized using the mode exponential distribution model. From Figure 6, it is clear that the potential of hydrogen (pH), solid content (in %), NaCN concentration (in ppm), and leaching time (in Hr)

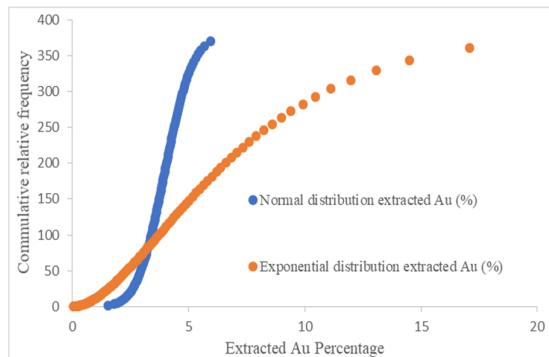


Figure 5. Comparison of normal and exponent cumulative relative frequency distribution Au (%).

6. Conclusions

Placer mining is a technique used traditionally in order to assess the gold grain texture. This methodology is employed to calculate the proportion of the extracted gold ore reserve. However, this technique is usually laborious, time-consuming, and associated with the availability of exploratory tool. The main objective of this work was to elucidate an extraction strategy for the probabilistic proportion model of gold content in cyanidation leaching. The cyanidation leaching tests were carried out at the laboratory, and 48 incomplete datasets were collected from numerous experiments. First, the incomplete data was accomplished by different replacement methods. Then the data was analyzed by the MC normal and exponential distribution functions in order to predict the probability of the measured distribution of gold ore deposits.

The conclusions of the current work are as follow:

- The constructed mean, median, mode, and successive data replacement methods show that they can process the datasets in a better way.
- The performance of the exponential mode model shows that the MC approach is more authentic and reliable for probabilistic estimation of gold ore recovery. As it can be seen in Table 8, the predicted exponential mode Au percentage is in the range of 0.000233–35.27975%, demonstrating that the proposed MC simulation executes the gold content distribution with a wide range.

are positively correlated, and the particle size (μm) is negatively correlated with the gold reserve percentage. Hence, the potential of hydrogen pH, solid content, and particle size are the most influential attributes to the estimation of gold ore deposits.

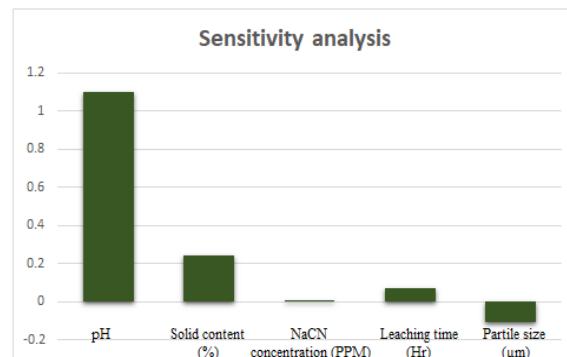


Figure 6. Sensitivity analysis effects of input variable on Au recovery (%).

- The maximum likelihood method was employed in order to compute the parameter of the model.
- Sensitivity analysis shows that pH is one of the most influential parameters in estimating the gold reserve content.
- In order to tackle the other alternative gold ore reserve estimation projects, this stochastic approach can be considered as a keystone.

Conflict of interest

The authors have no known conflicts of interest that will affect the work reported in this study.

Data availability

The data used in this study is available from authors if needed.

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References

- [1]. Ellis, L., Hlongwa, N., Kemp, H., Kershoff, G., Lemboe, C., Smit, B. and Van der Wath, N. (2011) First Quarter.
- [2]. Martens, E., Zhang, H., Prommer, H., Greskowiak, J., Jeffrey, M. and Roberts, P. (2012). In situ recovery of gold: Column leaching experiments and reactive transport modeling. *Hydrometallurgy*, 125: 16-23.

- [3]. Ramesh, A., Hasegawa, H., Sugimoto, W., Maki, T. and Ueda, K. (2008). Adsorption of gold (III), platinum (IV), and palladium (II) onto glycine modified crosslinked chitosan resin. *Bioresource technology*, 99 (9): 3801-3809.
- [4]. Wang, H. and Ren, Z.J. (2014). Bio-electrochemical metal recovery from wastewater: a review. *Water research*, 66, 219-232.
- [5]. Schodde, R.C. (2004). Discovery performance of the western world gold industry over the period 1985–2003. In Proceedings of PACRIM 2004 Congress.
- [6]. Vearncombe, J.R. and Phillips, G.N. (2020). The importance of brownfields gold exploration. *Mineralium Deposita*, 55 (2): 189-196.
- [7]. Chapman, R.J., Leake, R.C., Bond, D.P.G., Stedra, V. and Fairgrieve, B. (2009). Chemical and mineralogical signatures of gold formed in oxidizing chloride hydrothermal systems and their significance within populations of placer gold grains collected during reconnaissance. *Economic Geology*, 104 (4): 563-585.
- [8]. Townley, B.K., Héral, G., Maksaev, V., Palacios, C., de Parseval, P., Sepulveda, F. and Ulloa, C. (2003). Gold grain morphology and composition as an exploration tool: application to gold exploration in covered areas. *Geochemistry: Exploration, Environment, Analysis*, 3 (1): 29-38.
- [9]. Chapman, R.J., Mortensen, J.K. and LeBarge, W.P. (2011). Styles of lode gold mineralization contributing to the placers of the Indian River and Black Hills Creek, Yukon Territory, Canada as deduced from micro-chemical characterization of placer gold grains. *Mineralium Deposita*, 46 (8): 881-903.
- [10]. Lalov, A.V., Chefranov, R.M., Naumov, V.A., Naumova, O.B., LeBarge, W. and Dilly, R.A. (2017). Typomorphic features of placer gold of Vagran cluster (the Northern Urals) and search indicators for primary bedrock gold deposits. *Ore Geology Reviews*, 85: 321-335.
- [11]. Moles, N.R., Chapman, R.J. and Warner, R.B. (2013). The significance of copper concentrations in natural gold alloy for reconnaissance exploration and understanding gold-depositing hydrothermal systems. *Geochemistry: Exploration, Environment, Analysis*, 13 (2): 115-130.
- [12]. Omang, B.O., Suh, C.E., Lehmann, B., Vishiti, A., Chombong, N.N., Fon, A.N. and Shemang, E.M. (2015). Micro-chemical signature of alluvial gold from two contrasting terrains in Cameroon. *Journal of African Earth Sciences*, 112: 1-14.
- [13]. Youngson, J.H. and Craw, D. (1993). Gold nugget growth during tectonically induced sedimentary recycling, Otago, New Zealand. *Sedimentary geology*, 84 (1-4): 71-88.
- [14]. Jankovic, S. (1984). Metallogeny and mineral potential of Northern Pakistan (north of Indus Suture zone). A preliminary assessment.
- [15]. Halfpenny, R. and Mazzucchelli, R.H. (1999). Regional multi-element drainage geochemistry in the Himalayan Mountains, northern Pakistan. *Journal of Geochemical Exploration*, 67 (1-3): 223-233.
- [16]. Ali, L., Williamson, B.J., Moon, C.J., Shah, M.T. and Khattak, S.A. (2015). Distribution of gold in different size fractions of stream sediments as a guide to bedrock gold mineralization along the Shyok Suture Zone and adjacent areas of the Kohistan Island Arc, Pakistan. *Arabian Journal of Geosciences*, 8 (4): 2227-2235.
- [17]. Miandad, S., Shah, M.T., Khan, S.D. and Ahmad, L. (2014). Investigation for gold and base metals mineralization and petrochemical characteristics of the rocks of upper parts of Bagrot valley, Gilgit-Baltistan, Pakistan. *Journal of Himalayan Earth Sciences*, 47 (2): 29.
- [18]. Shah, M.T., Khan, S.D. and Ahmad, L. (2015). Investigation of gold and base metal mineralization and the petrochemical studies of the associated host rocks, Shagari Bala area, Skardu Gilgit-Baltistan, Pakistan. *Journal of Himalayan Earth Science*, 48 (1).
- [19]. Alam, M., Li, S.R., Santosh, M. and Yuan, M.W. (2019). Morphology and chemistry of placer gold in the Bagrote and Dainter streams, northern Pakistan: Implications for provenance and exploration. *Geological Journal*, 54 (3): 1672-1687.
- [20]. Alam, M., Li, S.R., Santosh, M., Shah, A., Yuan, M.W., Khan, H. and Zeng, Y.J. (2019). Morphological, thermo-electrical, geochemical and isotopic anatomy of auriferous pyrite from the Bagrote valley placer deposits, North Pakistan: Implications for ore genesis and gold exploration. *Ore Geology Reviews*, 112: 103008.
- [21]. Nasri, S., Kalate, A.N., Kahoo, A.R. and Monfared, M.S. (2020). New insights into the structural model of the Makran subduction zone by fusion of 3D inverted geophysical models. *Journal of Asian Earth Sciences*, 188: 104075.
- [22]. Radfar, A., Chakdel, A.R., Nejati, A. and Soleimani, M. (2019). New insights into the structure of the South Caspian Basin from seismic reflection data, Gorgan Plain, Iran. *International Journal of Earth Sciences*, 108 (2): 379-402.
- [23]. Idris, M.A. (2014). Probabilistic stability analysis of underground mine excavations (Doctoral dissertation, Luleå Tekniska Universitet).
- [24]. Wattimena, R.K., Kramadibrata, S., Sidi, I.D., Arif, I. and Azizi, M.A. (2012, January). Probabilistic analysis of single bench using new slope stability curves. In ISRM Regional Symposium-7th Asian Rock

- Mechanics Symposium. International Society for Rock Mechanics and Rock Engineering.
- [25]. Hajsadeghi, S., Asghari, O., Mirmohammadi, M., Afzal, P., and Meshkani, S.A. (2020). Uncertainty-volume fractal model for delineating copper mineralization controllers using geostatistical simulation in Nohkouhi volcanogenic massive sulfide deposit, Central Iran. Maden Tetkik ve Arama Dergisi, 161 (161): 1-11.
- [26]. Maleki, M. and Madani, N. (2016). Multivariate geostatistical analysis: An application to ore body evaluation. Iranian Journal of Earth Sciences, 8 (2): 173-184.
- [27]. Aslam, B., Maqsoom, A., Alaloul, W.S., Musarat, M.A., Jabbar, T., and Zafar, A. (2020). Soil erosion susceptibility mapping using a GIS-based multi-criteria decision approach: Case of district Chitral, Pakistan. Ain Shams Engineering Journal.
- [28]. Azizi, A., Ghaedrahmati, R., Ghahramani, N. and Rooki, R. (2016). Modelling and simulation of the cyanidation process of Aghdareh gold ore using artificial neural network and multiple linear regression. International Journal of Mining and Mineral Engineering, 7 (2): 139-154.
- [29]. Yilmaz, T., Seckin, G. and Yuceer, A. (2010). Modeling of effluent COD in UAF reactor treating cyanide containing wastewater using artificial neural network approaches. Advances in Engineering Software, 41 (7-8): 1005-1010.
- [30]. Zhang, J., Mao, Z.Z. and He, D.K. (2015). Real-time optimization based on a serial hybrid model for gold cyanidation leaching process. Minerals Engineering, 70: 250-263.
- [31]. Jun, Z., Hua, Y., Hongxia, Y., Zhongda, T. and Runda, J. (2019). Gold recovery modeling based on interval prediction for a gold cyanidation leaching plant. IEEE Access, 7, 71511-71528. [32]Paulin, F. and Santhakumaran, A. (2010). An Algorithm to Reconstruct the Missing Values for Diagnosing the Breast Cancer. Global Journal of Computer Science and Technology.
- [33]. Twala, B., Cartwright, M. and Shepperd, M. (2005, November). Comparison of various methods for handling incomplete data in software engineering databases. In 2005 International Symposium on Empirical Software Engineering, 2005. (pp. 10-pp). IEEE.
- [34]. Khotimah, B.K., Miswanto, M. and Suprajitno, H. (2020). Optimization of feature selection using genetic algorithm in naïve Bayes classification for incomplete data. Int. J. Intell. Eng. Syst, 13 (1): 334-343.
- [35]. Valdiviezo, H.C. and Van Aelst, S. (2015). Tree-based prediction on incomplete data using imputation or surrogate decisions. Information Sciences, 311: 163-181.
- [36]. Zhang, S., Jin, Z. and Zhu, X. (2011). Missing data imputation by utilizing information within incomplete instances. Journal of Systems and Software, 84 (3): 452-459.
- [37]. Yuen, K.V., Beck, J.L. and Katafygiotis, L.S. (2006). Efficient model updating and health monitoring methodology using incomplete modal data without mode matching. Structural Control and Health Monitoring: The Official Journal of the International Association for Structural Control and Monitoring and of the European Association for the Control of Structures, 13 (1): 91-107.
- [38]. Liu, F. (2011). Direct mode-shape expansion of a spatially incomplete measured mode by a hybrid-vector modification. Journal of Sound and Vibration, 330 (18-19): 4633-4645.
- [39]. Gromski, P.S., Xu, Y., Kotze, H.L., Correa, E., Ellis, D.I., Armitage, E.G. and Goodacre, R. (2014). Influence of missing values substitutes on multivariate analysis of metabolomics data. Metabolites, 4 (2): 433-452.
- [40]. Soleimani, M. and Shokri, B.J. (2016). Intrinsic geological model generation for chromite pods in the Sabzevar ophiolite complex, NE Iran. Ore geology reviews, 78: 138-150.
- [41]. Rajesh, G. and Sunoj, S.M. (2019). Some properties of cumulative Tsallis entropy of order α . Statistical Papers, 60 (3): 933-943.
- [42]. Karkevandi-Talkhooncheh, A., Sharifi, M. and Ahmadi, M. (2018). Application of hybrid adaptive neuro-fuzzy inference system in well placement optimization. Journal of Petroleum Science and Engineering, 166: 924-947.
- [43]. Koopialipoor, M., Nikouei, S.S., Marto, A., Fahimifar, A., Armaghani, D.J. and Mohamad, E.T. (2019). Predicting tunnel boring machine performance through a new model based on the group method of data handling. Bulletin of Engineering Geology and the Environment, 78 (5): 3799-3813.
- [44]. Aghajani-Bazzazi, A., Osanloo, M. and Azimi, Y. (2009, September). Flyrock prediction by multiple regression analysis in Esfordin phosphate mine of Iran. In Proceedings of the 9th international symposium on rock fragmentation by blasting. Granada, Spain (pp. 649-57).
- [45]. Wang, Y., Mazin, H.E., Xu, W. and Huang, B. (2016). Estimating harmonic impact of individual loads using multiple linear regression analysis. International Transactions on Electrical Energy Systems, 26 (4): 809-824.
- [46]. Liang, W., Sari, A., Zhao, G., McKinnon, S.D. and Wu, H. (2020). Short-term rock-burst risk prediction using ensemble learning methods. Natural Hazards, 104 (2): 1923-1946.

- [47]. Wattimena, R.K., Kramadibrata, S., Sidi, I.D., Arif, I. and Azizi, M.A. (2012, January). Probabilistic analysis of single bench using new slope stability curves. In ISRM Regional Symposium-7th Asian Rock Mechanics Symposium. International Society for Rock Mechanics and Rock Engineering.
- [48]. Azizi, M.A., Kramadibrata, S., Wattimena, R.K. and Sidi, I.D. (2013). Probabilistic analysis of physical models slope failure. Procedia Earth and Planetary Science, 6: 411-418.
- [49]. Solver, F. (2010). Premium solver platform. User Guide, Frontline Systems.
- [50]. EPA, U. (1997). Environmental Protection Agency. Guiding principles for Monte Carlo analysis. EPA/630/R-97/001.
- [51]. Ghasemi, E., Sari, M. and Ataei, M. (2012). Development of an empirical model for predicting the effects of controllable blasting parameters on fly-rock distance in surface mines. International Journal of Rock Mechanics and Mining Sciences, 52: 163-170.
- [52]. Mahdiyar, A., Hasaniapanah, M., Armaghani, D.J., Gordan, B., Abdullah, A., Arab, H. and Abd Majid, M.Z. (2017). A Monte Carlo technique in safety assessment of slope under seismic condition. Engineering with Computers, 33 (4): 807-817.
- [53]. Bianchini, F. and Hewage, K. (2012). Probabilistic social cost-benefit analysis for green roofs: A lifecycle approach. Building and Environment, 58, 152-162.
- [54]. Lima, H.R.B.R., Nascimento, D.S., Bispo, G.F.C., Teixeira, V.C., Valério, M.E.G. and Souza, S.O. (2014). Production and characterization of spodumene dosimetric pellets prepared by a sol-gel route. Radiation Physics and Chemistry, 104: 93-99.
- [55]. Morin, M.A. and Ficarazzo, F. (2006). Monte Carlo simulation as a tool to predict blasting fragmentation based on the Kuz-Ram model. Computers & geosciences, 32 (3): 352-359.
- [56]. Kamran, M. (2021). A Probabilistic Approach for Prediction of Drilling Rate Index using Ensemble Learning Technique. Journal of Mining and Environment.
- [57]. Wu, S., Wu, Z. and Zhang, C. (2019). Rock-burst prediction probability model based on case analysis. Tunneling and Underground Space Technology, 93: 103069.

مدل تصادفی ارتقاء محتوای طلا در لیچینگ سیانور با استفاده از شبیه سازی مونت کارلو

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چکیده:

این مقاله ایده و مفهوم جدیدی را برای برآورد میزان طلا در روش لیچینگ سیانید روشن می‌کند. روش سیانیداسیون به طور سنتی برای استخراج طلا استفاده می‌شود. با این حال این روش پر زحمت، وقت‌گیر و پرهزینه است و به در دسترس بودن و توانایی واحدهای پردازشی بستگی دارد. در این کار، تلاش می‌شود با شبیه سازی مبتنی بر روش مونت کارلو، محتوای سنگ طلا ارتقا داده شود. یک رویکرد عالی همیشه نیاز به کیفیت بالای مجموعه داده‌ها برای یک مدل خوب دارد. در مجموع ۴۸ مجموعه داده ناقص از منطقه شوغور، منطقه چیترال در خیر پختونخوا در پاکستان جمع آوری شده است. آزمایش شستشوی سیانیداسیون به منظور اندازه‌گیری درصد محتوای سنگ طلا انجام می‌شود. در این کار روشهای جایگزین میانگین، میانه، مدد و تکرار پی به گونه‌ای مورد استفاده قرار می‌گیرند که بتوانند مجموعه داده‌ها را با ویژگی‌های از دست رفته محاسبه کنند. از تجزیه و تحلیل رگرسیون چندگانه برای یافتن ارتباط بین پتانسیل غلظت یون هیدروژن (pH)، محتوای جامد (در درصد)، غلظت NaCN (در ppm)، زمان شستشو (در ساعت)، اندازه ذرات (در میکرومتر) و درصد اندازه گیری بازیابی طلا (در درصد) استفاده شد. علاوه بر این، از توزیع عادی و نمایی نیز برای پیش‌بینی عدم قطعیت در سنگ معدن طلا استفاده شد. عملکرد مدل نشان می‌دهد که روش مونت کارلو برای تخمین احتمال بازیابی سنگ طلا معتبرتر است. تجزیه و تحلیل حساسیت نشان می‌دهد که pH مهمترین پارامتر در برآورد ذخایر سنگ طلا است. این رویکرد تصادفی را می‌توان به عنوان پایه‌ای برای پیش‌بینی اکتشاف احتمالی ذخایر جدید طلا در نظر گرفت.

کلمات کلیدی: فرایند سیانیداسیون، اکتشاف، طلا، شبیه سازی مونت کارلو، تجزیه و تحلیل میزان حساسیت.