Shahrood University of Technology

Iranian Society of Mining Engineering (IRSME)

Journal of Mining and Environment (JME)

journal homepage: www.jme.shahroodut.ac.ir

THOMSON REUTERS

Application of Machine Learning Models for Predicting Rock Fracture Toughness Mode-I and Mode-II

Enayatallah Emami Meybodi*[1], Syed Khaliq Hussain[1], Mohammad Fatehi Marji[2], and Vamegh Rasouli[3]

1- *Department of Geology, Yazd University, Yazd, Iran.*
2- *Department of Mining and Metallurgical Engineering, Yazd University, Yazd, Iran*
3- *Department of Petroleum Engineering, University of North Dakota, North Dakota, USA.*

Article Info

Received 20 Jaunury 2022

Received in Revised form 28 April 2022

Accepted 27 May 2022

Published online 27 May 2022

DOI:10.22044/JME.2022.11596.2148

Keywords

Intact rock

Macro- and micro-parameters

Machine learning method

Rock fracture toughness

Abstract

In this work, the machine learning prediction models are used in order to evaluate the influence of rock macro-parameters (uniaxial compressive strength, tensile strength, and deformation modulus) on the rock fracture toughness related to the micro-parameters of rock. Four different types of machine learning methods, i.e. Multivariate Linear Regression (MLR), Multivariate Non-Linear Regression (MNLR), copula method, and Support Vector Regression (SVR) are used in this work. The fracture toughness of mode I and mode II (KIC and KIIC) is selected as the dependent variable, whereas the tensile strength, compressive strength, and elastic modulus are considered as the independent variables, respectively. The data is collected from the literature. The results obtained show that the SVR model predicts the values of KIC and KIIC with the determination coefficients (R2) of 0.73 and 0.77. The corresponding determination coefficient values of the MLR model and the MNLR model for KI and KII are R2 = 0.63, R2 = 0.72, and R2 = 0.62, 0.75, respectively. The copula model predicts that the value of R2 for KI is 0.52, and for KII R2=0.69. K-fold cross-validation testing method performs for all these machine learning models. The cross-validation technique shows that SVR is the best-designed model for predicting the fracture toughness mode-I and mode-II.

1. Introduction

Rock fracture toughness is a material property accounting for the fracture resistance of jointed rocks under the applied loads [1]. Rock fracture toughness is the critical stress intensity factor (SIF) measured experimentally in the laboratory, and can be used as one of the design parameters for various rock engineering problems, e.g. in rock fragmentation processes such as blasting operation and rock cutting [2]. In the modern fracture mechanics, the idea of linear elastic fracture mechanics (LEFM), which is based on the concept of SIF and fracture toughness, has been adopted to study the mechanism of failure and fracturing in brittle materials such as glass, rock, concrete, and other rock-like materials [3].The applied rock fracture mechanics is used in various fields such as civil, mining, petroleum, geological, and environmental engineering, e.g. in rock slope stability, tunneling and underground constructions, rock burst, and hydraulic fracturing [4]. In the literature on fracture mechanics and based on LEFM concepts, generally, three modes of loading cause the cracked body to start propagation from the crack tip due to high-stress concentration at the crack end. The three modes of loading are i) the opening mode or pure mode I, where its corresponding SIF KII, and iii) the out-of-plane shearing mode or pure mode II, where its corresponding SIF is KII. And iii) the out of plane shearing mode or pure mode III, where its corresponding SIF is KIII [5]. Many analytical, numerical, and experimental works have been carried out to estimate or measure the mode I and mode II fracture toughness of rocks [6]. The mode

Corresponding author: en.emami@yazd.ac.ir (E. Emami Meybodi).

I fracture toughness is the most versatile mode of fracture in use due to the relatively low tensile strength of rocks compared to those of the shear strength. However, the fracture problems related to the rock structure usually are associated with mode II and a mix-mode of loading mainly, mode I-II [7]; an example of this is the rock cutting by a wedge tool, where the fragmented rock chips are as result crack initiation and propagation below mix-mode I-II loading [8]. Numerous laboratory data shows that the mode-I fracture toughness linearly correlate with the tensile strength of the rock [9]. Although numerous experimental testing methods are developed for the estimation of the rock fracture toughness, they are time-consuming and costly; otherwise, using the sonic-log information from the field is proposed as the best relationship for the indirect determination of rock fracture toughness [10]. Chang Lee et al [11] have reported that most of the physio-mechanical properties of rocks (e.g. density, porosity, P-wave velocity, elastic modulus, uniaxial compressive strength, and Poisson’s ratio) and the P-wave velocity correlate with the mode I fracture toughness. They mentioned that the ductile materials presented more immense strain before reaching the failure state than those of brittle materials, as depicted schematically.

Different methods are used to determine the fracture toughness, which depends upon the type of loading direction on the crack plane [12]. The classical computational methods (finite element method or pure mathematics) are used to calculate fracture toughness [13]. The fracture mechanics is a complex and sensitive field; these classical methods are not suitable for calculation since they require highly advanced mathematical skills, and are a lot time-consuming. In the recent developments, the machine learning (ML) techniques have arisen as a promising means in various scientific domains, including some applications in geotechnical engineering [14]. A single machine learning method does not have the ability to provide the solution to all problems, especially in fracture behavior [15]. Various machine learning algorithms should be applied to solve the problem in fracture mechanics. The machine-learning methods increase the quantity and quality of the data available offer and new possibilities to handle complex problems covering the clustering, classification, and regression issues [16]. Recently, some researchers have applied machine learning methods in the field of fracture mechanics. Goswami et al. [17, 18] have developed new approaches for solving complex problems. Wiangkham et al. [19] have used artificial intelligence machine learning methods to determine the mixed-mode-I and mode-II fracture toughness, and the machine learning method provided the best result compared with the classical computational methods. Wang et al. [20] have used various machine learning approaches, including decision regression tree, random regression forest, extra regression tree, and fully-connected neural networks in order to predict the mode-I fracture toughness. Moreover, they have made improvements in the ISRM-suggested method of cracked chevron notched Brazilian disc (CCNBD) for rock specimens. Roy et al. [21] have predicted mode-I fracture toughness by using multiple regressions and soft computing methods such as an artificial neural network, fuzzy inference system, and adaptive neuro-fuzzy. Furthermore, for predicting KIC, they used tensile strength, P-wave velocity, and S-wave velocity. Fang and Fall [22] have predict the time, temperature, and sulfate ion's effect on the KI-mode and KII-mode in cemented backfill-rock interface, and also all specimens have been subjected to semicircular bend (SCB) tests. Karakul, H [1] has predicted KI and KII from tensile strength by applying the linear regressions, multivariate regressions, and Adaptive Neuro-Fuzzy Inference System (ANFIS) models. Moreover, KI, KII, and tensile strength were obtained using cracked chevron notched Brazilian disc (CCNBD), central cracked circular disk (CCCD), and Brazilian tests. Mahmoodzadeh et al. [23] KI using support vector regression with metaheuristic optimization algorithms, the support vector regression method are combined with six metaheuristic optimization models. Furthermore, they used 250 datasets including six input parameters and one output parameter (mode-I rock fracture toughness) are utilized in the models obtained through the CCNBD testing specimens suggested by the ISRM in the laboratory.

In the latest research work, only mode-I has been predicted so far in some cases, where KI and KII have been predicted; only the tensile strength parameter of the rock samples was used as the input data for estimation. In previous studies, parameters have been used for estimation, which requires unusual and time-consuming tests. It is the first time to use common macro-properties (uniaxial compressive strength, tensile strength, and, modulus of elasticity) as the input parameters for the estimation of fracture toughness, which is obtained in most rock mechanics projects. Furthermore, the purpose of this research work is to provide the optimal predictive models, and in this case, the correlation between the input variables is not an unfavorable parameter. In this regard, the K-fold cross-validation method has been used to determine the ability of models in forecasting, which confirms the capability of predictive models using these input parameters.

This study predicts the rock's most critical macro-parameters that may affect fracture toughness. For this purpose, various machine learning models are applied including Support Vector Regression (SVR), Multivariate Linear Regression (MLR), Multivariate Non-Linear Regression (MNLR), and the copula method, and their results are compared. The copula method is one of the most advanced soft computing methods that is used in the current study, for the prediction KI and KII. The K-fold cross-validation method is used to determine the ability of models in forecasting.

2. Macro-Parameters Affecting Fracture Toughness

The capacity of a material to take in energy and plastically deform and resist fracturing and propagation of the pre-existing cracks is referred to as the toughness modulus. The area below the rock's stress-strain curve before the ultimate strength is an index for absorption of energy, and it depends on the ultimate strength and elastic modulus. The elasticity modulus has a significant impact on the rock deformation and failure, while it is not being considered as an effective parameter in the rock brittleness calculations. Brittle rock failure begins with the loss of cohesion (cementation) between the grains at an early stage, followed by dilation and the mobilization of frictional resistance [24]. In the rock sample, behavior changes start at 30% to 50% during the peak stress, and the development of crack persists up to 70% to 85% during the peak stress. The concept of fracture toughness and strain energy is shown in (Figure1). Change of the tensile stress as a part of the crack opening, where the energy release rate reaches the area below the curve after the peak, (left-side of Figure A). The right-side of Figure B, the strain energy, which is the areas under the stress-strain curve, is used to define the energy upon failure [25]. The crack propagation in brittle material, e.g. rocks, is entirely dependent upon elastic energy. In the fracture mechanics, the elastic energy is the basic and essential energy for crack propagation. Irwin [26] gave a flexible solution of the energy required for the development of a crack on the crack tip, and showed that it did not depend on the state of stress if the plastic area across the crack tip was minimal in comparison to the crack length. The crack propagation is highly dependent on mode-II fracture toughness but the direct estimation of rock fracture toughness is difficult due to the limited number of available cores and consumption of a long time [27]. The numerical analysis method showed that the dominant mode of fracture, disregarding the shear stage, was tensional [28]. In Rock material, due to the presence of heterogeneity, porosity, bedding plane etc., the fracture toughness data is scattered, and a small number of specimens fail to provide versatile and reliable data. The three different modes of fracture toughness are shown in Figure 2.

Therefore, it is imperative to investigate the fracture toughness of rock statistically using several test samples [29]. Although the tensile and compressive rock strengths are relevant to elastic modulus, the elastic modulus results in brittleness and toughness cannot be ignored. In this research work, it is seen that the mode I and mode II fracture toughness are predicted better with three parameters of tensile and compressive rock strengths and Young's modulus (σt, σc and, E) as opposed to using only tensile strength. We estimated the mode I and mode II fracture toughness as a function of compressive and tensile strength and Young's modulus. The mechanical behavior of brittle and ductile fracture is shown in Figure 3.

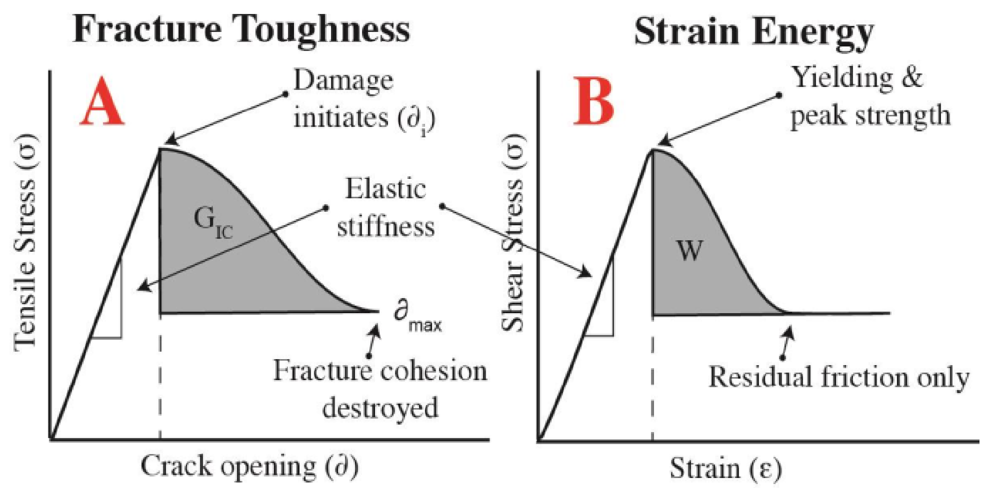


Figure 1. An example of similarities between fracture toughness (A) and strain energy dissipation (B). Figure A indicates an ordinary mode I (tensile opening) fracture toughness curve, in which the actual energy release rate (GIC) is used to predict KIC. Figure B indicates a typical strain-stress curve, in which the strain energy (W) is used to define the energy launched upon failure [25].

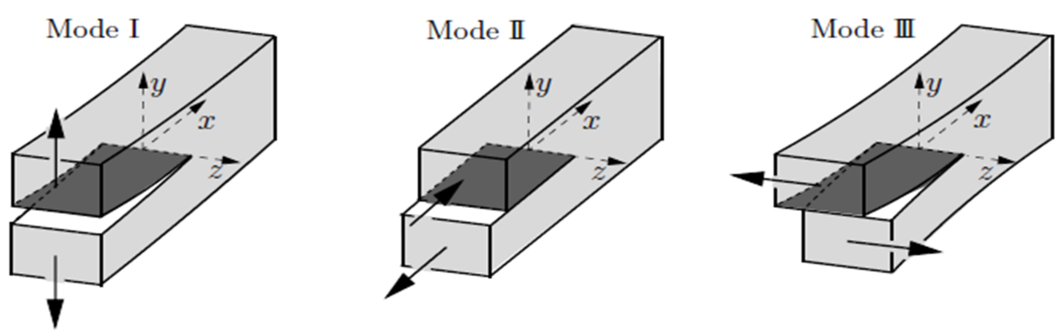


Figure 2. Three fundamental modes of crack propagation: mode I (tensile), mode II (in-plane shear), and mode III (anti-plane shear) [30].

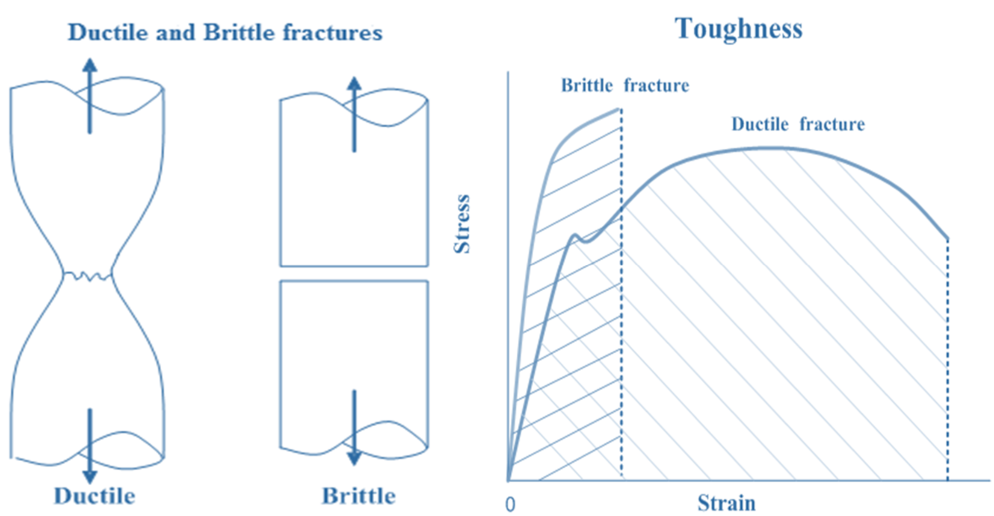


Figure 3. Mechanical behavior of brittle and ductile fractures [11].

3. Data Collection: Mechanical Properties of Rocks

The modes I and II fracture toughness (KIC, KIIC), tensile strength (σt), uniaxial compressive strength (σc), and elastic modulus (E) of some rock types collected from the literature are listed in Table 1. The values for tensile strength were derived from the Brazilian tests performed according to the ISRM suggested methods. The mode I and mode II fracture toughness values were estimated based on the Chevron Bend test and the Punch-via Shear test, respectively. The relationships between the input variables are investigated using R software. A Machine learning algorithm is applied to the available data in Table 1 to explore the impact of macro parameters (tensile strength, compressive strength, and elastic modulus) on fracture toughness as a micro-parameter of rock.

Table 1. The mechanical properties of some typical rocks

Rock Type	K_{IIC} (MPa.m$^{0.5}$)	K_{IC} (MPa.m$^{0.5}$)	σ_t (MPa)	σ_c (MPa)	E (GPa)	Reference
Welsh limestone	1.0	0.9	8.5	144.9	33.2	**[31]**
Coarse-grained sandstone	0.3	0.3	2.7	32.3	9.9	[31]
Fine-grained sandstone	0.4	0.4	3.3	58.4	14.6	[31]
Limestone	0.9	0.4	2.3	105.0	52.0	[32]
Marble	6.1	2.2	17.6	202.0	78.0	[33]
Sandstone	5.0	1.7	15.7	194.0	69.0	[33]
Granite	4.9	1.9	10.7	166.0	66.0	[33]
Aspo diorite	5.1	3.8	15.0	219.0	68.0	[34]
Aue granite	4.1	1.6	8.0	134.0	48.0	[34]
Mizunami granite	4.9	2.4	9.0	166.0	50.0	[34]
Carrara marble	3.1	2.4	7.0	101.0	49.0	[34]
Flechtingen sandstone	1.9	1.2	6.0	96.0	21.0	[34]
Rudersdorf limestone	2.3	1.1	5.0	40.0	22.0	[34]
Äspö diorite	4.4	3.8	14.9	211.0	76.0	[35]
Lac du Bonnet granite	6.4	2.5	14.8	165.0	68.0	[36]
Äspö diorite	2.0	1.0	10.0	224.0	60.0	[36]
Crystalline rock	3.1	1.7	8.0	115.0	37.0	[36]
Äspö diorite	4.7	3.3	14.8	165.0	68.0	**[37]**
Pegmatitic rock	3.3	2.0	12.0	115.0	55.0	**[38]**
Migmatitic gneiss	3.0	1.9	10.0	105.0	55.0	[38]
Migmatitic gneiss	3.9	3.1	14.0	123.0	55.0	[38]
Cement mortar	0.28	0.15	2.00	16.00	4.00	[38]
Cement mortar	0.8	0.5	5.0	68.0	28.0	[38]
Cement mortar	1.1	0.6	2.2	54.0	10.7	[38]

4. Data Analysis by Machine Learning Methods

The linear and non-linear multivariate regression analyses were applied to develop the prediction models to study the influence of the macro-parameter on the micro-parameter. Multiple linear regression (MLR) analysis was performed using the R-software. The backward method was selected to perform linear multivariate regression analysis. In this process, all impartial variables first enter into the equation, and the impact of all variables is estimated on the established variable. Less efficient variables exclude one after another, and in the end, those steps maintain till the test errors reach a sizeable degree of 10% [39]. Moreover, in this process, critical variables are acknowledged and continue to be in the equation. The rank order correlations (Pearson correlation) and descriptive statistics analysis in the form of a correlation matrix between the model inputs, KI and KII, are shown in Tables 2 and 3, respectively. The graphical presentation of the Pearson correlation is shown in Figure 4. Other machine learning methods are implemented in rock engineering-related problems [40]. The application of the support vector regression (SVR) models in solving non-linear problems is growing among the researchers who deal with the geotechnical issues. This approach focuses on forecasting analysis [41]. The SVR model is constructed based on the kernel function type of RBF. The reason for using the kernel function is its ability to transform our data from the non-linear to the linear form. SVR allows to find a fit, and then data map to the original space. There are several advantages for using a support vector machine (SVM) [42]. These advantages include a global and unique solution, a simple geometric interpretation, structural risk minimization, and a low prone to overfitting[43].

In this study, some statistical indices, including the mean absolute error (MAE), root mean square error (RMSE), and determination coefficient (R), are used to evaluate the performance of prediction models by comparing the predicted values with the actual values. The following (Equations 1 and 2) give expressions of MAE, and RMSE, respectively.

$$\mathrm{MAE} = \frac{1}{\mathrm{n}}\sum_{\mathrm{i=1}}^{\mathrm{n}} |(\mathrm{Yi} - \acute{\mathrm{Y}}\mathrm{i}\,) \tag{1}$$

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(Yi - \acute{Y}i')^2} \tag{2}$$

Here Yi is the predicted value, Yi is the measured value, and n is a number of all variables. The results of above statistical indices calculated for all models are presented in Table 8.

Table 2. Result summary of Descriptive statistics analysis.

	n	Mean	Sd	Median	Trimmed	Mad	Min	Max	Range	Skew	Kurtosis	se
KIIC	24	3.04	1.92	3.1	3.00	2.67	0.28	6.4	6.12	0.03	-1.38	0.39
KIC	24	1.70	1.09	1.7	1.64	1.11	0.15	3.8	3.65	0.36	-0.97	0.22
σ_t	24	9.10	4.91	8.75	9.05	6.67	2	17.6	15.6	0.06	-1.38	1.00
σ_c	24	125.82	61.36	119	126.42	69.68	16	224	208	-0.06	-1.17	12.52
E	24	45.73	22.72	51	46.48	25.20	4	78	74	-0.38	-1.26	4.64

Table 3. Pearson's correlation coefficients for model inputs for KI and KII.

Pearson correlation	KIIC	KIC	σ_t	σ_c	E
KIIC	1	0.817	0.878	0.749	0.844
KIC	0.817	1	0.825	0.687	0.77
σ_t	0.878	0.825	1	0.837	0.886
σ_c	0.749	0.687	0.837	1	0.889
E	0.844	0.77	0.886	0.889	1

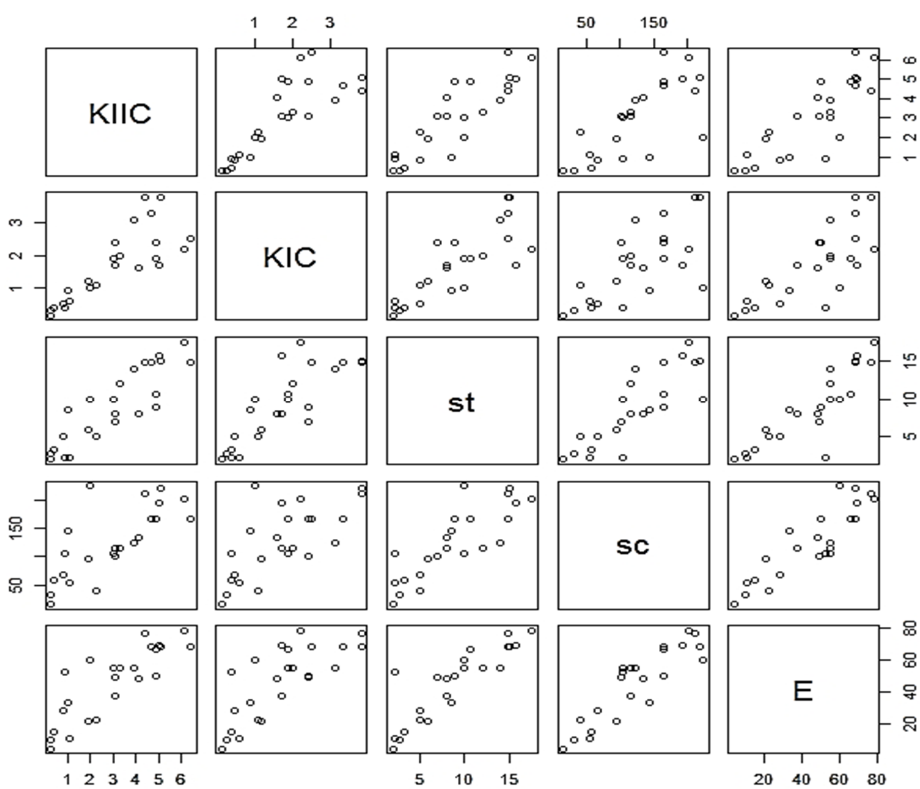


Figure 4. Graphical presentation of Pearson's correlation coefficients.

4.1. Predicting rock fracture toughness by MLR and MNLR

The multivariate linear regression (MLR) and multivariate non-linear regressions (MNLR) analyses were carried out to predict the values for the independent variable coefficients in the linear equation. The results of this regression analysis are present in Table 8. The values of R2 = 0.63 for KI and R2 = 0.72 for KII in MLR are shown in Figures 5 and 6, respectively. The results of non-linear regression analysis to predict the values of toughness are presented in Figures 7 and 8. The regression coefficients for non-linear multivariate regressions models for KI and KII are 0.62 and 0.75, respectively.

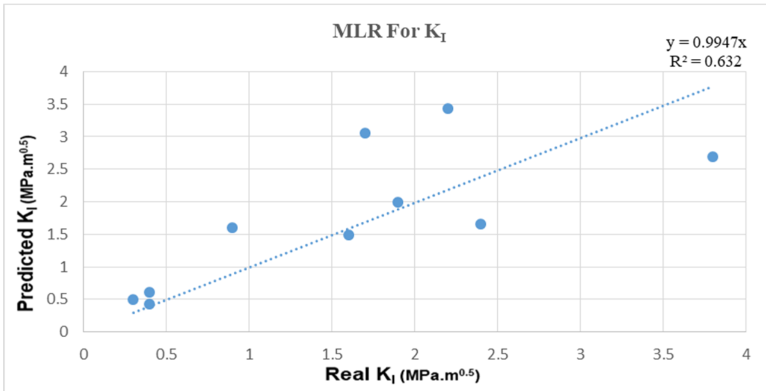


Figure 5. Scatter plots of predicted mode I toughness, K_I, from linear multivariate regressions.

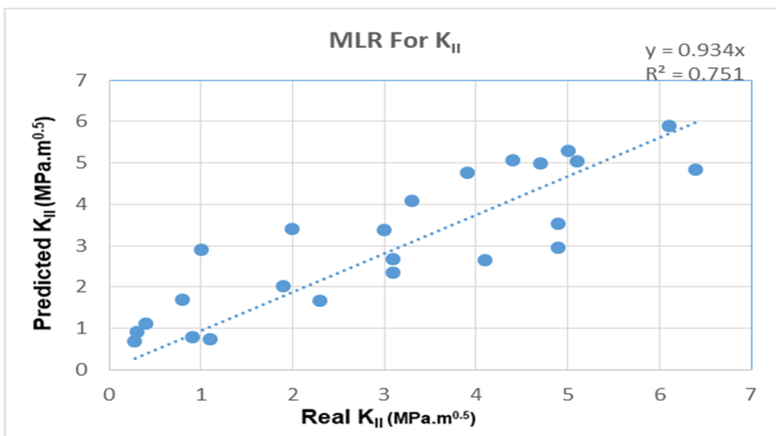


Figure 6. Scatter plots of predicted mode II toughness, K_{II}, from linear and multivariate regressions.

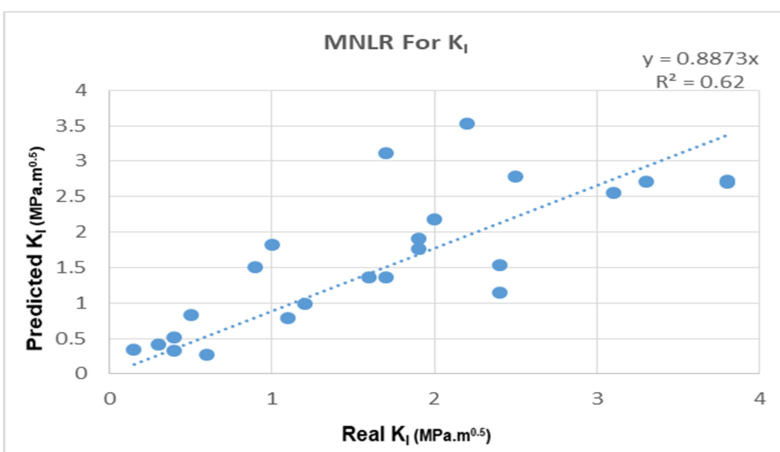


Figure 7. Scatter plots of predicted mode I toughness, KI, from non-linear multivariate regressions.

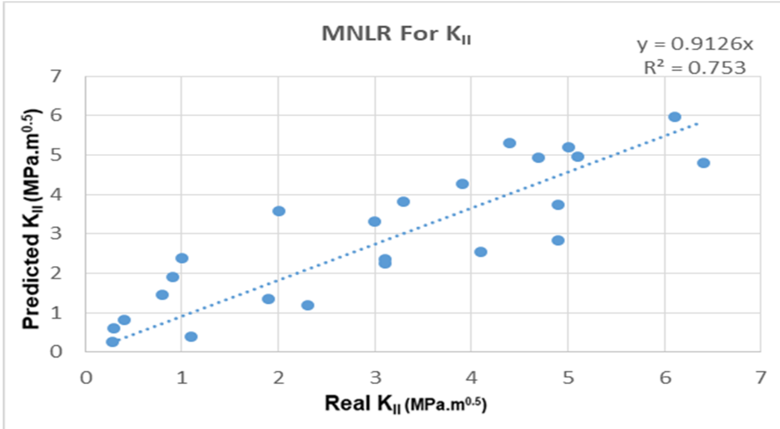


Figure 8. Scatter plots of predicted mode II toughness, K_{II}, from non-linear multivariate regressions.

4.2. Fracture toughness prediction using SVR

The model is turned into built by way of the e1071 package, which is the primary and maximum intuitive package in the R-software program. Compressive strength, tensile strength, elastic modulus, and fracture toughness are selected as the model inputs. The SVR model was built primarily based on the kernel function form of radius basis function (RBF). The reason for the use of the kernel function is its ability to transform the statistical data from the non-linear to the linear form. SVR is to find a fit, and then information map to the unique space. The SVR model results are present in Table 8. In the results presented in Figures 9 and 10, the models show a good ability to predict fracture toughness for KI and KII with corresponding determination factors of R2 = 0.74 and R2 = 0.78, respectively.

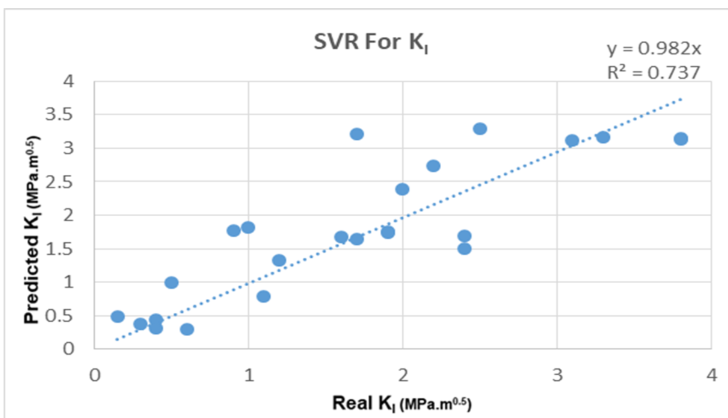


Figure 9. Scatter plots of predicted mode I toughness, K_I, SVR models.

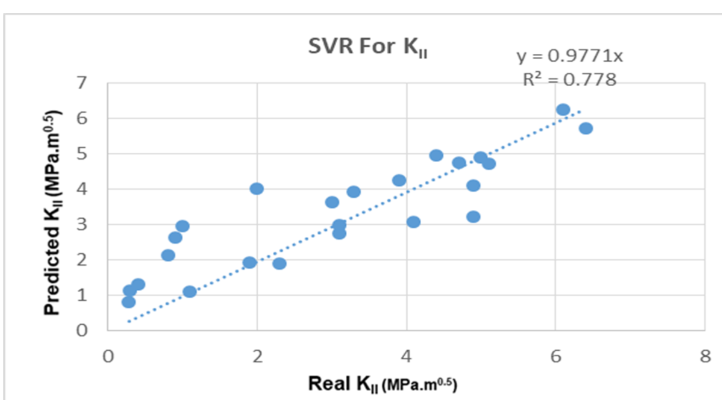


Figure 10. Scatter plots of predicted mode II toughness, K_{II}, SVR models.

4.3. Fracture toughness prediction using copula method

Copula is a function that combines a multivariate distribution function with its marginal distribution functions, commonly called marginals or simply margins. Copula is an excellent tool for modeling and simulating the correlated random variables. The copula method plays a significant role in the field of civil, geotechnical, and engineering geology due to its high accuracy [44]. The copula method is performed using the R-software to determine the fracture toughness mode-I and fracture toughness mode-II. For mode I, the copula method with only tensile strength is the best predictor model, and for mode II, copula with all three independent variables is the best predictor method. The copula method results are present in Table 8). The value of R2 = 0.52 for KI and R2 = 0.69 for KII in the copula method are shown in Figures 11 and 12, respectively.

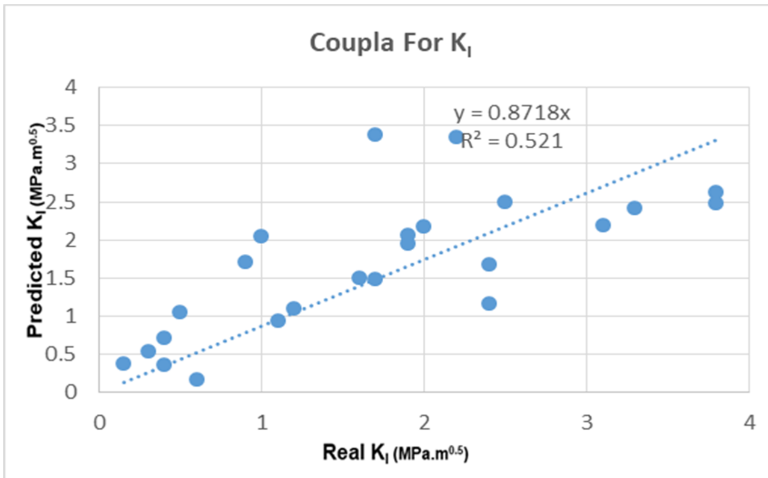


Figure 11. Scatter plots of predicted mode I toughness, K_I, from copula method.

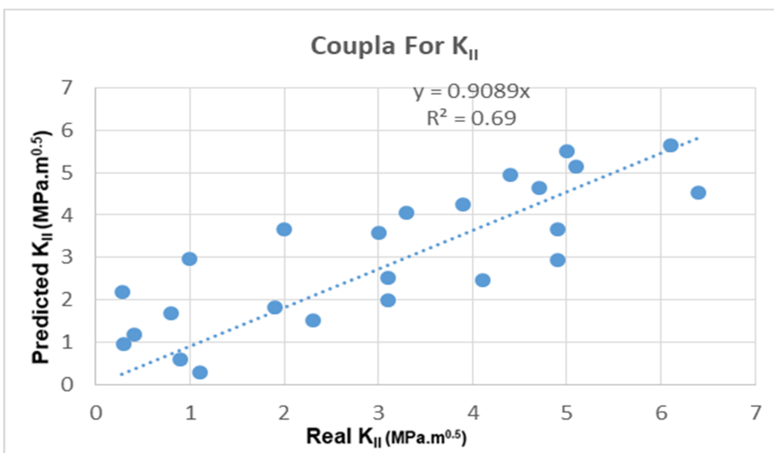


Figure 12. Scatter plots of predicted mode II toughness, KII, copula method.

5. Cross-Validation for Machine Learning Methods

One of the significant challenges in machine learning methods is to check the model accuracy of unseen data, to know whether the designed model is performing well or not. In order to evaluate the model accuracy, we should need to test it against those data points that are not present during the training of the model. Using R-programming language, one of the best methods for checking the accuracy of the machine learning model is the cross-validation (CV) method. Cross-validation is a standardized technique for testing the performance of a predictive model. In the process of cross-validation, a part of the dataset is saved, which will not be used in the model training. When the model is ready, this specific dataset is used for testing the purposes. The cross-validation method can be divided into two types: one is non-exhaustive cross-validation, which includes such as K-fold cross-validation, holdout method, repeated random sub-sampling validation, and exhaustive cross-validation includes such as leave-p-out cross-validation, leave-one-out cross-validation.

The K-fold cross-validation method is used to find the best-designed machine learning models for fracture toughness mode-I and fracture toughness mode-II. The K-fold cross-validation method is one of the most accurate and reliable methods for testing the machine learning models. This cross-validation technique divides the data into equal K subsets (folds). Besides these K-folds, one subset is used as a validation set, and the remaining is applied in the training model.

The K-fold cross-validation method has been used for Support Vector Regression (SVR), Copula Method, Multivariate Linear Regression (MLR), and Multivariate Non-Linear Regression (MNLR) to find the best machine learning model for fracture toughness mode-I and fracture toughness mode-II. For these machine learning methods, the fitting models are built by the K-fold cross-validation technique. A total of eighteen models are developed for fracture toughness mode-I. The results are shown in Table 4. There are six models for multivariate linear regression (model numbers 1 to 6), six models for Support Vector Regression (SVR) (model numbers 7 to 12), three models for Copula Method (model numbers 13 to 15), and three models for Multivariate Non-Linear Regression (MNLR) (model numbers 16 to 18). The details of Support Vector Regression (SVR) models for fracture toughness mode-I are in Table 5. A total of seventeen models are developed for fracture toughness mode-II. The results are shown in Table 6. There are six models for multivariate linear regression (model numbers 1 to 6), six models for Support Vector Regression (SVR) (model numbers 7 to 12), three models for Copula Method (model numbers 13 to 15), and two models for Multivariate Non-Linear Regression (MNLR) (models number 16 to 17). The detail of Support Vector Regression (SVR) models for fracture toughness mode-II are presented in table No 7. The result showed that the Support Vector Regression (SVR) is the best-designed model for fracture toughness mode-I and fracture toughness mode-II. In the K-fold cross-validation method for Support Vector Regression (SVR), darkness of color that ranges from 0 to 250 represents the best value for the required model. Furthermore, the darkness of color is processed to obtain the optimized value of cost and epsilon. For the fracture toughness mode-I, the value of cost seven and epsilon 0.04 (Figure 13). Moreover, for fracture toughness mode-II, the value of cost three and epsilon 0.6 (Fig. 14). The tensile strength has a direct effect on the fracture toughness mode-I and mode-II. In this regard, Support Vector Regression (SVR) is the more realistic and reliable model. The Multivariate Non-Linear Regression (MNLR) is the second-best Machine learning model to determine rock fracture toughness, which provide relationship in Equations 3 and 4 for fracture toughness mode-I and mode-II, respectively.

$$KIC = 0.13 \times st^{1.1336} \tag{3}$$

$$KIIC = 0.080 \times st^{0.735} \times E^{0.507} \tag{4}$$

Table 4. Cross-validation results for KI.

Model num	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
R^2	0.539	0.561	0.587	0.6	0.612	0.632	0.524	0.472	**0.64**	0.58	0.604	**0.737**	0.162	0.196	0.521	0.583	0.596	**0.62**
RMSE	0.735	0.715	0.692	0.679	0.669	0.651	0.753	0.798	**0.644**	0.701	0.685	**0.564**	1.138	1.078	0.75	0.7	0.695	**0.674**
MAE	0.597	0.578	0.553	0.539	0.515	0.498	0.608	0.648	**0.516**	0.591	0.524	**0.425**	0.879	0.836	0.571	0.565	0.536	**0.52**

Table 5. Cross-validation results for KII.

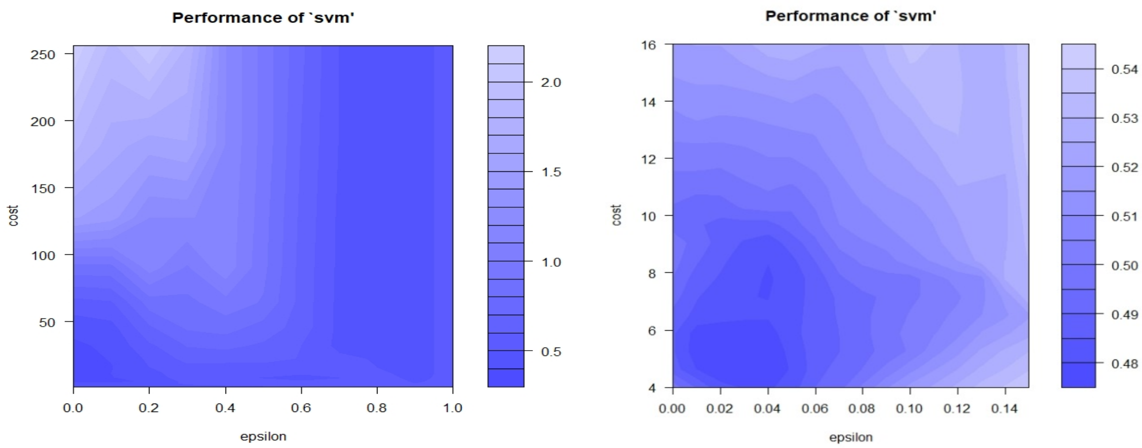
Model num	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17
R^2	0.683	0.689	0.726	0.731	0.741	**0.751**	**0.778**	0.652	0.738	0.638	0.741	0.721	0.637	0.69	0.664	0.731	**0.753**
RMSE	1.068	1.054	0.985	0.983	0.957	**0.938**	**0.94**	1.144	0.963	1.139	0.987	1.005	1.183	1.067	1.089	0.982	**0.944**
MAE	0.89	0.876	0.81	0.801	0.782	**0.759**	**0.723**	0.955	0.791	0.962	0.818	0.775	0.893	0.859	0.899	0.834	**0.77**

Table 6. Results of Support Vector Regression (SVR) model for fracture toughness mode-I.

Model	Micro-parameter (fracture toughness mode-I)	Macro-parameters (tensile strength, compressive strength, Young's modulus)	cost	Epsilon
SVR-model-7	KI	σt, σc, E	0.7	0.1
SVR-model-8	KI	σt, σc, E	13	0.6
SVR-model-9	KI	σt, E	2	0.2
SVR-model-10	KI	σt, E	1.4	0.4
SVR-model-11	KI	σt	190	0.2
SVR-model-12	KI	σt	7	0.04

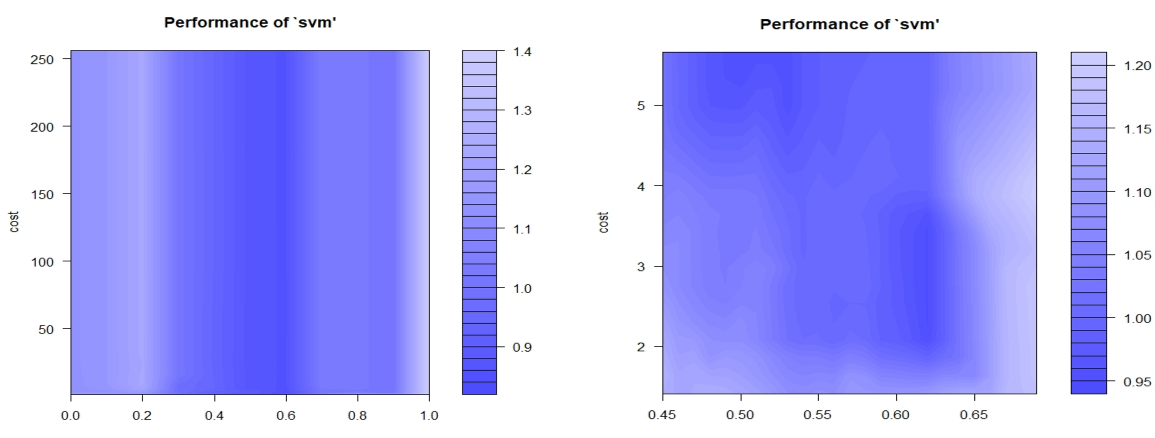
Table 7. Results of Support Vector Regression (SVR) model for fracture toughness mode-II.

Model	Micro-parameter (fracture toughness mode-II)	Macro-parameters (tensile strength, compressive strength, Young's modulus)	cost	Epsilon
SVR-model-7	KII	σt, σc, E	3	0.6
SVR-model-8	KII	σt, σc, E	3.6	0.7
SVR-model-9	KII	σt, E	1	0.25
SVR-model-10	KII	σt, E	1.4	0.4
SVR-model-11	KII	σt	15	0.6
SVR-model-12	KII	σt	1	0.2



a) First step for selecting optimized cost and epsilon **b) Final step for selecting optimized cost and epsilon**

Figure 13. Steps for determination of optimized parameters in Support Vector Regression (SVR) for fracture toughness mode-I with the value of cost seven and epsilon 0.04.



a) First step for selecting optimized cost and epsilon **b) Final step for selecting optimized cost and epsilon**

Figure 14. Steps for determination of optimized parameters in Support Vector Regression (SVR) for fracture toughness mode-II with the value of cost three and epsilon 0.6.

Figures 15 and 16, laboratory toughness mode-I and mode-II, are, respectively, compared with the best SVR and MNLR predictor models.

The overall performance of various models is presented in this work in prediction of the toughness values based on RMSE. Table 8 summarizes the comparison of all statistical indices (MSE, RMSE, and MAE) for different models.

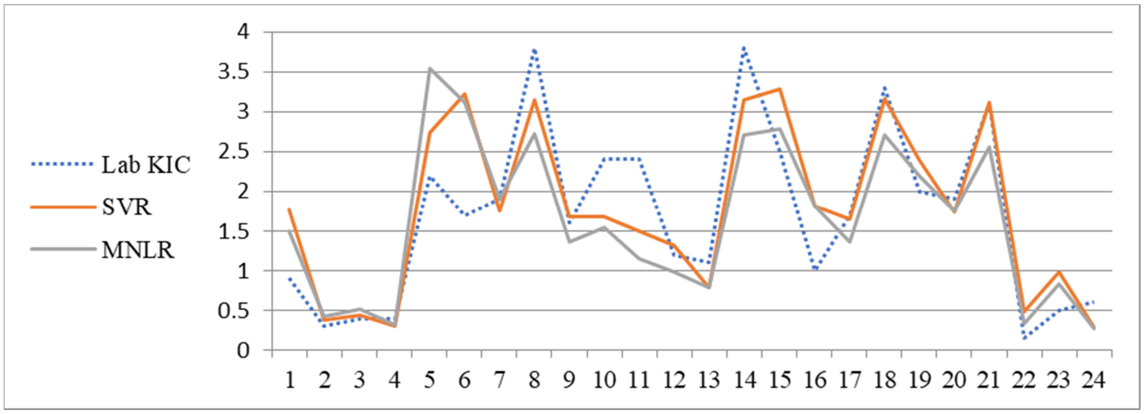


Figure 15. Comparison between data from best predictor models with real KI.

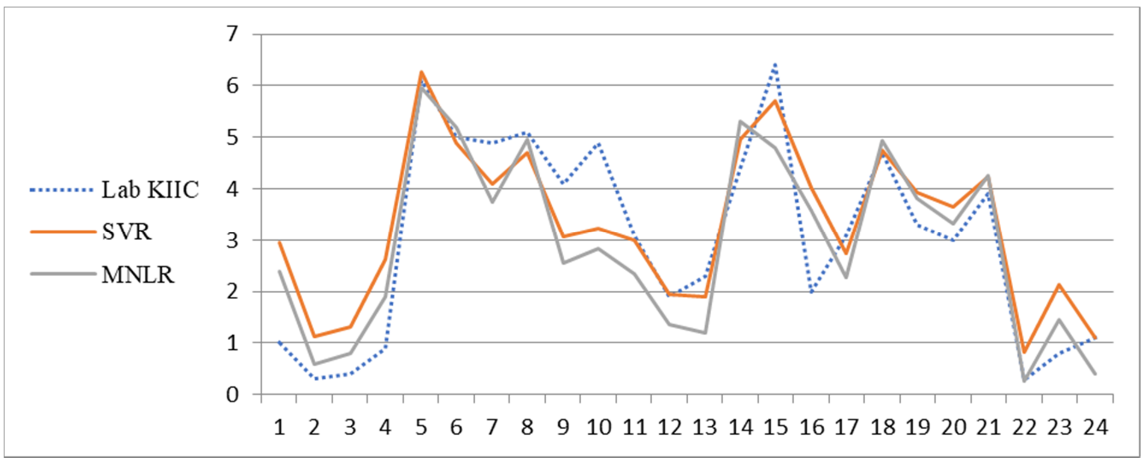


Figure 16. Comparison between data from best predictor models with real KII.

Table 8. Comparative table for all models

Model	Statistical indices for models		Characterizes of models		Total rank
	Mode I (K_I)	Mode II(K_{II})	Mode I (K_I)	Mode II (K_{II})	
MLR	R^2 = 0.63 RMSE =0.65 MAE = 0.57	R2 = 0.72 RMSE =0.93 MAE = 0.75	Analysis method: Backward	Analysis method: Backward	3
MNLR	R^2 = 0.62 RMSE = 0.67 MAE = 0.52	R^2 = 0.75 RMSE = 0.94 MAE = 0.77	Function: Power form	Function: Power form	2
SVR	R^2 = 0.73 RMSE = 0.56 MAE = 0.42	R^2 = 0.77 RMSE = 0.94 MAE = 0.72	SVM-type: eps-regression SVM-Kernel: radial cost: 7 epsilon:0.4 Number of Support Vectors:23	SVM-type: eps-regression SVM-Kernel: linear cost: 3 epsilon:0.6 Number of Support Vectors:23	1
Copula method	R^2 = 0.52 RMSE = 0.75 MAE = 0.43	R^2 = 0.69 RMSE = 1.18 MAE = 0.89	C-vine copula method	C-vine copula method	4

6. Conclusion

Rock fracture toughness is one of the most sensitive and complicated topics in various engineering fields. It is imperative to determine the effect of macro-parameters such as tensile strengths (σt), compressive strengths (σc), and Young's modulus on the different modes of fracture toughness. Therefore, modern and reliable machine learning models were used for this proposal.

Support Vector Regression (SVR) model provided more realistic and accurate results for rock fracture toughness. Rock fracture toughness mode I has a direct relationship with tensile strength. Furthermore, these results are very close to the reality of rock fracture toughness mode-I. The best predictor model for fracture toughness mode-II, the SVR model with tensile strengths (σt), compressive strengths (σc), and Young's modulus as independent components. The Multivariate Linear Regression (MLR) and Multivariate Non-Linear Regression (MNLR) provided good results as compared to the copula method for the rock fracture toughness. The copula method requires more data to achieve a good estimation model. Moreover, the results showed that the K-fold cross-validation was a beneficial method for determining the model with the best performance for prediction.

References

[1]. Karakul, H. (2021). Investigation of fracture properties of rocks under drilling fluid saturation. Environmental Earth Sciences. 80 (15): 1-16.

[2]. Abdolghanizadeh, K., Hosseini, M., & Saghafiyazdi, M. (2019). Effects of number of freeze-thaw cycles and freezing temperature on mode I and mode II fracture toughness of cement mortar. Journal of Mining and Environment. 10 (4): 967-978.

[3]. Marji, M.F. (2014). Numerical analysis of quasi-static crack branching in brittle solids by a modified displacement discontinuity method. International Journal of Solids and Structures. 51 (9): 1716-1736.

[4]. Behnia, M., Goshtasbi, K., Fatehi Marji, M. and Golshani, A. (2012). On the crack propagation modeling of hydraulic fracturing by a hybridized displacement discontinuity/boundary collocation method. Journal of Mining and Environment. 2 (1).

[5]. Hosseini, M. and Khodayari, A.R. (2018). Effects of temperature and confining pressure on mode II fracture toughness of rocks (Case study: Lushan Sandstone). Journal of Mining and Environment. 9 (2): 379-391.

[6]. Yao, W., & Xia, K. (2019). Dynamic notched semi-circle bend (NSCB) method for measuring fracture properties of rocks: Fundamentals and applications. Journal of Rock Mechanics and Geotechnical Engineering, 11(5), 1066-1093.

[7]. Wang, W., Zhao, Y., Teng, T., Zhang, C. and Jiao, Z. (2021). Influence of Bedding Planes on Mode I and Mixed-Mode (I–II) Dynamic Fracture Toughness of Coal: Analysis of Experiments. Rock Mechanics and Rock Engineering. 54 (1): 173-189.

[8]. Ghazvinian, A., Nejati, H.R., Sarfarazi, V. and Hadei, M.R. (2013). Mixed mode crack propagation in low brittle rock-like materials. Arabian Journal of Geosciences. 6 (11): 4435-4444.

[9]. Majdi, A. and Yazdani, M. (2021). Determination of Hydraulic Jacking Mechanism and Maximum Allowable Grout Pressure during Grout Injection in Anisotropic Rocks. Journal of Mining and Environment. 12 (2): 589-603.

[10]. Zhixi, C., Mian, C., Yan, J. and Rongzun, H. (1997). Determination of rock fracture toughness and its relationship with acoustic velocity. International Journal of Rock Mechanics and Mining Sciences. 34 (3-4): 49-e1.

[11]. Chang, S.H., Lee, C.I. and Jeon, S. (2002). Measurement of rock fracture toughness under modes I and II and mixed-mode conditions by using disc-type specimens. Engineering geology. 66 (1-2): 79-97.

[12]. Zeinedini, A., Moradi, M.H., Taghibeigi, H. and Jamali, J. (2020). On the mixed mode I/II/III translaminar fracture toughness of cotton/epoxy laminated composites. Theoretical and Applied Fracture Mechanics. 109: 102760.

[13]. Areias, P., Reinoso, J., Camanho, P.P., De Sá, J.C. and Rabczuk, T. (2018). Effective 2D and 3D crack propagation with local mesh refinement and the screened Poisson equation. Engineering Fracture Mechanics, 189, 339-360.

[14]. Aengchuan, P. and Phruksaphanrat, B. (2018). Comparison of fuzzy inference system (FIS), FIS with artificial neural networks (FIS+ ANN) and FIS with adaptive neuro-fuzzy inference system (FIS+ ANFIS) for inventory control. Journal of Intelligent Manufacturing. 29 (4): 905-923.

[15]. Balcıoğlu, H.E. and Seçkin, A.Ç. (2021). Comparison of machine learning methods and finite element analysis on the fracture behavior of polymer composites. Archive of Applied Mechanics. 91 (1): 223-239.

[16]. Abdallah, A. (2019). Prediction of the Soil Water Retention Curve from Basic Geotechnical Parameters by Machine Learning Techniques. In International Conference on Inforatmion technology in Geo-Engineering (pp. 383-392). Springer, Cham.

[17]. Goswami, S., Anitescu, C. and Rabczuk, T. (2020). Adaptive fourth-order phase field analysis using deep energy minimization. Theoretical and Applied Fracture Mechanics. 107: 102527.

[18]. Goswami, S., Anitescu, C., Chakraborty, S. and Rabczuk, T. (2020). Transfer learning enhanced physics informed neural network for phase-field modeling of fracture. Theoretical and Applied Fracture Mechanics, 106, 102447.

[19]. Wiangkham, A., Ariyarit, A. and Aengchuan, P. (2021). Prediction of the mixed mode I/II fracture toughness of PMMA by an artificial intelligence approach. Theoretical and Applied Fracture Mechanics. 112: 102910.

[20]. Wang, Y.T., Zhang, X. and Liu, X.S. (2021). Machine learning approaches to rock fracture mechanics problems: Mode-I fracture toughness determination. Engineering Fracture Mechanics, 253, 107890.

[21]. Roy, D.G., Singh, T.N. and Kodikara, J. (2018). Predicting mode-I fracture toughness of rocks using soft computing and multiple regression. Measurement. 126, 231-241.

[22]. Fang, K. and Fall, M. (2020). Insight into the mode I and mode II fracture toughness of the cemented backfill-rock interface: Effect of time, temperature and sulphate. Construction and Building Materials. 262: 120860.

[23]. Mahmoodzadeh, A., Nejati, H.R., Mohammadi, M., Ibrahim, H.H., Khishe, M., Rashidi, S. and Ali, H.F.H. (2022). Prediction of Mode-I rock fracture toughness using support vector regression with metaheuristic optimization algorithms. Engineering Fracture Mechanics, 264, 108334.

[24]. Barton, N. (2013). Shear strength criteria for rock, rock joints, rockfill and rock masses: Problems and some solutions. Journal of Rock Mechanics and Geotechnical Engineering. 5(4): 249-261.

[25]. Martin, C.D. (1997). Seventeenth Canadian geotechnical colloquium: the effect of cohesion loss and stress path on brittle rock strength. Canadian Geotechnical Journal. 34(5): 698-725.

[26]. Irwin, G.R. (1957). Analysis of stresses and strains near the end of a crack traversing a plate.

[27]. Jin, Y., Yuan, J., Chen, M., Chen, K. P., Lu, Y. and Wang, H. (2011). Determination of rock fracture toughness K IIC and its relationship with tensile strength. Rock mechanics and rock engineering. 44 (5): 621-627.

[28]. Sun, W., Du, H., Zhou, F. and Shao, J. (2019). Experimental study of crack propagation of rock-like specimens containing conjugate fractures. Geomechanics and Engineering. 17 (4): 323-331.

[29]. Aliha, M.R.M. and Ayatollahi, M.R. (2014). Rock fracture toughness study using cracked chevron notched Brazilian disc specimen under pure modes I and II loading–A statistical approach. Theoretical and Applied Fracture Mechanics, 69, 17-25.

[30]. Whittaker, B.N., Singh, R.N. and Sun, G. (1992). Rock fracture mechanics. Principles, design and applications.

[31]. Singh, R.N. and Sun, G.X. (1989). The relationship between fracture toughness hardness indices and mechanical properties of rocks. Nottingham University Mining Department Magazine;(UK), 41.

[32]. Al-Shayea, N.A., Khan, K. and Abduljauwad, S.N. (2000). Effects of confining pressure and temperature on mixed-mode (I–II) fracture toughness of a limestone rock. International Journal of Rock Mechanics and Mining Sciences. 37 (4): 629-643.

[33]. Rao, Q., Sun, Z., Stephansson, O., Li, C. and Stillborg, B. (2003). Shear fracture (Mode II) of brittle rock. International Journal of Rock Mechanics and Mining Sciences. 40 (3): 355-375.

[34]. Backers, T. (2005). Fracture toughness determination and micromechanics of rock under mode I and mode II loading (Doctoral dissertation, GeoForschungsZentrum Potsdam).

[35]. Andersson, J.C. (2007). Rock mass response to coupled mechanical thermal loading: Äspö Pillar Stability Experiment, Sweden (Doctoral dissertation, Byggvetenskap).

[36]. Shen, B., Stephansson, O., Rinne, M., Amemiya, K., Yamashi, R., Toguri, S. and Asano, H. (2011). FRACOD modeling of rock fracturing and permeability change in excavation-damaged zones. International Journal of Geomechanics. 11 (4): 302-313.

[37]. Rinne, M. (2008). Fracture mechanics and subcritical crack growth approach to model time-dependent failure in brittle rock. Teknillinen korkeakoulu.

[38]. Siren, T. (2011). Fracture mechanics prediction for Posiva's Olkiluoto spalling experiment (POSE) (No. POSIVA-WR--11-23). Posiva Oy.

[39]. Montgomery, D.C. and Runger, G.C. (2010). Applied statistics and probability for engineers. John Wiley & Sons.

[40]. Fathi, M., Alimoradi, A. and Hemati Ahooi, H.R. (2021). Optimizing Extreme Learning Machine Algorithm using Particle Swarm Optimization to Estimate Iron Ore Grade. Journal of Mining and Environment. 12 (2): 397-411.

[41]. Algaifi, H. A., Bakar, S.A., Alyousef, R., Sam, A.R.M., Alqarni, A.S., Ibrahim, M.H. and Salami, B.A. (2021). Machine learning and RSM models for

prediction of compressive strength of smart bio-concrete. Smart Structures and Systems. 28 (4): 535-551.

[42]. Alimoradi, A., Moradzadeh, A. and Bakhtiari, M.R. (2013). Application of Artificial Neural Networks and Support Vector Machines for carbonate pores size estimation from 3D seismic data. Journal of Mining and Environment. 4 (1): 1-14.

[43]. Kang, F. and Li, J. (2016). Artificial bee colony algorithm optimized support vector regression for system reliability analysis of slopes. Journal of Computing in Civil Engineering. 30 (3): 04015040.

[44]. Zhou, X., Zhang, G., Hu, S. and Li, J. (2019). Optimal estimation of shear strength parameters based on copula theory coupling information diffusion technique. Advances in Civil Engineering, 2019.

کاربرد مدل‌های یادگیری ماشین برای پیش‌بینی چقرمگی شکست حالت I و حالت II در سنگ

عنایت الله امامی میبدی[1*]، سید خالق حسین[1]، محمد فاتحی مرجی[2] و وامق رسولی[3]

۱- گروه زمین‌شناسی، دانشگاه یزد، یزد، ایران

۲- دانشکده مهندسی معدن و متالورژی، دانشگاه یزد، یزد، ایران

۳- دانشکده مهندسی نفت، دانشگاه داکوتای شمالی، آمریکا

ارسال ۲۰۲۲/۰۱/۲۰، پذیرش ۲۰۲۲/۰۵/۲۷

* نویسنده مسئول مکاتبات: en.emami@yazd.ac.ir

چکیده:

در این کار، از مدل‌های پیش‌بینی یادگیری ماشین به منظور ارزیابی تأثیر خواص کلان ماده سنگ (مقاومت فشاری تک محوری، مقاومت کششی و مدول الاستیسیته) بر ریز خواص چقرمگی شکست سنگ در حالت شکست کششی و برشی استفاده شده است. چهار نوع مختلف روش یادگیری ماشین، شامل رگرسیون خطی چند متغیره (MLR)، رگرسیون غیر خطی چند متغیره (MNLR)، روش مفصل مبنا (Copula) و روش رگرسیون بردار پشتیبان (SVR) مورد استفاده قرار گرفته است. چقرمگی شکست حالت I و حالت II (KIC و KIIC) به عنوان متغیر وابسته انتخاب می شود، در حالی که مقاومت کششی، مقاومت فشاری و مدول الاستیک به عنوان متغیرهای مستقل در نظر گرفته می شوند. داده ها از طریق مرور منابع جمع آوری شده است. نتایج به‌دست‌آمده نشان می‌دهد که مدل SVR مقادیر KIC و KIIC را به ترتیب با ضرایب تعیین ۰/۷۳و ۰/۷۷ پیش‌بینی می‌کند. مقادیر ضریب تعیین متناظر مدل MLR و مدل MNLR برای KIC و KIIC به ترتیب ۰/۶۳، ۰/۷۲ و ۰/۶۲، ۰/۷۵ است. مدل مفصل مبنا مقادیر ضریب تعیین را برای KIC، ۰/۵۲ و KIIC، ۰/۶۹ برآورد می‌کند. روش آزمون اعتبار متقابل چند لایه (KFCV) به منظور تعیین اعتبار روش‌ها و سنجش صحت نوع متغییرهای مستقل ورودی برای همه این مدل‌های یادگیری ماشین انجام شد. این روش اعتبارسنجی نشان می‌دهد که رگرسیون بردار پشتیبان (SVR) بهترین مدل طراحی‌شده برای پیش‌بینی چقرمگی شکست حالت-I و حالت II است.

کلمات کلیدی: ماده سنگ، خواص کلان و ریز خواص ماده سنگ، روش یادگیری ماشین، چقرمگی شکست سنگ.