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# Suitable Mining Method Selection using HFGDM-TOPSIS Method: a Case Study of an Apatite Mine

Myong Chun Pak[1], Un Chol Han[2*], and Dong Il Kim[3]

*1- Faculty of Mining Engineering, Kim Chaek University of Technology, Pyongyang, Democratic People's Republic of Korea;*
*2- School of Science and Engineering, Kim Chaek University of Technology, Pyongyang, Democratic People's Republic of Korea;*
*3- Department of Applied Mathematics, Kim Chaek University of Technology, Pyongyang, Democratic People's Republic of Korea*

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***Abstract***

Mining Method Selection (MMS) is the first and the most critical problem in mine design, and depends on some parameters such as the geo-technical and geological features and economic factors. The factors affecting MMS are determined by some mining experts, and the most suitable mining method is selected using the hesitant fuzzy group decision-making (HFGDM) and technique for order performance by similarity to the ideal solution (TOPSIS) method. These factors include the type of deposit, slope of deposit, thickness of orebody, depth below the surface, grade distribution, hanging wall Rock Mass Rating (RMR), footwall RMR, ore body RMR, recovery, capital cost, mining cost, annual productivity, and environmental impact. Firstly, we propose the group decision-making (GDM) method to determine the weights of multi-attributes based on the score function with the decision-makers' weights, in which the n-dimensional hesitant fuzzy environment take the form of hesitant fuzzy sets (HFS). Then we calculate the weights of these factors using the HFGDM method. A simple case study is also presented in order to illustrate the competence of this method. Here, we compare the seven mining methods for an Apatite mine, and select the optimal mining method using the TOPSIS method. Finally, the sub-level stope mining method is selected as the most suitable method to this mine.

## 1. Introduction

Mining method selection (MMS) is one of the most critical and problematic activities of mining engineering because the accuracy of choosing the process greatly affects its economic potential, and any mistake in decision-making imposes some irreparable finance to the owners [1]. The ultimate goals of the mining method selection are maximizing the company's profit and recovery of the mineral resources, and also providing a safe environment for the miners by selecting the suitable method with the least problems among the feasible alternatives. Selection of an appropriate mining method is a complex task that requires consideration of many technical, economic, social, and historical factors. The appropriate mining method is the method that is technically feasible for the ore geometry and ground conditions, while also being a low-cost operation. There is no single appropriate mining method for a deposit. Usually two or more feasible mining methods are possible in mines. Each method entails some inherent problems. Consequently, the optimal mining

Corresponding author: huch8272@star-co.net.kp (U. Chol Han).

method is one that offers the least problems in the mine. The approach of adopting the same mining method as that the neighboring operation is not always appropriate. However, this does not mean that one cannot learn from comparing mining plans of existing operations in the same district or of similar deposits. Each orebody is unique with its own properties, and engineering judgment has a great effect on the decision in such a versatile work like mining. Therefore, it seems clear that only experienced engineers who have improved his experience by working in several mines and gaining skills in different methods can make a logical decision about the mining method selection. Although experience and engineering judgment still provide a major input into the selection of a mining method, subtle differences in the characteristics of each deposit can usually be perceived only through a detailed analysis of the available data.

It becomes the responsibility of the geologists and engineers to work together to ensure that all factors are considered in the mining method selection process. One of the common techniques to select the optimal mining method is the Analytical Hierarchy Process (AHP). AHP is a widely used multiple criteria decision-making tool, firstly proposed by Saaty [2]. The traditional AHP method is problematic in that it uses an exact value to express the decision-maker’s opinion in a comparison of alternatives. Especially, the AHP method is often criticized due to its use of unbalanced scale of judgments and its inability to adequately handle the inherent uncertainty and imprecision in the pair-wise comparison process [3]. Ataei *et al*. [4] have used the analytic hierarchy process to choose the best mining method. Jamshidi *et al*. [5] have applied the analytic hierarchy process to choose the optimal underground mining method in the Jajarm bauxite mine. Bitarafan and Ataei [6] have selected an appropriate mining method in anomaly No. 3 of the Gol-Gohar iron mine using fuzzy multiple attribute decision-making method. Naghedehi *et al*. [7] have suggested the fuzzy AHP (FAHP) method for selection of a suitable mining method for Bauxite ore deposit in Iran.

Dehghani *et al*. [1] have chosen the most optimal mining method in the Gol-Gohar mine using the Grey and TODIM (an acronym in Portuguese, i.e. Tomada de Decisão Interativa Multicritério) decision-making techniques. Namin *et al*. [8] have proposed a new model to select the mining method based on the fuzzy TOPSIS. Samimi Namin *et al*. [9] have investigated the application of several decision-making techniques such as AHP, TOPSIS, and PROMETHEE to select an appropriate mining method in Iran. Also Bogdanovic *et al*. [10] have applied the PROMETHEE and analytic hierarchy process methods to determine an appropriate mining method in the Coka Marin mine in Serbia. Azadeh *et al*. [11] have presented a new method to select a mining method based on the improved Nicholas technique. Ataei *et al*. [12] have applied the Monte Carlo analytic hierarchy process method to select the best mining method in the Jajarm bauxite mine. Gelvez *et al*. [13] have used the analytic hierarchy process and the VIKOR methods to choose the optimal mining method in the coal mine in Colombia. Besides, Karimnia and Bagloo [14] have applied the analytical hierarchy process to choose the optimal extraction method in a salt mine in Iran. Yavuz [15] has used the AHP method to choose a suitable underground mining method for a lignite mine located in Istanbul. Chen *et al*. [16] have compared the results of the TOPSIS method with those for the AHP-VICOR method in the mining method selection problems. The results of this work showed that the proposed model could predict a mining method with more precision. Ataei *et al*. [17] have also used TOPSIS to do the same for the Jajarm mine in Iran. Nourali *et al*. [18] have used a Hierarchical Preference Voting System (HPVS) for the MMS problem that uses a Data Envelopment Analysis (DEA) model to produce weights associated with each ranking place.

However, the aforementioned operators and methods have some drawbacks as follow: An important topic in hesitant fuzzy group decision-making (HFGDM) problems is how to determine the weights of both attributes and decision-makers. All the aforementioned operators and methods only consider the situations where the attribute weights are completely known or partially known, and the decision-makers' weights are completely known. Furthermore, these weight vectors are provided by the decision-makers in advance, and therefore, are more or less subjective and insufficient.

Recently, some studies have been devoted to address this issue, and developed some completely unknown weight generation processes within the hesitant fuzzy environment. Hu *et al.* [19] have constructed the entropy weight model to determine the attributes weights based on the proposed entropy measures. Liu *et al*. [20] have taken advantage of the linear programming technique for multi-dimensional analysis of preference (LINMAP) to determine the attribute weights objectively in the hesitant fuzzy multiple attribute decision-making. Xu and Zhang [21] have established an optimization model based on the maximizing deviation method to determine the optimal relative weights of attributes under hesitant fuzzy environment. However, the above-mentioned weight generation methods only investigated multi-attributes single person decision-making with hesitant fuzzy information, and did not consider multi-criteria group decision-making (MCGDM) with hesitant fuzzy information. In addition, in a MCGDM with hesitant fuzzy information, because the experts have their own inherent value systems and consideration, and thus the disagreement among the experts are inevitable, in such a case, consensus turns out to be very important in group decision-making. The existing weight generation methods in Refs. [19-21] did not consider any consensus issue.

To address this issue, Zhang [22] has developed two non-linear optimization models for MCGDM problems with hesitant fuzzy information, one minimizing the divergence among the individual hesitant fuzzy decision matrices, and the other minimizing the divergence between each individual hesitant fuzzy decision matrix and the collective hesitant fuzzy decision matrix, from which two exact formulae were obtained to derive the decision-makers' weights and attributes, respectively. However, the operation and the methods in [22] did not consider that the decision-makers' weights mutually differed because the decision-makers had their own inherent value systems and consideration, and the weights of the attributes were related to the number of the decision-makers, and they decreased the computational complexity than the existing weighting methods in Refs. [19-21] but they increased the computational complexity in practical applications.

To address this issue, in this paper, we define n-dimensional hesitant fuzzy environment on number of the decision-makers, and develop the methods to determine the decision-makers' weights for each attribute and set of whole attributes based on simple average operation. Then we calculate the weights of the attributes based on the score function for each attribute and set of whole attributes with the decision-makers' weights, and we chose the optimal mining method in the apatite mine using TOPSIS decision-making technique. The main advantages of these methods in relation to the other prevalent methods are to apply the distance numbers, to consider the intensity of criteria changes, and high accuracy in decision-making. The outcome of such decision-making systems is to obtain the best results in the light of considering all the technical, economic, and safety criteria.

## 2. Methodology

## 2.1. HFGDM method

### 2.1.1. Hesitant fuzzy sets (HFS)

Torra [23] has proposed the notion of hesitant fuzzy sets to manage the situations in which several numerical values are possible for the definition of the membership of an element to a given set.

**Definition 1. [23]** Given a fixed set *X*, then a hesitant fuzzy set (HFS) on *X* is in terms of a function that when applied to *X* returns a subset of [0, 1].

To be easily understood, Xia and Xu [24], express HFS by mathematical symbol:

$$E = (< x, h_E(x) > | x \in X) \quad (1)$$

where $h_E(x)$ is a set of some values in [0, 1], denoting the possible membership degree of the element $x \in X$ to the set *E*. For convenience, Xia and Xu [24] have called $h = h_E(x)$ a hesitant fuzzy element (HFE), and *H* the set of all HFEs.

( i ) $h^c = \bigcup_{\gamma \in h} \{1 - \gamma\}$,

( ii ) $h_1 \cup h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 \vee \gamma_2\}$,

(iii) $h_1 \cap h_2 = \bigcup_{\gamma_1 \in h_1, \gamma_2 \in h_2} \{\gamma_1 \wedge \gamma_2\}$.

Xia and Xu [24] have defined the following comparison rules for HFEs:

**Definition 2. [24]** For a HFE *h*, $S(h) = \frac{1}{h_E} \sum_{\gamma \in h} \gamma$ is called the score function of *h*, where $h_E$ is the number of the elements in *h*. For two HFEs $h_1$ and $h_2$, if $S(h_1) > S(h_2)$, then $h_1 > h_2$, and if $S(h_1) = S(h_2)$, then $h_1 = h_2$.

### 2.1.2. Group decision-making methods to determine weights of multiple attributes under n-dimensional hesitant fuzzy environment

In this section, we propose the group decision-making methods to determine the weights of multiple attributes in case that the hesitant fuzzy environment is given as *n*-dimensional hesitant fuzzy environment by *n* decision-makers.

In real practical situations, the decision-makers' hesitant weights for each attribute may differ with one another because the decision-makers have different experiences and specialties. Therefore, in order to determine more reasonable decision-makers' weights for set of the whole attributes, it should be considered the decision-makers' hesitant weights for each attribute.

**To address this issue, suppose as follow:**

( i ) Importance degrees of the attributes differ with one another according to the alternatives.

( ii ) Evaluation levels of the decision-makers for set of the whole attributes are related to the evaluation levels of the decision-makers for each attribute.

(iii) Attribute weights for some alternatives are related to the number of the decision-makers.

Let $Z = \{z_1, z_2, \cdots, z_n\}$ be a set of the decision makers and $X = \{x_1, x_2, \cdots, x_m\}$ be a set of attributes (attribute set).

Let $h_j = \{h_{j_1}, h_{j_2}, \cdots, h_{j_n}\}$ be the hesitant evaluation for the importance weights of *j*-th attribute $x_j$ by $n$ decision-makers. Thus $h_j$ is an element in the $n$-dimensional hesitant fuzzy environment for $x_j$, and $H=\{h_1,h_2,\cdots,h_m\}$ is the set of all elements in the $n$-dimensional hesitant fuzzy environment for $x_j$ on $X=\{x_1,x_2,\cdots,x_m\}$.

According to the $n$-dimensional hesitant fuzzy environment, $h_j$ may be an element in $n$-dimensional hesitant fuzzy set, $n$-dimensional interval-valued hesitant fuzzy set or $n$-dimensional hesitant triangular fuzzy set. According to the opinions of $n$ decision-makers, the values of all $h_{jk}$ ; $k = 1, 2, \ldots, n$ may be the same. In case $h_{jk}$ ; $k = 1, 2, \ldots, n$ have the same values for $j$-th attribute $x_j$ , the hesitant degree for $j$-th attribute $x_j$ is the same hesitant degree for $h_{jk}$ .

In case the decision-makers evaluate the hesitant degrees for the attribute differently, we may have a single hesitant degree for the attribute as the mean of the different hesitant evaluation values.

It is possible to construct the set of elements in $n$-dimensional hesitant fuzzy environment for a single attribute according to the number of the decision-makers.

If $k$-th decision-maker evaluates the hesitant degree for $j$-th attribute $x_j$ as 0.46, 0.5 and 0.58, then we can change the hesitant degree into the mean value 0.513.

This main idea is based on that the varied hesitant evaluation values of decision-maker should be distributed to environs of mean value.

To determine weights of multiple attributes with decision-makers' weights under $n$-dimensional hesitant fuzzy set, we define the $n$-dimensional hesitant fuzzy set following as:

**Definition 3.** Given mapping $h_{E_n}$ that when applied to attribute set $X=\{x_1,x_2,\cdots,x_m\}$ returns a subset of $n$-dimensional values in [0,1] by set $Z=\{z_1,z_2,\cdots,z_n\}$ of decision-makers, then mapping $h_{E_n}$ determine the $n$-dimensional hesitant fuzzy set ($n$-DHFS) $E_n$ on $X$, and we call $h_{E_n}(x_j),\ x_j\in X(j=1,2,\cdots,m)$ a membership function of *n-DHFS* $E_n$.

To be easily understood, we express $n$-DHFS by the mathematical symbol:

$$E_n=(<x,h_{E_n}(x_j)>|x_j\in X,\ j=1,2,\cdots,m) \tag{2}$$

where $h_E(x)$ is a subset of some $n$-dimensional values in [0, 1], denoting the possible membership degree of the element $x_j\in X$ to the set $E_n$. For convenience, we call $h_j=h_{E_n}(x_j)$, $(j=1,2,\cdots,m)$ element of $n$-dimensional hesitant fuzzy set ($n$-DHFS) for $j$ attribute $x_j$ and $H_n$ the set of all $n$-DHFS.

Let the given element $h_j=\{h_{j_1},h_{j_2},\cdots,h_{j_n}\}$, $h_{j_k}\in[0,1]\ (j=1,2,\cdots,m,\ k=1,\cdots,n)$ of a *n-DHFE* for attribute $x_j$ permits duplication of $h_{jk}$ on the attribute set $X$, where $h_{jk}$ is related to an opinion of $k$-th decision-maker for $j$-th attribute $x_j$ .

In real situations, the hesitant opinion of the decision-makers for $j$-th attribute $x_j$ may differ with one another because the decision-makers may come from different fields, and thus have different experiences and specialties. Therefore, the decision-makers' weights may differ with one another for one attribute.

We can define the $k$-th decision-maker's weight for element $h_j$ of *n-DHFS* for $j$-th attribute $x_j$ as follows:

$$w'_{jk}=1-\left|\left(\frac{1}{n}\sum_{l=1}^{n}h_{jl}\right)-h_{jk}\right|,\quad (k=1,2,\cdots,n,\ j=1,2,\cdots,m) \tag{3}$$

where $w'_{j_k}\ (j=1,2,\cdots,m)$ is the weight of hesitant evaluation level degree of the $k$-th decision-maker to $h_j$ .

By considering the conditions $w_{j_k} \geq 0$ and $\sum_{k=1}^{n} w_{j_k} = 1\ (j = 1,2,\cdots,m)$, the $k$-th decision-maker's weight $w_{jk}$ to element $h_j$ of a *n-DHFS* for $j$-th attribute $x_j$ is determined as:

$$w_{jk} = w'_{jk} \Big/ \sum_{p=1}^{n} w'_{jp}$$

$$(k = 1,2,\cdots,n,\ j = 1,2,\cdots,m)$$

If the hesitant evaluation level degrees of the decision-makers are all the same, i.e. $h_{j1} = h_{j2} = \cdots = h_{jn}$, then $w_{jk} = 1; k = 1,2,\cdots,n$.

The main idea of (3) is that the hesitant evaluation level degree for the attribute approach to the mean value, the larger the decision-makers' weights for the attribute become.

**Definition 4.** The hesitant fuzzy score function with the weights to $h_j$ for $j$-th attribute $x_j$ is defined as follows:

$$S(h_j) = \frac{1}{n}\sum_{k=1}^{n} w_{j_k} h_{jk}\ ,\ (j = 1,2,\cdots,m) \qquad (4)$$

If the hesitant evaluation level degrees of the decision-makers are all the same, i.e. if $h_{j1} = h_{j2} = \cdots h_{jn} = h'_j$, $S(h_j) = h'_j$ is completed.

All the aforementioned operators and methods only consider the decision-makers' weights where an attribute is fix.

In real situations, the hesitant opinion of decision-makers for each other different attributes may differ with one another to the fact that the decision-makers may come from different fields, and thus have different experiences and specialties.

Therefore, the decision-makers' weights for every attributes on set of whole attributes may differ with one another.

Then we can define the weights of hesitant evaluation level degrees of decision-makers for $H_n$ to set $H_n = \{h_1, h_2, \cdots h_m\}$ of elements of all $n$-dimensional hesitant fuzzy set ($n$-DHFEs) as follows:

$$w'_k = 1 - \frac{1}{m}\left[\sum_{j=1}^{m} \left| S(h_j) - h_{jk} \right|\right], (k = 1,2,\cdots,n) \qquad (5)$$

Determining a weight $w_k$ of hesitant evaluation level degree of decision-maker for $H_n$, $w_{jk}$ of the $k$-th decision-maker to element $h_j$ of *n-DHFS* for $j$-th attribute $x_j$ $w_k \geq 0$ and $\sum_{k=1}^{n} w_k = 1$ might satisfy, then $w_k = w'_k \Big/ \sum_{p=1}^{n} w'_p\ (k = 1,2,\cdots,n)$ is completed.

**Definition 5.** Let given set $H_n = \{h_1, h_2, \cdots, h_m\}$ of all elements of $n$-dimensional hesitant fuzzy set ($n$-DHFEs) on attribute set $X = \{x_1, x_2, \cdots, x_m\}$.

We define the hesitant fuzzy score function with weights for attributes $x_j\ (j = 1,2,\cdots,m)$ on $H_n$ as follows:

$$S_j(H_n) = \frac{1}{n}\sum_{k=1}^{n} w_k h_{jk}\ , (j = 1,2,\cdots,m) \qquad (6)$$

If so, we can rank the $x_j\ (j = 1,2,\cdots,m)$ in descending order according to the values of $S_j(H_n)\ \ (j = 1,2,\cdots,m)$. Then we can decide the weights $W_{x_j}\ \ (j = 1,2,\cdots,m)$ of importance degree to each attribute on attribute set $X = \{x_1, x_2, \cdots, x_m\}$ as follow:

$$W_{x_j} = S_j(H_n) \Big/ \sum_{l=1}^{m} S_l(H_n)\ ,\ (j = 1,2,\cdots,m),\ W_{x_j} \geq 0\ \sum_{j=1}^{m} W_{x_j} = 1 \qquad (7)$$

### 2.1.3. Steps of determining weight of multiple attributes

Based on the above analysis, we next develop an approach to the GDM problem to determine weights of multiple attributes with $n$-dimensional hesitant fuzzy information, which is composed of the following steps:

Step 1. For a GDM problem to determine weights of multiple attributes with $n$-dimensional hesitant fuzzy information, the all decision-makers evaluate as hesitant value in [0,1] for each attribute $x_j (j=1,2,\cdots,m)$, and then $n$-dimensional hesitant fuzzy sets $h_j (j=1,2,\cdots,m)$ constructs for each attribute $x_j$ $(j=1,2,\cdots,m)$, and determine the set $H_n=\{h_1,h_2,\cdots,h_m\}$ of all $n$-DHFEs.

Step 2. Calculate the weight $w_{jk}$ of the $k$-th decision-maker to element $h_j$ of *n-DHFS* for $j$-th attribute $x_j$ using Equation (3), so that $w_{j_k} \geq 0$, $\sum_{k=1}^{n} w_{j_k} = 1$ $(j=1,2,\cdots,m)$ satisfy.

Step 3. Calculate the hesitant fuzzy score function values with weights for attributes $x_j$ of $h_j (j=1,2,\cdots, m)$ using Equation (4).

Step 4. Calculate the weights $w_k (k=1,2,\cdots,n)$ of hesitant evaluation level degrees of decision-makers for $H_n$ using Equation (5), so that $w_k \geq 0$ and $\sum_{k=1}^{n} w_k = 1$ satisfy.

Step 5. Calculate the hesitant fuzzy score function values with weights for attributes $x_j (j=1,2,\cdots,m)$ on $H_n$ by using Equation (6), and calculate the weights $W_{x_j} (j=1,2,\cdots,m)$ of importance degree to each attribute for attribute set $X=\{x_1,x_2,\cdots,x_m\}$ using Equation (7).

## 2.2. TOPSIS Method

Technique for ordering preference by similarity to an ideal solution (TOPSIS) is a classic Multi Attribute Decision Making (MADM) method developed by Hwang and Yoon [25]. TOPSIS helps the decision-makers develop issues to analyze, compare, and rank according to their alternate ratings.

TOPSIS is based on the concept of the closest alternative choice of a positive ideal solution (PIS) and furthest from the negative ideal solution (NIS). The sum of the highest values of each attribute is called a positive ideal solution (PIS). The sum of the lowest values of each attribute is called the negative ideal solution (NIS). Based on a comparison of the relative distance of PIS and NIS, alternative priority arrangements can be achieved [16, 26, 27]. TOPSIS advantages: (1) Human choice is represented by logical thinking, (2) The concept is simple and easy to understand, (3) The computing process can be easily programmed into a spreadsheet, (4) Be able to measure the relative performance of decision alternatives in simple mathematical form [28].

TOPSIS method includes six stages for solving decision-making problem [23, 29]:

**Step 1:** Converting decision-making matrix to a normalized matrix using Equation (8):

$$r_{ij} = \frac{a_{ij}}{\sum_{i=1}^{m} a_{ij}^2} \tag{8}$$

whereas $i$ = 1, 2, ..., $n$; and $j$ = 1, 2, ..., $m$

**Step 2:** Create a normalized weighted decision matrix.

Normalized weighted decision matrix.

$$y_{ij} = w_{x_j} r_{ij} \tag{9}$$

**Step 3:** Determines the matrix of positive ideal solutions (PIS) and the negative ideal solution matrix (NIS).

Normalized weights in the decision matrix ($y_{ij}$) are used to determine the positive ideal solution ($A^+$) and the negative ideal solution ($A^-$).

$$\begin{cases} A^+ = \left(y_1^+, y_2^+, \cdots y_n^+\right) \\ A^- = \left(y_1^-, y_2^-, \cdots y_n^-\right) \end{cases} \tag{10}$$

$$\begin{cases} y_j^+ = \left\{\left(\max_i y_{ij} \mid j \in J_1\right), \left(\min_i y_{ij} \mid j \in J_2\right)\right\} \\ y_j^- = \left\{\left(\min_i y_{ij} \mid j \in J_1\right), \left(\max_i y_{ij} \mid j \in J_2\right)\right\} \end{cases} \quad (11)$$

**Step 4:** Determine the distance between each alternative from a positive ideal solution matrix and a negative ideal solution matrix. The distance between the *i*-th alternative and the positive ideal solution as:

$$D_i^+ = \sqrt{\sum_{j=1}^{m} \left(y_j^+ - y_{ij}\right)} \quad (12)$$

The distance between $A_i$ alternatives with the negative ideal solution value is formulated as:

$$D_i^- = \sqrt{\sum_{j=1}^{m} \left(y_{ij} - y_j^-\right)} \quad (13)$$

**Step 5:** Calculating the closeness coefficient of each alternative from $D^+$ and $D^-$.

The closeness coefficient of alternative is given as:

$$C_i^+ = \frac{D_i^-}{D_i^+ + D_i^-} \quad (14)$$

**Step 6:** Ranking alternatives

The $C_i^+$ sequence is used to rank so that the best alternative is the shortest distance to the positive ideal solution and has the furthest distance to the negative-ideal solution.

## 3. Model of mining method selection in Apatite mine using HFGDM-TOPSIS method

### 3. 1 Description of studied site

In order to investigate the competence of this technique for the MMS problem, we chose an Apatite mine to conduct a case study, which is located in west of the DPR Korea. The three-dimension model of the orebody and main development workings including shaft and drifts is shown in Figure 1. As shown in this figure, entrances of main shaft are elevation +285 m, respectively, and level height for mining blocks is 50 m. As of now, ore mining and tunneling at the mine are carried out on levels 50 m and 100 m. The collapse of surface ground caused by mining activities is allowable, the effect of ground water is taken no account, because there are no rivers and geological faults, industry buildings and domestic houses in the mining area.

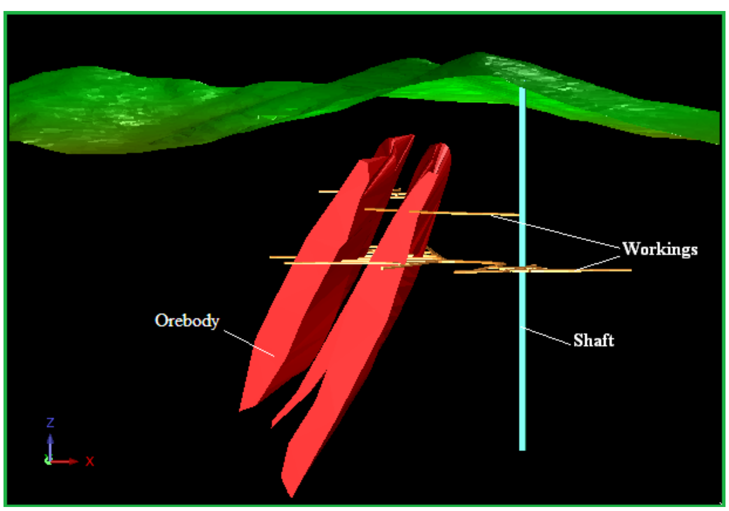


**Figure 1. 3-Dimension model of the orebody and main development workings in an Apatite mine.**

The physical and mechanics parameters such as deposit geometry (type of deposit, slope of deposit, thickness of orebody and depth below the surface) and rock mass characteristics are shown in Table 1.

**Table 1. Some information about apatite mine.**

| | | |
|---|---|---|
| **orebody** | Type of deposit | layer lattice |
| | Slope of deposit | 45~60° |
| | Thickness of orebody | 16~22 m |
| | Depth below the surface | 80~120 m |
| | Mineable reserve | 35,700,000 t |
| | Production rate | 500,000 t |
| **Geo-mechanical data** | Hanging wall Rock Mass Rating (RMR) | 35 |
| | Footwall RMR | 40 |
| | Orebody RMR | 55 |
| **Hydrogeology** | Hydrogeology conditions | Dry |

## 3.2. Model of mining method selection

For selecting the most economical and appropriate mining method using the HFGDM-TOPSIS method, in the first stage, all alternatives and decision attributes are determined.

Characteristics that have a major impact on the mining method selection include:

- Physical and mechanical characteristics of the deposit such as ground conditions of the ore zone, general shape, ore thickness, dip, plunge, depth below the surface, hanging wall, and footwall, grade distribution, and quality of resource. The basic components that define the ground conditions are: shear strength of rock material, natural fractures and discontinuities, orientation, length, spacing, and location of major geologic structures, *in situ* stress, hydrologic conditions, etc.
- Economic factors such as capital cost, mining cost, mineable ore tons, orebody grades, and mineral value.
- Technical factors such as mine recovery, dilution, flexibility of methods, machinery, and mining rate.
- Productivity factors such as annual productivity, equipment efficiency, and environmental considerations.

In this regard, in order to form the initial decision-making matrix, the parameter type of deposit, slope of deposit, thickness of orebody, depth below the surface, grade distribution, hanging wall RMR, footwall RMR, ore body RMR, recovery, capital cost, mining cost, annual productivity, and environmental impact were selected as the effective factors involved in choosing the mining method. Likewise, the mining methods including sub-level stoping, sub-level caving, block caving, cut and fill, shrinkage stoping, stope and pillar, and stull stoping were selected as the extraction options.

The hierarchical structure of the problem is shown in Figure 2.

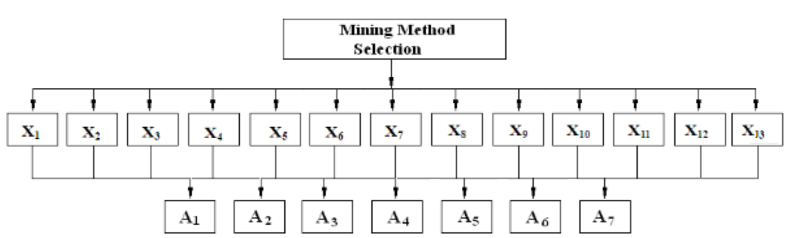


**Figure 2. Hierarchical structure of decision problem.**

whereas $X_1$-type of deposit, $X_2$- slope of deposit, $X_3$- thickness of orebody, $X_4$- depth below the

Corresponding author: huch8272@star-co.net.kp (U. Chol Han).

surface, $X_5$-hanging wall RMR, $X_6$-footwall RMR, $X_7$-ore body RMR, $X_8$-grade distribution, $X_9$-recovery, $X_{10}$-capital cost, $X_{11}$-mining cost, $X_{12}$-annual productivity, $X_{13}$-environmental impact; $A_1$-sub-level stoping, $A_2$-sub-level caving, $A_3$-block caving, $A_4$-cut and fill, $A_5$-shrinkage stoping, $A_6$-stope and pillar, $A_7$-stull stoping

## 4. Mining Method Selection Using HFGDM-TOPSIS Method

### 4.1. Determination of weight of attributes using HFGDM

Using HFGDM, the weights of 13-attributes are determined by 5-steps in Section 2.1.3.

**Step 1.** If determine set $H_5=\{h_1,h_2,h_3,h_4,h_5,h_6,h_7,h_8,h_9,h_{10},h_{11},h_{12},h_{13}\}$ of elements of 5-dimensional hesitant fuzzy set for set $X=\{x_1,x_2,x_3,x_4,x_5,x_6,x_7,x_8,x_9,x_{10},x_{11},x_{12},x_{13}\}$ of attributes and set $Z=\{z_1,z_2,z_3,z_4,z_5\}$ of decision-makers.

The decision-makers use the linguistic variables to evaluate the importance of attributes and the ratings of alternatives with respect to various attributes. In this work, to select the optional mining method for the studied mine, in order to illustrate the idea of HFGDM-TOPSIS, we deliberately transform the existing precise values to seven-levels, fuzzy linguistic variables; very low (VL), low (L), middle Low (ML), middle (M), middle high (MH), high (H), and very high (VH), where VL = [0, 0.1], L = [0.1, 0.3], ML = [0.3, 0.4], M = [0.4, 0.6], MH = [0.6, 0.7], H = [0.7, 0.9], and VH = [0.9, 1].

The importance linguistic values of the attributes determined by these five decision-makers are listed in Table 2.

**Table 2. Importance linguistic values of attributes ($x_j$) from five decision-makers ($z_i$).**

| Decision-makers | Importance linguistic values of attributes | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ |
| $z_1$ | M | VH | M | ML | H | M | MH | ML | VH | M | H | H | H |
| $z_2$ | ML | H | M | L | H | M | MH | ML | VH | M | H | VH | MH |
| $z_3$ | M | H | MH | L | MH | MH | H | ML | H | H | VH | H | H |
| $z_4$ | ML | H | MH | ML | H | M | H | L | VH | MH | VH | H | H |
| $z_5$ | M | VH | MH | L | H | M | MH | ML | VH | M | H | MH | H |

The importance values of attributes evaluated by five decision-makers are illustrated in Table 3.

**Table 3. Set $H_5=\{h_1,h_2,h_3,h_4,h_5,h_6,h_7,h_8,h_9,h_{10},h_{11},h_{12},h_{13}\}$ of elements of 5-DHFS.**

| Decision-makers | Importance linguistic values of attributes | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ |
| $z_1$ | 0.5 | 0.95 | 0.5 | 0.35 | 0.8 | 0.5 | 0.65 | 0.35 | 0.95 | 0.5 | 0.8 | 0.8 | 0.8 |
| $z_2$ | 0.35 | 0.8 | 0.5 | 0.2 | 0.8 | 0.5 | 0.65 | 0.35 | 0.95 | 0.5 | 0.8 | 0.95 | 0.65 |
| $z_3$ | 0.5 | 0.8 | 0.65 | 0.2 | 0.65 | 0.65 | 0.8 | 0.35 | 0.8 | 0.8 | 0.95 | 0.8 | 0.8 |
| $z_4$ | 0.35 | 0.8 | 0.65 | 0.35 | 0.8 | 0.5 | 0.8 | 0.2 | 0.95 | 0.65 | 0.95 | 0.8 | 0.8 |
| $z_5$ | 0.5 | 0.95 | 0.65 | 0.2 | 0.8 | 0.5 | 0.65 | 0.35 | 0.95 | 0.5 | 0.8 | 0.65 | 0.8 |

Here, the elements of 5-DHFS are as follow:

$$h_1 = (0.5, 0.35, 0.5, 0.35, 0.5),\ h_2 = (0.95, 0.8, 0.8, 0.8, 0.95),$$
$$h_3 = (0.5, 0.5, 0.65, 0.65, 0.65),\ h_4 = (0.35, 0.2, 0.2, 0.35, 0.2),$$
$$h_5 = (0.8, 0.8, 0.65, 0.8, 0.8),\ h_6 = (0.5, 0.5, 0.65, 0.5, 0.5),$$
$$h_7 = (0.65, 0.65, 0.8, 0.8, 0.65),\ h_8 = (0.35, 0.35, 0.35, 0.2, 0.35),$$
$$h_9 = (0.95, 0.95, 0.8, 0.95, 0.95),\ h_{10} = (0.5, 0.5, 0.8, 0.65, 0.5),$$
$$h_{11} = (0.8, 0.8, 0.95, 0.95, 0.8),\ h_{12} = (0.8, 0.95, 0.8, 0.8, 0.65),$$
$$h_{13} = (0.8, 0.65, 0.8, 0.8, 0.8)$$

**Step 2.** Calculate the weights $w_{jk}$ of the $k$-th decision-maker to element $h_j$ of 5-dimensional hesitant fuzzy set (5-DHFS) for $j$-th attribute $x_j$ using Equation (3), so that $w_{j_k} \geq 0$, $\sum_{k=1}^{5} w_{j_k} = 1$ $(j = 1, 2, \cdots, 13)$ satisfy. (Table 4)

**Table 4. Decision-makers' weights $w_{jk}$ $(j = 1, 2, \cdots 13,\quad k = 1, 2, 3, 4, 5)$ for $h_j$.**

| Decision-makers | Attributes | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $w_{1k}$ | $w_{2k}$ | $w_{3k}$ | $w_{4k}$ | $w_{5k}$ | $w_{6k}$ | $w_{7k}$ | $w_{8k}$ | $w_{9k}$ | $w_{10k}$ | $w_{11k}$ | $w_{12k}$ | $w_{13k}$ |
| $w_{j1}$ | 0.203 | 0.196 | 0.196 | 0.196 | 0.204 | 0.204 | 0.203 | 0.204 | 0.204 | 0.204 | 0.203 | 0.213 | 0.204 |
| $w_{j2}$ | 0.196 | 0.203 | 0.196 | 0.203 | 0.204 | 0.204 | 0.203 | 0.204 | 0.204 | 0.204 | 0.203 | 0.181 | 0.185 |
| $w_{j3}$ | 0.203 | 0.203 | 0.203 | 0.203 | 0.185 | 0.185 | 0.196 | 0.204 | 0.185 | 0.177 | 0.196 | 0.213 | 0.204 |
| $w_{j4}$ | 0.196 | 0.203 | 0.203 | 0.196 | 0.204 | 0.204 | 0.196 | 0.185 | 0.204 | 0.211 | 0.196 | 0.213 | 0.204 |
| $w_{j5}$ | 0.203 | 0.196 | 0.203 | 0.203 | 0.204 | 0.204 | 0.203 | 0.204 | 0.204 | 0.204 | 0.203 | 0.181 | 0.204 |

**Step 3.** Calculate the hesitant fuzzy score function values with weights for attributes $x_j$ of $h_j$ using Equation (4).

$$S(h_j) = (0.0882, 0.1718, 0.1182, 0.0518, 0.1545, 0.1055, 0.1418, 0.0645, 0.1845, 0.117, 0.1718, 0.16, 0.1545)$$

**Step 4.** Calculate the weights $w_k$ $(k = 1, 2, 3, 4, 5)$ of hesitant evaluation level degrees of decision-makers for $H_5$ using Equation (5), so that $w_k \geq 0$ and $\sum_{k=1}^{5} w_k = 1$ satisfy.

$$w_1 = 0.199,\ w_2 = 0.2134,\ w_3 = 0.1895,\ w_4 = 0.1943,\ w_5 = 0.2038$$

**Step 5.** Calculate the hesitant fuzzy score function values with weights for attributes $x_j$ on $H_5$ by using Equation (6) and calculate the weights $W_{xj}$ of importance degree to each attribute for set $X = \{x_1, x_2, \cdots, x_{13}\}$ of attributes using Equation (7).

$$S_j(h_5) = (0.0878, 0.1721, 0.1176, 0.0518, 0.1543, 0.1057, 0.1415, 0.0642, 0.1843, 0.1172, 0.1715, 0.1603, 0.1536)$$
$$W_{x_j} = (0.0522, 0.1023, 0.0699, 0.0308, 0.0918, 0.0628, 0.0841, 0.0382, 0.1096, 0.0697, 0.102, 0.0953, 0.0913) \quad (15)$$

## 4.2 Selection of optimal mining method using TOPSIS method

The optimal mining system is selected by the TOPSIS methods of 6-steps in Section 2.2.

Step 1, making of the decision matrix.

The decision matrix is made by calculating weight of the 7-alternatives (mining methods) for each attributes using the HFGDM method.

The importance linguistic values of the 7-alternatives for the type of deposit ($x_1$) determined by these five decision makers are listed in Table 5.

**Table 5. Importance linguistic values of 7-alternatives ($a_j$) for type of deposit ($x_1$) determined by 5-decision-makers ($z_i$).**

| Decision-makers | Importance linguistic values of 7-alternatives for the type of deposit ($x_1$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | M | VH | M | ML | H | M | MH |
| $z_2$ | ML | H | M | L | H | M | MH |
| $z_3$ | M | H | MH | L | MH | MH | H |
| $z_4$ | ML | H | MH | ML | H | M | H |
| $z_5$ | M | VH | MH | L | H | M | MH |

The importance values of 7-alternatives ($a_j$) for type of deposit ($x_1$) evaluated by five decision-makers are illustrated in Table 6. The weights ($a_{i1}$) of 7-altenatives calculated for the type of deposit ($x_1$) using 5-steps in Sec 2.1.3 are illustrated in Table 6.

**Table 6. Weights ($a_{i1}$) of 7-altenatives for the type of deposit ($x_1$).**

| Decision-makers | Importance linguistic values of 7-alternatives for the type of deposit ($x_1$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.8 | 0.8 | 0.65 | 0.8 | 0.8 | 0.35 | 0.8 |
| $z_2$ | 0.8 | 0.8 | 0.65 | 0.8 | 0.8 | 0.5 | 0.8 |
| $z_3$ | 0.95 | 0.8 | 0.8 | 0.8 | 0.8 | 0.35 | 0.65 |
| $z_4$ | 0.8 | 0.8 | 0.65 | 0.95 | 0.95 | 0.35 | 0.8 |
| $z_5$ | 0.8 | 0.65 | 0.65 | 0.95 | 0.8 | 0.5 | 0.8 |
| $a_{i1}$ | 0.1612 | 0.1496 | 0.1321 | 0.1668 | 0.1610 | 0.0797 | 0.1496 |

Like the preceding the weights ($a_{ij}$) of 7-altenatives calculated for each attributes ($x_j$) are illustrated in Tables 7-17.

**Table 7. Weights ($a_{i2}$) of 7-altenatives for slope of deposit ($x_2$).**

| Decision-makers | Importance linguistic values of 7-alternatives for slope of deposit ($x_2$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.95 | 0.95 | 0.65 | 0.8 | 0.8 | 0.35 | 0.65 |
| $z_2$ | 0.95 | 0.8 | 0.8 | 0.95 | 0.95 | 0.35 | 0.5 |
| $z_3$ | 0.8 | 0.8 | 0.8 | 0.95 | 0.95 | 0.35 | 0.5 |
| $z_4$ | 0.95 | 0.8 | 0.65 | 0.8 | 0.95 | 0.35 | 0.65 |
| $z_5$ | 0.8 | 0.95 | 0.65 | 0.8 | 0.8 | 0.5 | 0.65 |
| $a_{i2}$ | 0.1718 | 0.1662 | 0.1369 | 0.1659 | 0.1717 | 0.0734 | 0.1141 |

**Table 8. Weights ($a_{i3}$) of 7-altenatives for thickness of orebody ($x_3$).**

| Decision-makers | Importance linguistic values of 7-alternatives for thickness of orebody ($x_3$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.95 | 0.8 | 0.65 | 0.8 | 0.65 | 0.35 | 0.8 |
| $z_2$ | 0.95 | 0.65 | 0.65 | 0.8 | 0.8 | 0.2 | 0.65 |
| $z_3$ | 0.8 | 0.65 | 0.5 | 0.65 | 0.65 | 0.2 | 0.8 |
| $z_4$ | 0.8 | 0.65 | 0.5 | 0.8 | 0.65 | 0.2 | 0.8 |
| $z_5$ | 0.8 | 0.8 | 0.65 | 0.8 | 0.5 | 0.35 | 0.65 |
| $a_{i3}$ | 0.1876 | 0.1549 | 0.1284 | 0.1682 | 0.1422 | 0.0562 | 0.1625 |

**Table 9. Weights ($a_{i4}$) of 7-altenatives for depth below the surface ($x_4$).**

| Decision-makers | Importance linguistic values of 7-alternatives for depth below the surface ($x_4$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.8 | 0.5 | 0.35 | 0.8 | 0.95 | 0.65 | 0.65 |
| $z_2$ | 0.8 | 0.35 | 0.2 | 0.8 | 0.95 | 0.5 | 0.65 |
| $z_3$ | 0.8 | 0.35 | 0.2 | 0.95 | 0.8 | 0.65 | 0.8 |
| $z_4$ | 0.95 | 0.5 | 0.35 | 0.8 | 0.8 | 0.5 | 0.8 |
| $z_5$ | 0.8 | 0.5 | 0.35 | 0.8 | 0.95 | 0.65 | 0.8 |
| $a_{i4}$ | 0.1803 | 0.0949 | 0.0623 | 0.1807 | 0.1936 | 0.1278 | 0.1604 |

**Table 10. Weights ($a_{i5}$) of 7-altenatives for hanging wall RMR ($x_5$).**

| Decision-makers | Importance linguistic values of 7-alternatives for hanging wall RMR ($x_5$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.65 | 0.35 | 0.2 | 0.8 | 0.5 | 0.5 | 0.2 |
| $z_2$ | 0.8 | 0.35 | 0.2 | 0.65 | 0.8 | 0.35 | 0.2 |
| $z_3$ | 0.8 | 0.5 | 0.5 | 0.8 | 0.65 | 0.35 | 0.35 |
| $z_4$ | 0.8 | 0.5 | 0.35 | 0.8 | 0.8 | 0.5 | 0.35 |
| $z_5$ | 0.8 | 0.35 | 0.35 | 0.8 | 0.65 | 0.5 | 0.35 |
| $a_{i5}$ | 0.2100 | 0.1109 | 0.0855 | 0.2103 | 0.1851 | 0.1204 | 0.0778 |

**Table 11. Weights ($a_{i6}$) of 7-altenatives for footwall RMR ($x_6$).**

| Decision-makers | Importance linguistic values of 7-alternatives for footwall RMR ($x_6$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.65 | 0.5 | 0.5 | 0.8 | 0.8 | 0.35 | 0.35 |
| $z_2$ | 0.65 | 0.5 | 0.5 | 0.65 | 0.65 | 0.5 | 0.5 |
| $z_3$ | 0.8 | 0.8 | 0.5 | 0.65 | 0.65 | 0.35 | 0.35 |
| $z_4$ | 0.8 | 0.65 | 0.65 | 0.8 | 0.8 | 0.5 | 0.5 |
| $z_5$ | 0.65 | 0.65 | 0.65 | 0.65 | 0.8 | 0.5 | 0.5 |
| $a_{i6}$ | 0.1684 | 0.1469 | 0.1321 | 0.1688 | 0.1756 | 0.1041 | 0.1041 |

**Table 12. Weights ($a_{i7}$) of 7-altenatives for ore body RMR ($x_7$).**

| Decision-makers | Importance linguistic values of 7-alternatives for ore body RMR ($x_7$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.8 | 0.65 | 0.5 | 0.8 | 0.65 | 0.35 | 0.35 |
| $z_2$ | 0.95 | 0.5 | 0.35 | 0.8 | 0.8 | 0.35 | 0.5 |
| $z_3$ | 0.95 | 0.5 | 0.5 | 0.8 | 0.65 | 0.5 | 0.5 |
| $z_4$ | 0.8 | 0.65 | 0.5 | 0.95 | 0.8 | 0.5 | 0.2 |
| $z_5$ | 0.8 | 0.5 | 0.35 | 0.8 | 0.65 | 0.35 | 0.5 |
| $a_{i7}$ | 0.2038 | 0.1328 | 0.1040 | 0.1967 | 0.1682 | 0.0967 | 0.0978 |

**Table 13. Weights ($a_{i8}$) of 7-altenatives for grade distribution ($x_8$).**

| Decision-makers | Importance linguistic values of 7-alternatives for grade distribution ($x_8$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.8 | 0.5 | 0.35 | 0.65 | 0.8 | 0.8 | 0.5 |
| $z_2$ | 0.65 | 0.35 | 0.2 | 0.8 | 0.65 | 0.65 | 0.5 |
| $z_3$ | 0.8 | 0.5 | 0.35 | 0.65 | 0.65 | 0.65 | 0.65 |
| $z_4$ | 0.65 | 0.35 | 0.35 | 0.65 | 0.8 | 0.8 | 0.5 |
| $z_5$ | 0.65 | 0.5 | 0.5 | 0.8 | 0.8 | 0.65 | 0.5 |
| $a_{i8}$ | 0.1695 | 0.1044 | 0.0826 | 0.1704 | 0.1765 | 0.1698 | 0.1268 |

**Table 14. Weights ($a_{i9}$) of 7-altenatives for recovery ($x_9$).**

| Decision-makers | Importance linguistic values of 7-alternatives for recovery ($x_9$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.65 | 0.5 | 0.35 | 0.95 | 0.65 | 0.35 | 0.8 |
| $z_2$ | 0.65 | 0.5 | 0.35 | 0.95 | 0.65 | 0.2 | 0.65 |
| $z_3$ | 0.5 | 0.5 | 0.5 | 0.95 | 0.65 | 0.35 | 0.65 |
| $z_4$ | 0.5 | 0.35 | 0.35 | 0.8 | 0.5 | 0.35 | 0.8 |
| $z_5$ | 0.5 | 0.35 | 0.35 | 0.8 | 0.5 | 0.2 | 0.65 |
| $a_{i9}$ | 0.1451 | 0.1133 | 0.0987 | 0.2307 | 0.1525 | 0.0748 | 0.1849 |

**Table 15. Weights ($a_{i10}$) of 7-altenatives for capital cost ($x_{10}$).**

| Decision-makers | Importance linguistic values of 7-alternatives for capital cost ($x_{10}$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.65 | 0.65 | 0.5 | 0.35 | 0.95 | 0.5 | 0.35 |
| $z_2$ | 0.8 | 0.65 | 0.5 | 0.35 | 0.95 | 0.35 | 0.35 |
| $z_3$ | 0.65 | 0.65 | 0.5 | 0.5 | 0.8 | 0.5 | 0.5 |
| $z_4$ | 0.65 | 0.8 | 0.65 | 0.5 | 0.95 | 0.35 | 0.35 |
| $z_5$ | 0.5 | 0.65 | 0.5 | 0.35 | 0.95 | 0.35 | 0.5 |
| $a_{i10}$ | 0.1621 | 0.1694 | 0.1319 | 0.1017 | 0.2300 | 0.1023 | 0.1026 |

**Table 16. Weights ($a_{i11}$) of 7-altenatives for mining cost ($x_{11}$).**

| Decision-makers | Importance linguistic values of 7-alternatives for mining cost ($x_{11}$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.65 | 0.8 | 0.8 | 0.5 | 0.65 | 0.65 | 0.35 |
| $z_2$ | 0.8 | 0.8 | 0.8 | 0.65 | 0.5 | 0.5 | 0.35 |
| $z_3$ | 0.65 | 0.65 | 0.8 | 0.5 | 0.8 | 0.5 | 0.2 |
| $z_4$ | 0.65 | 0.8 | 0.8 | 0.65 | 0.65 | 0.5 | 0.35 |
| $z_5$ | 0.5 | 0.65 | 0.65 | 0.5 | 0.65 | 0.65 | 0.5 |
| $a_{i11}$ | 0.1562 | 0.1776 | 0.1853 | 0.1053 | 0.1571 | 0.1345 | 0.0840 |

**Table 17. Weights ($a_{i12}$) of 7-altenatives for annual productivity ($x_{12}$).**

| Decision-makers | Importance linguistic values of 7-alternatives for annual productivity ($x_{12}$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.8 | 0.8 | 0.8 | 0.65 | 0.5 | 0.65 | 0.5 |
| $z_2$ | 0.8 | 0.95 | 0.8 | 0.65 | 0.5 | 0.65 | 0.35 |
| $z_3$ | 0.95 | 0.8 | 0.95 | 0.8 | 0.5 | 0.8 | 0.35 |
| $z_4$ | 0.8 | 0.8 | 0.8 | 0.65 | 0.35 | 0.65 | 0.5 |
| $z_5$ | 0.8 | 0.8 | 0.8 | 0.65 | 0.35 | 0.65 | 0.5 |
| $a_{i12}$ | 0.1752 | 0.1761 | 0.1752 | 0.1434 | 0.0926 | 0.1434 | 0.0941 |

**Table 18. Weights ($a_{i13}$) of 7-altenatives for environmental impact ($x_{13}$).**

| Decision-makers | Importance linguistic values of 7-alternatives for environmental impact ($x_{13}$) | | | | | | |
|---|---|---|---|---|---|---|---|
| | $a_1$ | $a_2$ | $a_3$ | $a_4$ | $a_5$ | $a_6$ | $a_7$ |
| $z_1$ | 0.8 | 0.65 | 0.5 | 0.95 | 0.8 | 0.8 | 0.8 |
| $z_2$ | 0.8 | 0.5 | 0.5 | 0.95 | 0.8 | 0.8 | 0.65 |
| $z_3$ | 0.65 | 0.5 | 0.35 | 0.95 | 0.65 | 0.95 | 0.65 |
| $z_4$ | 0.8 | 0.35 | 0.35 | 0.8 | 0.8 | 0.8 | 0.8 |
| $z_5$ | 0.8 | 0.35 | 0.35 | 0.8 | 0.8 | 0.8 | 0.8 |
| $a_{i13}$ | 0.1580 | 0.0951 | 0.0833 | 0.1824 | 0.1580 | 0.1711 | 0.1521 |

According to Tables 6-18, the decision matrix is formed in Table 19.

**Table 19. Normalized decision matrix.**

| Alternatives | Attributes | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ |
| $a_1$ | 0.1612 | 0.1718 | 0.1876 | 0.1803 | 0.210 | 0.1684 | 0.2038 | 0.1695 | 0.1451 | 0.1621 | 0.1562 | 0.1752 | 0.1580 |
| $a_2$ | 0.1496 | 0.1662 | 0.1549 | 0.0949 | 0.1109 | 0.1469 | 0.1328 | 0.1044 | 0.1133 | 0.1694 | 0.1776 | 0.1761 | 0.0951 |
| $a_3$ | 0.1321 | 0.1369 | 0.1284 | 0.0623 | 0.0855 | 0.1321 | 0.1040 | 0.0826 | 0.0987 | 0.1319 | 0.1853 | 0.1752 | 0.0833 |
| $a_4$ | 0.1668 | 0.1659 | 0.1682 | 0.1807 | 0.2103 | 0.1688 | 0.1967 | 0.1704 | 0.2307 | 0.1017 | 0.1053 | 0.1434 | 0.1824 |
| $a_5$ | 0.161 | 0.1717 | 0.1422 | 0.1936 | 0.1851 | 0.1756 | 0.1682 | 0.1765 | 0.1525 | 0.230 | 0.1571 | 0.0926 | 0.158 |
| $a_6$ | 0.0797 | 0.0734 | 0.0562 | 0.1278 | 0.1204 | 0.1041 | 0.0967 | 0.1698 | 0.0748 | 0.1023 | 0.1345 | 0.1434 | 0.1711 |
| $a_7$ | 0.1496 | 0.1141 | 0.1625 | 0.1604 | 0.0778 | 0.1041 | 0.0978 | 0.1268 | 0.1849 | 0.1026 | 0.084 | 0.0941 | 0.1521 |

**Step 2:** According to Table 19, Equations (9) and (15), normalized weighted decision matrix is formed in Table 20.

**Table 20. Weighted decision matrix.**

| Alternatives | Attributes | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $x_6$ | $x_7$ | $x_8$ | $x_9$ | $x_{10}$ | $x_{11}$ | $x_{12}$ | $x_{13}$ |
| $a_1$ | 0.0084 | 0.0176 | 0.0131 | 0.0056 | 0.0193 | 0.0106 | 0.0171 | 0.0065 | 0.0159 | 0.0113 | 0.0159 | 0.0167 | 0.0144 |
| $a_2$ | 0.0078 | 0.0170 | 0.0108 | 0.0029 | 0.0102 | 0.0092 | 0.0112 | 0.0040 | 0.0124 | 0.0118 | 0.0181 | 0.0168 | 0.0087 |
| $a_3$ | 0.0069 | 0.0140 | 0.0090 | 0.0019 | 0.0078 | 0.0083 | 0.0088 | 0.0032 | 0.0108 | 0.0092 | 0.0189 | 0.0167 | 0.0076 |
| $a_4$ | 0.0087 | 0.0170 | 0.0118 | 0.0056 | 0.0193 | 0.0106 | 0.0166 | 0.0065 | 0.0253 | 0.0071 | 0.0107 | 0.0137 | 0.0167 |
| $a_5$ | 0.0084 | 0.0176 | 0.0099 | 0.0060 | 0.0170 | 0.0110 | 0.0142 | 0.0067 | 0.0167 | 0.0160 | 0.0160 | 0.0088 | 0.0144 |
| $a_6$ | 0.0042 | 0.0075 | 0.0039 | 0.0039 | 0.0110 | 0.0065 | 0.0081 | 0.0065 | 0.0082 | 0.0071 | 0.0137 | 0.0137 | 0.0156 |
| $a_7$ | 0.0078 | 0.0117 | 0.0114 | 0.0049 | 0.0071 | 0.0065 | 0.0082 | 0.0048 | 0.0203 | 0.0071 | 0.0086 | 0.0090 | 0.0139 |

**Step 3:** According to Table 20, Equations (10) and (11), PIS($A^+$) and NIS($A^-$) follow as:

$$A^{+} = \left(y_1^{+}, y_2^{+}, \cdots y_{13}^{+}\right) = \left(0.0087\ ,0.0176\ ,0.0131\ ,0.006\ ,0.0193\ ,0.011\ ,0.0171\ ,\ 0.0067\ ,0.0253\ ,0.016\ ,0.0189\ ,0.0168\ ,0.0167\right) \tag{16}$$

$$A^{-} = \left(y_1^{-}, y_2^{-}, \cdots y_{13}^{-}\right) = \left(0.0042\ ,0.0075\ ,0.0039\ ,0.0019\ ,0.0071\ ,0.0065\ ,0.0081\ ,\ 0.0032\ ,0.0082\ ,0.0071\ ,0.0086\ ,0.0088\ ,0.0076\right) \tag{17}$$

**Step 4:** According to Table 20, Equations (12), (13), (16) and (17), the distance between each alternative follows as:

$$D_i^{+} = \left(0.0208, 0.0523, 0.0702, 0.0238, 0.0304, 0.0832, 0.0719\right) \tag{18}$$

$$D_i^{-} = \left(0.0896, 0.0582, 0.0403, 0.0866, 0.08, 0.0273, 0.0386\right) \tag{19}$$

**Step 5:** According to Equations (14), (18) and (19), the closeness coefficient of each alternative follows as;

$$C_i^{+} = \left(0.8113, 0.5266, 0.3647, 0.7843, 0.7245, 0.2472, 0.3495\right) \tag{20}$$

According to the closeness coefficient of 7-alternatives, the order of these alternatives is $A_1>A_4>A_5>A_2>A_3>A_7>A_6$.

The sub-level stoping method is selected as its closeness coefficient has the highest value. In other words, the first alternative is closer to PIS and farther from NIS.

The result calculated from FAHP [7] using the above-mentioned model and conditions is as follows.

$$C_i = (0.757,\ 0.625,\ 0.547,\ 0.741,\ 0.712,\ 0.484,\ 0.536) \tag{21}$$

The order of these alternatives is $A_1 > A_4 > A_5 > A_2 > A_3 > A_7 > A_6$.

From the above calculating result, we can find that the proposed method in this study is in good agreement with the results obtained from FAHP.

## 5. Conclusions

MMS is one of the most important and the most essential of decisions of a mining project that have a significant influence on the all of the mine decision-making problems.

In this work, the best mining method for Apatite mine was selected using HFGDM-TOPSIS based on the viewpoints of the experts considering 13-criteria and 7-alternatives. After calculating the priority of the alternatives, the feasible mining methods for this mine were ranked. The results obtained showed that the sub-level stoping method with the priority of 0.8113 was the best for the studied mine.

The results indicated that by application of HFGDM-TOPSIS for the MMS problem, some difficulties related to the previous methods could be reduced. Moreover, the proposed approach could be applied simply in GDM with too many decision-makers and taken into account large amount of uncertain information. Hence, it is expected that this method will be applied to various problems of multi-criteria decision-making in mining engineering.

## Data Availability

The data used to support the findings of this work are available from the corresponding author upon request.

## Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

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# انتخاب روش استخراج مناسب با استفاده از روش HFGDM-TOPSIS: مطالعه موردی یک معدن آپاتیت

**میونگ چون پاک[۱]، اون چول هان،[۲*] و دونگ ایل کیم [۳]**

۱- دانشکده مهندسی معدن، دانشگاه صنعتی کیم چاک، پیونگ یانگ، جمهوری دموکراتیک خلق کره

۲- دانشکده علوم و مهندسی، دانشگاه صنعتی کیم چاک، پیونگ یانگ، جمهوری دموکراتیک خلق کره

۳- گروه ریاضیات کاربردی، دانشگاه صنعتی کیم چاک، پیونگ یانگ، جمهوری دموکراتیک خلق کره

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* نویسنده مسئول مکاتبات: *huch8272@star-co.net.kp*

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**چکیده:**

انتخاب روش معدن (MMS) اولین و اصلی‌ترین مشکل در طراحی معدن است و به پارامترهایی مانند ویژگی‌های ژئوتکنیکی و زمین شناسی و عوامل اقتصادی بستگی دارد. عوامل موثر بر MMS توسط برخی از کارشناسان معدن تعیین می‌شود و مناسب‌ترین روش استخراج با استفاده از تصمیم گیری گروه فازی مردد (HFGDM) و تکنیک برای عملکرد سفارش با شباهت به روش راه حل ایده آل (TOPSIS) انتخاب می‌شود. این عوامل عبارتند از نوع کانسار، شیب کانسار، ضخامت کانی، عمق زیر سطح، توزیع عیار، رتبه بندی جرم سنگ دیوار آویزان (RMR)، RMR دیواره سنگی، RMR سنگ درونگیر ماده‌ی معدنی، بازیابی، هزینه سرمایه‌گذاری، هزینه معدنکاری، بهره وری سالانه. ، و اثرات زیست محیطی. در ابتدا، ما روش تصمیم گیری گروهی (GDM) را برای تعیین وزن چند ویژگی بر اساس تابع امتیاز با وزن تصمیم گیرندگان پیشنهاد کردیم، که در آن محیط فازی n (HFGDM) بعدی به شکل مجموعه های فازی (HFS) است . سپس وزن این عوامل را با استفاده از روش HFGDM محاسبه شد. یک مطالعه موردی ساده نیز به منظور نشان دادن صلاحیت این روش ارائه شده است. در اینجا، هفت روش استخراج معدن آپاتیت را با هم مقایسه شده است و روش استخراج بهینه را با استفاده از روش TOPSIS انتخاب شد. در نهایت روش استخراج از طبقات زیرین به عنوان مناسب‌ترین روش برای این معدن انتخاب شده است.

**کلمات کلیدی:** انتخاب روش معدنکاری، تصمیم گیری گروه فازی مردد، تکنیک اجرای سفارش بر اساس شباهت به راه حل ایده آل، مجموعه های فازی مردد.

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