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Anisotropic Inverse Distance Weighting Method: An Innovative Technique for Resource Modeling of Vein-type Deposits

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Abstract

Geological modeling is an important step for the evaluation of natural resources. One option is to use a common geo-statistical modeling method such as Indicator Kriging (IK). However, there are specific problems associated with IK, the worthiest of attention is an order relation violation. Alternatively, some studies propose to use the Inverse Distance Weighting (IDW) method. Though again, there are certain limitations associated with the IDW geo-domain modeling application. In fact, the current IDW methodology does not cover the subject of anisotropic geo-domain modeling; thus it is only applicable for the isotropic cases. Therefore, this work proposes a previously unused geo-domain modeling–Anisotropic IDW, which underlies the concept of indicator variogram, allowing one to consider the spatial correlation of the domains. The experimental part in this work includes the comparison of anisotropic IDW, IK, and traditional IDW over the synthetic case study, which imitates a highly anisotropic geological behavior, and a more complicated real case study over a vein-type gold deposit from Kazakhstan. The case studies' results illustrate that the anisotropic IDW can model the geo-domains more accurately than IK and the traditional IDW.

1. Introduction

Three-dimensional geological modeling builds a representation of Earth's interior and exterior surfaces based on the surface and sub-surface geological and geo-physical observations [1]. Geological modeling is used for solving various problems, from managing natural resources to identifying hazards and others [2]. Concerning mining, the geological model plays a significant role, as it allows defining and interpreting the geological properties of a mineral deposit such as lithology, mineralization, alteration, and other properties [3], which later determine the economic value of a mineral deposit. The built geological model is then used to separately model the continuous variables (e.g. ore grades, minerals) inside each domain. Therefore, correct and trustworthy evaluation of the geological domains significantly impacts the final resource evaluation, and subsequently influences the downstream activities of a mining project.

There are two approaches for geological modeling. The most widespread method of traditional geological modeling is hand-contouring and wireframing; linking 2D sections to produce a 3D geological model. The process is called explicit modeling, in which a geo-modeler digitizes each section manually [4]. Explicit modeling is a laborious and time-consuming procedure. It requires profound knowledge and skills from the geo-modeler, as it incorporates the modeling of geological objects, which naturally have sophisticated geometry, vague boundaries, and many other complications [5]. Alternatively, there is an automatic approach–implicit modeling, which significantly alleviates the process of geomodelling by using mathematical functions for building the desired models [4]. This approach, for instance, offers the Inverse Distance Weighting (IDW) [6], Indicator Kriging (IK) [7], Radial Basis Function (RBF) [8], Signed Distance Function (SDF),

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Nearest Neighborhood, and other methods. The IK and IDW methods were selected for this study, due to its wide application in the mining industry and the latter as the method for further improvement. However, IK and IDW have disadvantages and limitations when applied for geological modeling. As for IK, firstly, the order relation is the main problem that this technique suffers from [9]. The sources of the order relation problems are negative indicator kriging weights and sometimes lack of data. In fact, IK estimates probabilities that sometimes are outside the interval of 0 and 1. The method may produce probabilities less than zero or more than one; thus a further order correction is required [10]. Secondly, the support effect is typical for IK because every estimated value is a weighted average of the nearby samples [11]; therefore, a support effect also may occur everywhere in an estimated model, which may lead to inaccurate geomodelling. Another difficulty in IK is that the method produces very smooth boundaries, which is frequently incompatible with the geological interpretation of the deposit.

As an alternative for IK, [12] introduced a methodology for geological modeling of categorical variables by IDW. However, the study is restricted to isotropic cases, and does not discover the problem of geological modeling in anisotropic spatial features of geological domains [13, 14].

Considering the disadvantages of IK and the gaps in the methodology for IDW, a new method, namely, anisotropic IDW, is proposed in this study. The proposed methodology in this study– anisotropic IDW is an expansion of the proposed method by [12] but integrates the search ellipsoid, which allows us to consider the spatial continuity and the anisotropy throughout the region.

This paper is outlined as what follows. Methodology introduces the methodology for geo-domain modeling by anisotropic IDW. Next section presents the results of anisotropic IDW testing over the synthetic and actual datasets and its comparison vs. traditional IDW and IK. Finally, the concluding notes about the conducted studies and further recommendations.

2. Methodology

2.1. Inverse Distance Weighting (IDW)

The conventional IDW method for modeling the continuous variables is based on calculating the distance and its corresponding inverse weights, where the influence of a sample point varies proportionally to the distance. The formula for the IDW method is presented as follows:

$$Z^*(\boldsymbol{u}) = \frac{\sum_{i=1}^{n} \frac{1}{d_i^p} \times Z(\boldsymbol{u}_i)}{\sum_{i=1}^{n} \frac{1}{d_i^p}} \tag{1}$$

where Z*(**u**) is an estimation output based on the values of *Z(u)* at the location in the search area; **u** is an estimation location; 1/d is a weight of the inverse distance (d) from the sample to the estimation point, and “p” is power value; n is the number of samples (e.g. borehole data) in this method, n includes all the data (unique neighborhood), and in some particular cases, n can be restricted to a moving neighborhood. [12] expanded the conventional IDW method to model the categorical variables based on the integration of indicator formalism [15]. In this proposed algorithm, the categorical variable should be converted to a matrix of K indicators:

$$I_k(\mathbf{u}) = \begin{cases} 1 \text{ if the event } k \text{ occurs at location } \mathbf{u} \\ 0 \text{ if not} \end{cases} \tag{2}$$

The method proceeds with an estimate of the distribution of uncertainty at the unsampled location. This produces the conditional probability distribution at the target location u including the estimated values for each indicator.

$$p_k^* = i^*(\boldsymbol{u}; k), k = 1, \dots, n_k \tag{3}$$

where i* is conventional IDW estimation, as explained in Eq. (1) for each indicator. Once the probabilities are obtained for all the categories, then the estimated category at the target location is inferred from the conditional distribution of probabilities based on the maximum probability of occurrence of those categories. In this method, n corresponds to all the sample points, where they are attended in the process of estimation p_k^*.

The proposed methodology in this study – anisotropic IDW is an expansion of the proposed method by [12] but integrates the search ellipsoid, which allows us to consider the spatial continuity and the anisotropy of the categories throughout the region. In our proposed approach, we restrict the n to a moving neighborhood or unique neighborhood, where its configuration can be derived from variogram analysis of the indicators. This gives more importance to the sample points along the directions of a larger range concerning the target location, similar to the assigning weights in kriging matrix.

The search ellipsoid plays a crucial role in modeling by anisotropic IDW. The search ellipsoid inference is based on the region's anisotropy investigation. There are several approaches to study the anisotropy such as variogram map, geological consideration, and calculation of the directional variograms with small tolerances. Variogram map [9] is based on calculating and visualizing the sample variogram in all directions, using the color map; thus the color map represents if there is an anisotropy in the region, and if the further sample variogram should be calculated according to the main anisotropy directions. Another approach is based on geological considerations by quantifying the geological structures' azimuth, dip, and plunge. Finally, calculating variogram in different directions is considered as a common practice to investigate the anisotropy in the region and find the maximum, minimum, and vertical anisotropies [16].

In order to clarify how the methodology works, the logic of the anisotropic IDW is presented as follows:

1) Supply the algorithm with required data, including IDW's power value (p), and K–nearest samples to consider in the neighborhood, and variogram model parameters. If K is equivalent to the total number of sample points, then a unique neighborhood is considered.
2) Set up the search ellipsoid.
3) Distinguish whether the samples are inside or outside the search ellipsoid.
4) Calculate the Euclidean distance from the estimation point to samples inside the search ellipsoid.
5) Select only a pre-defined number of the nearest neighbors (K parameter) for further estimation.
6) Calculate weights by IDW using Eq. (1).
7) Sum up the weights for each category and infer the probabilities using Eq. (2).
8) Estimate the category at the target location.

3. Results

This section presents the results of anisotropic IDW application over the synthetic and actual case studies.

3.1. Synthetic case study

The synthetic dataset was created by using plurigaussian simulation [17, 18]. For this purpose, a flag consisting of two domains is considered where one Gaussian variable can define the truncation threshold. To synthetically model the domains, a cubic variogram with geometric anisotropy is considered with maximum continuity along 45 degrees and minimum continuity along 135 degrees.

The reference map is one realization of a highly anisotropic geological model, where it shows maximum and minimum continuities along the azimuth of 45 and 135 degrees, respectively, in a 300 by 300 grid cell, mimicking a vein-type geological setting as presented in Figure 1 (a).

From the synthetic map, 300 samples (Figure 1 (b)) were randomly selected and served for training and validation. However, to compare the results, the synthetic samples are used to estimate the vein and background using traditional IDW [12], (which is an isotropic IDW) and IK. The synthetic map (Figure 1 (a)) is then used as the ground truth for comparison.

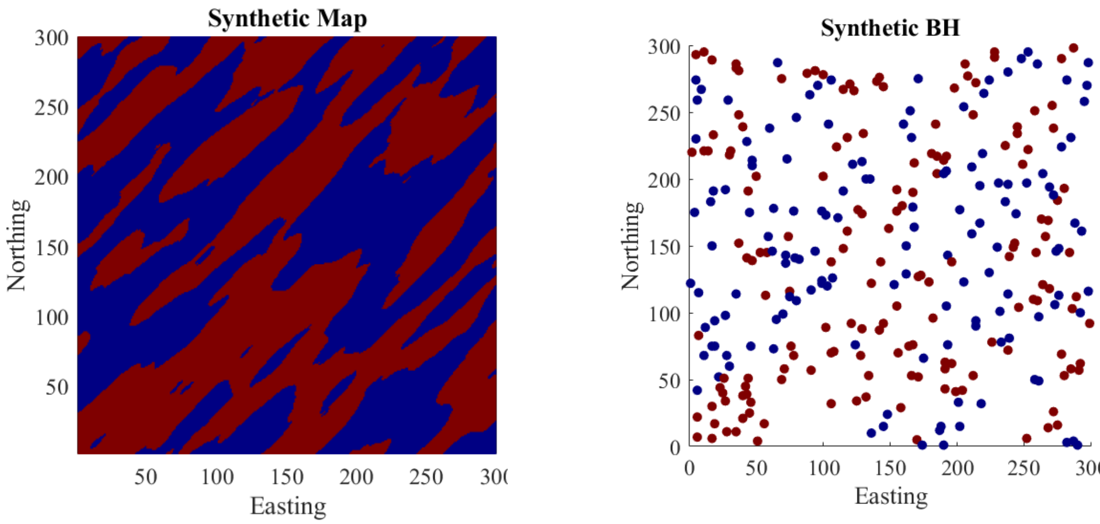
3.2. Geological modeling

The variogram is required for geological modeling by anisotropic IDW and IK; thus the indicator variogram was performed in different directions to investigate the anisotropy of the indicators in the region [19]. The directions are specified as North-East (45°) and North-West (315°) as it was expected (Figure 2). Since, we have only two indicators in this dataset, then one variogram analysis was performed over vein category:

$$\gamma(h) = 0.232sph(235m, 37m) + 0.031sph(37m, 21m) \quad (4)$$

Establishing the IK system also requires inference of variogram analysis. Therefore, the variogram analysis is used to implement IK considering simple kriging. The inferred anisotropic search ellipse to implement the IK and anisotropic IDW is considered as [235m, 37m, 1m] [45°, 0°, 0°]. However, traditional IDW uses a large circle with an infinite diameter. In order to make the analysis coherent throughout these three methods; 5, 10, 10, 25, 50 K number of samples are considered for establishing the neighborhood for each indicator. It is trivial that the selection of the nearest samples in anisotropic IDW and IK follows the identified search ellipse, while in traditional IDW, it corresponds to pure isotropy within a large circle covering the whole region.

The estimation results of geological modeling are presented in Figure 3 for the case of having five samples in the determined neighborhood. The rest of the maps for other cases are not presented to save the space.



(a) **(b)**

Figure 1. Synthetic map. (a)– reference map (blue zone – vein, red zone – background), (b) – randomly selected 300 samples.

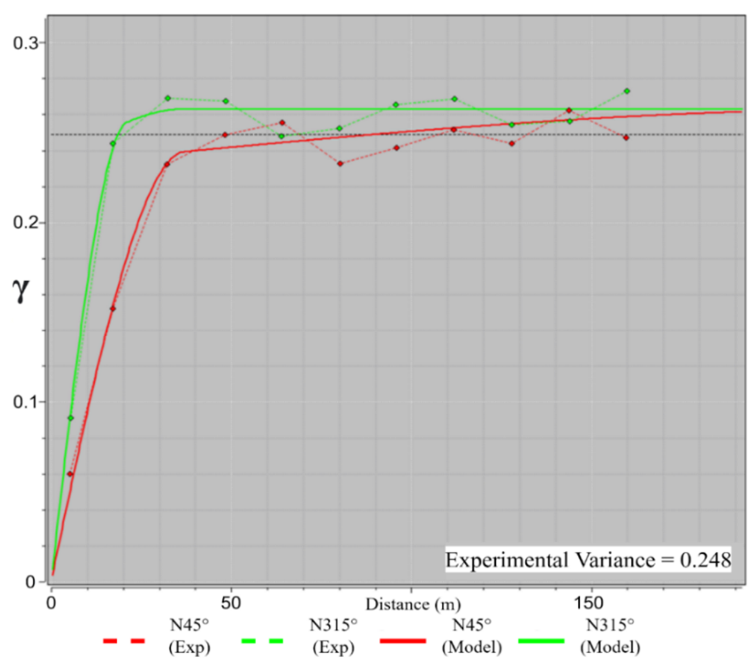


Figure 2. Indicator variogram for synthetic dataset (vein).

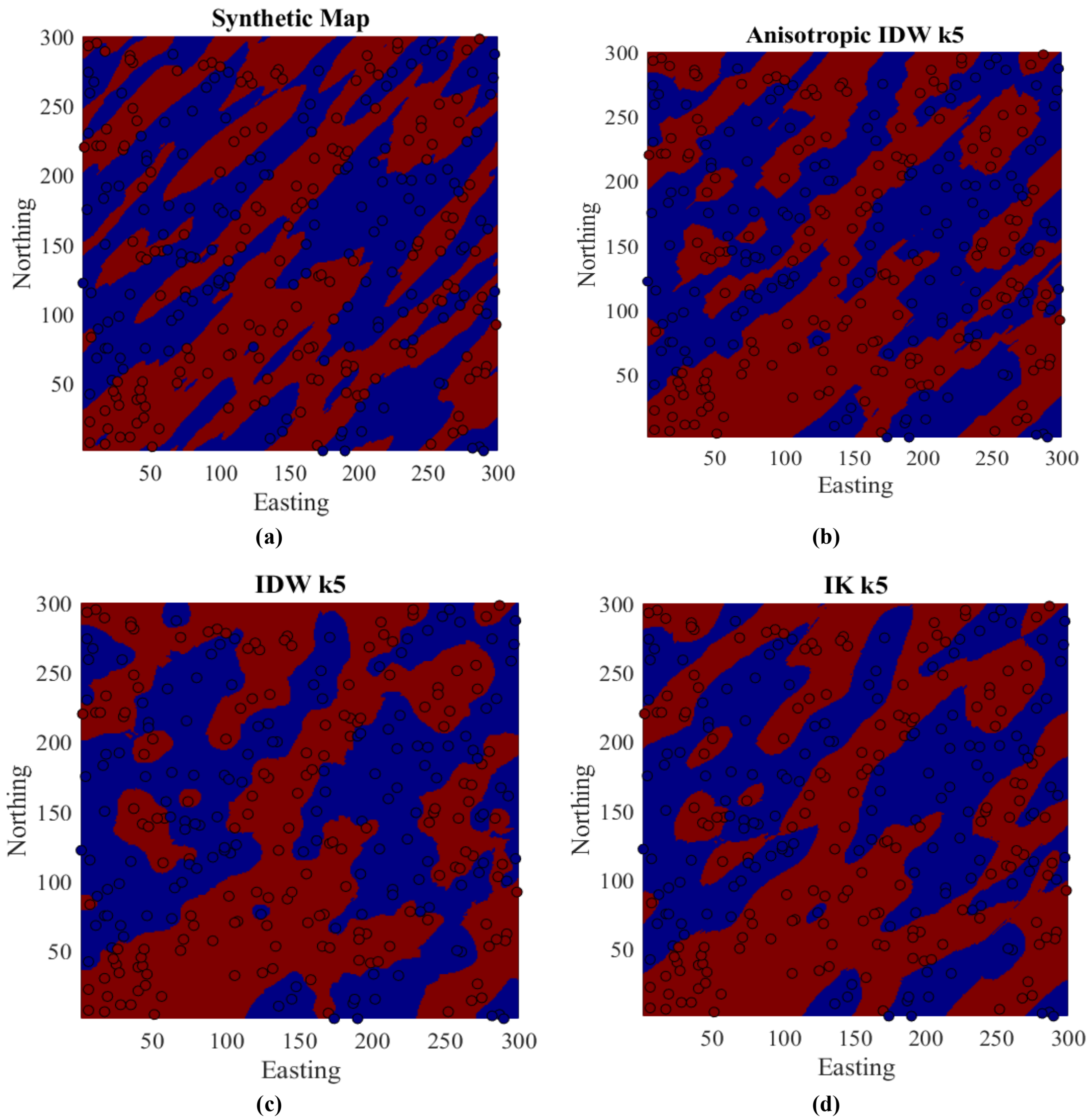


Figure 3. Geological modeling. (a)– reference map, (b)– anisotropic IDW, (c)– traditional IDW, and (d)– IK for k = 5 samples in the determined neighborhood.

From the visual assessment, the traditional IDW's performance is the worst because it failed to reproduce the anticipated anisotropy of the vein in the region. On the other hand, the anisotropic IDW and IK reproduced the underlying anisotropy. However, the IK result tends to smooth the boundaries between domains, while the anisotropic IDW represents a better shape of boundaries remarkably similar to the reference map. This visual inspection verifies that the anisotropic IDW produced the most trustworthy result because of reproducing the shape of boundaries, and perfectly reproducing the anisotropy of the categories in the region.

3.3. Evaluation of geological modeling

In order to evaluate the performance of geological modeling by anisotropic IDW, traditional IDW and IK, the cross-validation (CV), accuracy scores, and proportion reproduction of the original distribution were analyzed.

The CV and accuracy scores show how accurate the produced geo-domain model is. In this work, the Leave-One-Out-CV (LOOCV) [9] was used. The concept of LOOCV is to remove one observation at a time from the dataset and re-estimating the value from the remaining dataset; then the estimated and actual values are compared.

Once the predicted category is identified, then the accuracy score can measure how the estimation paradigm correctly predicted the actual observations at sample data locations:

$$AS = \frac{TP + TN}{TP + FP + FN + TN} \quad (5)$$

where TP is true positive, i.e. these are correctly predicted positive values; on the other hand, the actual ore domain is predicted as ore domain; TN is a true negative, i.e. these are correctly predicted negative values; on the other hand, the actual waste domain is predicted as the waste domain; FP is false positive, i.e. these are incorrectly predicted values, on the other hand, the actual waste domain is predicted as ore domain; FN is a false negative, i.e. the incorrectly predicted values; on the other hand, the actual ore domain predicted as a waste domain.

The CV and accuracy score results are presented in Table 1 for different K numbers of samples.

Table 1. Accuracy scores; ANS IDW: anisotropic IDW, IDW: traditional IDW, and IK: indicator kriging.

K	Accuracy score (%)		
	ANS IDW	IDW	IK
3	81.0	76.0	80.3
5	79.3	75.3	80.3
10	75.3	76.0	81.7
25	75.3	75.7	80.0
50	75.7	76.3	81.3
Average	77.3	75.9	80.7

All methods showed an accuracy score of more than 75% with little difference between them; thus the accuracy scores of all methods are somewhat acceptable, yet slightly better in IK.

In order to compare the calculated proportions and determine which one of the geological modeling methods reproduces the original distribution more accurately, the relative error for each reproduction of proportions was calculated:

$$Relative\ Error = \frac{measured - real}{real} * 100\% \quad (6)$$

where measured and real refer to the reproduced proportion and original declustered proportion, respectively. The results are presented in Figure 4.

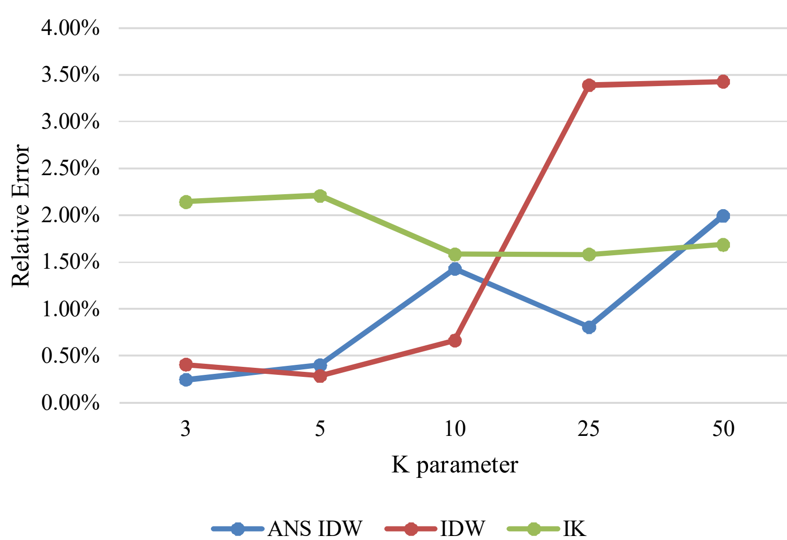


Figure 4. Relative error for reproduction of original declustered proportion for vein domain.

The relative error for the vein domain (Figure 4) shows that all geological modeling methods produce acceptable results; the difference between the methods is around 2 to 3 percent. However, relative error as depicted in this figure, shows that when the K number of samples in the neighborhood was small (less than 10), then IK produced the highest error, and not much difference can be seen between anisotropic IDW and traditional IDW. This is, nevertheless, different when the K number of samples is large (more than 10). In this case, traditional IDW yields the highest error for the reproduction of original declustered proportions.

All things considered, the anisotropic IDW showed an acceptable performance, being on the same level compared to traditional IDW and IK in terms of accuracy scores and reproduction of the original proportions, while showing much better visualization; thus the anisotropic IDW is successfully applied for modeling over the highly anisotropic synthetic dataset.

4. Real case study

Borehole dataset of an actual vein-type gold deposit was used for the real case study. The drilling campaign possess 32 boreholes, including information about geographical coordinates, gold grade (measured in ppm), and domain (vein and wall rock). The name and location of this gold deposit cannot be revealed for confidentiality reasons. The coordinates and gold grades were rescaled by a constant scale factor to keep the confidentiality. The location map of this mineral deposit is presented in Figure 5 (North view). In this deposit, the vein and wall rock are considered ore and gangue domains, respectively. The amount of Au in vein is much higher than in wall rock, the reason why identification of ore and gangue in this study corresponds to vein and wall rock. The form of mineralization in vein also is highly variable as a result of variations in host rocks and the conditions of mineral deposition.

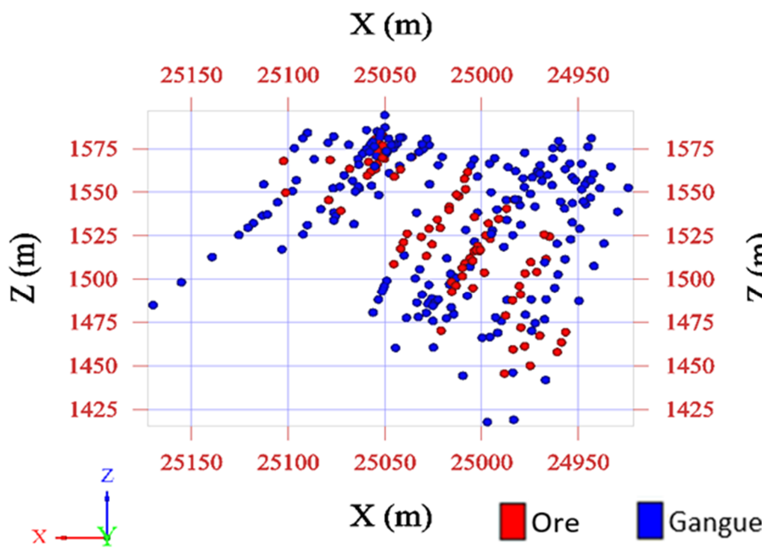


Figure 5. Location map of a vein-type gold deposit (North view).

Following the same process as in the synthetic case study, the idea is to build the geological domains by anisotropic IDW, traditional IDW, and IK, and then evaluate the produced geological models using the same evaluation criteria as in the synthetic case study, and then finally come up with resource estimation of gold in this deposit.

5. Geological modeling

To build the geological domains by using the anisotropic IDW and IK, as already discussed, variogram analysis needs to be implemented. Therefore, the indicator variogram was conducted in different directions to investigate the anisotropy in the region with small tolerances. The results showed that the maximum, medium and minimum continuities are along azimuth 0 degrees (North direction), azimuth 45 degrees (East direction), and vertical direction (dip = 90 degrees), respectively (Figure 6):

$$\gamma(h) = 0.179Sph(10000m, 50m, 3m) + 0.063Sph(500m, 350m, 63m) + 0.042Sph(30m, 11m, 9m) \quad (7)$$

Once the variogram is inferred, the next step is to produce the estimation results for geological domains. For implementing anisotropic IDW and IK, a search ellipse of [350m 50m 30m] and [0° 0° 0°] is considered, and a circle (pure isotropy) is considered for traditional IDW with infinity diameter. In order to establish this neighborhood; different K sample numbers of 5, 10, 25, 50, and 100 are considered for further comparison. The 3D block dimension is 5m by 5m by 5m that totally 199,800 blocks cover the entire region.

Figure 7 presents one section (No. 31) toward the north view of the geological models obtained from anisotropic IDW, traditional IDW, and IK. This section was selected because it clearly showed the difference in geological modeling, i.e. it is seen how differently the geomodelling methods produce the vein – ore domain. For both anisotropic IDW and traditional IDW, the power value in the formula (Eq. (1)) was set to two.

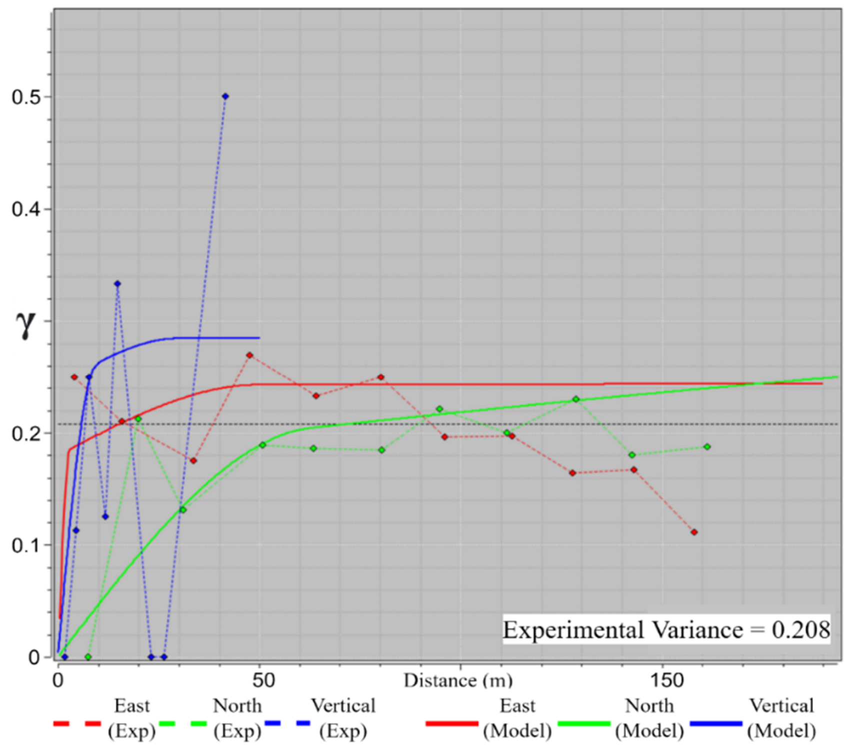


Figure 6. Indicator variogram. Dashed line– experimental variogram, solid line– variogram model.

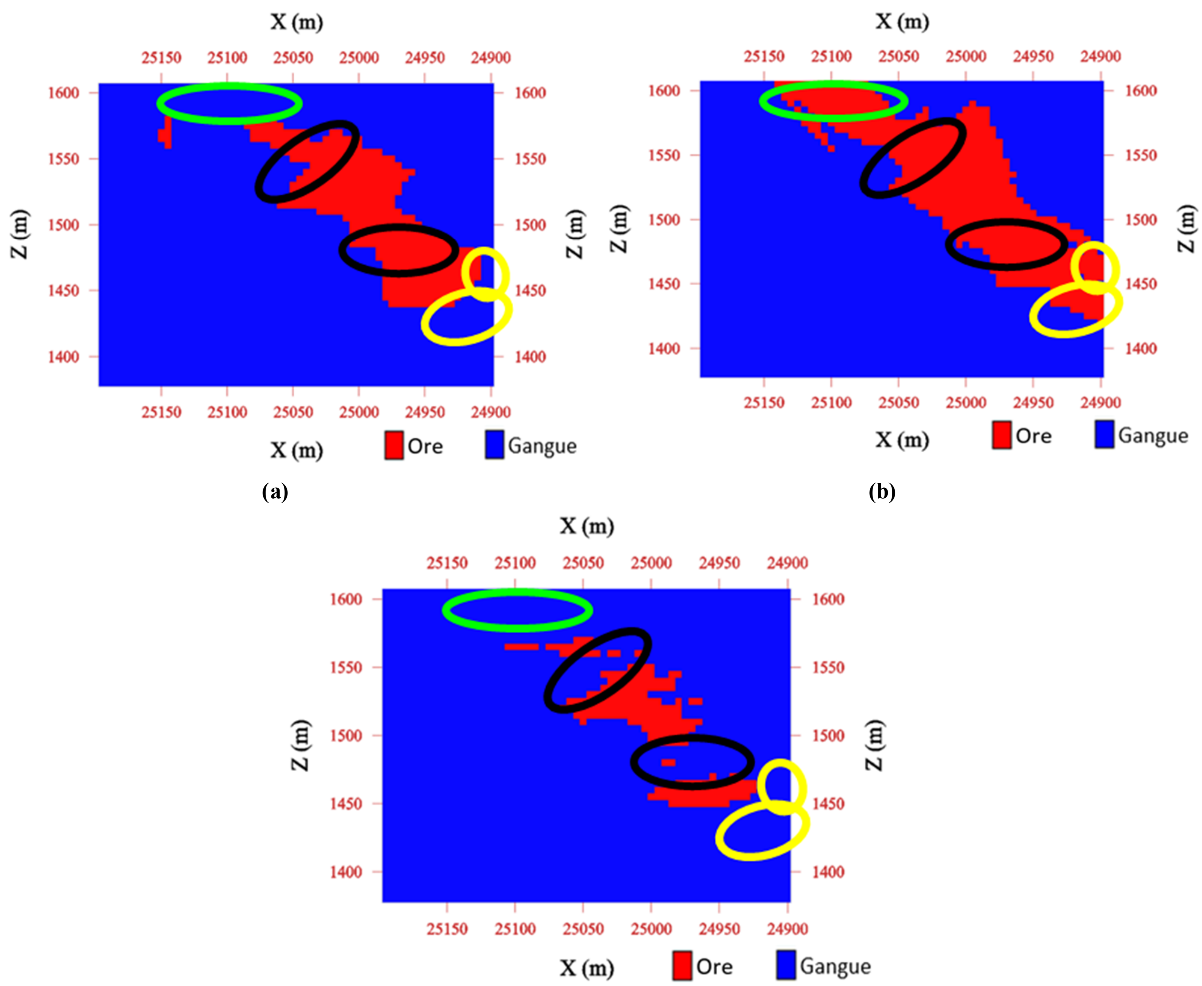
The visual assessment of the geological modeling is divided into three parts: top, middle, and bottom parts of the vein (Figure 7), highlighted with green, black, and yellow, respectively. These areas were selected because they showed significant differences, reproducing the ore domain.

According to geological clarifications from the mine site, the surface is covered by sedimentary rocks, which abruptly cut the top part of the vein. The anisotropic IDW and IK produced no footprint of the vein in the surface, highlighted green in Figures 7 (a) and (c). In contrast, the traditional IDW produced an extra ore domain, i.e. a non-existing part of the vein close to the surface, Figure 7 (b), highlighted green, which is incompatible with the geological interpretations.

Anisotropic IDW and traditional IDW reproduced almost the same shape for the middle part of the vein, highlighted black in Figures 7 (a) and (b). At the same time, IK showed discontinuities in the vein, presented in Figure 7 (c), which does not correspond to the geological information of this structure.

Regarding the bottom part of the vein, the anisotropic IDW and IK show the same vein shape, Figures 7 (a) and (c) indicate the sharp end of the vein. In contrast, the traditional IDW indicates the continuity of vein (Figure 7 (b)). From the geologic perspective, the conventional IDW shows a satisfying shape of the structure, which corresponds to the geological behavior of the vein in this deposit, but the extension as shown in yellow areas cannot be confirmed due to a lack of samples in this region. Therefore, it is recommended to expand the drilling program to identify whether the areas highlighted yellow in Figure 7 belong to the vein or background.

Summarizing the visual assessment, the anisotropic IDW showed an agreeable reproduction of the ore domain, except for the bottom part of the vein (Figure 7) highlighted yellow, which requires additional drilling for a more comprehensive interpretation of geological modeling.



(c)

Figure 7. Geo-domain models. View – North (Section No. 31); (a) anisotropic IDW, (b) traditional IDW, and (c) IK, for k = 10 samples in the determined neighborhood.

6. Evaluation of geological modeling

This section presents the evaluation results of geological modeling over the actual dataset. The same measures that were used for the evaluation are the same as in the synthetic case study.

The accuracy scores are presented in Table 2. The average accuracy score of traditional IDW is slightly below 80%, 1 percent above the anisotropic IDW. The IK's shows the lowest accuracy scores in an average of 72 percent.

Table 2. Accuracy scores.

K	Accuracy score (%)		
	ANS IDW	IDW	IK
5	78.5	78.9	65.6
10	79.3	78.5	72.4
25	78.2	80.7	73.5
50	77.8	79.6	74.6
100	78.2	78.2	74.6
Average	78.4	79.2	72.1

In order to compare the calculated proportions and determine which of the geological modeling methods reproduces the original distribution more accurately, the relative proportion's percentage error was calculated. The declustered proportion was calculated from borehole dataset and served as ground truth for percentage error calculations.

From Figure 8, it is clear that the anisotropic IDW has the lowest error irrespective of K number of observations in the determined neighborhood, with a lowest value of 11.5 percent for K = 5. As for the other cases, only for the K = 10 the traditional IDW catch up with the anisotropic IDW; as for the rest cases, both the conventional IDW and IK were far behind the anisotropic IDW.

Summarizing all geological modeling evaluation measures, the anisotropic IDW showed acceptable accuracy scores, which are on the same level as traditional IDW, while outperforming IK. As for the reproduction of the original distribution, the anisotropic IDW showed the best performance, as it had the lowest relative error.

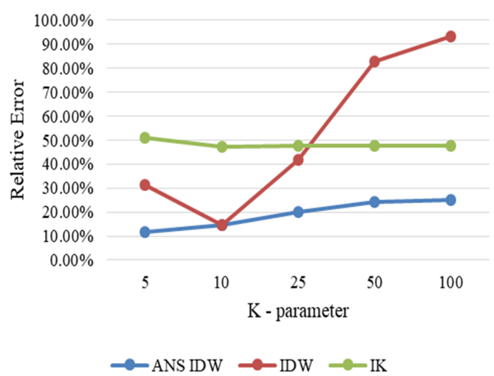


Figure 8. Relative error against the K nearest number of samples in the neighborhood for the vein domain.

Next, exploratory analysis is performed over the gold grades. In the first step, the outliers are detected and handled. Outliers significantly impact the statistics like mean and standard variation [20]; therefore, to distinguish the outliers, the boxplot analysis was performed; then the outliers were capped; according to the boxplot in Figure 10, the upper limit for the gangue domain is 6 ppm, while for the ore domain is 13 ppm.

Table 3 presents the statistics before and after the capping for the ore and waste domains.

Finally, declustering is a requirement to alleviate the impact of the clustered collected samples, as the data is rarely collected uniformly. However, the difference between the clustered and declustered statistical parameters was minimal in this gold deposit; therefore, the declustering technique was skipped, and for the remaining of the study, the original dataset was used.

7. Modeling of gold grade

Once the geological domains are built, the next step is to estimate the gold grades. A contact analysis was performed to examine whether the gold grade can be estimated separately in each geo-domain (ore and gangue). The aim is to determine the contact behavior between the adjacent domains, i.e. whether the difference in grades is significant – hard contact, or minor – soft contact [20]. The results of contact analysis presented in Figure 9 show a hard contact, i.e. the grade difference between the ore and gangue is significant; therefore, the resource estimation should be performed separately for each domain [21].

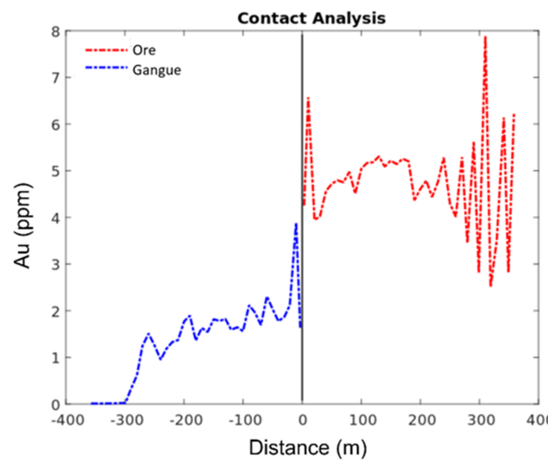


Figure 9. Contact analysis.

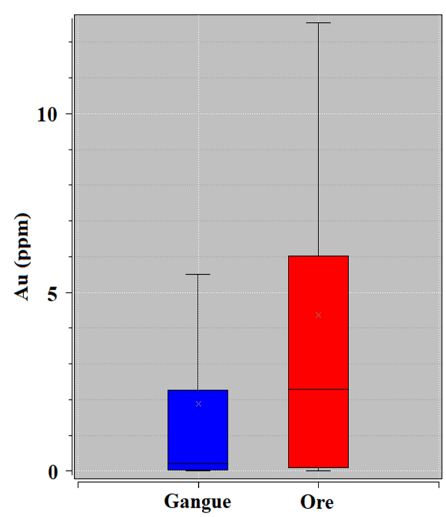


Figure 10. Boxplot analysis for waste and ore domains.

8. Recoverable resources

A resource evaluation (i.e. quantification of recoverable resources) for the gold grade in an entire deposit was implemented by estimating the gold in each domain separately. In order to compare the final results of gold resource modeling, conventional IDW was used to estimate the gold grade inside each built geo-domain separately, for which they are obtained from anisotropic IDW, traditional IDW, and IK.

Section 31 from the north view was selected (Figure 11, (a)– anisotropic IDW, (b)– traditional IDW, and (c)– IK) to demonstrate the difference between the geological modeling methods. The primary areas of difference are highlighted with colored shapes (Figure 11). The vein's top, middle, and bottom parts are highlighted green, black, and yellow, respectively. These differences in resource estimation presented in Figure 11 are inherited from the geological models, and a detailed explanation is given in geological modeling section.

Table 3. Statistical parameters calculated before and after capping.

№ of samples	Unit	Min.	Max.	Mean	Std Dev.	Q25	Q50	Q75
Gangue								
197	ppm	0.00	47.17	1.88	4.66	0.03	0.20	2.26
Gangue capped								
197	ppm	0.00	6.00	1.31	2.04	0.03	0.20	2.26
Ore								
82	ppm	0.01	50.85	4.37	7.34	0.09	2.27	6.02
Ore capped								
82	ppm	0.01	13.00	3.52	3.84	0.09	2.27	6.02

The results of recoverable resources for a vein are presented in Figure 12 and Tables 4, 5, and 6. A bulk density of 2.7 t/m^3 was used during the calculations. Figure 12 shows the resource estimation graphs, which help to analyze the potential amount of mineable materials; (a) – the grade-tonnage curve, which presents two main characteristics, firstly, the cumulative tonnage vs. different cutoff grades (COG) and the mean grade vs. different COG; as for the (b), the same concept, except it presents the predicted metal quantity vs. different COG.

The formulas of recovery resources are as follow:

The real tonnage for each block can be calculated as:

$$Real\ tonnage = volume(m^3) \times density(t/_{m^3}) \quad (8)$$

$$Metal\ quantity = real\ tonnage(t) \times\ grade \quad (9)$$

And the mean grade above cut-off is just a simple arithmetic averaging of the blocks that have the estimated grades above the particular cut-off.

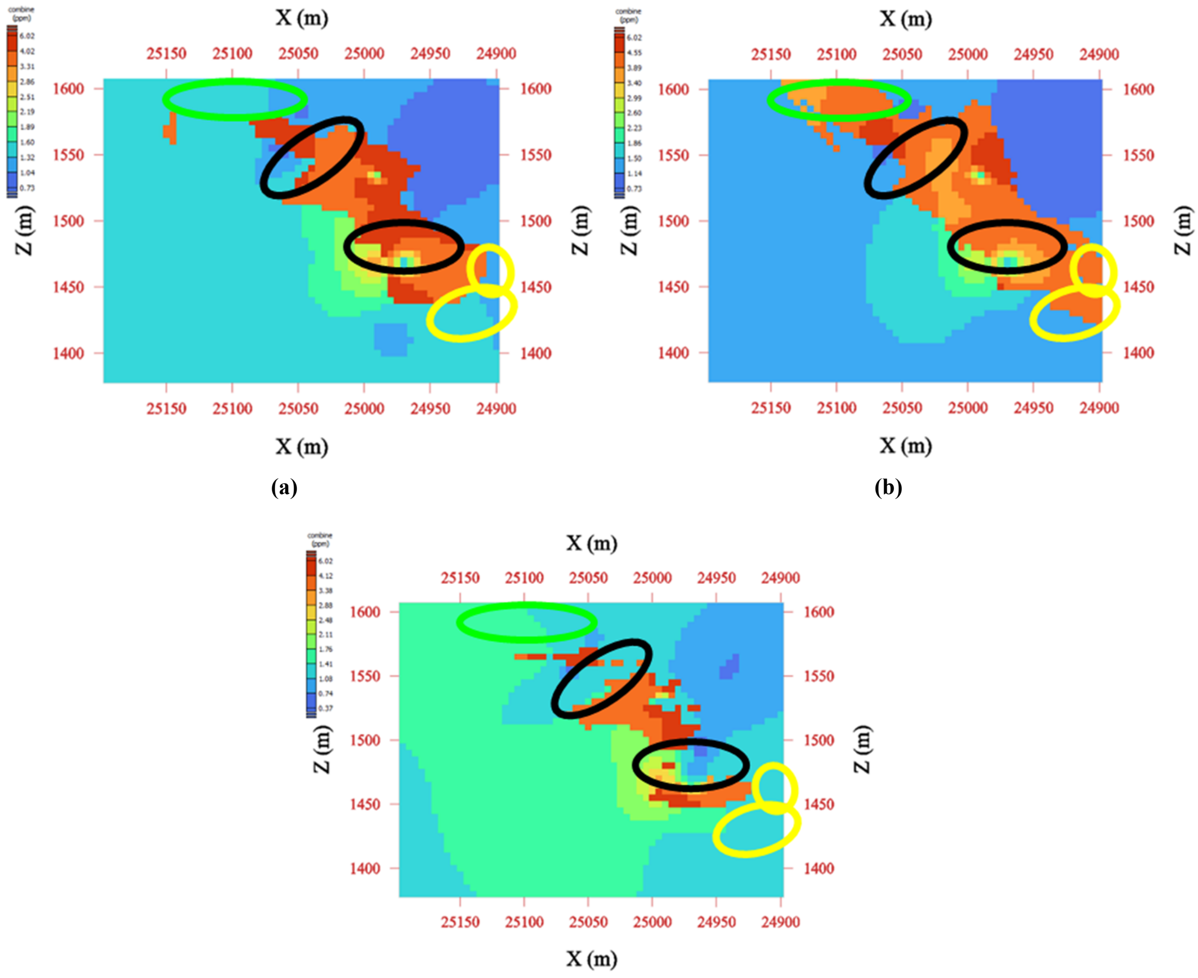


Figure 11. Resource estimation. North view, section – No. 31. (a) – for the Anisotropic IDW model, (b) for the Traditional IDW model, and (c) – for the IK model.

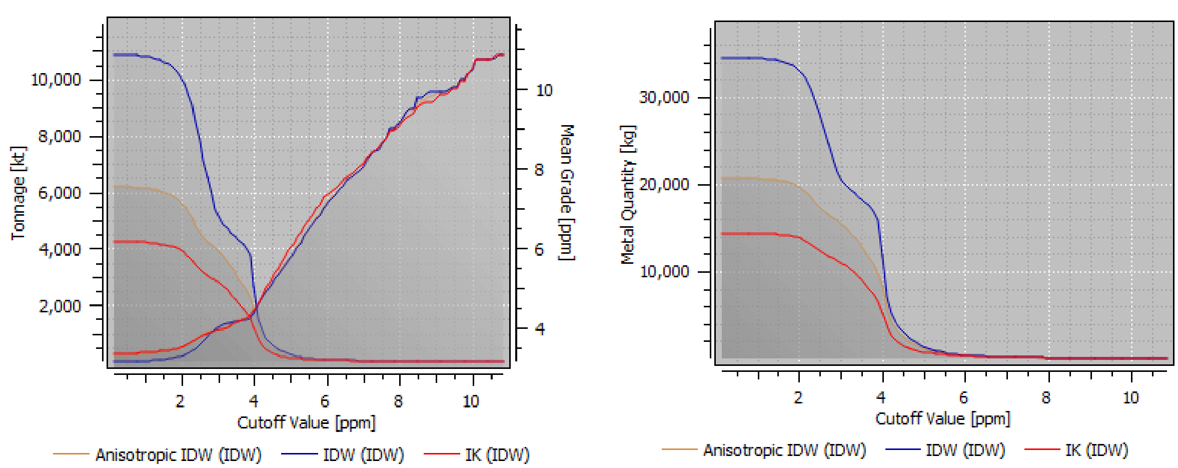


Figure 12. Recoverable resource graphs (only for vein domain): (a) tonnage vs. cut-off and mean grade vs. cut-off curves, and (b) metal quantity vs. cutoff.

Table 4. Grade tonnage table for ANS-IDW model

Cut-off (ppm)	Tonnage (kt)	Metal (kg)	Mean (ppm)
0.15	6173.55	20627.9	3.34
1.15	6103.08	20566.4	3.37
2.15	5390.28	19270.9	3.58
3.15	3713.85	14953.5	4.03
4.15	1052.46	4929.74	4.68
5.15	158.76	947.924	5.97
6.15	43.74	317.516	7.26

Table 5. Grade tonnage table for IDW model.

Cut-off (ppm)	Tonnage (kt)	Metal (kg)	Mean (ppm)
0.15	10860.4	34448.2	3.17
1.15	10776	34374.1	3.19
2.15	9625.84	32259.5	3.35
3.15	4837.72	19787.9	4.09
4.15	1313.89	6132.82	4.67
5.15	188.325	1121.19	5.95
6.15	50.625	366.383	7.24

Table 6. Grade tonnage table for IK model.

Cut-off (ppm)	Tonnage (kt)	Metal (kg)	Mean (ppm)
0.15	4261.61	14345.1	3.37
1.15	4197.15	14289.9	3.4
2.15	3789.79	13550.5	3.58
3.15	2647.35	10584.1	4
4.15	702.675	3286.16	4.68
5.15	103.95	642.941	6.19
6.15	37.4625	278.006	7.42

As it can be seen in Figure 12 and Tables 4, 5, and 6, the traditional IDW shows the unrealistic tonnage and metal quantity compared to others; it happens due to the areas highlighted with green and yellow shapes, where the excessive vein domain is produced (Figure 11 (b)). However, it was proved in the geological modeling section that the green shapes belong to the gangue domain, while the yellow bodies require further, more detailed evaluation; therefore, the traditional IDW tends to overestimate the gold in this deposit. IK shows the correct visualization of resources in the top part of the vein, Figure 11 (c), highlighted green, corresponds to the reality (details in the geological modeling section). The bottom part of the vein requires further evaluation. In the middle part of the vein, Figure 11 (c), the IK produced discontinuities in the vein domain, which contradicts the geological interpretation of this vein, i.e. it has significant breaks in the vein, highlighted in black shapes. Since IK did not produce the continuous form of the vein in the middle part of the vein correctly; then the resource estimation by IK tends to underestimate the gold in this deposit, presented in Figures 12 (a) and (b), where the IK has the lowest predicted tonnage and metal quantity. The resource estimation results of the anisotropic IDW model show the most reliable and realistic ones due to the correct vein reproduction in the top and middle parts of the vein. Though, it requires a more thorough evaluation of the bottom part, highlighted yellow in Figure 11 (a), where there is potential for additional resources. The predicted tonnage and metal quantity for anisotropic IDW in Figures 12 (a) and (b) lies in between the traditional IDW and IK, where the former tends to overestimate while the latter tends to underestimate, thus, producing the optimum and most reliable resource evaluation for gold in this deposit.

8. Conclusions

Up to now, no studies have been conducted for geomodelling discrete variables by anisotropic IDW. The carried-out research filled the gap in applying the anisotropic IDW for geological modeling. The case study results showed that (1) the anisotropic IDW could be successfully applied for geological modeling in anisotropic conditions; (2) the proposed methodology outperforms the traditional IDW and IK, while not suffering from the order relation problem; (3) the proposed methodology might be used as a practical guide for geomodelling of veins by anisotropic IDW.

Summarizing the results of the synthetic case study, the anisotropic IDW showed acceptable results compared to the traditional IDW and IK; then, a more complex case study over the vein-type gold deposit was conducted. Summarizing the results of the real case study: the anisotropic IDW outperformed the traditional IDW and IK in terms of visual assessment, evaluation measures, and resource estimation results. Furthermore, the anisotropic IDW in real conditions performed even better than in the synthetic case study. Finally, the case studies confirmed the successful application of anisotropic IDW for geo-domain modeling in anisotropic conditions. The current workflow works with two categories, but it is versatile, and easily can be updated to work with any number of categories. The proposed method is also applicable for continuous data, where there is an important anisotropy in the region.

Concerning the recommendations for further research: (1) during the case studies, binary indicators were used; therefore, it is recommended to test the methodology over more than two categories for further research studies and evaluate its performance. (2) as the fundamental concept of anisotropic IDW is indicator variogram analysis, it is recommended to test the methodology in a non-stationarity condition, i.e., where the indicator variogram has linear behavior, to evaluate whether the proposed methodology is capable of producing acceptable results in such conditions.

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Appendix

Workflow for implementing the proposed algorithm in this study, anisotropic IDW for categorical variable:

1- Convert the categorical data into indicators based on the number of category (the categorical variable can represent the estimation domains).

2- Variogram analysis over the indicators and infer the linear model of regionalization for each indicator separately.

3- Using the proposed method, implement the anisotropic IDW to estimate each indicator separately at target blocks. In this step, a moving neighborhood can be taken into account based on the variogram inferred from each indicator

4- Infer the candidate category at target block by referring to the maximum probability obtained from step 3.

5- Obtain the final model of geological domains and implement the final grade modeling at each domain separately

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روش وزن‌دهی با فاصله معکوس ناهمسانگرد: یک تکنیک ابتکاری برای مدل‌سازی منابع رسوبات رگه‌ای

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چکیده:

مدل سازی زمین شناسی گام مهمی برای ارزیابی منابع طبیعی است. یکی از گزینه‌ها استفاده از یک روش مدل سازی زمین آماری رایج مانند کریجینگ شاخص (IK) است. با این حال، مشکلات خاصی در ارتباط با IK وجود دارد. برخی از مطالعات استفاده از روش وزن دهی معکوس فاصله (IDW) پیشنهاد می‌کنند. اگرچه باز هم، محدودیت‌های خاصی در رابطه با برنامه مدل‌سازی دامنه جغرافیایی IDW وجود دارد. در واقع، روش IDW فعلی موضوع مدل‌سازی ژئو دامنه ناهمسانگرد را پوشش نمی‌دهد. بنابراین فقط برای موارد همسانگرد قابل استفاده است. بنابراین، این پژوهش یک مدل‌سازی Geo-domain-IDW ناهمسانگرد را پیشنهاد می کند که پیش‌تر استفاده نشده، این مدل زیربنای مفهوم واریوگرام شاخص است و به شخص اجازه می‌دهد تا همبستگی فضایی دامنه‌ها را در نظر بگیرد. بخش تجربی در این کار شامل مقایسه ناهمسانگرد IDW، IK، و IDW سنتی نسبت به مطالعه موردی مصنوعی، که یک رفتار زمین‌شناسی بسیار ناهمسانگرد را تقلید می‌کند، و یک مطالعه موردی واقعی پیچیده‌تر بر روی یک کانسار طلا از نوع رگه‌ای از قزاقستان است را انجام داده است. نتایج مطالعات موردی نشان می‌دهند که IDW ناهمسانگرد می‌تواند ژئو دامنه‌ها را با دقت بیشتری نسبت به IK و IDW سنتی مدل‌سازی کند.

کلمات کلیدی: وزن‌دهی معکوس فاصله ناهمسانگرد، وزن‌دهی معکوس فاصله، متغیر طبقه‌بندی، مدل‌سازی زمین‌شناسی، مدل‌سازی ضمنی زمین.