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Statistical Analysis and Optimization of Drilling Process using Response Surface Methodology and Experimental Data

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Abstract

It is well-established that the response surface methodology (RSM) is commonly employed to establish the differences between the predicted values and those observed experimentally. This study mainly goals on the impact of four drilling factors including weight on the bit (WOB), the rotating rapidity of the bit, RPM, cutting angle β, and rock resistance on the penetration rate of the drilling tool. In this examination, three kinds of limestone rocks were considered. The planned assessments were carried out at three stages of the considered four input variables. The statistical analysis was realized using both RSM approach and analysis of variance (ANOVA). This analysis allowed us to develop the appropriate penetration model with a higher determination coefficient of 96.19%, which demonstrates the high correlation between the predicted and experimental data, and consequently, it can be concluded that the obtained model is highly suitable for the prediction of the penetration rate. Also from variance analysis, the results obtained show that rotational speed, RPM, and weight on the bit (WOB) parameters, as well as the nature of the rock, which is determined by the rock compressive resistance, having a significant effect on the penetration rate; however, the rake angle has little effect. Finally, the optimal parameters were determined to find the best possible penetration rate of the drilling tool.

1. Introduction

Ever since the world recognized the vital demand for hydrocarbons in terms of utilization and substantial financial investment resources, oil exploration and exploitation have emerged as pivotal aspects in the advancement of technology and profit growth. Furthermore, it is widely acknowledged that the oil and gas sector is placing greater emphasis on optimizing the drilling process design to lower operational expenses while enhancing operational efficiency [1]. Rotary blasting hole drills are widely used on a global scale in surface mineral extraction for the purpose of waste removal. The accurate estimation of penetration rate for rotary drill rigs has significant significance within the context of rock drilling, particularly in the fields of geology and petroleum technology [2, 3]. The accurate estimation of the penetration rate has significant importance in the process of mine construction. The assessment of total drilling expenses may be accomplished by the use of predictive formulas [4]. Furthermore, the use of a prediction formula may be employed to determine the optimal drilling rig type that is most suitable for certain situations. Rotary tricone bits including tungsten carbide inserts are widely favored as the primary drilling tools for deep holes with substantial diameters in extensive surface mining processes [5]. The exploration rate has seen an upward trend over time as a result of the use of more powerful drills and improved management of operational

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factors. This has subsequently led to a rise in mining output and a decrease in drilling expenses.

In the contemporary day, the practice of deep drilling has significant importance, and is widely advocated within the oil and gas sector. Nevertheless, this technique is not without its drawbacks, mostly stemming from the considerable depth involved and the intricate process of tool replacement, compounded by the abnormalities encountered within the layers of the formation. This scenario mostly results in incongruous outcomes, therefore giving rise to various mechanical abnormalities that subsequently contribute to a reduction in the tool's depth of penetration. In this particular environment, there is a shared interest among industry experts and academics regarding the design and development of novel drilling techniques with the aim of enhancing drilling operation performances [6, 7]. Improving the efficiency of the drilling operation and achieving higher performance levels require the optimization of several drilling parameters. These parameters include the weight of the drill bit, the rotational speed of the drilling apparatus, the rock's resistance, and the properties of the drilling mud. This optimization primarily revolves around achieving the highest drilling rate while minimizing costs and the mass of the rock drillable indicator [8, 9]. Much awareness has been compensated to improve the quality of the drilling procedure. Garnier and Van Lingen [10] were interested in certain phenomena that could affect drilling processes. Response surface methodology (RSM) is one of the best ways to make it possible to understand and model such phenomena. RSM aims to explore the correlation systematically and efficiently between the input factors and response variables to optimize procedures, products or systems while minimizing the need for extensive experimentation and resources [11].

RSM is considered a crucial part of the experimental design to develop new processes and improve their performance. This methodology was also developed to improve products and systems to enhance the load component and reduce process response instability [12]. In general, RSM comprises a collection of statistical and mathematical techniques that prove highly effective for analyzing and addressing issues in which multiple factors impact the response variable. Its goal is to enhance this response [13, 14]. The goal of RSM is to find the optimal empirical design with the fewest possible design repetitions. Its use in empirical design dates back to the late 1990s [15]. This technique has been used by numerous researchers such as Panagiotis et Angelos [16]. To explore how the process parameters of fiber laser percussion drilling affect the geometric characteristics of 1.0 mm thick Inconel 718, experimentations were conducted using RSM by Moradi and Mohazabpak [17]. The primary aim of this study is to formulate mathematical simulations for the expectation of propulsion force and cutting torque in the context of drilling operations. Salehnezhad *et al*. [18] utilized RSM to design and improve the properties of drilling mud. By utilizing the box-Behnken design within the framework of RSM, Zhang [19] conducted multiple laser drilling experiments. These experiments aimed to determine the certain energy of rock by varying three key empirical factors: laser power, irradiative time, and spot diameter. Alakbari *et al*. [20] introduced novel statistical empirical correlations for prediction through the application of RSM. RSM was employed to establish mathematical relationships between factors and responses, as well as to elucidate the interactions among variables. Surekha *et al*. [21] have tried to find the effect of aluminum powder on the electrical discharge machining (EDM) of EN-19 alloy steel. Through the use of surface response modeling, a connection has been made between the responses, on the one hand, and the procedure's operational factors, on the other.

The selection of a PDC, polycrystalline diamond, drilling tool was made in the present research work due to its many benefits and extensive range of applications. The drilling exams were done in accordance with the empirical design. Hence, our research work was primarily centered on investigations undertaken within this particular environment. In the research work conducted by Capik and Batmunkh [22], experiments were conducted to examine the correlation between the rate of bitwear and various physical and mechanical features of rocks. The study also investigated the impact of drill ability, abrasive qualities, and fragility of rocks on the wear rate of the bit. The use of statistical analysis was employed to create equations that enable the estimation of the bit wear rate using rock characteristics as the basis for estimation. In their study, Modi *et al*. [23] highlighted that optimization represents one of the most effective techniques employed in the manufacturing industry to enhance product quality while reducing costs. Through the use of Taguchi's technique, this study demonstrates an efficient way for the optimization of drilling parameters based on a single answer. In addition, the study investigates how the material removal rate (MRR) is affected by the input process parameters, notably spindle speed and tool diameter. To build a variety of structures in sectors such as aerospace and automotive, that require precise machining holes to meet with strict geometric tolerances, it is usual practice to drill a large number of holes. One group of researchers who have an interest in pursuing this line of inquiry is Aamir *et al*. [24]. Their work is centered on enhancing the performance of the drilling specifications and drilling procedures, and they make use of the empirical plan approaches, most notably the Taguchi approach, to do this. Venkateshwarlu *et al*. [25] used statistical methods to determine the importance and influence of machining factors. These methods included orthogonal arrays and ANOVA. In this study, drilling operations were conducted on titanium alloy under various machining conditions, encompassing wet and dry conditions with the utilization of a cryogenically treated drill.

The literature review has shown the significance of using statistical analysis approaches in investigating the behavior of drilling operations. Additionally, the study demonstrated the efficacy of the experimental design approach, namely the surface response technique, in effectively addressing cost minimization and performance improvement issues with a high level of accuracy. Nevertheless, it is crucial to underscore that previous studies have not thoroughly examined all drilling parameters, specifically regarding the fluctuation of the cutting angle and its effects on the efficacy of other parameters, as well as its influence on the overall drilling process's quality. This highlights the significance of the current work and its ability to provide useful insights into comprehending the behavior of drilling parameters. Hence, the present research work was conducted with the aim of addressing this disparity, minimizing operational expenses, and enhancing overall efficiency.

The present examination proposes to analyze the influences of different factors such as the weight on the bit (WOB), the rapidity of rotation, RPM, the angle cutting β, and the rock resistance (Cs) on the development of penetration rate, ROP, to reduce operational costs and increase the efficiency of the drilling operation. The optimization of drilling parameters was based on empirical data and processed and analyzed statistically to arrive at the method to improve (rationalize) the design of drilling procedures. The original goal of this study is to employ the response surface methodology of experimental design, RSM, to process and analyze the experimental data. This includes developing a model that establishes relations between drilling parameters through empirically derived formulas and determining the optimal penetration rate depending on the optimal amounts of the examined factors.

2. Experimental Procedures

An original experimental test rig was developed to simulate the real operating conditions of the drilling development. The specially designed vertical drilling rig was built on the one hand, to simulate the process of rotary drilling and, on the other hand, to measure the rate of penetration (ROP) parameter. The schematic diagram of the developed device is shown in Figure 1, and it is made up of the following parts:

1. An electric motor is a source of rotary motion that reaches 1490 rpm and a power of 1.5 KW.
2. A variable speed drive to give the desired speed factor. Three levels of speeds have been estimated (118 rpm, 135 rpm, and 152 rpm).
3. A hydraulic cylinder is the source of the force exerted on the drilling tool against the rock; in this experiment, the chosen compression forces are 80 kgf, 120 kgf, and 160 kgf.
4. A rock stabilizer is the reserve part to tie the rock so that the rock is fixed well to obtain better results. During the tests, three types of rocks were used.
5. Cutter; the drilling rig used to perform experiments consists of a single cutter. The cutter is marked for three cutting angles of 3°, 8°, and 45°.

The characteristics of the used drill rig are summarized in Table 1.

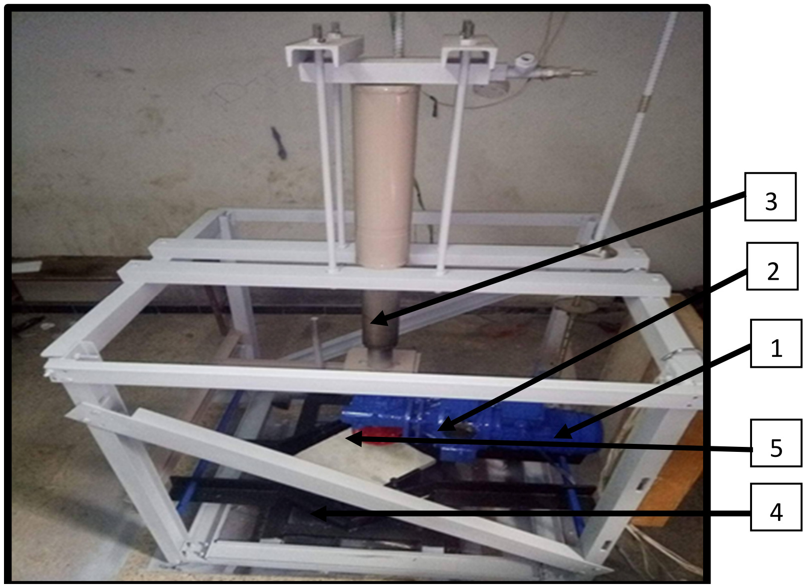


Figure 1. The empirical setup.

Table 1. Drill properties.

Drilling characteristics	
Highest power of the engine	1.5 kw
Highest weight of the trephine	250 kgf
Maximum rotational speed	220 rpm
Diameter of the bit	13 mm

The bit used to perform the tests is of the PDC type. Figure 2 shows the applied forces on PDC drilling tool. The mode of destroying the rock using a PDC tool is presented in Figure 3.

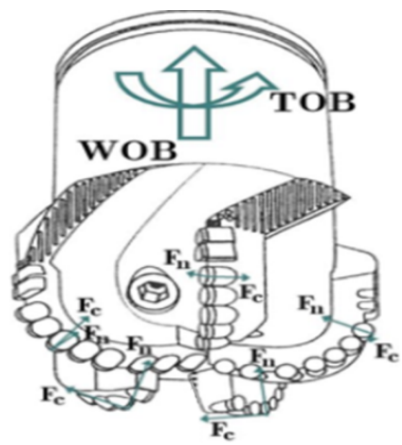


Figure 2. PDC drill tool and forces on tool [14].

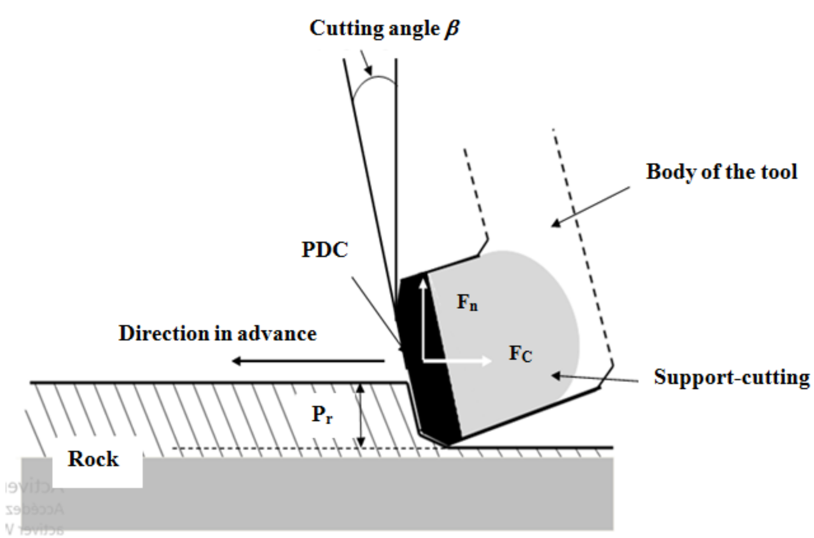


Figure 3. Mode of destroying the rock using a PDC tool.

The cutting angle in the context of a PDC (Polycrystalline Diamond Compact) tool refers to the angle at which the cutting edge or face of the PDC cutter contacts the material being drilled, as demonstrated in Figure 3 [26]. Figure 4 establishes the experimental cutting tool consisting of a single cutter and specifically designed to provide study data on the cutting angle.

Figure 4. PDC (Polycrystalline Diamond Compact) drill tool was designed for this experiment.

The study was conducted using three kinds of limestone and marble sampled from diverse locations in Algeria. The geometrical configuration of the used rocks is displayed in Figure 5. Throughout sample gathering, every block was checked to identify macroscopic faults, and consequently, guarantee the utilization of fracture-free samples. The bit penetration rate was performed for the three chosen rocks. The blocks have a size of about 30 cm, 30 cm × 4 cm.

Rock A **Rock B** **Rock C**

Figure 5. Used rocks (A) from El Ghedir Quarry, (B) from Hadjar Soud's quarry, and (C) from Felfla quarry.

Table 2 presents the resistance of rocks to compression. The compressive strength is established by identifying a sample on the hydraulic press table and then applying a load until the sample is completely crushed. Thereafter, the endurance is then evaluated by applying the subsequent equation (Equation (1)) [27]:

$$Cs = \frac{F}{S}; {kgf}/{cm^2} \quad (1)$$

with: Cs is the compressive resistance, F is the force exerted on the rock, and S is the surface.

Table 2. Resistance of rocks to compression.

Rocks	Resistance to compression
Rock A	1550 kgf/cm^2
Rock B	750 kgf/cm^2
Rock C	640 kgf/cm^2

During experiments, three levels were provided for each parameter. The quantities chosen for each study factor are given in Table 3. The choice of parameters was made according to the capabilities of the drilling rig as well as the behavior of the response after the preliminary tests.

Table 3. Factors and selected levels.

Factors	Unit	Code	levels		
			1	2	3
Angle de coupe	$degree$	β	3	8	45
Weight on the bit	kgf	WOB	80	120	160
Rotation speed	rpm	RPM	118	135	152

3. Test Planning according to RSM

RSM is recognized as one of the primary approaches that can be used in experimental data processing. It serves, on the one hand, to determine the mathematical formulation that shows the relationship between input variables and studied responses [28]. For the response surface model, it is highly recognized that the variance analysis (ANOVA) is easier to perform. Additionally, depending on the RSM strategy, it is important to note that the fundamental drawback of the traditional empirical design approaches is the huge number of trials that are required whenever the number of operational variables increases. This is something that should be taken into consideration. In the course of our inquiry, we have taken an active part in the RSM approach, which is a way of empirical design that cuts down on the number of tests that are carried out as a result of using the traditional empirical design. The ability to collect important components in a short amount of time and at a relatively cheap cost is, as a result, the primary benefit of using this methodology.

Firstly, it is necessary to define the different input quantities (factors) that affect the output (the response) and adopt a plan of experiments, then apply a regression assessment with the RSM quadratic model. Using ANOVA allows us to analyze statistically the individual input factors to determine the main factors that impact the response. Thereafter, the situation of the obtained RSM model can be determined to see if the RSM model needs filtering variables or not. Finally, we proceed to the optimization and validation of the expected performance characteristics [29, 30]. The quantity of interest Y as a function of the various factors is represented according to the following formula (Equation 2) [31]:

$$Y = \varphi(\beta, WOB, RPM, Cs) \quad (2)$$

with φ is the response function.

The regression analysis process [32] is a statistical approach to finding the correlation between experimental data that depend on several measured factors, required to create an appropriate mathematical prototypical that verifies the connection between the different levels of each test factor and an object function [33, 34].

For the mathematical modeling of the studied problem, a non-linear quadratic prototypical is used to link the studied response to the factors. The mathematical function established on RSM approach has the form of a polynomial of the second degree (Equation 3):

$$Y = b_0 + \sum_{i=1}^{n} b_i\, x_{iu} + \sum_{i}^{n} b_{ii}\, x_{iu}^2 + \sum_{\underbrace{i=1}_{i<j}}^{n} b_{ij}\, x_{iu} x_{ju} + \varepsilon \quad (3)$$

where x_{iu} and x_{ju} represent the coded factors (β, WOB, RPM, Cs), and ε is the deviation (i.e. the adjustment error for the regression model). b_0 , b_i , b_{ij} , b_{ii} are the coefficients of the mathematical model where b_0 is a constant or free term of the regression equation, b_i is the coefficients of the linear terms, b_{ii} is the coefficients of quadratic terms and b_{ij} are the coefficients of the cross-product terms. Table 4 brings together 27 runs associated with the four factors where each factor has three levels.

Analysis of variance (ANOVA) is an effective method employed for analyzing experimental results. ANOVA is thus employed to identify the effect of input factors on the desired studied response and allows for providing an interpretation of the output data [26, 27]. The statistical magnitude of adjusted quadratic prototypes is determined by the p and F quantities of the ANOVA. The p-value is the probability of obtaining the consequences detected in an examination (or additional maximum results), it should be ranged between 0 and 1. It is possible to evaluate the results obtained as follows:

- If $p > 0.05$, the factor is not meaningful.
- If $p < 0.05$, the factor is meaningful.

The summation of squares (SS) is employed to approximate the square of the deviation from the average. The sum of squares is given in Equation 4:

$$SS_f = \frac{N}{N_{nf}} \sum_{i=1}^{N_{nf}} (\bar{y}_i - \bar{y})^2 \quad (4)$$

where:

$\bar{y} = \frac{1}{N} \sum_{i=1}^{N} y_i$ is the mean of the response,

$\bar{y}_i$ is the mean of the response observed during the experiment, wherein the component f succeeds the i^{th} level, N is the overall quantity of tests, and N_{nf} is the level of every component f.

The average squares are calculated through the relation presented in Equation 5:

$$M_{Si} = \frac{SS_i}{df_i} \quad (5)$$

d_{fi} is the freedom degree.

To verify the competence of the representation, we use the ratio F. The evaluated F-quantities must be greater than those acquired from Table 5. The F values can be calculated by the relation given by equation 6:

$$F_i = \frac{MS_i}{MS_e} \quad (6)$$

where MS_e is the average square of errors.

When the model's estimated F quantity is higher than its tabular F quantity, one may say with a degree of confidence of 95% that the model is acceptable. Assuming p estimates are lower than 0.05 (or when there is an acceptable level of 95%), the resulting models are regarded as statistically noteworthy. As a result, it has been shown that the words used in the model do in fact have a considerable impact on the answers. The quality of the fit may be quantified using a statistic known as the coefficient of perseverance, or R^2, which can be characterized as the ratio of the explained variance to the overall variation. In other terms, if R^2 approximates to unity, data gained via the quadratic model is closer to the experimental data [35, 36]. It can be calculated as follows:

$$R^2 = \frac{\sum (y_i - \bar{y})^2}{\sum (\bar{y}_i - \bar{y})^2} \quad (7)$$

The penultimate column of the ANOVA tables depicts the contribution of every input component (percentage, cont. %) to the total variation, thus showing the degree of impact on the studied response [37, 38]. The contribution parameter can be calculated as follows:

$$CONT.\% = \frac{SS_f}{SS_T} \times 100 \quad (8)$$

4. Data Processing

In this investigation, the experimental rig visualized in Figure 1 was employed to determine the penetration rate, ROP under the effect of several geometrical and mechanical parameters like the weight on the tool, WOB, rotational velocity, RPM, cutting angle (β), and compressive strength of rocks (R_c).

Table 4 gives the main penetration rate, ROP results of the drilling tool obtained according to the planned experiments. It is worth noting that all planned experiments are performed in equal periods (i.e. 8 seconds for each test). During each test, the advancement (depth) of the tool in the rock is measured. Thereafter, the penetration rate, ROP is decided by the following expression:

$$ROP = L/t_d \quad (9)$$

where L is the depth (micrometer depth), and t_d is the penetration time (the time identified during the tests is 8 seconds).

The experimental results obtained were analyzed using the MINITAB17 software. The ANOVA results for the penetration rate, ROP, at the confidence level of 95% are visualized in Table 5.

Table 4. Penetration rate, ROP, results from the effects of input factors.

Runs	Factors				Response	Runs	Factors				Response
	RPM (rpm)	WOB (kgf)	β (°)	Cs (kgf/cm^2)	ROP (mm/min)		RPM (rpm)	WOB (kgf)	β (°)	Cs (kgf/cm^2)	ROP (mm/min)
1	135	160	8	640	2.82	15	118	80	8	750	1.19
2	118	120	3	750	1.01	16	118	120	45	750	1.43
3	135	120	45	1550	1.71	17	118	120	8	640	1.96
4	135	80	8	640	1.86	18	135	120	8	750	1.51
5	152	120	8	1550	1.67	19	135	80	3	750	0.95
6	135	120	45	640	1.88	20	135	120	3	640	1.86
7	135	120	8	750	1.51	21	152	160	8	750	2.91
8	135	80	8	1550	0.48	22	152	120	8	640	2.53
9	135	160	8	1550	1.56	23	135	80	45	750	1.60
10	152	120	3	750	1.69	24	135	160	3	750	1.70
11	135	120	3	1550	0.49	25	118	120	8	1550	0.81
12	152	80	8	750	1.68	26	152	120	45	750	3.14
13	135	120	8	750	1.51	27	135	160	45	750	2.96
14	118	160	8	750	1.72						

Table 5. Analysis of variance for penetration rate, ROP.

Source	DF	Adj SS	Adj MS	F-value	P-value	Cont.%	Remarks
Model	14	11.9980	0.857002	21.62	0.000		
Linear	4	0.7743	0.193581	4.88	0.014		
RPM	1	0.3422	0.342232	8.63	0.012	8,96	Substantial
WOB	1	0.2270	0.227031	5.73	0.034	5,95	Substantial
B	1	0.0474	0.047390	1.20	0.296	1,24	Unsubstantial
Cs	1	0.5704	0.570373	14.39	0.003	14,94	Significant.
Square	4	1.5125	0.378122	9.54	0.001		
RPM*RPM	1	0.3139	0.313931	7.92	0.016	8,22	Substantial
WOB*WOB	1	0.2219	0.221934	5.60	0.036	5,81	Substantial
B*B	1	0.2756	0.275633	6.95	0.022	7,22	Substantial
Cs*Cs	1	0.5387	0.538743	13.59	0.003	14,11	Substantial
Way Interaction	6	0.7908	0.131793	3.33	0.036		
RPM*WOB	1	0.1225	0.122500	3.09	0.104	3,21	Unsubstantial
RPM*B	1	0.3875	0.387461	9.78	0.009	10,15	Substantial
RPM*Cs	1	0.0064	0.006380	0.16	0.695	0,17	Unsubstantial
WOB*B	1	0.1009	0.100855	2.54	0.137	2,64	Unsubstantial
WOB*Cs	1	0.0137	0.013693	0.35	0.568	0,36	Unsubstantial
B*Cs	1	0.1745	0.174486	4.40	0.058	4,57	Unsubstantial
Error	12	0.4756	0.039637			12,46	
Lack-of-fit	10	0.4756	0.047564	*	*		
Pure error	2	0.0000	0.000000				
Total	26	12.4737					

DF: degree of freedom; SS: summation of squares; MS: adjusted mean squares

The conducted examination is based on the study of variances and significance degrees (p-value). The statistical analysis mainly indicates that the model used is well-fitted since the sum of squares due to the error ($SS = 0.47$) can be considered very small compared to the total sum of squares ($SS_{TOTAL} = 12.47$). The calculation of the determination coefficient R^2 is an index to find the quality of the fit of the obtained model. R^2 is thus a coefficient indicating the degree of correspondence between the detected data and the quantities expected by the statistical model. If R^2 approaches to unity, the experiments and predictions data are sufficiently correlated, and the predicted model is reliable. From this investigation, the obtained higher value of the determination coefficient (i.e. $R^2 = 0.96$) explains 96% of the variability in the response and shows clearly the high correlation between the predicted and experimental data, that is only 4% represent the influence of other factors or other variables not included in the model.

It is noteworthy that the majority of the terms of the model have a remarkable contribution to the evolution of the penetration rate, according to the percentage of the contribution (i.e. cont.%). Results obtained also indicate that the rock resistance factor represents the highest statistical significance with 4.5728% contribution, which reflects the influence of the rock nature on the response. Rotational speed also has a significant contribution to total ROP variations with 2.7433%, while exerted weight explains 1.898% of the changes in response.

The cutting angle β makes a small contribution to the fluctuation of the response ROP with a percentage of 0.3799%. Furthermore, all the interactions between the different factors have a weak contribution to the variations of the response, except the interaction between RPM and β by 3.1065 %.

The quadratic term of the compressive strength ($Cs * Cs$) has also a significant contribution to the change in response with a percentage of 4.3186%. This is confirmed by the complementary finer analysis based on the P-value, which determines the degrees of the significance of the factors. P-quantities less than 0.05 imply that the prototype terms are meaningful. The results presented in Table 5 show that Cs, RPM, WOB, as well as the interaction of RPM and β ($RPM * \beta$) and the quadratic term of resistance to compression ($Cs * Cs$) are significant model terms, so they have a significant effect on the response (ROP).

5. Final Equation in terms of Actual Parameters

Equation 10 represents the quadratic model formula for the Rate of Penetration (ROP) response:

$$\begin{aligned} OP = {} & 24.96 - 0.2451\,RPM - 0.0577\,WOB - 0.0481\,B - 0.01114\,Cs + 0.000842\,RPM * RPM \\ & + 0.000128\,WOB * WOB - 0.001378\,B * B + 0.000004\,Cs * Cs + 0.000257\,RPM \\ & * WOB + 0.000748\,RPM * B + 0.000004\,RPM * Cs + 0.000162\,WOB * B \\ & + 0.000003\,WOB * Cs + 0.000016\,B * Cs \end{aligned} \tag{10}$$

The influence of different input parameters on the output response that describes the behavior of the drilling process according to the mathematical model based on RSM approach has been established, and the results are depicted in Figure 6. Also a three-dimensional graphical representation of the interaction effect of two factors is drawn in Figure 7.

It should be interpreting the contour plot to understand how changes in the two factors influence the response variable. Figure 6 shows and confirms that the interaction effect of WOB and RPM is the most influential on the development of ROP, where the areas of steep incline in the contour lines indicate that regions where small changes in the WOB and RPM are led to significant changes in the response ROP. It should be noted that all the effects of the interactions that contain the compressive resistance factor Cs influence disproportionately on the development of the response ROP.

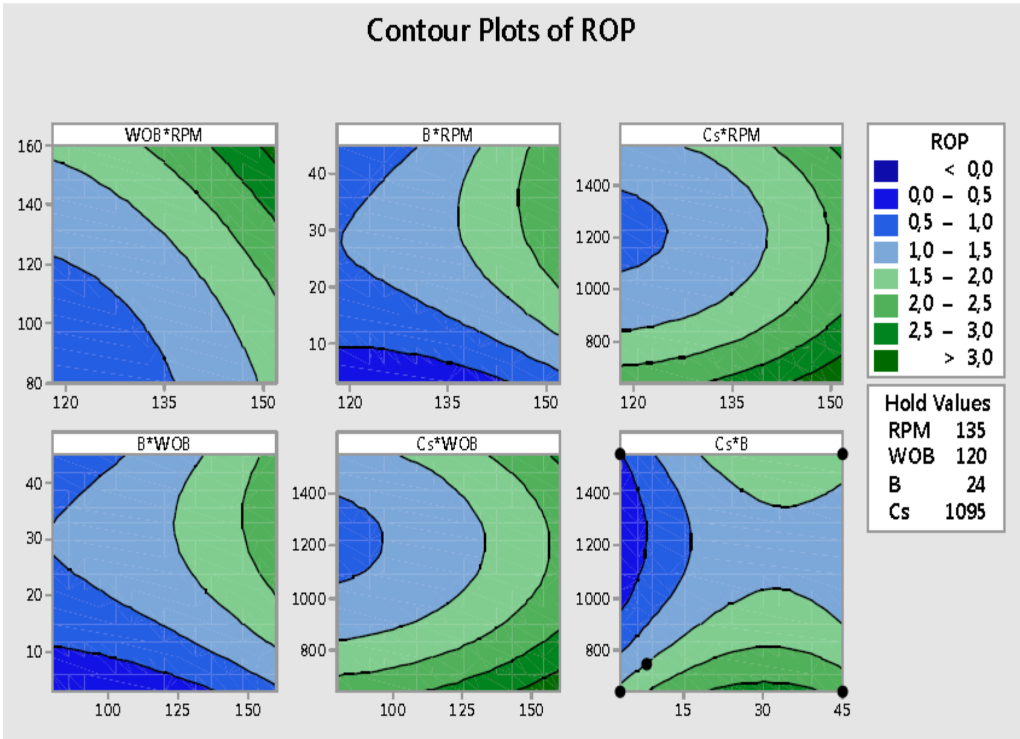
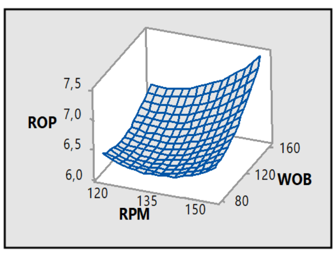


Figure 6. Contour-plots of penetration rate, ROP.



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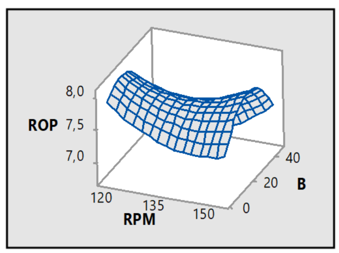

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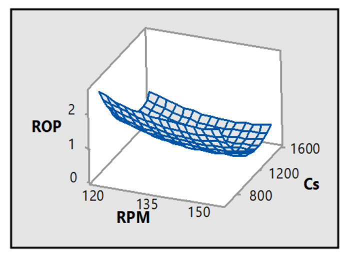

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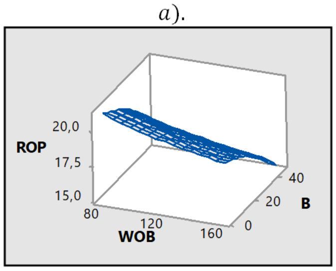

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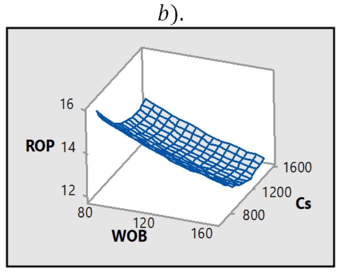
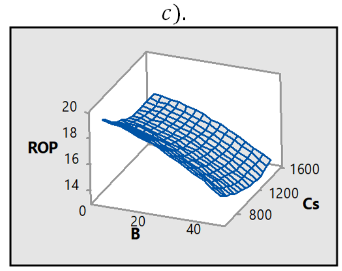


e).



f).

Figure 7. 3D plots of ROP response versus input variables.

From Figure 7(a), it can be seen that the maximal penetration rate, ROP, is obtained with the combination of the highest values of weight, WOB and rotational speed, RPM. As drawn in Figure 7(b), the maximal ROP is achieved with a higher RPM value and middle cutting angle (B). Also results obtained in Figure 7(c) reveal that the maximal ROP can be gained with the lowest values of both RPM and rock resistance (Cs). Figures 7(d) and 7(e) depict that the maximal ROP values are determined with an association of lowest values of WOB and B, and WOB and Cs, respectively. Finally, the maximal ROP has also been achieved with the lowest values of both B and Cs parameters, as displayed in Figure 7(f). Finally, the maximal ROP has also been achieved with the lowest values of both B and Cs parameters, as displayed in Figure 7(f).

6. Results and Discussion

As shown in the above sections, statistical data processing using ANOVA with a determination of the influence of each input variable and its interactions on the output response was done. Also the response surface plots obtained are generated taking into account the effect of two parameters simultaneously; the third term is considered constant. The optimization and modeling are performed using RSM outputs.

Figure 8 displays the normal probability of the residuals of the obtained models. Results obtained show that the fit errors are distributed within reasonable proximity to the reference straight line. Also it is highly noticed that residuals have a normal distribution.

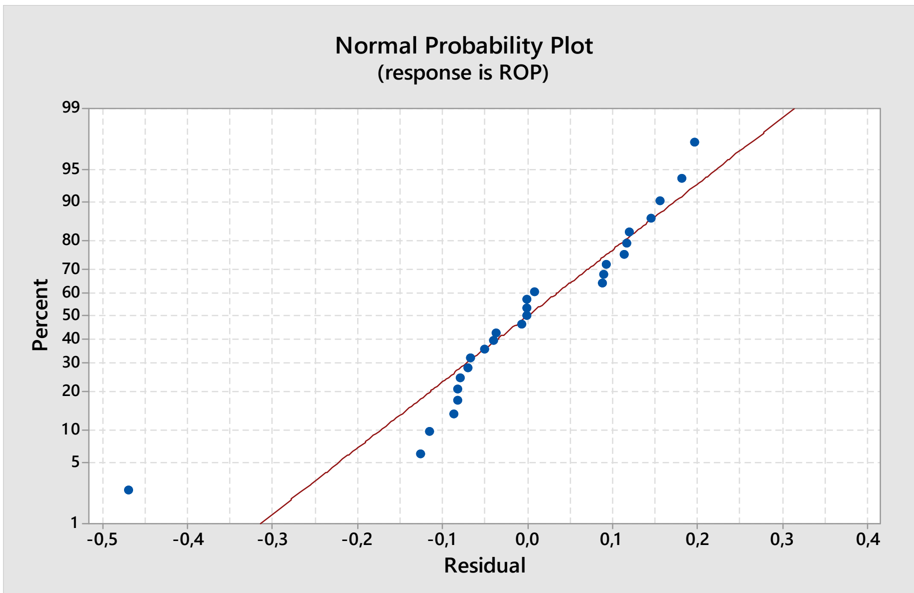


Figure 8. Normal probability plot of the residuals for penetration rate, ROP.

The adjustment evaluation of this model can be identified more precisely using the results obtained by calculating the percentage of error between the measured and the predicted value for each of the experiments carried out.

Table 6 shows that the greatest adjustment error made (around 54.81% for an observed response of 0.49) corresponds to experiment 11. Also the error percentages of experiments 25 and 8 reached 49.94 and 24.7725, respectively; they are considered relatively high. This was to be expected because it is impossible for the response predicted by the model to be perfectly fitted to all the values obtained during experiments. Nevertheless, 25 out of 27 experiments give well-adjusted results with predicted responses with error percentages below 20% (Cetin *et al*., 2011), so the error percentages are within acceptable limits, confirming that the fit is very good.

Figure 9 describes the regression model that allows us to estimate the correlation of the experimental data. This consists in finding the relation, making it possible to explain the behavior of the penetration rate, ROP, response measured as a function of the predicted response.

The dispersion of the measured response is a measure of the correlation; if the dispersion is small, the regression analysis is appropriate to describe the variation of this response, and if the dispersion is high, the regression analysis is not appropriate.

Table 6. Percentage of errors between the measured and the predicted response.

runs	Actual ROP	Predicted ROP	Percentage of error	Runs	Actual ROP	Predicted ROP	Percentage of error
1	2.82	2.5987	8.5158%	15	1.19	1.2533	5.0506
2	1.01	1.0419	3.0617%	16	1.43	1.2713	12.4832
3	1.71	1.6175	5.7187%	17	1.96	1.8324	6.9635
4	1.86	1.7242	7.8761	18	1.51	1.4559	3.7159
5	1.67	1.3991	19.3624	19	0.95	0.8969	5.9203
6	1.88	2.3091	18.5829	20	1.86	1.6196	14.8431
7	1.51	1.4559	3.7159	21	2.91	2.9043	0.1962
8	0.48	0.3847	24.7725	22	2.53	2.5676	1.4644
9	1.56	1.4776	5.5766	23	1.60	1.3881	15.2654
10	1.69	1.6650	1.5015	24	1.70	1.7329	1.89
11	0.49	0.3165	54.8183	25	0.81	0.5402	49.9444
12	1.68	1.6539	1.5780	26	3.14	2.9624	5.9951
13	1.51	1.4559	3.7159	27	2.96	2.7685	6.9171
14	1.72	1.8046	4.6880				

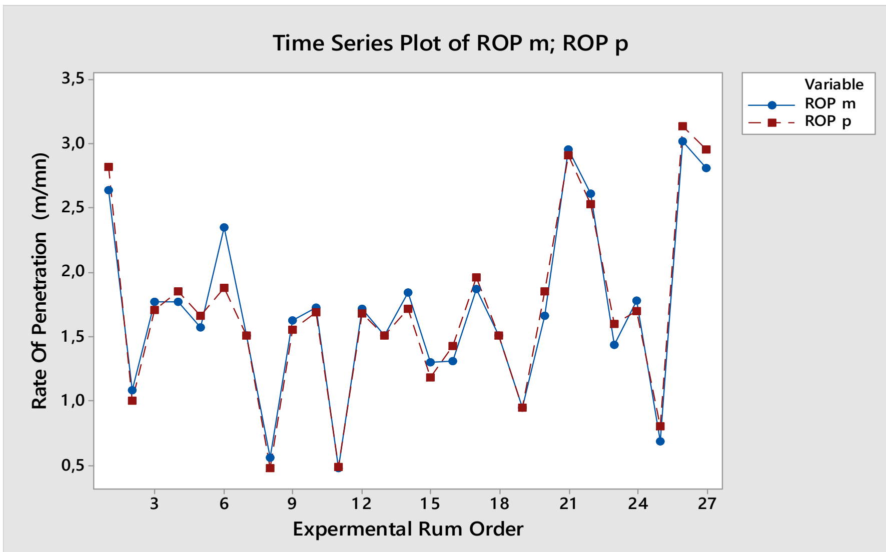


Figure 9. Measured and predicted values of penetration rate, ROP.

Results obtained reveal that the points that represent the measured response are almost coincident on the benchmark curve, which represents the predicted response, which reflects a great convergence between the two curves in terms of change of values. The comparison between the measured response and the response predicted by the model confirms that the fit is of very good quality. Therefore, we conclude that the model is suitable and useful for predicting the penetration rate, ROP, behavior.

The comparison of the prior research serves the purpose of evaluating the scientific value of our study [7.20, 27.30]. This assessment is based on the data used, the methodologies adopted, and the results obtained. When comparing prior studies in the field of drilling, a consistent theme emerges; all these studies share a common primary objective, which is to effectively manage drilling operations with the goal of cost minimization. The approach I was employed to treat the research problem similar to that of previous studies. Upon juxtaposing the current study with prior research endeavors employing similar analysis and modeling techniques in the drilling field, it becomes evident that our study's model adaptation coefficient (R^2) surpasses those obtained in the previous studies. This observation underscores the high quality of the results we have achieved, the model's efficiency, and its capacity for providing confident predictions.

Given the precision of the obtained results, they are deemed reasonable and relatively realistic, making them applicable, especially in environments featuring rocks of a similar nature such as oil well drilling. Furthermore, the inclusion of additional factors such as the hydraulic parameters of the drilling process, particularly the mud flow rate, may enhance ability the potential to significantly improve the comprehensive analysis of ROP behavior.

7. Response Optimization

The final step in our investigation is to find the optimal values for the studied problem, which are the values of weight, rotational speed, drill ability, as well as rake angle, leading to maximizing the feed rate. To find the maximum of a function of several variables, it must find any point canceling the partial derivatives.

If we set $\hat{y}$ the predicted response, and x_1, x_2, x_3, x_4 the four factors, we obtain the system of the following functions:

$$\frac{\partial \hat{y}}{\partial x_1}(x_1\ x_2\ x_3\ x_4) = 0 \tag{11.1}$$

$$\frac{\partial \hat{y}}{\partial x_2}(x_1\ x_2\ x_3\ x_4) = 0 \tag{11.2}$$

$$\frac{\partial \hat{y}}{\partial x_3}(x_1\ x_2\ x_3\ x_4) = 0 \tag{11.3}$$

$$\frac{\partial \hat{y}}{\partial x_4}(x_1\ x_2\ x_3\ x_4) = 0 \tag{11.4}$$

The solutions of this system are the optimal values that lead to maximizing the ROP. Figure 10 shows the results of solving the equations according to the RSM method using the MINITAB software.

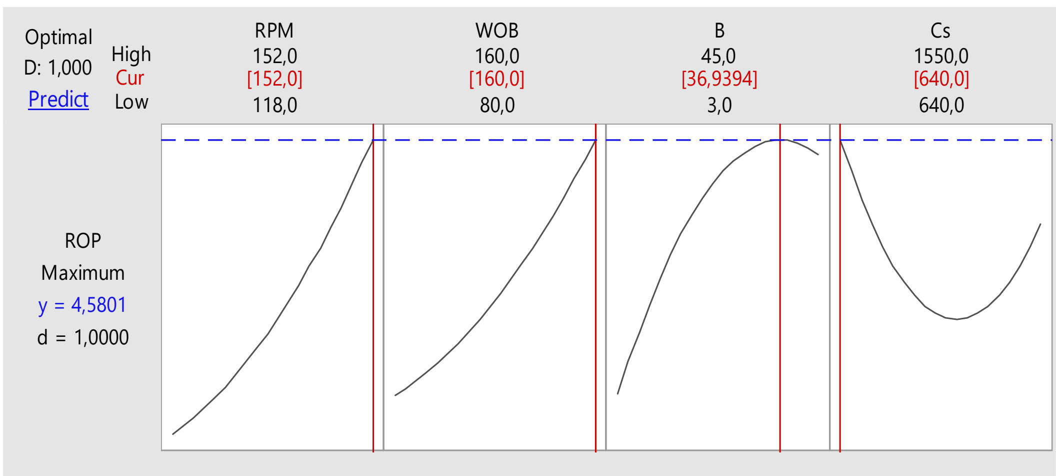


Figure 10. Response optimization plot for penetration rate in rotary drilling.

The optimized penetration response area was plotted using the MINITAB software, as shown in Figure 10. The desirability function has been selected to find a suitable factor value. Figure 10 displays that the objective optimization of the penetration rate was achieved with a rotation speed of 152 rpm, a bit weight of 160 kgf, an angle of cut $\beta' = 36.9394 \approx 37^{\circ}$, and compressive strength of rocks 640 kgf/cm^2. It can be also possible to estimate the maximum penetration rate, gained for optimum conditions, at 4.5801 mm. Particularly, the optimized response curve makes it possible to obtain an optimal penetration rate by redefining the values of the process parameters in the experimental interval. The optimization results are tabulated in Table 7.

Table 7. Response optimization parameters (global solution).

Factors	Code	Optimization parameters	Penetration rate (RMS model)
Rotation speed	RPM_3	152 rpm	4.5801
Weight on the bit	WOB_3	160 kgf	
Cutting angle	β'	37°	
Resistance of rocks to compression	Cs	640 kgf/cm^2	

8. Conclusions

To our best knowledge, in the literature, there are no research works that treat and optimize the drilling process statistically via response surface methodology. Our investigation can be highly considered an original contribution to understanding the drilling process and optimizing the rate of penetration (ROP). The current research work was thus mainly focused on optimizing the penetration rate against variations in different functional parameters such as weight on the bit, bit rotational speed, formation type, and cutting angle during a drilling operation. The Response Surface Methodology (RSM) was adopted to optimize the penetration rate of the drilling tool. A mathematical model simulating the behavior of a drilling system has been developed. The study was based on a statistical regression analysis of experimental data. It was found that the model developed is reliable and in good agreement with experimental observations. Based on the RSM methodology, it can be concluded that:

- The different parameters have a noticeable effect on the drilling penetration rate. It should be noted that the effect of the interaction of RPM and WOB is the factor that has the greatest influence on ROP compared to other factors.
- Regarding the estimation of the different parameters of the model, it is interesting to note that the parameter of Cs has an important significant effect on the changes in the penetration rate, with values of 4.5728%.
- During this research work, the RSM methodology was successfully employed to optimize the rate of penetration (ROP). We have found interesting results with high performance and cost minimization. We have tried to find the optimal values that give a more efficient drilling operation (i.e. the best rate of penetration (ROP). The best ROP was thus obtained for the following operating input parameters:

β', WOB_3, RPM_3, and Cs_1, with $\beta' \approx 37°$, RPM_3 = 152 rpm, WOB_3 = 160 kgf, and Cs_1 = 640 kgf/cm^2.

- The quadratic mathematical model was developed in confidence intervals of 96.1863%, for the prediction of the penetration rate, ROP. The results show that the chosen model is well-adjusted, and therefore, it is very useful for determining the predicted response.

It's important to note that the study was conducted on rocks of limestone and marble nature, and changing the rock type may yield different results. Hence, additional research is necessary to obtain further results, allowing for a more comprehensive evaluation of the impact of various parameters on the drilling process's performance.

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تحلیل آماری و بهینه‌سازی فرآیند حفاری با استفاده از روش‌شناسی سطح پاسخ و داده‌های تجربی

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چکیده:

به خوبی ثابت شده است که روش سطح پاسخ (RSM) معمولاً برای ایجاد تفاوت بین مقادیر پیش‌بینی‌شده و مقادیر مشاهده‌شده تجربی استفاده می‌شود. این مطالعه عمدتاً تأثیر چهار عامل حفاری شامل وزن روی بیت (WOB)، سرعت چرخش بیت، RPM، زاویه برش β، و مقاومت سنگ بر نرخ نفوذ ابزار حفاری را هدف قرار می‌دهد. در این بررسی سه نوع سنگ آهکی در نظر گرفته شد. ارزیابی‌های برنامه‌ریزی شده در سه مرحله از چهار متغیر ورودی در نظر گرفته شده انجام شد. تجزیه و تحلیل آماری با استفاده از روش RSM و تحلیل واریانس (ANOVA) انجام شد. این تجزیه و تحلیل به ما اجازه داد تا مدل نفوذ مناسب با ضریب تعیین بالاتر ۹۶.۱۹٪ را توسعه دهیم که نشان دهنده همبستگی بالا بین داده‌های پیش‌بینی شده و تجربی است و در نتیجه می‌توان نتیجه گرفت که مدل به دست آمده برای پیش‌بینی بسیار مناسب است. نرخ نفوذ. همچنین از تحلیل واریانس، نتایج به‌دست‌آمده نشان می‌دهد که سرعت چرخش، RPM و وزن روی پارامترهای بیت (WOB) و همچنین ماهیت سنگ که توسط مقاومت فشاری سنگ تعیین می‌شود، تأثیر قابل‌توجهی بر نفوذ دارند. نرخ؛ با این حال، زاویه چنگک تاثیر کمی دارد. در نهایت، پارامترهای بهینه برای یافتن بهترین نرخ نفوذ ممکن ابزار حفاری تعیین شد.

کلمات کلیدی: بهینه سازی، داده های تجربی، پارامترهای حفاری، پارامترهای بهینه، روش سطح پاسخ.