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Simultaneous Optimization of Mine Production Rate and Cut-off Grade using Particle Swarm Optimization

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Abstract

The production rate and cut-off grade are two critical variables in the design and planning of open-pit mines. Generally, the production rate depends on the reserve amount, which is influenced by the cut-off grade. Additionally, the cut-off grade is affected by the production cost, which is influenced by the production rate and product price. A conventional approach optimizes each variable individually, and neglects the trade-off between production rate and cut-off grade, leading to a sub-optimal solution. This work aimed to address the simultaneous optimization of the production rate and cut-off grade and provided a novel solution for this problem. In this context, a non-linear mathematical model was developed. The Particle Swarm Optimization (PSO) algorithm was used due to the model's non-linear nature and the continuous decision variables. Implementing the model for a typical copper mine showed that the suggested model resulted in a concurrent optimization of production rate and cut-off grade. The maximum NPV of 1.153 billion dollars occurred at a production rate of 15.66 Mt/y, and a cut-off grade of 0.64%. Additionally, a sensitivity analysis was conducted for key factors such as product price, discount rate, and maximum capital cost.

1. Introduction

The profitability of a mining project is influenced by two key factors, namely production rate and cut-off grade [1]. The production rate is determined based on the amount of mineral` reserve, and is measured using a series of predefined cut-off grades. The cut-off grade is commonly calculated based on the operating cost and product price. The product price is out of the control, but the operating cost has shown a direct relationship with the production rate. Therefore, there is a trade-off between these factors, and the optimization of the project requires considering simultaneously both the production rate and cut-off grade. The optimization of production rate is a critical issue, as it determines mine opening size, mining equipment, processing capacity, and the number of human workforces [2, 3].

Generally, a higher production rate needs a higher capital cost. However, it lowers the operating cost and the mine life. With a reduced production rate, the capital costs and revenues show a remarkable decrease, but the mine life would be prolonged. When the production rate of the reserves be too high, the operating lifetime would be very short in order to cause an adequate return on the capital investment. Furthermore, an extremely high rate of production requires a significant capital cost, which may not be cost-effective for the investors, especially when regional infrastructure is inadequate. If the production rate is too low compared with the ore reserve tonnage, the resulting operating profit may be too small to recover the capital cost, as it is expected to be returned during the initial years of the operation [4, 5].

Typically, an optimum production rate exists between the two extremes that balances negative and positive cash flows to maximize the project's

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profitability [6, 7]. The production rate is affected by many factors including reserve, cut-off grade, unit costs, and selling price. Except for the product price, the other factors are related to the production rate and change. For example, the enhancement in the production rate results in a reduction in unit cost, causing a decrease in the cut-off grade, and an increase in the reserve. The conventional techniques that optimize each variable independently can lead to sub-optimal solutions, as trade-offs between these factors are neglected.

This work aimed to develop a novel model to optimize simultaneously the production rate and cut-off grade of metal open-pit mines. In this context, a non-linear optimization model was presented, and a Particle Swarm Optimization (PSO) algorithm was implemented to solve the model. The necessity of this work becomes particularly evident in the early stages of open-pit mine feasibility studies, while more advanced multi-period and dynamic cut-off grade calculations are essential for detailed production planning. The reason behind is that such analyses are often impractical during the initial phases of project evaluation. In these preliminary stages, it is crucial to estimate production capacity, which is directly influenced by the ore reserves defined by a single cut-off grade. Given the interconnected relationship between cut-off grade, ore reserves, and production capacity, multiple scenario analyses may be required to identify the optimal capacity. Our proposed optimization approach sought to streamline this process by providing a systematic method for determining the optimal production rate and cut-off grade, thereby reducing the need for extensive scenario analysis.

2. Literature Review

Many studies have been conducted considering the optimization of production rate and cut-off grade. Taylor presented an empirical formula to select the mining production rate based on the survey of 30 projects [8]. However, this approach has several disadvantages such as the ignorance of operational and financial parameters [4, 9]. Dowd addressed the issue through dynamic programming, assuming that costs and prices could be accurately predicted [10]. Wells suggested a method to maximize the ratio of the positive present value of cash flow to its negative counterpart [6]. Cavender sought to optimize the production rate based on the Net Present Value (NPV) maximization, regarding cash flow and option pricing techniques. However, this model was not widely used due to the lack of practical operating constraints [11]. Hajdasiński critically evaluated Z. Li's model using the Net Future Value (NFV) for optimization. His results demonstrated that the NPV was a more appropriate basis for such models. It showed that NFV made time variables undefined and rendered Li's sensitivity analysis and optimization results misleading. These results underscored the necessity of using NPV to obtain significant optimum solutions in mine production capacity optimization [12]. Smith investigated the techniques and variables affecting the selection of the most suitable production rate. The results suggested that the cut-off grade should be included to calculate the production rate [2]. Abdul Sabour proposed a model according to a marginal analysis, which suggested that setting marginal cost equal to marginal revenue could lead to an optimum production rate [13]. Smith and Abdul Sabour indicated that the production rate associated with the maximum NPV might not always be optimum and recommended considering a range of production rates. The upper level of the range corresponded to the maximum value of NPV, and the lower level coincided with the production rate that yielded to double the capital cost [2].

Ordin and Ordin studied the optimization of the design capacity of a mine considering the investment risk. The study employed mathematical and economic models to evaluate critical factors such as ore grade, production costs, and market conditions. This approach facilitated informed decision-making under varying economic scenarios, finally enhancing the economic value of mining operations [14]. Elkington and Durham desegregated pushback and production capacity optimization in open-pit mines, aiming to maximize NPV. They proposed a mathematical model to optimize simultaneously the mining and processing capacities including intermediate and ultimate pushback selection, the determination of scheduling, cut-off grade, and stockpiling [15]. Ordin et al. developed a dynamic lag modeling approach to optimize the mine design capacity, considering the discount of cash flows and variations in technical and economic parameters over the mine's operational life. This method was applied into the diamond placer mines of Solur and Vostochny in the Republic of Sakha (Yakutia), demonstrating the improved accuracy and reliability for capacity optimization by incorporating macroeconomic dynamics [16]. Zuo et al. developed a multi-disciplinary optimization model for underground metal mines, considering income, safety, and environmental impact.

Through an adaptive optimization algorithm, the model was applied to a lead and zinc mine, achieving a production scale of 1.25 Mt/y, and improving profits, safety, and environmental impacts [17]. Kizilkal and Dimitrakopoulos presented an interactive and non-linear model to estimate the optimum production rate under financial uncertainty [18]. Malli et al. developed a model to determine the optimum production capacity and mine life of open-pit mining, considering NPV and haulage costs, geotechnical features, and slope stability. Their findings highlighted that higher production capacity and steeper slope angles not only increased NPV, but also raised initial investment costs and financial risks. These results underscored the importance of balancing production capacity, mine life, and investment in mining feasibility assessments [19]. Runge supported this idea by proposing that an increase in the production rate would lead to a reduction in mining costs due to the economy of scale [9]. Akishev et al. investigated advanced techniques to evaluate the production capacity and life of open-pit diamond mines. They highlighted the importance of the accurate evaluation methods to optimize mine planning and resource application. By integrating geological, economic, and operational factors, the study aimed to enhance prediction accuracy. The article tried to refine these evaluation methods to improve decision-making processes, ensuring more effective and sustainable mining operations [20]. Salama et al. conducted a study on the effect of financial analysis of increased mining rates on underground mining through simulation and mixed-integer programming [21]. Arteaga et al. investigated the trade-off between shovel utilization and mining rate in open-pit mining. They showed that productivity was relevant to the optimum mining rate [22]. Magda investigated the effect of utilizing mine production capacity on unit production costs, emphasizing that costs were minimized when production matched the capacity. Two key indices-rate of capacity utilization and fixed costs per unit-are crucial for cost reduction. Increasing the capacity utilization rate and reducing the fixed costs could effectively decrease the unit production costs, providing a basis for the mining company restructuring programs [23]. Neingo et al. studied the effectiveness of three production rate estimation methods for the South African platinum mines by comparing estimated rates from rules of thumb with actual reported production rates. Findings revealed significant variations up to 218% among the estimation methods and weak correlations with actual production, suggesting that deposit size and geometry alone were insufficient for an accurate estimation. These results underlined the necessity for the robust mathematical models that incorporated multiple constraints to optimize the production rates [24]. Souza et al. presented a mathematical formula to optimize the mining production rate to achieve a maximum profit, similar to the study by Abdul Sabour [25]. Nyandwe et al. concentrated on optimizing the production rate of a copper mine based on the feasibility study of three production plans. The optimum production rate was selected through criteria such as investment, cost, engineering complexity, and economic advantage [26]. Sohrabi et al. investigated the optimum production rate of the Sari Gunay gold mine in Iran under price uncertainty. Using the Taylor and Zwiagin's methods across different scenarios, they compared the influence of price certainty and uncertainty on the mine's NPV. The findings highlighted that scenarios accounting for price uncertainty, particularly those using the binomial tree method, yielded higher and more stable NPV values. These results underscored the importance of incorporating price uncertainty into mining project evaluations to mitigate risks and optimize financial outcomes [27]. Liu et al. developed a production capacity model for open-pit coal mines, linking the working face length and annual advancing speed to the production capacity. Applied to the Baorixile open-pit Coal Mine in Inner Mongolia, China, the remaining unmined areas were divided into four regions. The optimum production capacities were determined for the mining districts, ensuring efficient resource extraction and operational continuity [28].

The investigation of the mentioned studies underscored the significance of optimizing production rates and delineated various methodologies to achieve this purpose. Nevertheless, some authors emphasized that the optimization of the production rate and cut-off grade should be carried out simultaneously to obtain reliable results [6, 7, 29, 30] Since .mination of the cut-off grade is influenced by the production rate, any changes in the cut-off grade requires a recalculation of the extractable reserves, production rate, and cost estimates [30]. Many studies have addressed the determination of cut-off grade for mining projects by incorporating it into the production planning optimization. In a review paper, Asad discussed the issue of cut-off grades [31], meanwhile, the area of interest in this work focused on the cut-off grade that dealt with the production rate in more details or considered the production rate in the optimization of the cut-off grade.

Vickers presented a graphical method that used marginal analysis to determine the cut-off grade. The technique produced a constant schedule of cut-off grades over the life of the operation [32]. Lane's model considered not only the grade-tonnage distribution, but also the production capacities of different components of the mining operation. This model supported the overall objective of a mining operation by maximizing the NPV, while considering the limitations of mining, processing, and refining capacities [31, 33, 34]. Nieto and Bascetin utilized a multiyear Generalized Reduced Gradient (GRG) iterative factor to determine a cut-off grade strategy that maximized NPV over several years. By integrating the economic and geological factors into the GRG iterative factor, the approach aimed to improve the accuracy and efficiency of the cut-off grade determination. Finally, their strategy proposed iteratively adjusting cut-off grades based on the multi-year GRG factor, increasing long-term profitability and decision-making in mining ventures [35]. Nieto and Bascetin introduced a new method to determine the optimum cut-off grade policy to maximize NPV in the mining projects. This method integrated an optimization factor, considering the economic, operational, and geological parameters. Through outlining the implementation steps and emphasizing potential benefits, their approach offered a systematic and effective means to optimize NPV by selecting the most appropriate cut-off grade policy [36]. Asad integrated the operating costs and commodity price variation into the Lane's model. The results proposed a more practical cut-off grade policy that considered these variations [37]. He et al. proposed a novel technique to optimize cut-off grades by combining neural network nesting with the genetic algorithm method. Their findings acknowledged the complexity and non-linearity of the traditional model, and proposed an evolutionary approach to find solutions [38]. Gholamnejad incorporated the cost of waste dump rehabilitation into the profit function of the Lane's model. This adjustment led to a variation in the relationship between benefits and costs, and caused a shift in the optimum cut-off grade [39, 40]. King proposed a modification to the cut-off grade and profit equations by separating the cost of mining ore and waste into two distinct parts. This adjustment showed that some mining operations incurred lower costs when blasting waste, and the transportation costs for ore and waste were always different [41]. Abdollahisharif et al. studied the optimum cut-off grade with variable capacities in open-pit mining. They modified the Lane's algorithm to account for variable processing capacities, and determine the optimum cut-off grade in open-pit mines [42]. Khodayari and Jafarnejad improved the Lane's model by maximizing the annual metal production. Their mathematical formulation was built upon the balancing cut-off grade concept of the Lane's model [43]. Gama introduced a simplified method that enabled the mining companies to optimize their production strategies efficiently. The technique focused on the key factors such as cut-off grade determination, production rate optimization, and economic considerations. By employing this method, the mining companies streamlined their decision-making process, enhanced profitability, and achieved better financial outcomes. Generally, they provided a practical and accessible framework to maximize profits in open-pit mining operations [44]. Hustrulid et al. and Rendu discussed the applicability of the Lane's model in both open-pit and underground mining scenarios, emphasizing the importance of stockpiling policies through various case studies [5, 45]. Johnson et al. proposed a mathematical algorithm based on partial differential equations that dynamically determined the cut-off grade strategy to manage market uncertainty. They applied an algorithm to a real mine case study. According to their findings, the decision to send a mining block to either the processing flows or waste dump, depended not only on commodity price or ore grade, but also on the grade of future mining blocks, processing costs, and mining and processing capacities [46]. Dagdelen and Kawahata highlighted the significance of strategic mine planning and the optimization of cut-off grades to enhance the profitability. They emphasized on the integration of geological, economic, and operational variables to formulate resilient strategies in line with long-term goals. The article explored diverse methodologies and tools to optimize the cut-off grades, considering ore attributes, processing expenses, and market fluctuations. Finally, it emphasized on the pivotal role of strategic planning and cut-off-grade optimization in fostering sustainable value within mining endeavors [47]. Ganguli et al. provided an overview of a methodological approach to optimize mine scheduling and cut-off grades through Mixed-Integer Linear Programming (MILP). This technique aimed to enhance the efficiency and profitability of mining operations by systematically allocating resources and determining the optimum extraction sequence, considering economic constraints and geological factors. They discussed the application of MILP in mine planning, highlighting its ability to address complex decision-making scenarios and improve the overall performance. By integrating mine scheduling and cut-off grade optimization, mining companies achieved better resource utilization and maximized economic returns [48]. Ahmadi and Bazzazi conducted a study on determining the optimum cut-off grade of open-pit mines using the Imperialist Competitive Algorithm (ICA) and Particle Swarm Optimization (PSO) as the efficient meta-heuristic optimization methods [49, 50]. Fathollahzadeh et al. presented a MIP model to schedule open-pit mining operations incorporating the grade engineering techniques. This method enhanced the efficiency by pre-processing materials to deliver only high-grade ore to the mineral processing plant. It optimized the resource use. The model aimed to maximize NPV by adhering to the operational constraints [51]. Sotoudeh et al. introduced a novel technique to establish the cut-off grade in the underground metalliferous mining operations. It emphasized on the integration of pre-concentration systems to increase sustainability and resource utilization. The proposed model that was applied to a sub-level stopping operation, demonstrated significant improvements in profitability, efficiency, and sustainability by lowering the cut-off grade and optimizing ore processing [52]. Wells used the Present Value Ratio (PVR) as an optimization criterion to identify the optimum combination of cut-off grade and production rate. The PVR was computed by dividing the present value of positive cash flows using the present value of negative cash flows, indicating economic viability when the ratio exceeded 1. While Wells accounted for the simultaneous optimization of production rate and cut-off grade, they observed that the influence of the economy of scale on capital and operating costs was overlooked [6]. The Park's method, as described in previous work [29], could maximize overall mine profit by identifying the optimum set of production rates and cut-off grades, which would use a trial-and-error approach that might not always yield optimum outcomes.

In this work, the static cut-off grade and production rate were optimized for the early stage of feasibility studies. The objective of this work was to introduce a model that optimized both the production rate and cut-off grade, considering the effect of the economy of scale. As the model involved non-linear equations and continuous decision variables, the Particle Swarm Optimization (PSO) algorithm was implemented.

3. Statement of Problem and Mathematical Formulation

As mentioned before, the cut-off grade, average grade, reserve amount, and unit cost are the key factors that affect the production rate. A change in the cut-off grade affects not only the reserve, but also the production rate and unit costs. The selection of an appropriate production rate is determined through an iterative process. First, the reserve tonnage and average grade are determined based on a preliminary cut-off grade. Then, assuming this reserve, a possible production rate is considered, and the revenue, capital, and operating costs are estimated for the entire project. A new cut-off grade is calculated based on these costs and revenues, and the corresponding reserve tonnage is estimated. Considering economic indicators, the iterative process must be repeated until the most suitable production rate is achieved.

This iterative process is time-consuming, and requires the change-and-see scenario evaluations. Therefore, a more systematic and standard optimization model was the main objective of the current study. The problem was first formulated using the standard mathematical modeling language. Due to the dependency between the decision variables (cut-off grade and production rate), the model was non-linear. Therefore, a Particle Swarm Optimization (PSO) algorithm was implemented to solve the model, and obtain a near-optimum solution. In this algorithm, a set of possible cut-off grades and production rates were generated. For each of these combinations, NPV, as the objective value was computed, the cut-off grade and production rate corresponding to the maximum NPV value were reported as the near-optimum solution. The scheme of the research methodology has been shown in Figure 1.

3.1. Notations

The notations used in the current model are as follows:

Decision Variables

g: Cut-off grade

q: Production rate (Mt/y)

Dependent Variables

G_g: Average grade of the reserve assuming cut-off grade g such that $G_g = f(g)$ & $G_g > 0$

R_g: Total tonnage of the reserve assuming cut-off grade g (tone) such that $R_g = f(g)$ & $R_g > 0$

C_q^{oc}: Operating cost (mining and milling) for the production rate of q ($/tone) such that $C_q^{oc} = f(q)$ & $C_q^{oc} > 0$. The operating costs of mining include both the expenses associated with ore and waste extraction, i.e. the stripping ratio is involved.

C_q^{cc}: Capital cost (mining and milling) for the production rate of q ($) such that $C_q^{cc} = f(q)$ & $C_q^{cc} > 0$

N: Years of production such that $N = \frac{R_g}{q}$.

Parameters

P: Price of commodity

ρ: Recovery of the operations (mining and milling)

ξ : Discount rate

C^{max} : Maximum available capital spending

Objective Function

Maximize

$$\left(\sum_{n=1}^{N} \frac{q \times P \times G_g \times \rho - q \times C_q^{oc}}{(1+\xi)^\wedge n}\right) - C_q^{cc} \quad (1)$$

The above objective function is seeking a cut-off grade and a production rate that maximizes the NPV.

subject to

$$C_q^{cc} < C^{max} \quad (2)$$

According to this constraint, the capital cost must remain below the maximum allowable amount. As highlighted by Smith, maximizing NPV will lead to a very high and impractical production rate, requiring a remarkable capital investment. This constraint is applied to address this shortcoming [2].

$$\mathrm{N} \geq N_{min} \quad (3)$$

Based on this constraint, the mine life must always be higher or equal to an acceptable minimum. The minimum life of the mine must be determined in advance according to the experimental and analytical criteria.

$$q > 0 \ \& \ g > 0 \quad (4)$$

Assumption

The production rate of the mining operation and the mineral processing plant are equal, limiting the amount of ore extraction to the plant's needs, and there is no stockpile.

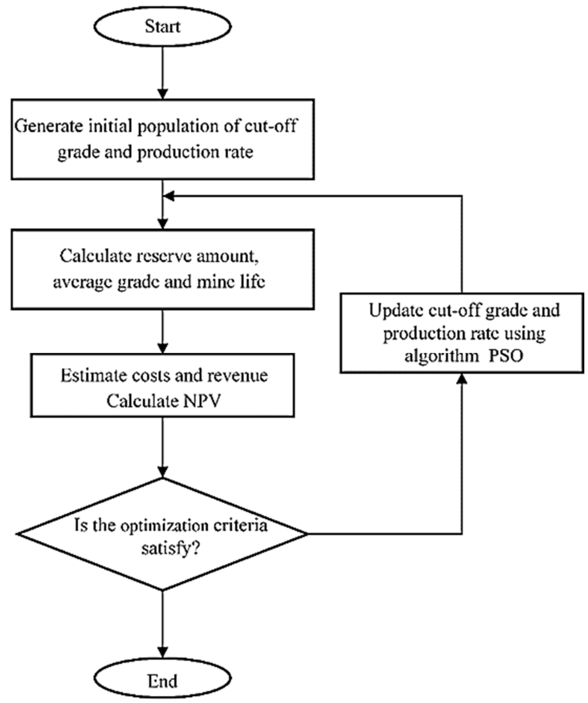


Figure 1. The scheme of this study`s methodology.

3.2. Particle swarm optimization algorithm

Particle swarm optimization (PSO) is inspired by the collective behaviors of birds and fish. This approach is characterized by a population-based methodology with random weighting. Its optimization capability arises from localized interactions between the individuals. In PSO, swarm particles collaborate to explore and exploit the search space, aiming to reach the global optimization solution. The fundamental premise of PSO is that each particle within the swarm, exploring, and exploiting the search space retains knowledge of its initial velocity, having the best local position, and the best global position within the entire swarm. Leveraging this information, each swarm member continuously updates its velocity and position. Unlike the conventional methods, PSO can efficiently tackle various problem types with minimal or no need for adaptation, as it does not rely on the problem-specific features. However, it instead employs a parallel, cooperative exploration of the search space through a population of individuals [53].

In the context of a d-dimensional search space with N swarm particles, where the position attribute of the ith particle is denoted by $X_i = (x_{i1}, x_{i2}, \dots, x_{id})$, and the velocity attribute is denoted by $V_i = (v_{i1}, v_{i2}, \dots, v_{id})$, the velocity attribute reflects the particle's speed at the ith position. This velocity is updated using the individual experienced local-best position attribute, given by $P_{Li} = (P_{Li1}, P_{Li2}, \dots, P_{Lid})$. The best position is represented by $P_g = (P_{g1}, P_{g2}, \dots, P_{gd})$. Each swarm particle adjusts its velocity and position in the search space, moving toward the global minimum position. This adjustment is governed by the following equations:

$$\mathrm{v}_{\mathrm{i(k+1)}} = \omega \times \mathrm{v}_{\mathrm{ik}} + \mathrm{c}_1 \times rand_1 \times (P_{Li} - x_{ik}) + c_2 \times rand_2 \times \left(P_{gi} - x_{ik}\right) \quad (5)$$

$$x_{i(k+1)} = x_{ik} + v_{i(k+1)} \quad (6)$$

where k varies from 1 to d, and ω is the inertia parameter coefficient. c_1 and c_2 are parameter coefficients for acceleration, having values higher than zero. In this work, these parameters and the size of particles were calculated based on the Taguchi method [54] and were chosen equal to 0.2, 0.8, 0.2, and 1000, respectively. $rand_1$ and $rand_2$ are the randomly generated values between 0 and 1. The individual experience, i.e. cognitive term is determined by term $\mathrm{c}_1 \times rand_1 \times (P_{Li} - x_{ik})$, whereas the social term due interaction among the particles is determined by term $c_2 \times rand_2 \times \left(P_{gi} - x_{ik}\right)$.

The cognitive term assists particles in exploring the search space, while the social term aids in exploiting the search space. As indicated in Equations (4) and (5) above, it is evident that swarms continually alter their positions by adjusting their instantaneous velocities, relying on the information about their own previously optimum positions and the overall group's best position [55].

The mathematical model of this study included an objective function that was non-linear and depended on two decision variables, namely the cut-off grade and the production rate. Additionally, the total mineable reserve, the years of production, the average grade, and capital and operating costs changed due to variations in these decision variables. Therefore, five dependent variables were considered in this work that must be computed when the value of the decision variables was known. Accordingly, this problem was presented as a constrained non-linear optimization problem with a non-linear dependency relationship between the decision variables. To the best of our knowledge, there were no exact solution techniques available for this type of problem. Therefore, a metaheuristic technique was necessary for a quick and efficient solution. As the decision variables were continuous in this problem, the Particle Swarm Optimization (PSO) technique was used. PSO is a population-based optimization algorithm inspired by the social behaviors exhibited by birds, fish, and insects in dynamic communication. Developed and introduced by James Kennedy and Russell Eberhart in 1995, the technique has since been successfully applied to a variety of optimization problems including those that are constrained and non-linear [56].

In summary, our proposed model provided a static cut-off grade calculated for a single period. In more advanced models and with minor changes, this model could make it possible to calculate a multi-period cut-off grade, where the cut-off grades changed over time. Despite these changes, the current methodology and most similar studies were based on a single grade-tonnage curve that ignored the real spatial distribution of ore tonnage in the mine. A more accurate calculation could still be achieved by considering the exact grade-tonnage distribution using a block model. This sophisticated cut-off grade optimization represents the next step of our current work. Specifically, the current metaheuristic has been developed for integration into future advanced simultaneous production scheduling and cut-off grade optimization, which involves mathematically complex soft computing techniques. However, in this paper only the nonlinear relationship between production rate and cut-off grade optimization, the particle swarm algorithm and economic functions as the core engine of a sophisticated dynamic optimization strategy has been investigated.

Algorithm 1. Pseudo code of the PSO algorithm.

Input: Economic and technical parameters

Initialize a population (swarm) of particles with random initial positions $x_{i,0}$ (including cut-off grade, production rate and life of mine) and random velocities $v_{i,0}$ in the search space. The random values are within the acceptable ranges for these parameters.

Initialize each particle's personal best position ($Pli_{1,0}$) to $x_{i,0}$ as its initial position.

Calculate the fitness value of each particle (NPV from Equation 1) at its initial position $x_{i,0}$ and determine the initial global best position (Pgi).

While (k < maximum number of generations)

    For all particles do

        Update the particle's velocity and position using Equations [5] and [6].

            Calculate the fitness value of each particle at its current position $x_{i,k}$

            If fitness ($x_{i,k}$) is better than the fitness $Pl_{i,k-1}$

                $Pl_{i,k} \leftarrow x_{i,k}$

            End If

            If fitness $Pl_{i,k}$ is better than the fitness Pg_{k-1}

                $Pg_{k} \leftarrow Pl_{i,k}$

            End If

    End For

End While

4. Case Study

In this research work, the Sungun copper deposit, the second-largest copper mine of Iran was studied. It is known for its substantial copper reserves and relatively high production capacity. The Sungun copper deposit is known as porphyry copper mineralization, and is mined through the open-pit method [57].

While designing a deposit, several cut-off grade values are considered. Based on the grade distribution curve, the corresponding ore reserve tonnage and the average grade are determined for a specific cut-off grade. Therefore, for the cut-off grade as a variable, the average grade and ore reserve tonnage could be represented as a grade-tonnage curve. Figure 2 shows the grade-tonnage curve for the Sungun reserve. For example, assuming a cut-off grade of 0.25% Cu, the total ore tonnage was 420 Mt with an average grade of 0.58%. The best fits were computed below to use this curve in the optimization model:

Average grade function:	$G_g = f_1(g) = 1.011g + 0.325$	(7)
Reserve function (Mt):	$R_g = f_2(g) = 896.67 \times EXP(-3.062 \times g)$	(8)

Since the change in the production rate affected the operating and capital costs, regression models were developed using cost data from COSTMINE [58]. The purpose was to relate the production rates to these costs. These models and the fitted curves have been shown in Figures 3 and 4. The increased production rate resulted in a reduction in the unit operating costs and enhanced the economies of scale. However, the higher production rate also necessitated additional equipment and machinery, leading to an increase in capital costs.

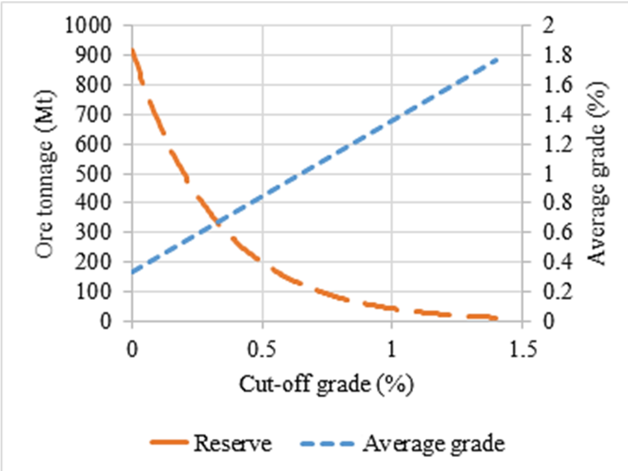


Figure 2. Grade-Tonnage curve.

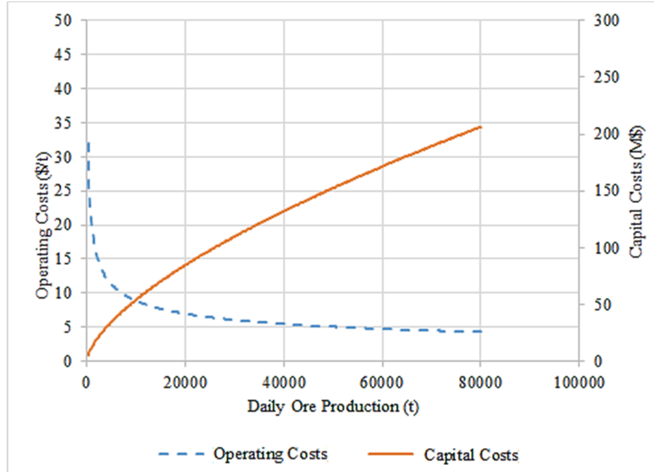


Figure 3. Mining costs versus production rate.

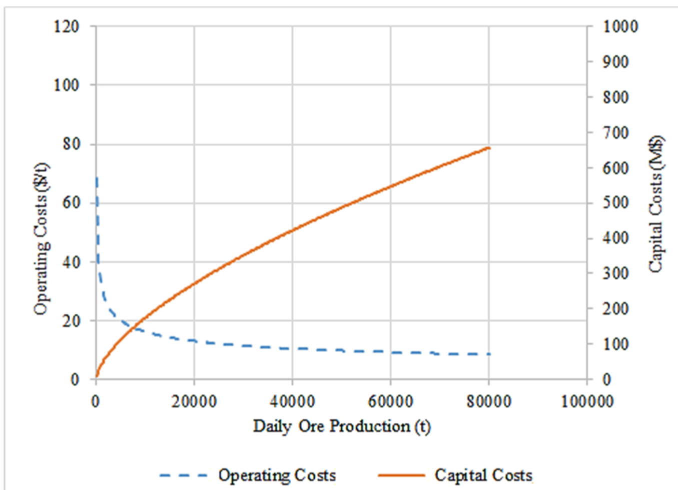


Figure 4. Milling costs versus production rate.

According to previous studies, the technical and economic parameters were determined, as reported in Table 1.

Table 1. Input data.

Parameter	Symbol	Unit	Amount
Copper price	P	USD/ton	7500
Maximum capital cost	C^{max}	Billion dollars	1
Recovery	ρ	Percentage	80
Discount rate	ξ	Percentage	10
Number of working days per year	-	-	260
Reserve	R_g	Mt	897

5. Results and Discussion

In this paper, calculating the optimum cut-off grade, and the production rate for open-pit mines were studied through developing a mathematical model and implementing a solution technique on a real mine. The current case study involved a copper mine with a linear grade distribution. The final product was cathode copper, assuming a constant and deterministic product price in the objective function.

The PSO algorithm was implemented in the MATLAB software to solve this problem and maximize the NPV, enabling the simultaneous optimization of both the cut-off grade and production rate. In the PSO algorithm, the particle velocity in each iteration consists of two components: the first component is the current velocity of the particle, and the second component is the weighted sum of the particle's personal best position, and the best position among its neighbors in the search space. Without the second component, the algorithm would only perform a local search around the best particle, and could not explore large regions of the search space. Conversely, without the first component, the algorithm would lose its ability to converge to a solution. By combining these two components, the PSO algorithm of the study attempted to balance between the local and global searches. One thousand particles (representing the cut-off grade and production rate in this optimization problem) were randomly selected to optimize the function. The inertia weight was set to 0.8. Additionally, the C1 and C2 parameters were adjusted to generate multiple solutions during program execution and evaluate the results. Although it is not critical for PSO convergence, setting these parameters could expedite convergence. After several program executions, it was achieved that setting C1 = C2 = 0.2 was more helpful to obtain a better solution. The termination criterion of the algorithm was set to 1000 iterations. The model`s output variables included cut-off grade, production rate, average grade, mine life, and other variables, as summarized in Table 2. The results indicated that with a cut-off grade of 0.64% and a production rate of 15.66 Mt/y, the NPV was maximized at 1.153 billion dollars. It was the economic cut-off grade, covering both capital and operating costs for mining and processing. This result varied from the marginal cut-off grade to satisfy only processing operating costs. Figure 5 shows the optimization of NPV using the PSO algorithm across different iterations. All computations were performed on an ASUS K52J with five 2.53 GHz cores and 8 GB RAM. The computational time was about 30 seconds.

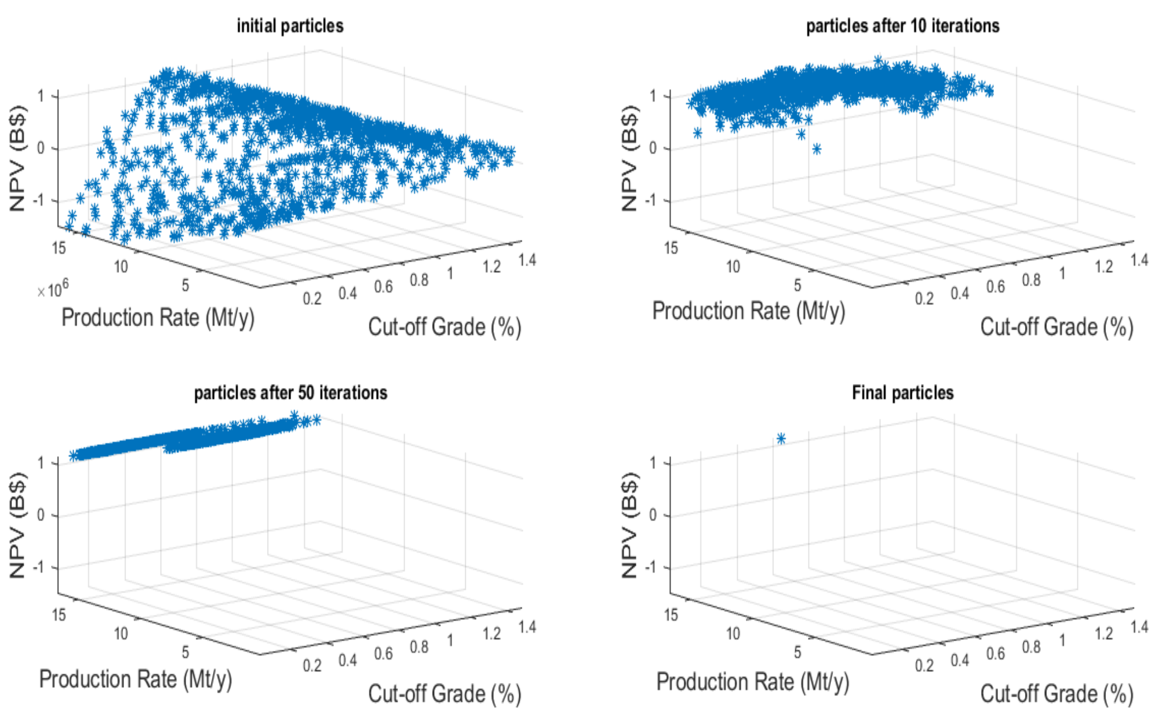
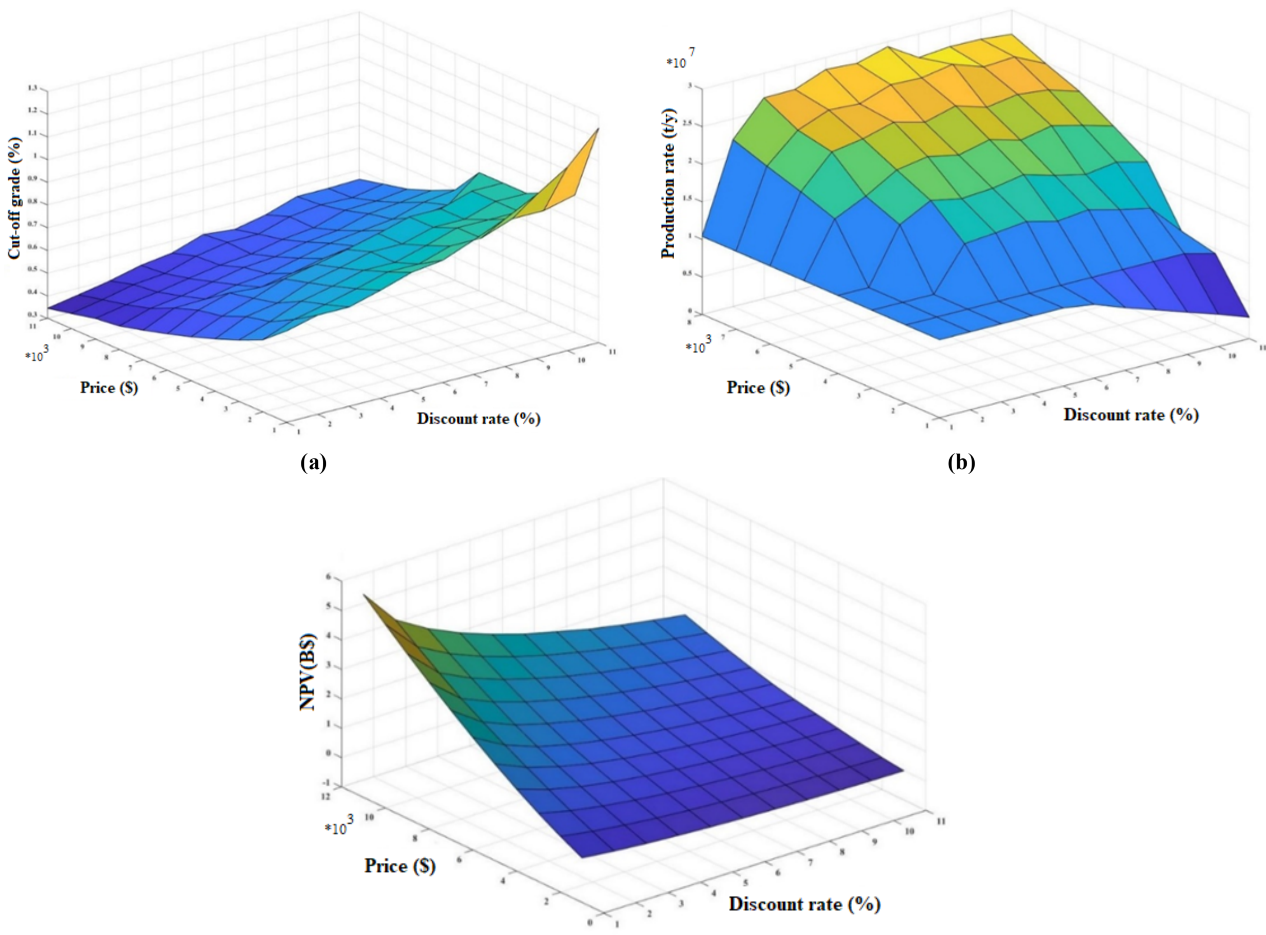


Figure 5. Optimization of NPV using the PSO algorithm.

Table 2. Model output.

Variable	Unit	Amount
Cut-off grade	Percent	0.64
Production rate	Mt/y	15.66
NPV	M$	1153
Mining operating cost	$/t	5.13
Processing operating cost	$/t	11.52
Total capital cost (mining and processing)	M$	936
Average grade	Percent	0.97
Ore tonnage	Mt	128

In the following section, a sensitivity analysis was conducted on various parameters. The metal price varied between 5,000 and 10,000 dollars, and the discount rate varied between 0 to 20 % to analyze their sensitivity. As shown in Figure 6, in general, as the metal price increased and the discount rate decreased, the cut-off grade decreased (Figure 6a), while the production rate (Figure 6b) and NPV (Figure 6c) increased.



(c)

Figure 6. Price and discount rate sensitivity analysis: a) Cut-off grade, b) Production rate, c) NPV.

The region's inadequate infrastructure may cause a challenge for the investors due to the significant capital costs required for high production rates. Therefore, considering constraint (2), the impact of the maximum capital cost (C^{max}) was explored by changing C^{max} from 0.1 to 2 billion dollars. As shown in Figure 8, the cut-off grade decreased with an increase in C^{max}, until a certain point where it remained constant. Conversely, the production rate and NPV increased with an increase in C^{max}, until they reached a constant level, one billion dollars in this case.

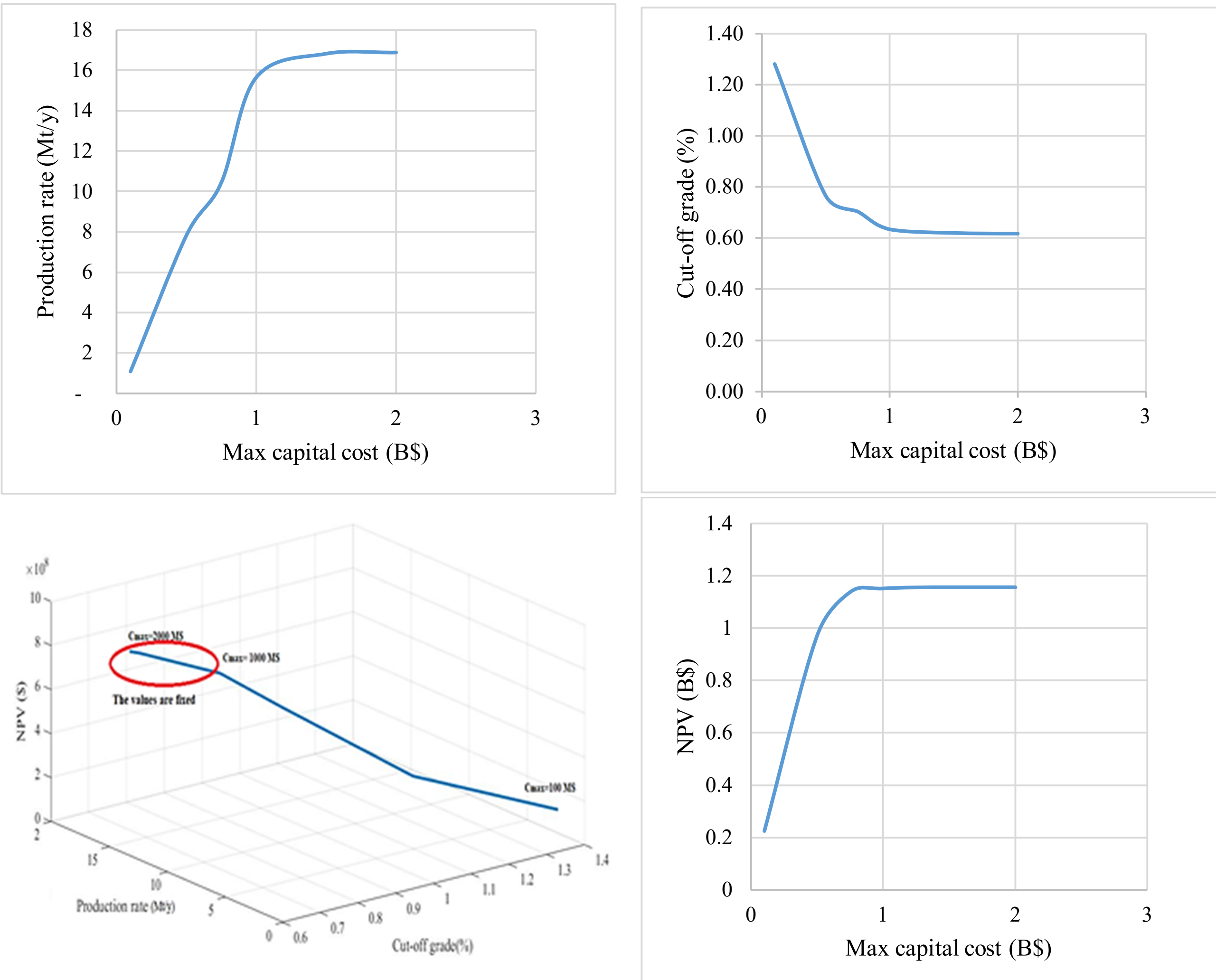


Figure 7. Sensitivity analysis of the maximum capital cost (C^{max}).

From Figure 8, the optimum cut-off grade for maximum NPV was 0.64%, representing the balance point between the mineable reserve and the profit margin. When the cut-off grade increased to higher values (such as 0.9%), the amount of the mineable reserves was decreased, resulting in a lower production rate and NPV. Conversely, the lower cut-off grades (such as 0.5%) led to a reduction in the average grade and, subsequently, a decrease in the profit margin.

Furthermore, the reduction rate in NPV after its peak was more pronounced for a cut-off grade of 0.9% compared to the cut-off grade of 0.5%. When the cut-off grade was 0.9%, the mineable reserve decreased significantly, and the NPV remained sensitive to the changes in production rate. Furthermore, at a cut-off of 0.5%, the cost-saving benefits of economy of scale in both capital and operating costs were substantial due to the high volume of mineable reserve.

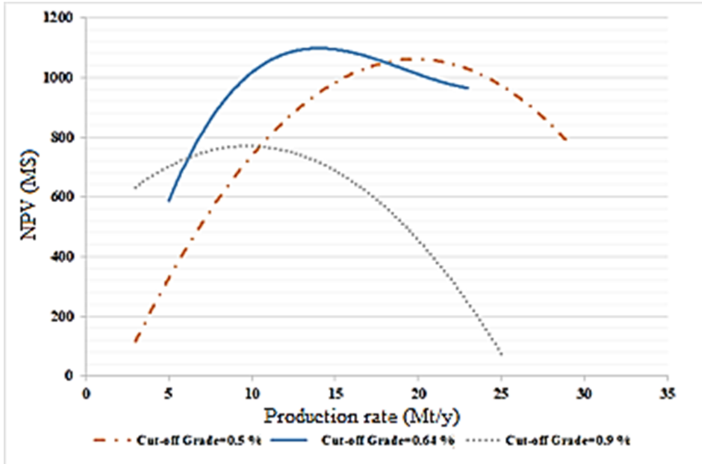


Figure 8. Sensitivity analysis of the production rate and cut-off grade.

6. Conclusions

Addressing the challenges to optimize the cut-off grade and production rate in open-pit mine planning was the essential objective of this work. Both mentioned variables were considered simultaneously to achieve an optimum result, with the outcomes heavily reliant on significant parameters used to construct the model. These parameters included deposit characteristics, mining and processing cost relationships, product price, and time value of money. In this context, a novel framework was proposed to use the computationally efficient particle swarm optimization algorithm to solve the long-term production scheduling problem in open-pit mines. The PSO algorithm showed a reliable ability to solve non-linear and continuous problems, and could effectively solve this model. The data from an open-pit copper mine containing approximately 897 million tons was used to investigate the efficiency of the proposed model. Our findings showed that for a cut-off grade of 0.64% and a production rate of 15.66 Mt/y, the NPV could be maximized to $1.153 billion. This cut-off grade represented the economic threshold, covering both capital costs and operational expenses during mining and processing. The average grade and reserves were computed from the tonnage-grade curve to simplify the problem. A fixed product price was assumed without considering its fluctuations. It is suggested to develop a block model approach and incorporate price uncertainty for future studies to improve the robustness of the results. These studies can lead to a more accurate representation of the deposit's characteristics and improve the reliability of the economic analysis.

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بهینه‌سازی هم‌زمان نرخ تولید معدن و عیارحد با استفاده از الگوریتم بهینه‌سازی ازدحام ذرات

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چکیده:

نرخ تولید و عیار حد دو متغیر تاثیرگذار اصلی در طراحی و برنامه‌ریزی تولید معادن روباز هستند. به طور کلی، نرخ تولید به میزان ذخیره بستگی دارد که خود نیز تحت تأثیر عیار حد است. از طرف دیگر، عیار حد نیز به هزینه تولید وابسته است، که به نوبه خود تحت تأثیر نرخ تولید و قیمت محصول قرار دارد. رویکردهای سنتی معمولاً این متغیرها را به صورت جداگانه بهینه‌سازی می‌کنند و ارتباط متقابل بین نرخ تولید و عیار حد را نادیده می‌گیرند. این امر منجر به راه‌حل‌های غیر بهینه می‌شود. این پژوهش بر بهینه‌سازی همزمان نرخ تولید و عیار حد متمرکز بوده و راه‌حلی نوآورانه برای این مسئله ارائه کرده است. در این راستا، یک مدل ریاضی غیرخطی توسعه داده شد. به دلیل ماهیت غیرخطی مدل و متغیرهای تصمیم‌گیری پیوسته، از الگوریتم بهینه‌سازی ازدحام ذرات استفاده شده است. اجرای این مدل برای یک معدن مس نشان داد که مدل پیشنهادی امکان بهینه‌سازی همزمان نرخ تولید و عیار حد را فراهم می‌سازد. بر این اساس، بیشینه ارزش خالص فعلی به میزان ۱.۱۵۳ میلیارد دلار در نرخ تولید ۱۵.۶۶ میلیون تن در سال و عیار حد ۰.۶۴ درصد به دست آمد. علاوه بر این، تحلیل حساسیت برای عوامل کلیدی نظیر قیمت محصول، نرخ تنزیل و حداکثر مقدار سرمایه در دسترس انجام شد.

کلمات کلیدی: عیار حد، نرخ تولید، بهینه‌سازی ازدحام ذرات، معدنکاری روباز.