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Estimation of ore grades using Archimedean copulas in a copper deposit in Peru

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Abstract

Traditional geostatistical methods such as kriging exhibit limitations by assuming linear and symmetric dependencies, which can lead to smoothed estimates and the loss of local variability. To address these issues, this study applies Archimedean copulas (Clayton, Gumbel, and Frank) for the estimation of copper ore grades in a deposit located in Peru. A total of 5,654 composites, each 15 meters in length, were obtained from 185 diamond drill holes. The data were transformed to a uniform scale to allow for copula fitting. Dependence structures were modeled by lag distance, with the dependence parameter fitted using fifth-degree polynomials, and three-dimensional conditional estimation was implemented. Results indicate that ordinary kriging yielded RMSE = 0.161, MAE = 0.104, R2 = 0.692, and a correlation of 0.861. The Clayton copula slightly improved these metrics (RMSE = 0.154, MAE = 0.101, R2 = 0.717, R = 0.871), while the Gumbel copula captured higher local variability (RMSE = 0.161, MAE = 0.116, R2 = 0.692, R = 0.855). The Frank copula achieved the best performance with RMSE = 0.137, MAE = 0.090, R2 = 0.778, and R = 0.905. In conclusion, Archimedean copulas significantly enhance geostatistical estimation by better capturing spatial dependence, offering a robust alternative to classical geostatistical methods.

1. Introduction

In geostatistics, a fundamental challenge lies in using a limited set of spatial observations to predict the value of a variable at unsampled locations. For decades, kriging has been the conventional tool to address this problem, providing an unbiased estimate of the weighted average of the samples [1]. In this technique, the weights assigned to each sample are determined by minimizing the estimation variance, which depends on the spatial configuration of the samples and the covariance model that describes the spatial dependence structure of the variable [2]. However, by representing this dependence solely through covariance function, kriging introduces an oversimplification that may be inadequate for natural phenomena characterized by complex or nonlinear spatial structures [3]. Moreover, a significant limitation of kriging is that the estimation variance is independent of the observed values. That is, replacing an extreme value with one closer to the mean does not affect the resulting estimation variance, which may reduce the model's ability to accurately reflect the true variability of the phenomenon [4]. These shortcomings have prompted the search for alternative approaches capable of more effectively representing spatial dependence, particularly when attempting to incorporate both the sampled values and their spatial relationships.

A powerful alternative to traditional covariance-based models are copulas functions that allow the construction of joint distributions from univariate marginal distributions transformed into uniform variables over the [0,1] interval. Copulas have

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gained widespread acceptance in disciplines such as finance, civil engineering, and hydrology due to their ability to capture nonlinear dependencies and asymmetric tail behaviors between variables [5–9]. Various families of theoretical copulas exist, including elliptical copulas such as the Gaussian and Student's t copulas [10–13], and Archimedean copulas, a particularly attractive class due to their mathematical simplicity and their capacity to model asymmetric dependencies [14, 15]. This class includes the Gumbel-Hougaard [16], Clayton [17], Ali-Mikhail-Haq [18], Frank [19], and Joe [20] copulas, all of which are constructed from a generator function known as the Archimedean generator [21]. Marshall and Olkin [22] used inverse Laplace transformations to construct these generators, and Genest and MacKay [23] later expanded the theoretical framework by allowing more general conditions for Archimedean generators. The application of copulas has been extended to the field of geostatistics, notably with the spatial copula proposed by Bárdossy [24], which marked a milestone by allowing both point distances and sample values to influence the estimation weights. Additional developments include chi-squared copulas [25], pair-copula constructions [26–28], v-transformed copulas [29], and convex combinations of Archimedean copulas [3, 30, 31].

The incorporation of copulas into geostatistics has been explored in various studies with promising results. Kazianka and Pilz [32] applied copulas for the spatial interpolation of primary variables, overcoming kriging's limitations in representing uncertainty, which in this case depends not only on the spatial configuration but also on the observed values themselves [33, 34]. In recent years, multiple authors have expanded and refined the application of copulas to geostatistical problems. Sohrabian [31] proposed the use of convex combinations of Archimedean copulas instead of bivariate Gaussian functions to improve spatial interpolation. [35] integrated categorical information and secondary data into a Gaussian copula interpolation model applied to groundwater monitoring. [36] developed a copula-based multiple-indicator kriging model for non-Gaussian spatial data. Dinda and Samanta [29] evaluated the efficiency of copula-based simulations for resource estimation, showing advantages over disjunctive and multigaussian kriging. Kazianka and Pilz [37] applied copulas in environmental monitoring to model the probability of exceeding critical thresholds.

In mining, the use of copulas has also shown significant progress. Addo et al. [38] applied D-vine copulas to predict copper recovery from indirect testing. Dinda et al. [39] used v-transformed copulas in combination with MCMC simulation for lithological classification in a copper deposit in India. Sohrabian and Tercan [40] proposed a new multiple-point simulation method based on copulas for deposits with irregular drill spacing, and in subsequent work [41], they demonstrated the effectiveness of Gaussian copulas over traditional techniques such as ordinary kriging. Hernández et al. [42] applied copulas to the estimation of metallurgical recovery using conditional quantile regression. Krysa et al. [43] modeled the economic value of copper and silver considering local variability through copulas. Xu and Zhu [44] introduced a Hubbert-type copula for modeling mineral co-production. Furthermore, [45] applied linear combinations of Archimedean copulas (LCAC) to estimate copper grades in Iran, and Akbari Gharalari et al. [46] demonstrated that the use of Archimedean copulas in grade estimation allowed for a greater number of blocks to be classified as high-quality reserves compared to ordinary kriging. Recent efforts have also focused on addressing spatial uncertainty in resource classification and mine design. Sotoudeh et al. [47] employed sequential Gaussian simulation in underground stope layout optimization, showing that accounting for grade uncertainty via simulation significantly enhances decision-making robustness. Parhizkar et al. [48, 49] proposed probabilistic models for improving grade reconciliation, identifying systematic and random uncertainties as primary drivers of discrepancies between estimated and actual grades.

Machine learning approaches have further advanced lithological classification and grade modeling. Farhadi et al. [50] evaluated ensemble models such as stackingC and Adaboost, reporting outstanding classification performance based on geochemical data. Fathi et al. [51] introduced a hybrid approach combining extreme learning machines and particle swarm optimization, which achieved highly accurate grade estimation results in an Iranian iron ore deposit. The integration of geophysical data in geostatistics has also proven beneficial. Afzal et al. [52] used IP and resistivity data in combination with drilling and applied co-kriging and regression models, demonstrating that IP data substantially improves estimation accuracy and reduces borehole requirements. Additional studies have shown that using auxiliary variables, such as magnetic susceptibility, in co-kriging and co-simulation can reduce estimation error and improve classification reliability, as observed in the Darreh-Ziarat iron deposit [53]. In addition, a combination of drilling data with induced polarization (IP) and electrical resistivity (Rs) data was used to estimate grades and mineral resources at the Abassabad copper mine in Iran [54].

Simulation-based techniques have been further explored by Monjezi et al. [55], who compared SGS and OK and found that conditional simulation better reproduced the true grade distribution, yielding more realistic mineable volumes and economic outcomes. Tahernejad et al. [56] confirmed that incorporating grade uncertainty through simulation reduces financial risk in open-pit planning. In the field of intelligent exploration, Ghasemitabar et al. [57] developed a borehole simulation framework using Python, clustering algorithms, and gradient boosting, achieving over 91% accuracy in ore block prediction while reducing exploratory costs.

Despite the growing interest in the application of copulas in geostatistics and mining, there remains a lack of comparative studies that evaluate the performance of different Archimedean copulas for ore grade estimation under real-world metallic deposit conditions. It has not been rigorously documented whether copulas such as Clayton, Frank, and Gumbel offer concrete statistical advantages over ordinary kriging in terms of estimation accuracy and the ability to capture nonlinear spatial dependence. This lack of evidence limits the adoption of these models in mining practice. In response, the present study applies and compares ore grade estimation using three classical Archimedean copulas and ordinary kriging in a copper deposit in Peru, evaluating their performance through quantitative metrics and providing empirical evidence on the applicability of Archimedean copulas in mining geostatistics.

The remainder of this article is structured as follows. Section 2 describes the study methodology. Section 3 presents the results and discussion based on the case study of a copper deposit in Peru. Finally, Section 4 provides conclusions and recommendations for future research.

2. Methodology

2.1. Copula fundamentals

Copulas are mathematical functions that enable the modeling of dependence between random variables independently of their marginal distributions. Formally, if Z is a continuous random variable with cumulative distribution function F_Z, its transformation to a uniform scale is defined as show in Eq. (1) [20]:

$$U = F_Z(Z) \tag{1}$$

Under this transformation, the copula $C(u_1, u_2)$ describes the dependence structure between two variables through the relationship, as shown in Eq. (2):

$$F(Z_1, Z_2) = C\big(F_Z(Z_1), F_Z(Z_2)\big) = C(u_1, u_2), \tag{2}$$

where Z_1, Z_2 are continuous random variables, F_Z are their marginal distribution functions, and u_1, u_2 are the transformed variables on the uniform scale. The copula also enables the derivation of the conditional density between variables, which is key in probabilistic estimation, as shown in Eq. (3):

$$f(u_2|u_1) = \frac{\partial C(u_1, u_2)}{\partial u_2}, \tag{3}$$

where $f(u_2|u_1)$ is the conditional density of u_2 given u_1. $\partial C/\partial u_2$ is the partial derivative of the copula with respect to u_2. This approach allows the modeling of dependence relationships without imposing linear or Gaussian assumptions, thereby overcoming the limitations of the classical covariance-based model [3, 24].

2.2. Spatial application of the copula approach

In the context of this study, the variable Z represents the copper grade recorded at different spatial locations within the deposit. To facilitate modeling via copulas, each value Z is transformed to the uniform scale U using its empirical distribution function. Subsequently, data pairs (U_1, U_2) are generated, where U_1 is the transformed value at a reference location, and U_2 is the value corresponding to a neighboring point separated by a lag distance h.

For each lag h, a copula is fitted to describe the dependence between the transformed values. The estimated conditional density function is defined as shown in Eq. (4):

$$f(U_2|U_1 = u_1; h) \tag{4}$$

This density represents the probability of observing U_2 at a target location given that U_1 is known at an adjacent location separated by distance h, thereby capturing the structure of conditional spatial dependence [31, 34].

2.3. Selection of Archimedean copulas

Archimedean copulas form a particularly useful family for spatial applications due to their simple construction using univariate generator functions and their ability to model nonlinear and asymmetric dependencies [14, 22, 23]. They are defined as shown in Eq. (5):

$$C(u_1, u_2) = \varphi^{-1}\big(\varphi(u_1) + \varphi(u_2)\big), \qquad (5)$$

where φ is a strictly decreasing generator function and φ^{-1} is its generalized inverse. In this study, three widely studied Archimedean copulas were selected for their ability to model different patterns of spatial continuity: Clayton, Gumbel, and Frank [45, 46].

The Clayton copula is well suited for modeling strong lower-tail dependence, meaning it is effective at capturing the joint occurrence of low values. Its parameter θ controls the strength of this dependence: as $\theta \to 0$, the copula approaches independence; larger values of θ indicate stronger lower-tail dependence. This copula does not capture upper-tail dependence, making it asymmetric [31, 46]. The Gumbel copula, by contrast, specializes in upper-tail dependence, capturing the tendency of high values to co-occur. The parameter θ begins at 1 (independence) and increases with dependence strength. It does not model lower-tail dependence, making it asymmetric in the opposite direction compared to Clayton [3]. The Frank copula is one of the few Archimedean copulas that is symmetric, i.e., it does not favor high or low values, making it suitable for situations where the dependence between variables is uniform across the value range. It is also flexible, as it can model both positive and negative dependence depending on the sign of θ [28, 29]. The key properties of the Archimedean copulas used in this study are summarized in Table 1.

Table 1. Properties of the Archimedean copulas used

Copula	$\varphi_\theta(t)$	θ	Copula function $C_\theta(u_1, u_2)$	λ_L	λ_U
Clayton	$\frac{1}{\theta}(t^{-\theta} - 1)$	$\theta \in (0, \infty)$	$C(u_1, u_2) = (u_1^{-\theta} + u_2^{-\theta} - 1)^{-1/\theta}$	$2^{-\frac{1}{\theta}}$	0
Gumbel	$(-\ln t)^\theta$	$\theta \in [1, \infty)$	$C(u_1, u_2) = \exp\left(-\left[(-ln\, u_1)^\theta + (-ln\, u_2)^\theta\right]^{\frac{1}{\theta}}\right)$	0	$2 - 2^{\frac{1}{\theta}}$
Frank	$-\ln\left(\frac{e^{-\theta t} - 1}{e^{-\theta} - 1}\right)$	$\theta \in \mathbb{R} \setminus \{0\}$	$C(u_1, u_2) = -\frac{1}{\theta} ln\left(1 + \frac{(e^{-\theta u_1} - 1)(e^{-\theta u_2} - 1)}{e^{-\theta} - 1}\right)$	0	0

where λ_L represents the lower tail dependence, λ_U the upper tail dependence, θ denotes the range of the dependence parameter, and $\varphi_\theta(t)$ is the generator function.

Although other Archimedean copulas such as Joe and Ali-Mikhail-Haq are theoretically valid options, they were excluded from this study based on both conceptual and practical considerations. The Joe copula, like Gumbel, captures only upper-tail dependence and would have introduced redundancy without offering additional modeling benefits. In contrast, the Ali-Mikhail-Haq copula has a limited range of dependence and demonstrated numerical instability during preliminary calibration tests with the dataset used. Therefore, the selection of Clayton, Gumbel, and Frank copulas was made to cover a wide range of dependence structures (lower-tail, upper-tail, and symmetric), ensuring mathematical tractability and alignment with the geostatistical characteristics of the data.

2.4. Prediction steps

After conducting a statistical analysis of the original database (composite samples) [58–60], the following steps were carried out:

i. Transform original data to the standard uniform scale. Each observed copper grade value $Z(x_i)$ was transformed to $u_i \in (0,1)$ using its empirical distribution function $U(x_i) = F_Z(Z(x_i))$.

ii. Define lag distances and tolerance. Lags h were defined from 10 m to 300 m in 10 meter increments, with a tolerance of ± 5 m. For each lag, all sample pairs $U_1, U_2)$ approximately separated by that distance were identified.

iii. Fit Archimedean copulas for each lag. For each lag distance, Clayton, Gumbel, and Frank copulas were fitted using maximum likelihood estimation. The dependence parameter $\theta(h)$ was estimated and interpolated using a fifth-degree polynomial, producing a continuous function $\theta(h)$ for each copula.

iv. Compute the conditional density at the estimation point. Given a target point x_0, the $k = 5$ nearest neighbors were identified. For each pair (u_i, u_o), the conditional density was evaluated using the interpolated $\theta(h)$ and the specific copula formula.

v. Integrate the density to obtain the expected value. The conditional density was numerically integrated over the uniform domain $u_o \in (0.0001, 0.9999)$. Using the inverse empirical quantile function $F^{-1}(u_0)$, the conditional expectation of copper grade from each neighbor was calculated, as shown in Eq. (6):

$$\hat{z}_{0i} = \int_0^1 F^{-1}(u_0) * f(u_0|u_i)du_0 \qquad (6)$$

vi. Estimate the final value using a weighted average. The conditional values $\hat{z}_{0i}$ were combined using an inverse distance-weighted average, as shown in Eq. (7):

$$\hat{Z}(x_0) = \sum_{i=1}^{k} w_i * \hat{z}_{0i}, \quad w_i = \frac{1/h_i}{\sum_{j=1}^{k} 1/h_j} \qquad (7)$$

The final estimated value was then back-transformed to the original data space, preserving the empirical distribution structure of the mineral grades in the deposit.

The workflow of the methodology is illustrated in Figure 1.

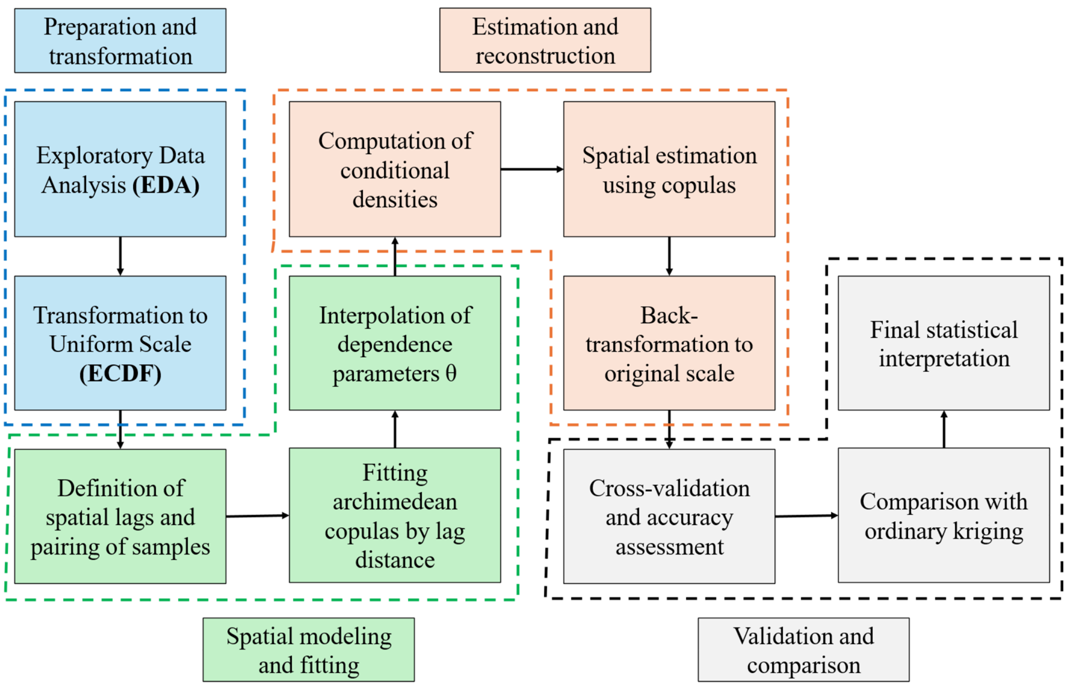

Figure 1. Research workflow

3. Results

3.1. Study area and geological context

The copper deposit analyzed in this study is located in the central highlands of Peru at an average elevation of approximately 4,600 meters above sea level. The geological framework of the area is complex, comprising five dominant lithological units: magnetite skarn, granodiorite, dacitic porphyry, calcareous sedimentary rocks, and volcanic formations. The mineralization is primarily composed of copper (Cu), accompanied in places by molybdenum (Mo). High-grade copper zones are predominantly hosted within the magnetite skarn and granodiorite units, while the dacitic porphyry, calcareous sediments, and volcanic rocks exhibit moderate to lower levels of mineralization. The deposit belongs to a porphyry-skarn metallogenic system, where mineralization occurs as disseminations, veinlets, and massive sulfide replacements, controlled by hydrothermal activity and local structural features.

3.2. Data preparation and transformation

This study was conducted in a copper porphyry deposit located in the central highlands of Peru, at approximately 4,600 meters above sea level. The study area exhibits complex geology, characterized by five main lithological units: granodiorite, dacitic porphyry, magnetite skarn, calcareous sediments, and volcanic units. The primary mineralization consists of copper (Cu) and molybdenum (Mo), predominantly hosted in the granodiorite and magnetite skarn, where the highest-grade zones are concentrated. The dataset comprises 185 diamond drill holes, drilled on an average spacing of 30 meters and reaching a maximum depth of 480 meters. From these drill holes, a total of 5,654 composite samples, each 15 meters long, were generated, as shown in Figure 2.

The histogram of copper grades in Figure 3 shows a positively skewed distribution, which is typical of geochemical data for mineral content [61, 62]. This asymmetry reflects the presence of multiple geological populations, including zones where mineralization is concentrated in narrow high-grade bodies, a pattern consistent with porphyry-style deposits [63].

Table 2 summarizes the descriptive statistics of the main variables. The mean copper grade is 0.43%, with a maximum of 2.95%, a standard deviation of 0.29, a variance of 0.08, and a skewness of 1.21.

To enable the application of the copula-based approach, copper grade data were transformed into a uniform distribution on the interval [0,1] using an empirical cumulative distribution function (ECDF), as stated by Sklar's theorem [20]. Figure 4 compares the original Cu distribution (left) with the transformed uniform distribution (right). This transformation eliminates skewness and ensures the data meet the uniformity requirement for copula modeling.

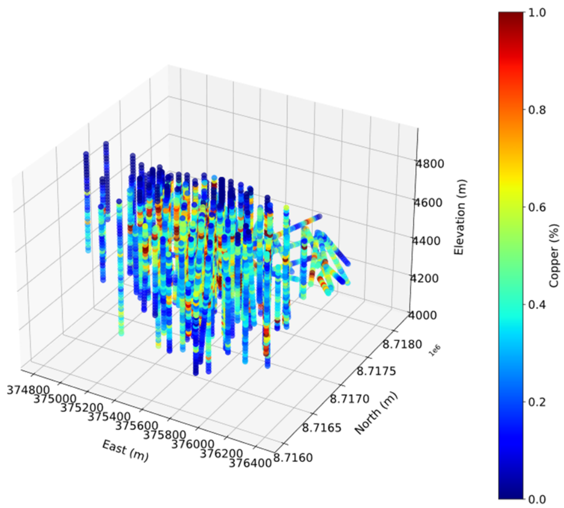

Figure 2. 3D map of drill hole locations

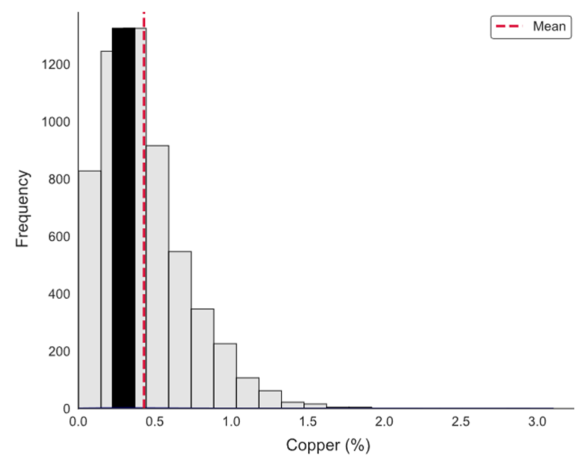

Figure 3. Histogram of 15 m Cu composite samples

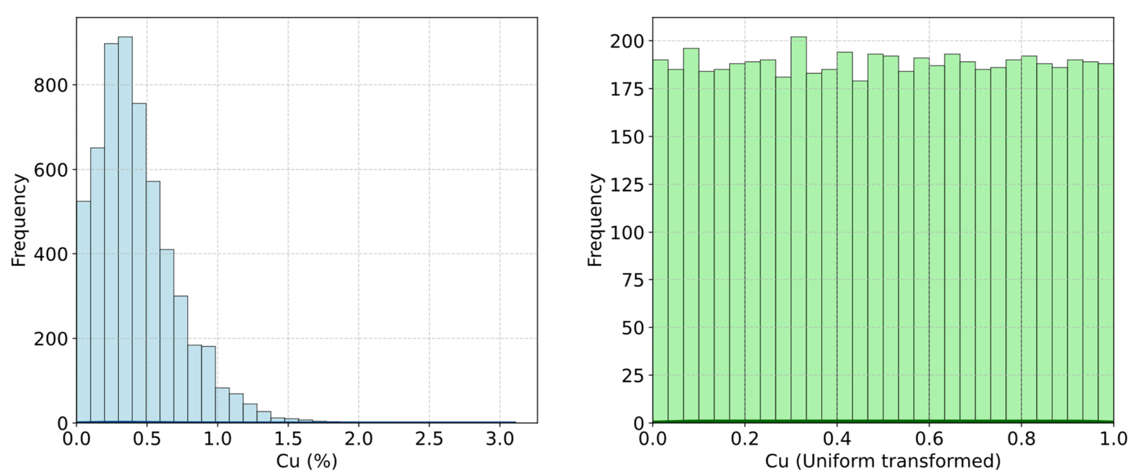

Figure 4. Copper grade distributions. Left: original Cu data. Right: transformed uniform Cu data

Table 2. Descriptive statistics for 15 m composites

Feature	East (m)	North (m)	Elevation (m)	Copper (%)
Count	5,654	5,654	5,654	5,654
Mean	375,606.25	8,717,015.68	4,473.54	0.43
Std dev	307.24	393.54	169.54	0.29
Minimum	374,821.06	8,716,003.08	4,050.35	0.002
25%	375,393.42	8,716,738.40	4,340.07	0.23
50%	375,602.29	8,716,995.80	4,462.81	0.38
75%	375,824.99	8,717,271.73	4,607.49	0.58
Maximum	376,414.81	8,718,153.15	4,902.14	2.95
Variance	94,394.22	154,875.70	28,743.49	0.08
Skewness	0.01	0.19	0.07	1.21

3.3. Spatial modelling and fitting

The analysis of spatial dependence in copper grades began with the construction of the omnidirectional experimental semivariogram. A lag structure of 15 intervals was defined, each 40 meters in length with a tolerance of ± 20 meters. The parameters of the fitted model are presented in Table 3. Figure 5 shows the model fit (red line) over the experimental values (black points), revealing that most of the spatial variability is concentrated within the first 150 meters.

Three-dimensional distances between sampling points were calculated using X, Y, and Z coordinates. Lags were defined from 10 to 300 meters in 10 m increments, with a tolerance of ± 5 meters. For each lag, scatter plots of paired values were generated (Figure 6). At short lags (30 to 80 m), a clear dependence is observed, indicated by the slope and clustering of points, which tends to diminish with increasing distance.

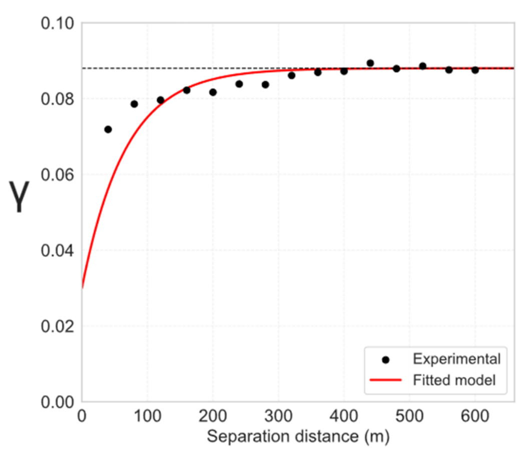

Figure 5. Experimental Cu semivariogram (black dots) with fitted model (red line)

To quantify this dependence, Clayton, Gumbel, and Frank copulas were fitted to each set of lagged pairs. The empirically obtained dependence parameters $\theta(h)$ are shown in Figure 7, revealing a systematic decrease of θ with distance, which reflects the progressive weakening of spatial dependence. This behavior was successfully modeled using fifth-degree polynomials, with excellent goodness of fit (R^2=0.98) in all three cases. Notably, the initial θ value for the Frank copula (>6) was significantly higher than for Gumbel or Clayton, suggesting a stronger initial spatial dependence captured by this copula.

Table 3. Fitted variogram parameters

Feature	Value
Model	Exponential
Nugget	0.03
Sill	0.058
Total sill	0.088
Range	200
Lags	15
Lag size	40
Tolerance	20

A deeper probabilistic interpretation was carried out through the estimation of the conditional density functions $f(u_2|u_1;h)$, evaluated for fixed values $u_1 = \{0.25, 0.50, 0.75\}$ across different lag distances. For the Gumbel copula (Figure 8), a distinct pattern of upper-tail dependence is evident. The density curves show that for high u_1 values (e.g., 0.75), there is a higher conditional probability of also observing high u_2 values. This behavior is consistent with spatial continuity in high-grade zones and aligns with the geological interpretation of the deposit, where copper-rich sectors are structurally clustered within mineralized domains. As the lag increases, the intensity of this dependence decreases gradually, but the structure remains visible up to at least 70 meters, supporting its applicability to conventional block modeling scales.

The Clayton copula (Figure 9) reveals a concentration of density in the lower tail. The curves show that when $u_1 = 0.25$, it is much more likely that u_2 will also adopt low values, reflecting the spatial clustering tendency of low-grade zones.

The Frank copula (Figure 10) exhibits a symmetric dependence structure. The curves for different u_1 values do not show a clear preference for upper or lower tails but do reveal a well-defined probabilistic interaction. This behavior translates into greater flexibility to model smooth transitions or zones lacking a dominant spatial trend, which is common in geologically heterogeneous contexts or gradual alteration zones. As suggested by Addo et al. [28, 64], the Frank copula is particularly suitable for deposit areas where mineralization transitions are gradual or lack directional anisotropy.

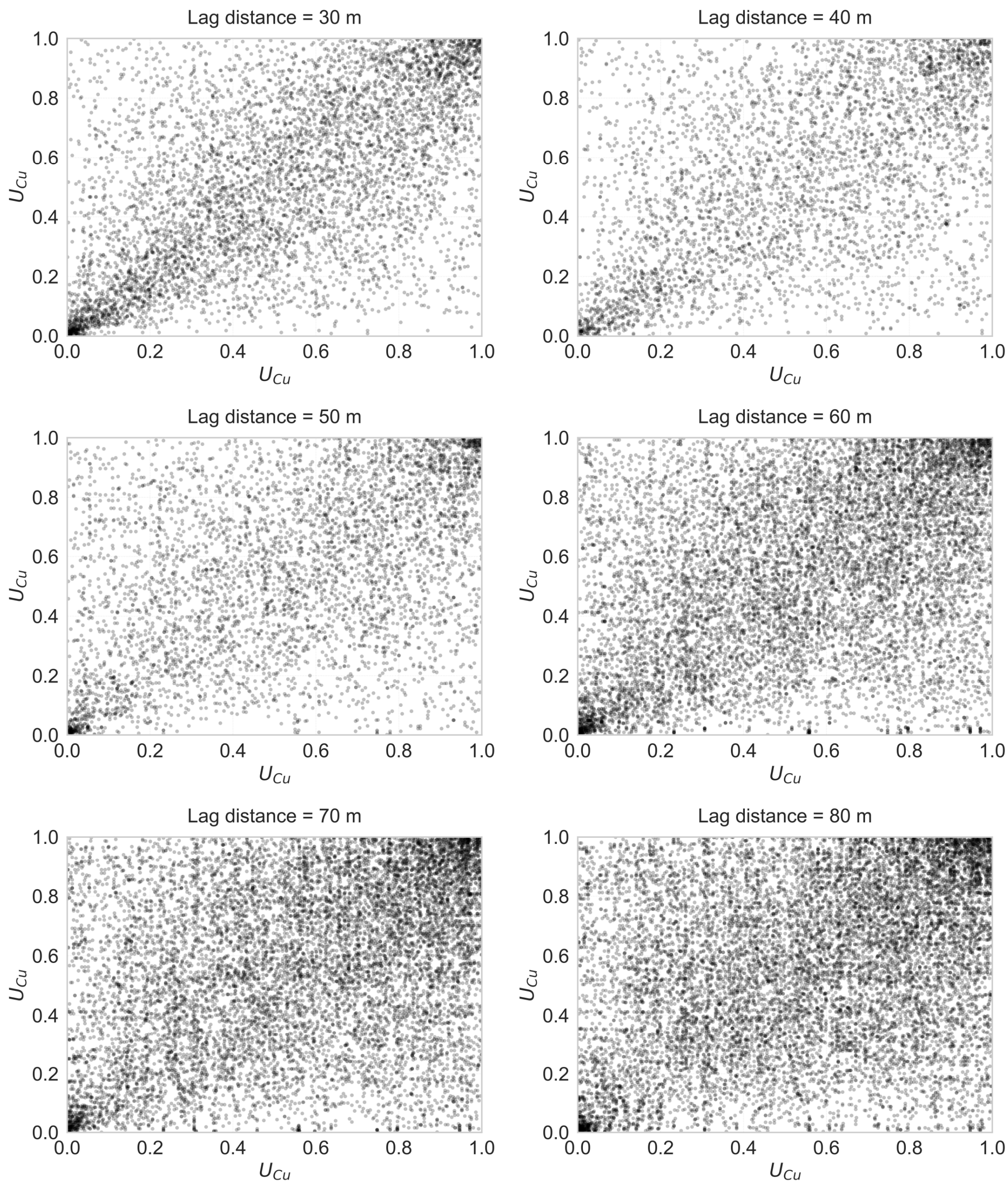

Figure 6. Lagged scatter plots of Cu at different distances

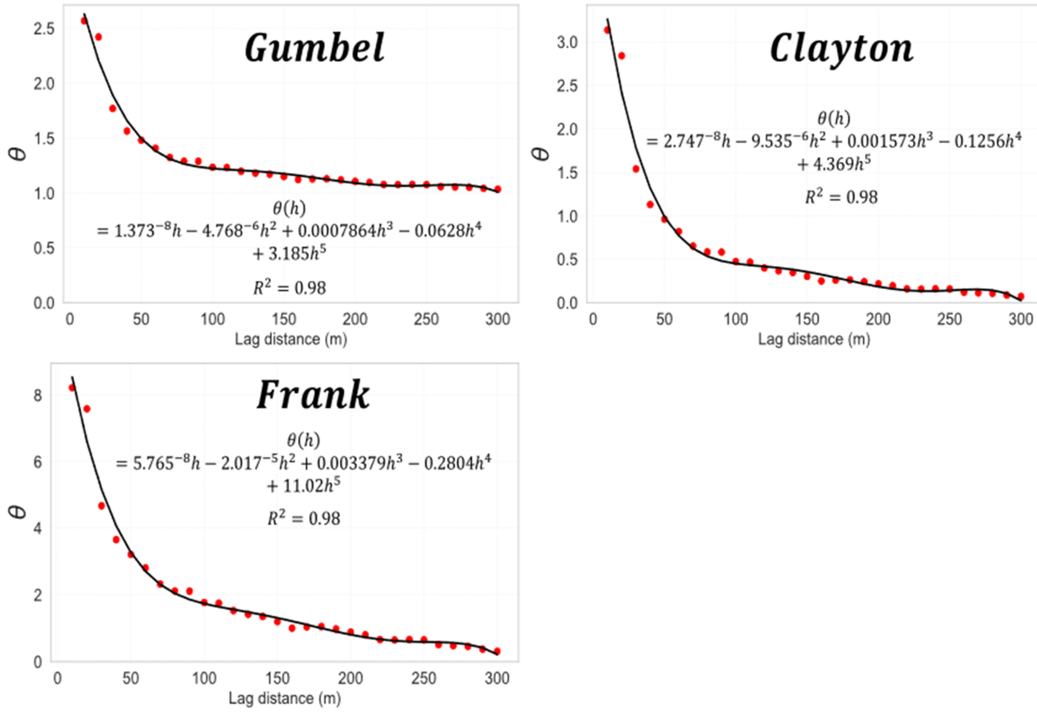

Figure 7. Best empirical copula parameters (red dots) at different lags and fitted models (black line)

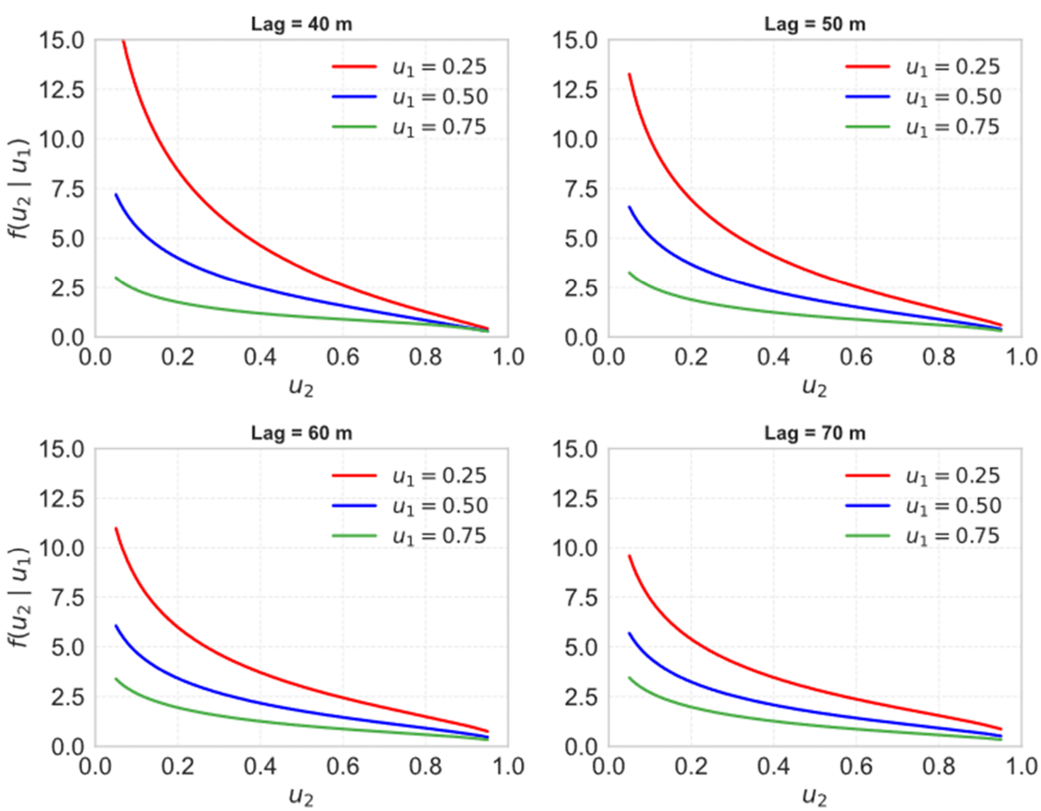

Figure 8. Conditional density of the Gumbel copula at different lag distances

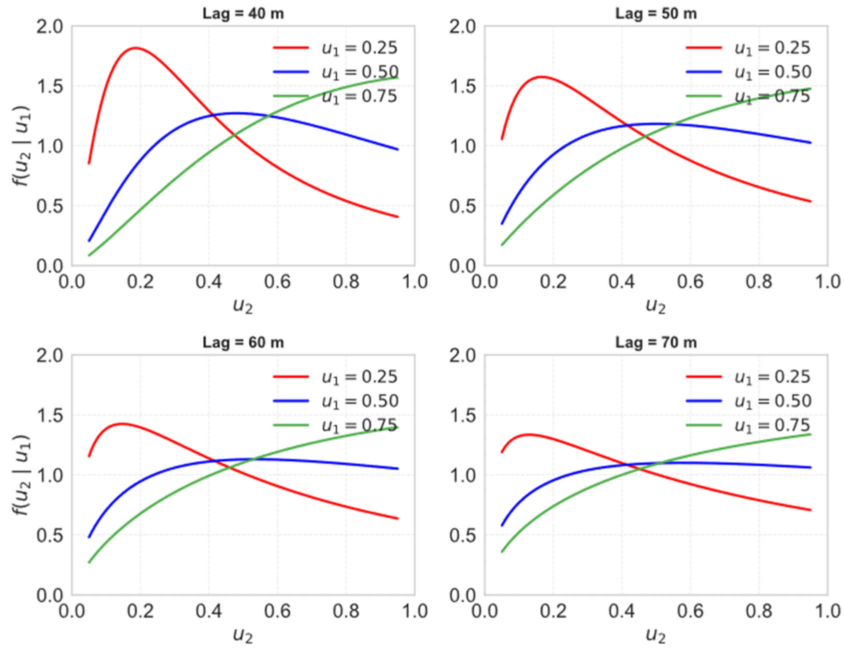

Figure 9. Conditional density of the Clayton copula at different lag distances

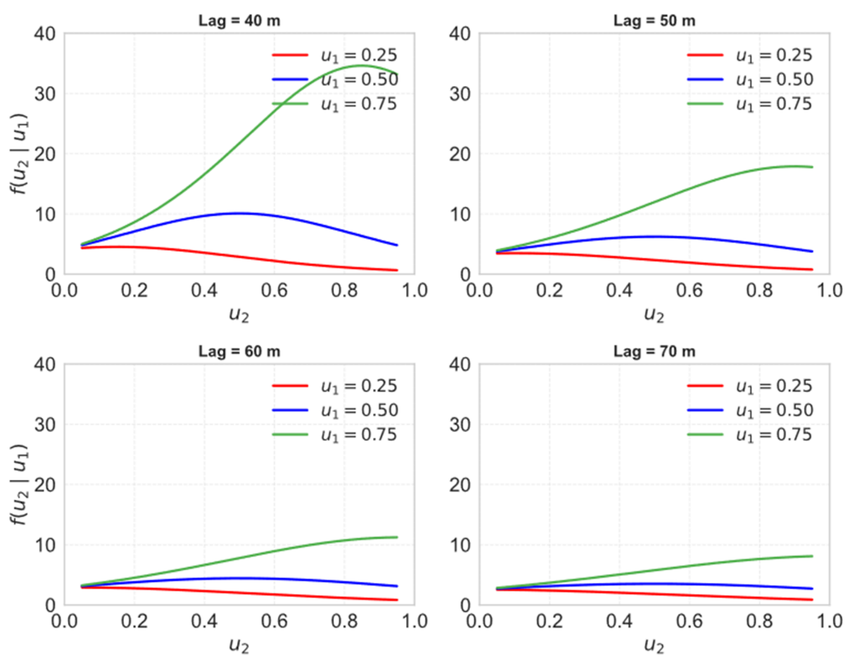

Figure 10. Conditional density of the Frank copula at different lag distances

3.4. Ore grade estimation

The comparison between different spatial estimation techniques reveals significant differences in how patterns of continuity and geological heterogeneity within the deposit are represented. While ordinary kriging tends to excessively smooth both high and low-grade zones due to its linear nature and exclusive reliance on covariance structures [1, 65], copula-based estimates provide a more accurate representation of local variability. The Gumbel copula highlights high-grade zones due to its affinity for upper-tail dependence [3], whereas the Clayton copula improves the detection of low-grade areas by capturing lower-tail structures [46]. Meanwhile, the Frank copula provides a symmetric and balanced estimate, making it well suited for settings with moderate spatial continuity [28]. These differences underscore the value of the copula-based approach for enhancing estimation accuracy in mineral deposits (see Figure 11). This figure corresponds to a horizontal cross-section extracted at a fixed Northing coordinate (Y = 8,716,600 m), selected because it intersects zones of contrasting mineralization and lithological complexity. It provides a representative view of how each method captures local variability in both high- and low-grade areas.

3.5. Validation of results and comparative analysis

To assess the robustness of the estimation methods, a leave-one-out cross-validation (LOOCV) procedure was employed. For each sample, the value was temporarily removed and estimated using the remaining data, applying the corresponding interpolation method (ordinary kriging or copula-based). This ensures that each prediction is made independently of the target value, offering a reliable measure of model performance. Although LOOCV does not fully account for spatial autocorrelation, the large and spatially distributed dataset helps mitigate this effect.

The results of the cross-validation analysis (Table 4) show that the Frank copula-based method delivers the best overall performance, achieving the highest coefficient of determination (R^2=0.778) and strongest linear correlation (0.905), along with the lowest mean absolute error (MAE = 0.090) and root mean square error (RMSE = 0.137). These results suggest that the Frank copula, due to its ability to model symmetric dependencies, is well suited to the spatial structure of the deposit [28, 29]. In contrast, ordinary kriging displayed a positive bias (0.028) and lower relative accuracy (R^2=0.692), confirming its limitations in capturing nonlinear relationships [1]. The Gumbel copula, although slightly biased, achieved a higher correlation than OK, while the Clayton copula provided a balance between low bias and low error, showing intermediate performance. These findings reinforce the advantage of copulas for improving geostatistical prediction quality in the presence of complex spatial dependencies [66]. The graphical representation is shown in Figure 12.

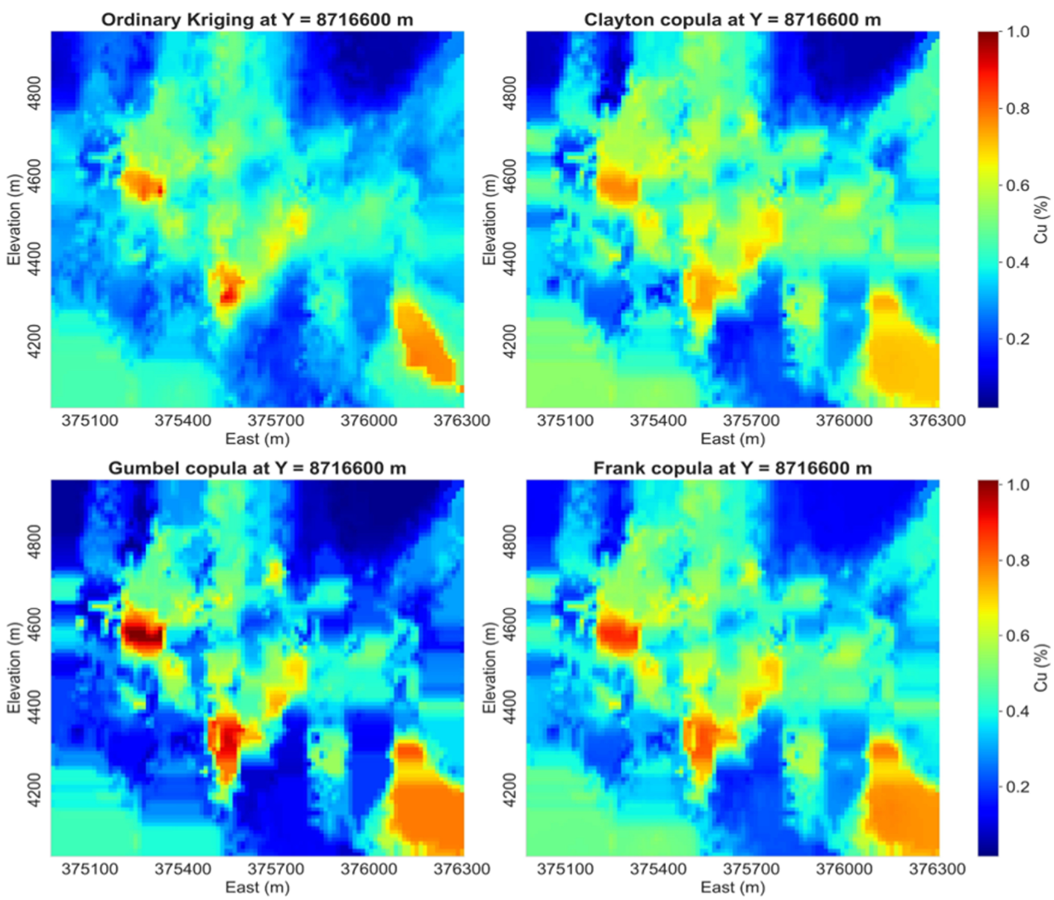

Figure 11. Comparison of estimated copper grades using different methods at a cross-section with fixed North coordinate (Y=8,716,600 m)

Table 4. Validation metrics for estimations

Metric	OK	Clayton	Gumbel	Frank
Bias	0.028	-0.004	0.057	-0.004
Std. deviation of error	0.158	0.154	0.150	0.136
RMSE	0.161	0.154	0.161	0.137
MAE	0.104	0.101	0.116	0.090
R2	0.692	0.717	0.692	0.778
Correlation	0.861	0.871	0.855	0.905

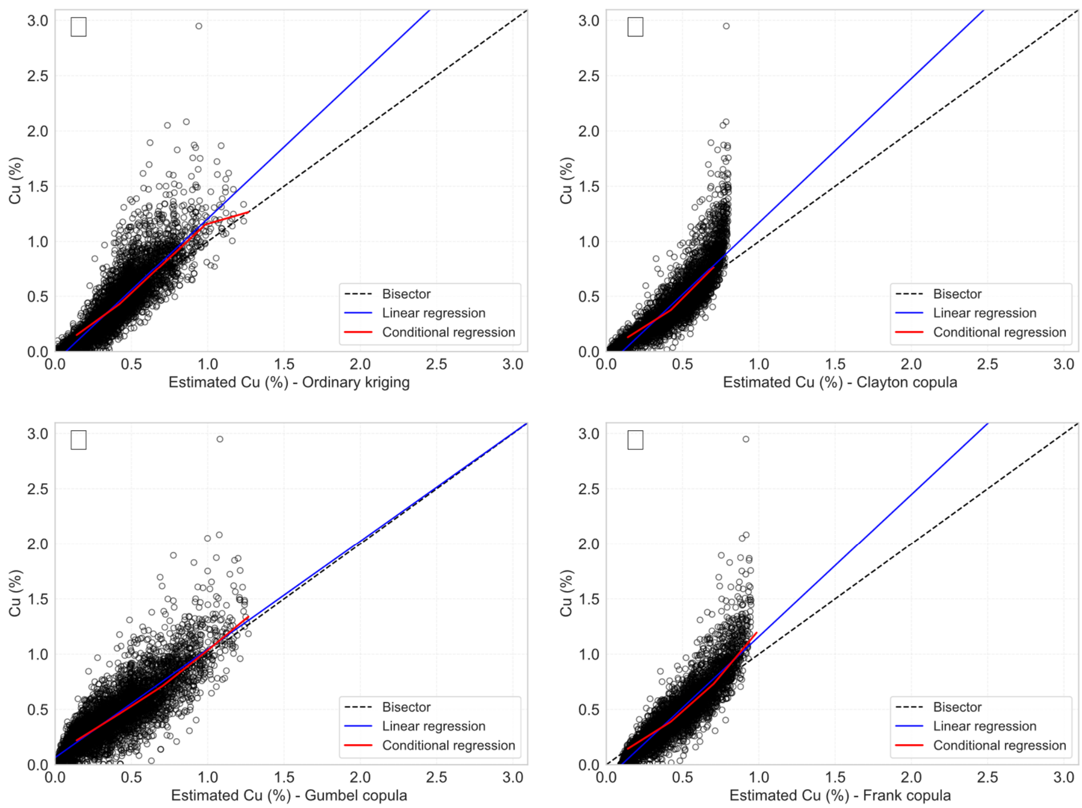

Figure 12. Copper grade estimation validation. A: Ordinary kriging. B: Clayton copula. C: Gumbel copula. D: Frank copula

A comparative analysis using box plots (Figure 13) shows that ordinary kriging systematically underestimates maximum values and compresses dispersion, a common effect known as “smoothing” [67–70]. In contrast, the Gumbel copula better preserves distribution spread, including extreme values consistent with its affinity for upper-tail dependence [30]. The Clayton copula maintains a mean close to the actual and lower variability, which is useful in quality control or classification contexts. The Frank copula provides a well-centered and balanced distribution, supporting its robustness in modeling mixed dependence patterns [71, 72]. This behavior confirms the ability of Archimedean copulas to adapt estimation to varying spatial dependence structures.

The mean local variance provides a measure of each method’s ability to capture spatial heterogeneity. The Gumbel copula yielded the highest mean local variance (0.0131), indicating less smoothing and better preservation of natural variability especially useful for modeling high-grade mineralization with upper-tail characteristics [31, 41]. In contrast, ordinary kriging, Clayton, and Frank copulas showed lower variances (0.0096, 0.0082, and 0.0084, respectively), with Clayton being the most conservative. This result suggests that, while Gumbel may be more expressive, its use must be balanced with sensitivity to outliers. Overall, the analysis confirms that copulas offer greater flexibility in controlling the propagation of spatial uncertainty compared to traditional covariance-based methods (see Figure 14 and Table 5).

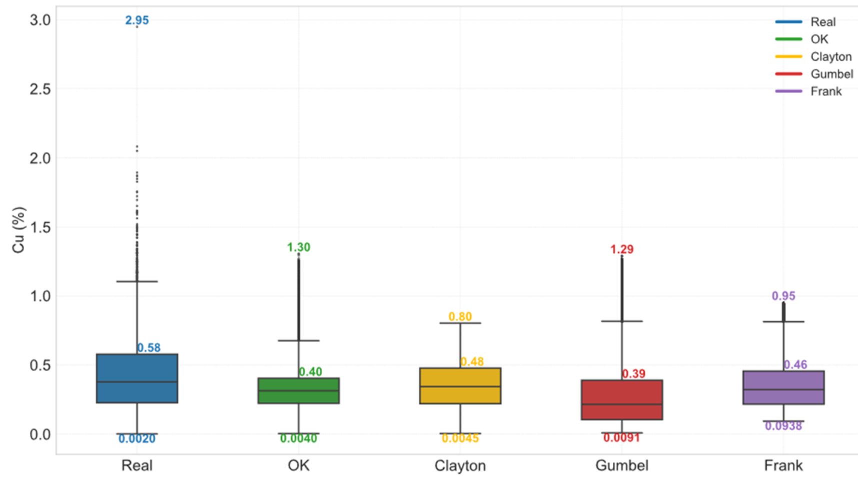

Figure 13. Comparison of ore grade estimators versus real values

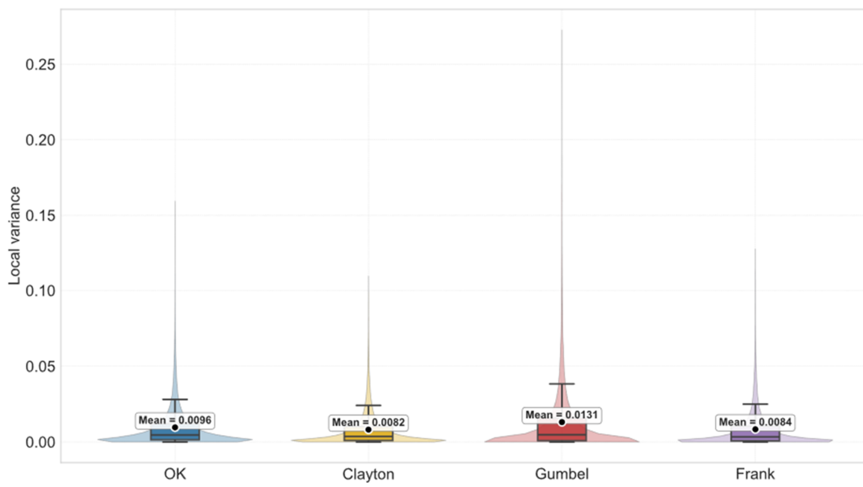

Figure 14. Comparison of local variance in copper grade estimation

Table 5. Descriptive statistics of local variance summary

Model	Mean	Std. Dev	Minimum	Q1	Median	Q3	Maximum
OK	0.0096	0.014	0.0	0.0015	0.0045	0.012	0.159
Clayton	0.0082	0.012	0.0	0.0010	0.0036	0.010	0.110
Gumbel	**0.0131**	**0.022**	**0.0**	**0.0009**	**0.0047**	**0.016**	**0.273**
Frank	0.0084	0.013	0.0	0.0007	0.0034	0.010	0.127

3.6. Statistical validation of estimation results

To assess whether the differences observed between the copula-based estimations and ordinary kriging are statistically significant, formal statistical tests were conducted. An analysis of variance (ANOVA) was applied to the absolute estimation errors of each method (Clayton, Gumbel, and Frank copulas) using ordinary kriging as a reference. The results (Table 6) show that the differences in mean errors are statistically significant at the 5% level ($F = 45{,}513.59$, $p < 0.001$), indicating that at least one method significantly differs from the others.

To further examine the differences, Fisher's F-tests were applied to compare the variances of each copula method with OK. All comparisons showed statistically significant differences ($p < 0.001$), with Gumbel displaying the highest variance ($F = 1.89$), followed by Clayton ($F = 1.33$) and Frank ($F = 1.27$) (Table 7). These results validate that the copula-based models yield estimations with statistically different dispersion and central tendencies compared to ordinary kriging. The statistical significance supports the robustness of the observed improvements in modeling local variability and spatial heterogeneity.

Table 6. Results of ANOVA for absolute estimation errors

Parameters	Value
Test	ANOVA
Groups compared	Gumbel, Clayton, Frank
F-statistic	45,513.59
p-value	0.00
Significant (α=0.05)	Yes

Table 7. Fisher's F-tests for variance comparison with ordinary kriging

Comparison	Variance 1	Variance (OK)	F-stat	p-value	Significant (α=0.05)
Gumbel vs OK	0.0431	0.0228	1.89	< 0.001	Yes
Clayton vs OK	0.0304		1.33	< 0.001	Yes
Frank v OK	0.0291		1.27	< 0.001	Yes

3.7. Improvements in ore grade estimation using copulas

The incorporation of Archimedean copulas into the geostatistical estimation of copper grades enabled a more realistic representation of the spatial structure of the deposit compared to ordinary kriging. Whereas the latter relies on a single, symmetric covariance function, copulas offer the advantage of capturing more complex and asymmetric relationships between neighboring points, an especially valuable feature in formations with high lithological heterogeneity and the presence of extreme grade values. In this case, the use of Archimedean copulas led to the following improvements:

i. Copulas allow modeling of complex relationships between sample pairs that do not follow Gaussian or symmetric patterns something ordinary kriging is not capable of capturing effectively [23, 73].

ii. Unlike kriging, which tends to smooth the data, copulas better preserve extreme values and the empirical shape of the distribution

iii. Lower errors (RMSE and MAE) were achieved with copula-based methods, especially with the Frank copula, demonstrating a quantitative improvement in prediction accuracy.

iv. The ability to fit θ parameters by distance and copula type allows for more realistic spatial variation modeling, overcoming the limitations of a single variogram model [3].

v. Copulas such as Gumbel and Clayton can focus on upper or lower-tail dependence, which is useful in different geological scenarios [28, 38, 64].

4. Conclusions

This study demonstrates that the use of Archimedean copulas in the geostatistical estimation of copper grades significantly improves prediction quality compared to ordinary kriging. The Frank copula achieved the best overall performance, with a R^2 of 0.778, an RMSE of 0.137, and a correlation of 0.905 outperforming the corresponding values obtained using kriging (R^2=0.692; RMSE = 0.161; corr. = 0.861). Additionally, the Gumbel and Clayton copulas effectively captured upper- and lower-tail dependence structures, respectively, preserving geologically relevant spatial patterns. The local variance analysis further showed that Gumbel maintained the highest spatial heterogeneity (mean variance = 0.0131), while Frank provided a balanced and robust estimate. These improvements support the use of copulas as effective tools for modeling nonlinear spatial structures, particularly in deposits with complex lithologies and highly variable grade distributions.

Despite their advantages, copula-based approaches require greater computational effort, specific parameter fitting for each lag, and careful validation to avoid overfitting especially in domains with limited data availability. Moreover, this study focused on bivariate copulas; a natural extension would be to explore multivariate dependence structures using vine copulas or higher-order conditional copulas. Future research could also evaluate the integration of this approach into stochastic simulations and multi-element contexts, where interdependencies between grades are critical for informed, integrated mining decisions. Moreover, considering that the deposit contains more than one economically significant element such as molybdenum, which is briefly mentioned, future work could explore multivariate extensions of the copula framework. Approaches such as vine copulas or conditional copulas offer flexible and tractable tools for modeling joint spatial dependencies between multiple variables. These methods could support integrated resource modeling, allowing simultaneous estimation of correlated attributes such as Cu and Mo, while preserving their distinct marginal distributions and capturing complex interdependencies beyond linear correlation.

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تخمین عیار سنگ معدن با استفاده از توابع مفصل ارشمیدسی در یک کانسار مس در پرو

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کاپولاهای ارشمیدسی
تخمین زمین آماری
وابستگی مکانی

چکیده

روش‌های زمین‌آماری سنتی مانند کریجینگ با فرض وابستگی‌های خطی و متقارن، محدودیت‌هایی را نشان می‌دهند که می‌تواند منجر به تخمین‌های هموار و از دست دادن تغییرپذیری محلی شود. برای پرداختن به این مسائل، این مطالعه از توابع مفصل ارشمیدسی (کلیتون، گامبل و فرانک) برای تخمین عیار سنگ معدن مس در یک کانسار واقع در پرو استفاده می‌کند. در مجموع ۵۶۵۴ ترکیب، هر کدام به طول ۱۵ متر، از ۱۸۵ گمانه مته الماسی به دست آمد. داده‌ها به مقیاس یکنواخت تبدیل شدند تا امکان برازش توابع مفصل فراهم شود. ساختارهای وابستگی با فاصله تأخیر مدل‌سازی شدند، و پارامتر وابستگی با استفاده از چندجمله‌ای‌های درجه پنجم برازش شد و تخمین شرطی سه‌بعدی اجرا شد. نتایج نشان می‌دهد که کریجینگ معمولی RMSE = 0.161، MAE = 0.104، R2 = 0.692 و همبستگی ۰.۸۶۱ را به دست آورد. مفصل Clayton این معیارها را کمی بهبود بخشید (RMSE = 0.154، MAE = 0.101، R2 = 0.717، R = 0.871)، در حالی که مفصل Gumbel تغییرپذیری محلی بالاتری را ثبت کرد (RMSE = 0.161، MAE = 0.116، R2 = 0.692، R = 0.855). مفصل Frank با RMSE = 0.137، MAE = 0.090، R2 = 0.778 و R = 0.905 بهترین عملکرد را به دست آورد. در نتیجه، مفصل‌های Archimedean با ثبت بهتر وابستگی مکانی، تخمین زمین‌آماری را به طور قابل توجهی افزایش می‌دهند و جایگزینی قوی برای روش‌های زمین‌آماری کلاسیک ارائه می‌دهند..

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