Journal of Mining and Environment (JME) Vol. 18, No. 4, 2026, 1493-1528

Shahrood University of Technology

Journal of Mining and Environment (JME)

Journal homepage: www.jme.shahroodut.ac.ir

Iranian Society of Mining Engineering (IRSME)

Achieving the Optimal Blasting Pattern Using Multi-Criteria Decision-Making Models (Case Study: Miduk Copper Mine)

Majid Noorian-Bidgoli*, and Mohammad Reza Rezaei

Department of Mining Engineering, Faculty of Engineering, University of Kashan, Kashan, Iran

Article Info

Received 8 July 2025

Received in Revised form 21 July 2025

Accepted 7 September 2025

Published online 7 September 2025

DOI: 10.22044/jme.2025.16486.3223

Keywords

Multi-criteria decision-making (MCDM)

Blasting pattern

Drilling

Optimization

Miduk copper mine

Abstract

Drilling and blasting are crucial operations in open-pit mining, aimed at optimizing rock fragmentation, minimizing negative effects like backbreak and flyrock, and reducing costs, while enhancing efficiency and minimizing environmental and infrastructure impacts. This study focuses on optimizing drilling and blasting patterns at the Miduk copper mine using Multi-Criteria Decision-Making (MCDM) methods. The primary objectives were to achieve optimal fragmentation, minimize specific charge and drilling costs, and reduce undesirable phenomena like backbreak and flyrock caused by blasting. A total of 52 blasting patterns implemented at the mine were evaluated using various MCDM techniques, including TOPSIS, ELECTRE, VIKOR, and COCOSO. By constructing decision matrices and ranking the alternatives in each method, the most suitable blasting pattern was identified. The Copeland method was further applied to integrate the results from the decision models and establish a consensus on the final ranking of blasting patterns based on the criteria. The study's innovation lies in the application of advanced MCDM techniques to optimize drilling and blasting patterns, as well as the integration of results to enhance the decision-making process's accuracy. The optimal blasting pattern (M_Patt_03) was found to feature a burden of 6.5 meters, a spacing of 8 meters, and a borehole diameter of 150 millimetres, offering the best balance of fragmentation, charge efficiency, and drilling costs, while minimizing backbreak and flyrock. This study demonstrates the effectiveness of MCDM methods in optimizing complex engineering challenges in surface mining, providing a comprehensive framework for evaluating multiple criteria simultaneously and enabling more informed and balanced decision-making.

1. Introduction

Efficient surface mining operations depend heavily on proper rock fragmentation. Optimizing the size of blasted material is essential for facilitating handling and transportation [1]. However, excessively fine materials can cause environmental issues, such as dust, leading to additional operational costs [2-3].

Determining the optimal rock size post-blasting is a complex challenge. The ideal size of fragmented rocks is typically dictated by the type of loading machinery and the crusher's input size [4-5], both of which directly influence fragmentation costs, production efficiency, and the effectiveness of subsequent loading, hauling, and crushing operations [6]. Therefore, rock fragmentation quality plays a crucial role in determining the overall extraction costs in open-pit mining [7-8].

A wide range of methodologies has been employed over the years to optimize blasting patterns, including empirical techniques [9], numerical modeling [10-11], and advanced methods [12-15]. However, many studies have failed to account for all the factors influencing blasting operations simultaneously, leading to incomplete evaluations.

Previous research has shown that selecting the right blasting pattern can reduce costs, improve

Corresponding author: noriyan@kashanu.ac.ir (M. Noorian-Bidgoli)

mining efficiency, and minimize undesirable outcomes such as backbreak [16-17], flyrock [18-20], and ground vibration [21]. Therefore, optimizing the drilling and blasting pattern is essential for achieving effective fragmentation and reducing associated costs.

This study applies multi-criteria decision-making (MCDM) methods—specifically, TOPSIS, ELECTRE, VIKOR, and COCOSO—to determine the optimal blasting pattern at the Miduk copper mine. Unlike traditional empirical approaches, this method offers a systematic, mathematical framework for optimization, enhancing both accuracy and efficiency in blasting design. To consolidate the results from the four MCDM methods, the Copeland method is used for integration and ranking, ensuring a more reliable final decision. Five key parameters—rock fragmentation, specific charge, specific drilling, backbreak, and flyrock—are evaluated simultaneously, providing a more holistic analysis compared to previous studies that often consider only one or two parameters. Real data from 52 completed blasting patterns at the Miduk mine are utilized, demonstrating the practical applicability of the proposed approach. The findings reveal the optimal blasting pattern, offering a practical solution for improving blasting operations at this mine. The method presented here can also serve as a guide for optimizing blasting operations in similar mining contexts.

The novelty of this study lies in the application of advanced Multi-Criteria Decision-Making (MCDM) methods to optimize drilling and blasting patterns at the Miduk copper mine. This approach represents a shift from traditional empirical methods, offering a more structured, data-driven, and systematic framework for decision-making. Moreover, the integration of results from multiple MCDM models using the Copeland method adds another layer of reliability, ensuring more accurate decision-making.

2. Factors affecting blasting results

In blasting operations, the energy released during the detonation of explosives is primarily responsible for rock fragmentation and breakage. Various indices related to the rock, such as drillability and blastability, are commonly used to predict and assess fragmentation.

Blastability refers to the rock's response to the energy released during the explosion of the explosive material. This property is influenced by factors such as the rock's mechanical properties, the type of explosive used, and the charging method. A clear understanding of blastability allows for an estimation of the required explosive material for effective rock mass blasting [22].

Among the critical factors affecting blasting are the inherent properties of the rock—such as strength, specific gravity, porosity, rock type and composition, and the presence of fractures. These factors, which are generally uncontrollable, influence the blastability and the overall capacity of the rock mass [23-24].

In contrast, controllable factors in blasting include parameters that can be adjusted by the designer of the drilling and blasting pattern. By modifying these factors, blasting results can be improved. These factors include the blasthole diameter, burden, spacing between rows of holes, bench height in mining, sub-drilling depth, stemming length, type of explosive material, specific charge, and the arrangement of blast holes [25].

To optimize blasting results in surface mining, the rock fragmentation condition after the blast must be evaluated. For example, the geometric shape of the muck pile following the blast, which depends on the controllable blasting parameters, can significantly impact the performance of loading and hauling equipment [26].

Following blasting, the formation of toes in surface mining operations may occur if the bench floor is either too high or too low. Research has indicated that adjusting parameters like burden, sub-drilling, and inter-row delay time, while concentrating explosive charge at the bottom of the hole, can prevent toe formation [27].

A key objective in designing an optimal blasting pattern is the prevention of backbreak, which can occur due to weak geological structures, excessive stemming length, or improper hole spacing. To avoid backbreak, it is critical that the explosion shears the rock mass at the last row of holes, ensuring that the fracturing does not extend beyond the final row of holes [28].

Another significant concern in blasting is flyrock, which refers to the ejection of rock fragments into the air. Flyrock can cause injury to personnel and damage to equipment. Factors such as insufficient burden, excessive explosive charge, reduced stemming length, and weak geological surfaces contribute to flyrock. Addressing these issues is essential for improving blasting safety and efficiency [29].

Fragmentation degree, which determines the size distribution of blasted rocks, is another critical factor. It can be quantitatively described using sieve aperture sizes, such as D80, D50, or D20. For example, D80 represents the sieve size through which 80% of the material passes, while 20% cannot pass [30].

In terms of economic evaluation, the specific drilling and specific charge are key parameters. Specific drilling refers to the amount of drilling required to extract one ton or one cubic meter of rock [31]. The specific drilling (PS) can be determined using the following equations:

$$PS = \frac{\left(\frac{H}{cos\beta} + J\right)}{\left(\frac{B}{cos\beta} \times S \times H\right)} \tag{1}$$

$$PS = \frac{\left(\frac{H}{cos\beta} + J\right) \times (250 \times \pi \times J)}{\left(\frac{B}{cos\beta} \times S \times H\right)} \tag{2}$$

In these equations, β represents the angle of the blast hole, S denotes the spacing, J indicates the sub-drilling, B is the burden (distance between a blasthole and the nearest free face), and H refers to the bench height.

Another critical parameter influencing the economic evaluation of drilling and blasting patterns in mines is the specific charge [32]. The specific charge refers to the amount of explosive material required to extract one ton or one cubic meter of rock. The specific charge can be calculated using the following equation:

$$q = \frac{Q}{B \times S \times L} \tag{3}$$

In this equation, Q represents the amount of explosive used, B is the burden, S is the spacing between blast holes, and L is the total length of the blast hole. Generally, the stronger the rock, the greater the amount of explosive material required, which in turn increases the specific charge consumed.

Over the years, numerous researchers have proposed empirical relationships to determine drilling and blasting patterns. However, due to the inherent complexity of blasting operations in mines, these relationships have often failed to address all the issues related to blasting. Typically, because some parameters influencing blasting operations are omitted or overlooked, these relationships lack comprehensiveness. Consequently, after their application, adjustments to the initial drilling and blasting design are often required through trial-and-error methods to improve and optimize the blasting pattern [33-36].

3. Multi-Criteria Decision-Making (MCDM)

The decision problem in the context of Multi-Criteria Decision-Making (MCDM) refers to the challenge of selecting the most suitable alternative from a set of potential choices, based on the evaluation of multiple criteria, while considering the decision maker's preferences. Given a set of alternatives A=$\{a_1, a_2, ..., a_n\}$ and a set of criteria C=$\{c_1, c_2, ..., c_m\}$, where each alternative a_i is evaluated based on its performance against each criterion c_j, the decision problem is to identify the best alternative (or a set of best alternatives) from A that satisfies the decision-maker's preferences. The decision-making process can be formalized as follows:

Alternatives: A finite set A=$\{a_1, a_2, ..., a_n\}$ of possible decision alternatives.

Criteria: A finite set C=$\{c_1, c_2, ..., c_m\}$ of criteria or objectives that are used to evaluate the alternatives.

Evaluation Function: A performance matrix $P \in R^{(n\times m)}$ where the element p_{ij} represents the performance score of alternative a_i with respect to criterion c_j.

Decision Maker's Preferences: A preference structure that expresses the importance of each criterion and how the alternatives should be ranked. This can be represented through:

- Weighting Function: A vector $w \in R^m$ where each weight w_j corresponds to the relative importance of criterion c_j.
- Preference Relations: A set of rules or preferences that indicate how the decision maker values trade-offs among criteria, typically expressed as a utility function or through pairwise comparisons.

Decision Method: An MCDM technique or method (such as Weighted Sum Model, Analytic Hierarchy Process (AHP), or Technique for Order Preference by Similarity to Ideal Solution (TOPSIS)) used to aggregate the performances of the alternatives and rank them according to the decision maker's preferences.

Objective: The goal is to identify the optimal alternative $a^* \in A$ by maximizing or minimizing a utility function $U(a)$, which aggregates the performances and preferences across all criteria. Depending on the specific MCDM method, this function can take different forms. In this case, a^* represents the optimal or best alternative selected after evaluating all alternatives based on the decision criteria. $\in A$ indicates that a^* is an element of the set A, meaning that the selected optimal alternative is a member of the set of available alternatives A. In simpler terms, a^* denotes the best choice or the set of best choices that has been identified from the set A after considering all the decision criteria and applying the appropriate MCDM method.

In summary, the MCDM decision problem involves finding the most suitable alternative by considering multiple criteria that are relevant to the decision at hand, and applying an appropriate method to aggregate these criteria in line with the decision-maker's preferences.

Given that multiple parameters influence blasting results in an open-pit mine, methods capable of accounting for all or most of the relevant criteria and evaluating them simultaneously should be employed to determine the optimal drilling and blasting pattern. Multi-Criteria Decision-Making (MCDM) methods are widely used approaches that can help achieve the optimal pattern, with the goal of reducing costs and minimizing the adverse effects of blasting [37].

Multi-Criteria Decision-Making (MCDM) or Multi-Criteria Decision-Analysis (MCDA) is a precise method for identifying the best solution by utilizing both qualitative and quantitative criteria [38]. Unlike classical operations research methods such as linear and nonlinear programming, which optimize based on a single criterion, MCDM methods allow for the simultaneous optimization of multiple criteria [39].

In MCDM methods, several indicators or objectives are considered for optimization [40]. If an indicator is used as a criterion in the decision-making process, this is referred to as multi-index decision-making. In this approach, multiple options or solutions are analyzed, and they are ranked based on specific criteria [41-42].

On the other hand, if the multiple criteria refer to objectives, the method is known as Multi-Objective Decision-Making (MODM). In this method, several objectives are simultaneously examined in order to optimize the problem. Then, by selecting a measurement scale, the objectives are evaluated [43].

To use various MCDM methods effectively, the objective must first be established. If the goal is to determine the weight of the decision-making criteria, methods such as the Analytic Hierarchy Process (AHP) [44], Analytic Network Process (ANP) [45], Best-Worst Method (BWM) [46], Step-wise Weight Assessment Ratio Analysis (SWARA) [47], or expert judgment can be applied.

On the other hand, if the aim is to select the optimal option, methods such as TOPSIS (Technique for Order Preference by Similarity to Ideal Solution) [48], VIKOR (Vlse Kriterijumsk Optimizacija Kompromisno Resenje) [49], ELECTRE (Elimination and Choice Translating Reality) [50], and COCOSO (Combined Compromise Solution) [51] can be utilized. In most of these methods, a decision matrix is first created, and then, based on each method's guidelines, the options are ranked according to the criteria, leading to the selection of the most appropriate option.

However, when multiple decision-making methods are employed, result integration and consensus methods—such as the Borda method or Copeland method [52]—can be used at the end of the process to rank and select the optimal solution.

In this study, the criteria were carefully chosen to reflect the key performance indicators that directly influence the efficiency and effectiveness of drilling and blasting operations in open-pit mining. The selection process was based on the primary objectives of optimizing the blasting pattern, improving rock fragmentation, minimizing undesirable blasting effects, and reducing operational costs. Specifically, the following criteria were considered:

Rock Fragmentation: This is a critical factor as it directly impacts subsequent mining activities, such as material handling and processing. Efficient fragmentation reduces energy consumption and improves downstream operations.

Specific Charge and Drilling Requirements: These criteria are essential for evaluating the economic efficiency of the blasting pattern. Reducing the specific charge and drilling requirements leads to cost savings in mining operations.

Backbreak and Flyrock: These are undesirable effects that can result in safety hazards and additional cleanup costs. Minimizing these impacts is vital for ensuring safe and effective blasting operations.

The criteria were selected based on both their direct relevance to the objectives of the study and their ability to be quantitatively evaluated through available data from the 52 executed blasting patterns at the Miduk copper mine. Each criterion was chosen to ensure that the decision-making process accurately reflects the complex trade-offs inherent in blasting pattern optimization. The aim was to provide a balanced and objective approach for selecting the most suitable blasting pattern, aligned with both operational and economic goals, through these criteria.

The weights for the criteria were assigned using expert judgment from blasting specialists in this study. Opinions from these specialists were gathered to evaluate the relative importance of each criterion, ensuring that the assigned weights accurately reflected their significance in relation to the study's objectives.

In the next stage, the decision matrix was constructed by evaluating each alternative (blasting pattern) against the selected criteria. Data from 52 executed drilling and blasting patterns at the Miduk copper mine were used for this analysis. Each blasting pattern was assessed based on its performance across five key parameters: rock fragmentation, specific charge, specific drilling, backbreak, and flyrock. For each criterion, the performance of the alternatives was recorded in a matrix format, where each row represented a different blasting pattern and each column represented a different criterion. The values in the matrix were based on empirical data and expert evaluations. In cases where necessary, data was normalized or scaled to ensure consistency and comparability across the criteria. In the following steps, this matrix served as the basis for applying multiple MCDM methods, such as TOPSIS, ELECTRE, VIKOR, and COCOSO, to rank the alternatives and identify the most suitable blasting pattern.

4. Optimization of the blasting pattern in Miduk copper mine

The Miduk copper mine is situated 42 kilometers northeast of Shahrebabak city, to the west of Kerman province, and 132 kilometers northwest of the Sarcheshmeh copper mine (Figure 1). The mine's porphyry copper deposit dates back to the Cenozoic (metallogenic) era. Currently, the mine is being exploited using the open-pit method, where mineral excavation is performed through drilling and blasting.

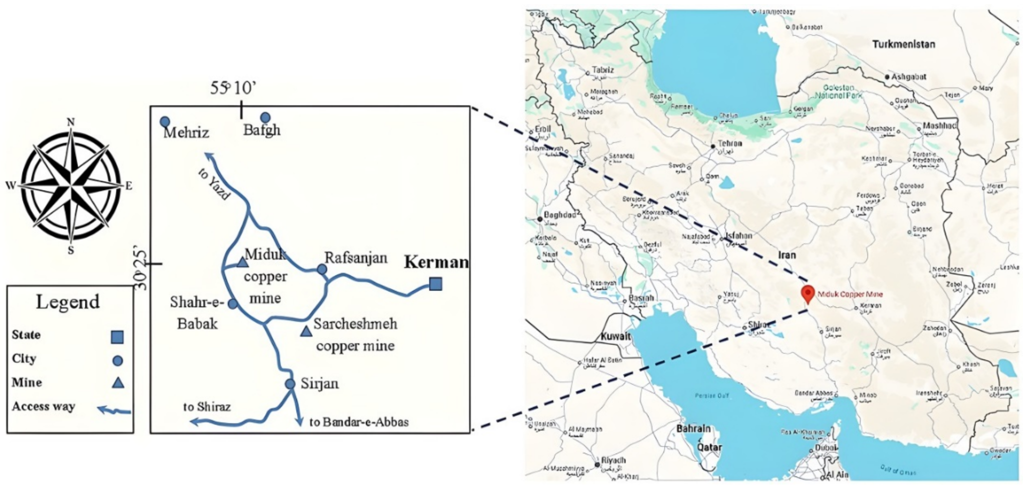

Figure 1. Geographical location of the Miduk copper mine

Considering that a portion of the ore in the Miduk copper mine is directed to the low-grade material and leaching stockpile, the size distribution of the blasted material (fragmentation) must be optimized to avoid hindering the leaching operation. The goal is to maximize the efficiency of the production process in the leaching unit. Additionally, in light of the existing blasting operational issues in the mine, such as backbreak and flyrock, this research aims to optimize the blasting operations to achieve an optimal blasting pattern. This will result in desirable fragmentation and reduce the aforementioned operational problems.

Information on the blast blocks in the mine was recorded in two stages: before and after the blasting operation. Ultimately, a database containing 52 drilling and blasting patterns from this mine was analyzed, and the data was compiled (Table 1). In this table, for each blasting pattern (Pattern ID), values such as burden (B), spacing (S), and blast hole diameter (D) are provided, along with the powder factor, drilling factor, backbreak, flyrock, and the degree of rock fragmentation resulting from the explosion (based on D_{80}).

Table 1. Data of drilling and blasting patterns in the Miduk copper mine

Pattern ID	B (m)	S (m)	D (mm)	Powder factor (kg/m^3)	Drilling factor (m/m^3)	Flyrock (m)	Backbreak (m)	D_{80} (mm)
M_Patt_01	7.5	9.5	200	0.12	0.014	133.9	3.3	118.5
M_Patt_02	7.5	9.5	200	0.12	0.014	133.9	3.3	118.5
M_Patt_03	6.5	8	150	0.12	0.019	100.4	3.9	73.7
M_Patt_04	5.5	7	150	0.12	0.026	100.4	1.2	201.0
M_Patt_05	5.5	7	150	0.12	0.026	100.4	1.3	201.7
M_Patt_06	7.5	9.5	203	0.13	0.014	135.9	3.4	176.0
M_Patt_07	6.5	8	230	0.23	0.019	153.9	4.1	177.0
M_Patt_08	6.5	8	230	0.24	0.019	525.2	4.0	176.6
M_Patt_09	6.5	8	200	0.23	0.019	425.2	2.5	130.0
M_Patt_10	6.5	8	165	0.16	0.019	350.8	2.6	110.0
M_Patt_11	5.5	7	150	0.12	0.026	100.4	2.0	95.3
M_Patt_12	5.5	7	150	0.12	0.026	100.4	2.0	121.5
M_Patt_13	5.5	7	150	0.16	0.026	100.4	1.5	123.0
M_Patt_14	6.5	8	200	0.12	0.019	133.9	2.5	193.2
M_Patt_15	6.5	8	200	0.16	0.019	228.3	2.4	95.7
M_Patt_16	5.5	7	150	0.17	0.026	171.3	1.4	116.0
M_Patt_17	5.5	7	150	0.17	0.026	171.3	0.7	116.0
M_Patt_18	6.5	8	200	0.17	0.019	267.7	2.5	115.2
M_Patt_19	5.5	7	150	0.17	0.026	200.8	1.5	111.5
M_Patt_20	5.5	7	150	0.17	0.026	200.8	1.6	115.2
M_Patt_21	6.5	8	165	0.13	0.019	129.9	2.9	112.4
M_Patt_22	7.5	9.5	200	0.12	0.014	133.9	3.0	167.1
M_Patt_23	6.5	8	170	0.13	0.019	133.9	2.7	131.8
M_Patt_24	5.5	7	150	0.12	0.026	100.4	1.4	112.6
M_Patt_25	5.5	7	150	0.17	0.026	200.8	1.1	176.4
M_Patt_26	5.5	7	150	0.13	0.026	118.1	0.6	81.9
M_Patt_27	6.5	8	200	0.22	0.019	393.7	3.2	124.4
M_Patt_28	5.5	7	150	0.26	0.026	118.1	1.7	122.0
M_Patt_29	5.5	7	150	0.33	0.026	118.1	1.5	123.3
M_Patt_30	6.5	8	200	0.55	0.019	385.8	2.5	87.8
M_Patt_31	7.5	9.5	200	0.3	0.014	133.9	3.2	137.9
M_Patt_32	5.5	7	150	0.31	0.026	100.4	2.6	143.8
M_Patt_33	5.5	7	150	0.33	0.026	118.1	1.8	123.0
M_Patt_34	5.5	7	150	0.31	0.026	100.4	5.0	82.0
M_Patt_35	5.5	7	225	0.35	0.026	150.6	1.7	93.0
M_Patt_36	6.5	8	200	0.41	0.019	228.3	2.3	111.7
M_Patt_37	6.5	8	200	0.41	0.019	228.3	2.0	111.7
M_Patt_38	5.5	7	150	0.23	0.026	47.2	1.4	166.0
M_Patt_39	5.5	7	150	0.34	0.026	70.9	1.3	155.0
M_Patt_40	5.5	7	150	0.35	0.026	70.9	1.5	130.0
M_Patt_41	6.5	8	165	0.28	0.019	78.0	2.5	121.4
M_Patt_42	6.5	8	230	0.36	0.019	108.7	2.4	114.0
M_Patt_43	5.5	7	150	0.31	0.026	100.4	0.9	107.5
M_Patt_44	6.5	8	165	0.3	0.019	110.4	2.8	107.8
M_Patt_45	5.5	7	150	0.33	0.026	118.1	1.3	80.2
M_Patt_46	6.5	8	200	0.55	0.019	385.8	2.5	119.1
M_Patt_47	5.5	7	150	0.34	0.026	129.9	2.0	81.7
M_Patt_48	7.5	9.5	200	0.32	0.014	157.5	3.2	96.0
M_Patt_49	5.5	7	150	0.43	0.026	200.8	1.1	176.4
M_Patt_50	5	6	150	0.53	0.033	289.4	0.7	86.1
M_Patt_51	5	6	150	0.53	0.033	289.4	0.7	166.2
M_Patt_52	5.5	7	150	0.41	0.026	200.8	1.5	111.5

Given the limitations of the empirical methods mentioned earlier and the operational challenges associated with blasting, this section focuses on optimizing the blasting pattern of the Miduk copper mine using multi-criteria decision-making models such as TOPSIS, ELECTRE, COCOSO, and VIKOR. Subsequently, the Copeland aggregation method is employed to rank the blasting patterns resulting from each model. By comparing and aggregating the rankings, a final ranking is derived to identify the optimal blasting pattern.

In evaluating the drilling and blasting pattern, this research considers several key criteria, including rock fragmentation, specific charge, specific drilling, backbreak, and flyrock. Since lower values of specific charge and specific drilling indicate more economically favourable blasting, these two parameters are treated as negative evaluation indicators. Additionally, since less impact on the extraction faces is preferred, backbreak and flyrock are also considered negative indicators in this evaluation.

It is important to note that the D_{80} parameter was used to assess the degree of fragmentation. Since more fragmentation reduces the energy required in the mineral processing stage, this parameter is regarded as a positive evaluation indicator. The acceptable fragmentation size in this mine ranges from 65 mm to 210 mm.

A summary of the operational definitions for the five criteria used in the decision-making process of this study, including descriptions, units of measurement, value ranges, and normalization methods, is provided in Table 2.

Table 2. Technical definitions and measurement specifications for decision-making criteria

Criterion	Description	Unit	Range	Normalization
Fragmentation	Average particle size after blasting	mm	73.7-201.73	Min-max
Specific charge	Explosive weight per volume of rock blasted	kg/m³	0.12-0.55	Min-max
Specific drilling	Drilled length per volume of blasted rock	m/m³	0.014-0.033	Min-max
Backbreak	Length of overbreak behind last row of holes	M	0.6-5	Min-max
Flyrock	Maximum distance of thrown rock from blast site	M	47.2-525.2	Min-max

4.1. TOPSIS method for MCDM

In this section, the TOPSIS method is applied to evaluate the blasting pattern. This method is based on two core concepts: the ideal solution and similarity to the ideal solution. The ideal solution represents the best possible outcome in every aspect, which, in practice, typically does not exist. Therefore, the goal is to approach this ideal as closely as possible.

To measure how closely an option resembles the ideal and anti-ideal solutions, the distance of that option from both the ideal and anti-ideal solutions is calculated. In the next step, the options are evaluated and ranked based on the ratio of the distance from the anti-ideal solution to the total distance (sum of the distance from the ideal solution and the anti-ideal solution). The steps in this method are as follows:

Step 1: Create the decision matrix (D)

In this step, a matrix consisting of 5 columns and 52 rows is constructed (Table 3). In this matrix, the columns represent the criteria, and the rows represent the blasting patterns. The criteria include:

C1: Fragmentation index

C2: Specific charge

C3: Specific drilling

C4: Backbreak

C5: Flyrock

Step 2: Normalizing the decision matrix

The objective of this step is to make the decision matrix dimensionless. To achieve this, the norm normalization method is applied. In this method, the following equation is used to convert the decision matrix into a dimensionless form:

$$n_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}} \tag{4}$$

At the end of this step, the dimensionless decision matrix (N) is obtained, as described in Equation 5. The results of the normalization process are presented in Table 4.

$$N = \left[n_{ij}\right] \tag{5}$$

Table 3. Decision matrix (D) created in the TOPSIS method

	C1	C2	C3	C4	C5
A1	118.5	0.300	0.014	3.3	133.9
A2	118.5	0.310	0.014	3.3	135.0
A3	73.7	0.300	0.019	3.9	100.4
A4	201.0	0.300	0.026	1.2	100.4
A5	201.7	0.300	0.026	1.3	100.4
A6	176.0	0.300	0.014	3.4	135.9
A7	177.0	0.300	0.019	4.1	153.9
A8	176.6	0.600	0.019	4.0	525.2
A9	130.0	0.575	0.019	2.5	425.2
A10	110.0	0.575	0.019	2.6	350.8
A11	95.3	0.300	0.026	2.0	100.4
A12	121.5	0.300	0.026	2.0	100.4
A13	123.0	0.300	0.026	1.5	100.4
A14	193.2	0.300	0.019	2.5	133.9
A15	95.7	0.400	0.019	2.4	228.3
A16	116.0	0.400	0.026	1.4	171.3
A17	116.0	0.400	0.026	0.7	171.3
A18	115.2	0.425	0.019	2.5	267.7
A19	111.5	0.425	0.026	1.5	200.8
A20	115.2	0.425	0.026	1.6	200.8
A21	112.4	0.325	0.019	2.9	129.9
A22	167.1	0.300	0.014	3.0	133.9
A23	131.8	0.325	0.019	2.7	133.9
A24	112.6	0.300	0.026	1.4	100.4
A25	176.4	0.425	0.026	1.1	200.8
A26	80.2	0.325	0.026	1.3	118.1
A27	119.1	0.525	0.019	2.5	385.8
A28	81.7	0.350	0.026	2.0	129.9
A29	96.0	0.325	0.014	3.2	157.5
A30	176.4	0.425	0.026	1.1	200.8
A31	86.1	0.525	0.033	0.7	289.4
A32	166.2	0.525	0.033	0.7	289.4
A33	111.5	0.425	0.026	1.5	200.8
A34	118.5	0.300	0.014	3.3	133.9
A35	118.5	0.310	0.014	3.3	135.0
A36	73.7	0.300	0.019	3.9	100.4
A37	201.0	0.300	0.026	1.2	100.4
A38	201.7	0.300	0.026	1.3	100.4
A39	176.0	0.300	0.014	3.4	135.9
A40	177.0	0.300	0.019	4.1	153.9
A41	176.6	0.600	0.019	4.0	525.2
A42	130.0	0.575	0.019	2.5	425.2
A43	110.0	0.575	0.019	2.6	350.8
A44	95.3	0.300	0.026	2.0	100.4
A45	121.5	0.300	0.026	2.0	100.4
A46	123.0	0.300	0.026	1.5	100.4
A47	193.2	0.300	0.019	2.5	133.9
A48	95.7	0.400	0.019	2.4	228.3
A49	116.0	0.400	0.026	1.4	171.3
A50	116.0	0.400	0.026	0.7	171.3
A51	115.2	0.425	0.019	2.5	267.7
A52	111.5	0.425	0.026	1.5	200.8

Table 4. Dimensionless decision matrix (N) created in the TOPSIS method

	C1	C2	C3	C4	C5
A1	15.0	0.033	0.001	0.6	12.3
A2	15.0	0.035	0.001	0.6	12.5
A3	5.8	0.033	0.002	0.9	6.9
A4	43.1	0.033	0.004	0.1	6.9
A5	43.4	0.033	0.004	0.1	6.9
A6	33.0	0.033	0.001	0.7	12.6
A7	33.4	0.033	0.002	1.0	16.2
A8	33.3	0.133	0.002	0.9	188.9
A9	18.0	0.122	0.002	0.4	123.8
A10	12.9	0.122	0.002	0.4	84.3
A11	9.7	0.033	0.004	0.2	6.9
A12	15.7	0.033	0.004	0.2	6.9
A13	16.1	0.033	0.004	0.1	6.9
A14	39.8	0.033	0.002	0.4	12.3
A15	9.8	0.059	0.002	0.3	35.7
A16	14.3	0.059	0.004	0.1	20.1
A17	14.3	0.059	0.004	0.0	20.1
A18	14.1	0.067	0.002	0.4	49.1
A19	13.2	0.067	0.004	0.1	27.6
A20	14.1	0.067	0.004	0.1	27.6
A21	13.5	0.039	0.002	0.5	11.6
A22	29.8	0.033	0.001	0.5	12.3
A23	18.5	0.039	0.002	0.4	12.3
A24	13.5	0.033	0.004	0.1	6.9
A25	33.2	0.067	0.004	0.1	27.6
A26	7.2	0.039	0.004	0.0	9.6
A27	16.5	0.112	0.002	0.6	106.1
A28	15.9	0.039	0.004	0.2	9.6
A29	16.2	0.039	0.004	0.1	9.6
A30	8.2	0.102	0.002	0.4	101.9
A31	20.3	0.033	0.001	0.6	12.3
A32	22.1	0.033	0.004	0.4	6.9
A33	16.1	0.039	0.004	0.2	9.6
A34	7.2	0.033	0.004	1.4	6.9
A35	9.2	0.033	0.004	0.2	15.5
A36	13.3	0.059	0.002	0.3	35.7
A37	13.3	0.059	0.002	0.2	35.7
A38	29.4	0.019	0.004	0.1	1.5
A39	25.6	0.028	0.004	0.1	3.4
A40	18.0	0.028	0.004	0.1	3.4
A41	15.7	0.028	0.002	0.4	4.2
A42	13.9	0.028	0.002	0.3	8.1
A43	12.3	0.033	0.004	0.0	6.9
A44	12.4	0.033	0.002	0.5	8.4
A45	6.9	0.039	0.004	0.1	9.6
A46	15.1	0.102	0.002	0.4	101.9
A47	7.1	0.045	0.004	0.2	11.6
A48	9.8	0.039	0.001	0.6	17.0
A49	33.2	0.067	0.004	0.1	27.6
A50	7.9	0.102	0.007	0.0	57.3
A51	29.5	0.102	0.007	0.0	57.3
A52	13.2	0.067	0.004	0.1	27.6

Step 3: Determining the weights of indicators

In this step, the weights of the indicators are determined by performing a pairwise comparison between the indicators. The final weights are then assigned to each indicator. For this purpose, expert opinions in the field of blasting can be utilized. In this research, after conducting the pairwise comparison and calculating the weights, the inconsistency rate was found to be less than 0.1, which indicates the logical consistency of the experts' opinions. The weights of the indicators derived in this stage are presented in Table 5.

Table 5. Indicators weights in the TOPSIS method

W1_Criterion_1	W2_Criterion_2	W3_Criterion_3	W4_Criterion_4	W4_Criterion_5
0.398	0.214	0.175	0.122	0.091

Step 4: Determining the weighted dimensionless matrix (V)

In this step, to obtain the weighted dimensionless matrix (V), the dimensionless matrix from the previous steps is multiplied by the square matrix (W), where the main diagonal entries represent the weights of the indicators, and all other elements are set to zero:

$$V = N \times W_{nn} \tag{6}$$

According to this method, this step was performed using MATLAB software, and its output is presented as the following matrix calculations, as shown in Table 6.

$$V = \begin{bmatrix} 15.0 & 0.033 & 0.001 & 0.6 & 12.3 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ . & . & . & . & . \\ 13.2 & 0.067 & 0.004 & 0.1 & 27.6 \end{bmatrix} \times \begin{bmatrix} 0.398 & 0 & 0 & 0 & 0 \\ 0 & 0.214 & 0 & 0 & 0 \\ 0 & 0 & 0.175 & 0 & 0 \\ 0 & 0 & 0 & 0.122 & 0 \\ 0 & 0 & 0 & 0 & 0.091 \end{bmatrix}$$

Step 5: Determining the positive ideal solution (PIS) and the negative ideal solution (NIS)

In this step, the positive ideal solution (PIS) and the negative ideal solution (NIS) are defined as follows:

$\mathbf{v}_j^+$: Vector of the best values for each indicator

$\mathbf{v}_j^-$: Vector of the worst values for each indicator

For this evaluation, the best values for positive indicators are the largest values, and for negative indicators, the smallest values. Conversely, the worst values for positive indicators are the smallest values, and for negative indicators, the largest values. The obtained values for the positive and negative ideal solutions are presented in Table 7.

Table 7. Positive and negative ideal solution values in the TOPSIS method

	C1	C2	C3	C4	C5
V_j^+	17.3	0.0284	0.001	0.003	0.139
V_j^-	0.0040	0.0002	0.1757	17.198	0.0040

Step 6: Determining the distance of each option from the positive and negative ideals

In this step, the Euclidean distance of each option from the positive ideal (d_i^+) and the negative ideal (d_i^-) are calculated using the following equations:

$$d_i^+ = \sqrt[2]{\sum_{j=1}^{n}\left(v_{ij} - v_j^+\right)^2}$$
$$d_i^- = \sqrt[2]{\sum_{j=1}^{n}\left(v_{ij} - v_j^-\right)^2} \tag{7}$$

Step 7: Determining the relative closeness of each option to the ideal solution

In this step, the relative closeness of each option (cl) to the ideal solution (best pattern) is calculated by the following equation:

$$cl_i = \frac{d_i^-}{d_i^- + d_i^+} \tag{8}$$

This step was performed using Excel software, and the results are presented in Table 8.

Table 6. Weighted dimensionless matrix (V) created in the TOPSIS method

	C1	C2	C3	C4	C5
A1	5.964	0.007	0.000	0.076	1.117
A2	5.964	0.008	0.000	0.076	1.136
A3	2.307	0.007	0.000	0.107	0.628
A4	17.158	0.007	0.001	0.011	0.628
A5	17.283	0.007	0.001	0.012	0.628
A6	13.155	0.007	0.000	0.080	1.151
A7	13.305	0.007	0.000	0.117	1.477
A8	13.248	0.028	0.000	0.115	17.197
A9	7.172	0.026	0.000	0.046	11.272
A10	5.139	0.026	0.000	0.049	7.672
A11	3.855	0.007	0.001	0.028	0.628
A12	6.264	0.007	0.001	0.028	0.628
A13	6.425	0.007	0.001	0.016	0.628
A14	15.850	0.007	0.000	0.043	1.117
A15	3.891	0.013	0.000	0.040	3.251
A16	5.715	0.013	0.001	0.014	1.829
A17	5.715	0.013	0.001	0.003	1.829
A18	5.634	0.014	0.000	0.044	4.468
A19	5.276	0.014	0.001	0.015	2.513
A20	5.634	0.014	0.001	0.018	2.513
A21	5.365	0.008	0.000	0.058	1.052
A22	11.857	0.007	0.000	0.062	1.117
A23	7.380	0.008	0.000	0.051	1.117
A24	5.384	0.007	0.001	0.013	0.628
A25	13.209	0.014	0.001	0.009	2.513
A26	2.850	0.008	0.001	0.003	0.870
A27	6.568	0.024	0.000	0.072	9.664
A28	6.321	0.008	0.001	0.020	0.870
A29	6.451	0.008	0.001	0.016	0.870
A30	3.275	0.022	0.000	0.046	9.281
A31	8.074	0.007	0.000	0.073	1.117
A32	8.786	0.007	0.001	0.048	0.628
A33	6.424	0.008	0.001	0.023	0.870
A34	2.856	0.007	0.001	0.176	0.628
A35	3.673	0.007	0.001	0.020	1.414
A36	5.296	0.013	0.000	0.038	3.251
A37	5.296	0.013	0.000	0.028	3.251
A38	11.701	0.004	0.001	0.013	0.139
A39	10.203	0.006	0.001	0.012	0.313
A40	7.177	0.006	0.001	0.016	0.313
A41	6.262	0.006	0.000	0.045	0.379
A42	5.519	0.006	0.000	0.042	0.736
A43	4.903	0.007	0.001	0.006	0.628
A44	4.931	0.007	0.000	0.057	0.760
A45	2.731	0.008	0.001	0.011	0.870
A46	6.023	0.022	0.000	0.045	9.281
A47	2.835	0.010	0.001	0.029	1.052
A48	3.916	0.008	0.000	0.075	1.546
A49	13.209	0.014	0.001	0.009	2.513
A50	3.150	0.022	0.001	0.004	5.220
A51	11.731	0.022	0.001	0.004	5.220
A52	5.276	0.014	0.001	0.016	2.513

Table 8. Relative closeness of each pattern to the ideal solution based on the TOPSIS method

	d_i^+	d_i^-	cl_i	Rank
A1	3.7860	24.3940	0.8656	15
A2	3.7910	24.3944	0.8655	16
A3	0.5003	24.3801	0.9799	1
A4	14.8591	24.3799	0.6213	50
A5	14.9838	24.3799	0.6193	51
A6	10.8957	24.3953	0.6913	44
A7	11.0799	24.4107	0.6878	46
A8	20.2654	29.7182	0.5946	52
A9	12.1491	26.7701	0.6878	45
A10	8.0475	25.4894	0.7600	36
A11	1.6243	24.3799	0.9375	6
A12	3.9876	24.3799	0.8594	22
A13	4.1473	24.3799	0.8546	25
A14	13.5789	24.3939	0.6424	49
A15	3.4920	24.5651	0.8755	14
A16	3.8036	24.4278	0.8653	20
A17	3.8036	24.4278	0.8653	19
A18	5.4603	24.7459	0.8192	33
A19	3.8019	24.4825	0.8656	17
A20	4.0876	24.4825	0.8569	23
A21	3.1925	24.3907	0.8843	12
A22	9.6002	24.3940	0.7176	41
A23	5.1664	24.3931	0.8252	32
A24	3.1154	24.3799	0.8867	11
A25	11.1578	24.4825	0.6869	47
A26	0.9104	24.3847	0.9640	4
A27	10.4344	26.1461	0.7148	42
A28	4.0802	24.3847	0.8567	24
A29	4.2084	24.3847	0.8528	27
A30	9.1929	26.0111	0.7389	38
A31	5.8497	24.3940	0.8066	34
A32	6.4974	24.3800	0.7896	35
A33	4.1816	24.3847	0.8536	26
A34	0.7558	24.3805	0.9699	2
A35	1.8687	24.4071	0.9289	7
A36	4.3150	24.5651	0.8506	29
A37	4.3149	24.5650	0.8506	28
A38	9.3945	24.3779	0.7218	40
A39	7.8983	24.3769	0.7553	37
A40	4.8736	24.3769	0.8334	30
A41	3.9628	24.3774	0.8602	21
A42	3.2677	24.3830	0.8818	13
A43	2.6421	24.3799	0.9022	9
A44	2.6969	24.3828	0.9004	10
A45	0.8448	24.3847	0.9665	3
A46	9.8683	26.0111	0.7250	39
A47	1.0555	24.3897	0.9585	5
A48	2.1385	24.4134	0.9195	8
A49	11.1578	24.4825	0.6869	48
A50	5.1509	24.8821	0.8285	31
A51	10.7067	24.8821	0.6992	43
A52	3.8019	24.4825	0.8656	18

Step 8: Ranking of options

In this step, the option with the larger *cl* value is considered superior and is selected. The results obtained at the end of this stage are shown in Table 9. Based on these results, Pattern 3 (ID: M_Patt_03) has been identified as the best blasting pattern using the TOPSIS model.

Table 9. Ranking of patterns based on the TOPSIS method

Pattern ID	B (m)	S (m)	D (mm)	Rank
M_Patt_01	7.5	9.5	200	15
M_Patt_02	7.5	9.5	200	16
M_Patt_03	6.5	8	150	1
M_Patt_04	5.5	7	150	50
M_Patt_05	5.5	7	150	51
M_Patt_06	7.5	9.5	203	44
M_Patt_07	6.5	8	230	46
M_Patt_08	6.5	8	230	52
M_Patt_09	6.5	8	200	45
M_Patt_10	6.5	8	165	36
M_Patt_11	5.5	7	150	6
M_Patt_12	5.5	7	150	22
M_Patt_13	5.5	7	150	25
M_Patt_14	6.5	8	200	49
M_Patt_15	6.5	8	200	14
M_Patt_16	5.5	7	150	20
M_Patt_17	5.5	7	150	19
M_Patt_18	6.5	8	200	33
M_Patt_19	5.5	7	150	17
M_Patt_20	5.5	7	150	23
M_Patt_21	6.5	8	165	12
M_Patt_22	7.5	9.5	200	41
M_Patt_23	6.5	8	170	32
M_Patt_24	5.5	7	150	11
M_Patt_25	5.5	7	150	47
M_Patt_26	5.5	7	150	4
M_Patt_27	6.5	8	200	42
M_Patt_28	5.5	7	150	24
M_Patt_29	5.5	7	150	27
M_Patt_30	6.5	8	200	38
M_Patt_31	7.5	9.5	200	34
M_Patt_32	5.5	7	150	35
M_Patt_33	5.5	7	150	26
M_Patt_34	5.5	7	150	2
M_Patt_35	5.5	7	225	7
M_Patt_36	6.5	8	200	29
M_Patt_37	6.5	8	200	28
M_Patt_38	5.5	7	150	40
M_Patt_39	5.5	7	150	37
M_Patt_40	5.5	7	150	30
M_Patt_41	6.5	8	165	21
M_Patt_42	6.5	8	230	13
M_Patt_43	5.5	7	150	9
M_Patt_44	6.5	8	165	10
M_Patt_45	5.5	7	150	3
M_Patt_46	6.5	8	200	39
M_Patt_47	5.5	7	150	5
M_Patt_48	7.5	9.5	200	8
M_Patt_49	5.5	7	150	48
M_Patt_50	5	6	150	31
M_Patt_51	5	6	150	43
M_Patt_52	5.5	7	150	18

Based on the characteristics of blasting Pattern 3, it can be concluded that this pattern performs more favourably in terms of fragmentation, specific charge, and specific drilling compared to other patterns. However, in terms of backbreak and flyrock, it is not considered among the best. Additionally, the results indicate that the weights of the indices align well with the relative importance of each criterion. The findings also show that Patterns 8, 5, and 4 are among the worst performing patterns.

4.2. ELECTRE method for MCDM

In this section, the ELECTRE method is applied to evaluate the blasting pattern. This method relies on concordance and discordance sets, where all options are evaluated through non-ranked comparisons, eliminating ineffective choices.

The steps for the ELECTRE method are similar to the TOPSIS method up to the point of obtaining the dimensionless decision matrix (matrix V). After determining matrix V, the steps for performing calculations and ranking the blasting patterns are as follows:

Step 1: Concordant and discordant sets

In this step, all options are evaluated against all criteria. The concordant set of options k and I, denoted by $S_{k,l}$, includes all the criteria in where option A_k is more desirable than option A_l. To determine this desirability, the nature of the decision-making criteria must be considered, specifically whether they have a positive or negative aspect. Accordingly, if the criterion in question has a positive aspect:

$$S_{k,l} = \{j \mid v_{kj} \geq v_{ij}\} \;\;, \; j = 1, \dots, n \tag{9}$$

If the criterion in question has a negative aspect:

$$S_{k,l} = \{j \mid v_{kj} \leq v_{ij}\} \;\;, \; j = 1, \dots, n \tag{10}$$

The discordant set $D_{k,l}$ also includes criteria for where option A_k has less desirability than option A_l. This means that for a positive criterion:

$$D_{k,l} = \{j \mid v_{kj} < v_{ij}\} \;\;, \; j = 1, \dots, n \tag{11}$$

And with a negative index:

$$D_{k,l} = \{j \mid v_{kj} > v_{ij}\} \;\;, \; j = 1, \dots, n \tag{12}$$

This stage has also been performed using Excel software, and the results are summarized in Tables 10 and 11.

Table 10. The concordant sets obtained by the ELECTRE method

$S_{1\,2} = (1,2,3,4)$	$S_{2\,1} = (1,2,3,4)$	$S_{3\,1} = (1,2,5)$	$S_{52\,1} = (1,4)$
$S_{1\,3} = (2,3,4)$	$S_{2\,3} = (3,4)$	$S_{3\,2} = (1,2,5)$	$S_{52\,2} = (1,4)$
$S_{1\,4} = (1,2,3)$	$S_{2\,4} = (1,3)$	$S_{3\,4} = (1,2,3,5)$	$S_{52\,3} = (4)$
$S_{1\,5} = (1,2,3)$	$S_{2\,5} = (1,3)$	$S_{3\,5} = (1,2,3,5)$	$S_{52\,4} = (1,3)$
$S_{1\,6} = (1,2,3,4)$	$S_{2\,6} = (1,3,4,5)$	$S_{3\,6} = (1,2,5)$	$S_{52\,5} = (1,3)$
$S_{1\,7} = (1,2,3,4)$	$S_{2\,7} = (1,3,4,5)$	$S_{3\,7} = (1,2,3,4,5)$	$S_{52\,6} = (1,3)$
$S_{1\,51} = (1,2,3,5)$	$S_{2\,51} = (1,2,3,5)$	$S_{3\,51} = (1,2,3,5)$	$S_{52\,50} = (2,3,5)$
$S_{1\,52} = (2,3,5)$	$S_{2\,52} = (2,3,5)$	$S_{3\,52} = (1,2,3,5)$	$S_{52\,51} = (1,2,3,5)$

Table 11. The discordant sets obtained by the ELECTRE method

$D_{1\,2} = (5)$	$D_{2\,1} = (5)$	$D_{3\,1} = (3,4)$	$D_{52\,1} = (2,3,5)$
$D_{1\,3} = (1,5)$	$D_{2\,3} = (1,2,5)$	$D_{3\,2} = (3,4)$	$D_{52\,2} = (2,3,5)$
$D_{1\,4} = (4,5)$	$D_{2\,4} = (2,4,5)$	$D_{3\,4} = (4)$	$D_{52\,3} = (1,2,3,5)$
$D_{1\,5} = (4,5)$	$D_{2\,5} = (2,4,5)$	$D_{3\,5} = (4)$	$D_{52\,4} = (2,4,5)$
$D_{1\,6} = (5)$	$D_{2\,6} = (2)$	$D_{3\,6} = (3,4)$	$D_{52\,5} = (2,4,5)$
$D_{1\,7} = (5)$	$D_{2\,7} = (2)$	$D_{3\,7} = (\;)$	$D_{52\,6} = (2,4,5)$
$D_{1\,51} = (4)$	$D_{2\,51} = (4)$	$D_{3\,51} = (4)$	$D_{52\,50} = (1,4)$
$D_{1\,52} = (1,4)$	$D_{2\,52} = (1,4)$	$D_{3\,52} = (4)$	$D_{52\,51} = (4)$

Step 2: Concordant and discordant sets

At this stage, based on the information from the previous step, the concordant (I) and discordant (NI) matrices are determined. The concordant matrix is a 52×52 square matrix with no elements on its diagonal. The other elements of this matrix are obtained by summing the weights of the criteria belonging to the concordant set, using the following equation:

$$I_{k,l} = \sum w_j \quad , \; j \in A_{k,l} \tag{13}$$

The criterion ($I_{k,l}$) represents the relative importance of A_k compared to A_l. The value of this criterion ranges between zero and one. The higher this value, the greater the superiority of A_k over A_l.

The discordant matrix, like the concordant matrix, is a 52×52 square matrix with no elements on its main diagonal. The other elements of this matrix are derived from the normalized weighted matrix. These elements are determined using the following equation:

$$NI_{kl} = \frac{Max \left|v_{kl} - v_{lj}\right| \; , \; j \in D_{k,l}}{Max \left|v_{kj} - v_{lj}\right| \; , \; j \in \; all \; indicators} \tag{14}$$

In any case, this criterion measures the ratio of the undesirability of the discordant set to the total discordance in the criteria. The concordant and discordant matrices are presented in Tables 12 and 13, respectively. This stage was carried out using the concordant and discordant sets obtained from the previous stage, through Excel software.

It should be noted that due to the length of these matrices, only a summarized portion of each matrix is shown in Figures 2 and 3.

Table 12. Concordant matrix obtained by the ELECTRE method

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23
A1	--	1	0.51	0.79	0.79	1	1	1	0.88	0.48	0.39	0.79	0.79	0.88	0.48	0.48	0.48	0.48	0.48	0.48	0.39	0.88	0.88
A2	0.69	--	0.3	0.57	0.57	0.79	0.79	1	0.88	0.48	0.18	0.57	0.57	0.57	0.48	0.48	0.48	0.48	0.48	0.48	0.39	0.57	0.79
A3	0.7	0.7	--	0.88	0.88	0.7	1	1	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.7	0.88
A4	0.43	0.43	0.43	--	1	0.43	0.43	0.43	0.43	0.43	0.6	0.6	0.6	0.43	0.43	0.6	0.48	0.43	0.6	0.6	0.43	0.43	0.43
A5	0.43	0.43	0.43	0.48	--	0.43	0.43	0.43	0.43	0.43	0.6	0.6	0.6	0.43	0.43	0.6	0.48	0.43	0.6	0.6	0.43	0.43	0.43
A6	0.39	0.39	0.51	0.79	0.79	--	1	1	0.48	0.48	0.39	0.39	0.39	0.79	0.48	0.48	0.48	0.48	0.48	0.48	0.39	0.39	0.39
A7	0.21	0.21	0.39	0.79	0.79	0.21	--	0.48	0.48	0.48	0.39	0.39	0.39	0.79	0.48	0.48	0.48	0.48	0.48	0.48	0.39	0.21	0.39
A8	0	0	0.18	0.57	0.57	0	0.69	--	0.18	0.18	0.18	0.18	0.18	0.57	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0	0.18
A9	0.12	0.12	0.3	0.57	0.57	0.52	0.69	1	--	0.51	0.18	0.18	0.18	0.57	0.18	0.18	0.18	0.18	0.18	0.18	0.3	0.52	0.69
A10	0.52	0.52	0.3	0.57	0.57	0.52	0.69	1	0.88	--	0.18	0.57	0.57	0.57	0.18	0.57	0.57	0.57	0.57	0.57	0.69	0.52	0.69
A11	0.82	0.82	0.43	0.88	0.88	0.82	0.82	0.82	0.82	0.82	--	1	0.88	0.82	0.82	0.88	0.88	0.82	0.88	0.88	0.82	0.82	0.82
A12	0.43	0.43	0.43	0.88	0.88	0.82	0.82	0.82	0.82	0.43	0.6	--	0.88	0.82	0.43	0.48	0.48	0.43	0.48	0.48	0.43	0.82	0.82
A13	0.43	0.43	0.43	0.88	0.88	0.82	0.82	0.82	0.82	0.43	0.6	0.6	--	0.82	0.43	0.48	0.48	0.43	0.48	0.6	0.43	0.82	0.82
A14	0.43	0.43	0.51	0.79	0.79	0.43	0.6	0.6	0.6	0.6	0.39	0.39	0.39	--	0.48	0.48	0.48	0.6	0.48	0.48	0.51	0.43	0.6
A15	0.52	0.52	0.3	0.57	0.57	0.52	0.69	1	1	1	0.18	0.57	0.57	0.69	--	0.79	0.79	1	0.79	0.79	0.69	0.52	0.69
A16	0.52	0.52	0.12	0.57	0.57	0.52	0.52	0.82	0.82	0.43	0.3	0.69	0.69	0.52	0.43	--	0.88	0.43	0.6	0.6	0.12	0.52	0.52
A17	0.52	0.52	0.12	0.69	0.69	0.52	0.52	0.82	0.82	0.43	0.3	0.69	0.69	0.52	0.43	1	--	0.43	0.6	0.6	0.12	0.52	0.52
A18	0.52	0.52	0.3	0.57	0.57	0.52	0.69	1	1	0.6	0.18	0.57	0.57	0.57	0.18	0.57	0.57	--	0.39	0.79	0.3	0.52	0.69
A19	0.52	0.52	0.12	0.57	0.57	0.52	0.52	0.82	0.82	0.43	0.3	0.69	0.69	0.52	0.21	0.57	0.57	0.82	--	1	0.52	0.52	0.52
A20	0.52	0.52	0.12	0.57	0.57	0.52	0.52	0.82	0.82	0.43	0.3	0.69	0.57	0.52	0.21	0.57	0.57	0.82	0.48	--	0.12	0.52	0.52
A21	0.61	0.61	0.3	0.57	0.57	0.61	0.79	1	0.88	0.48	0.18	0.57	0.57	0.66	0.48	0.88	0.88	0.88	0.48	0.88	--	0.61	0.88
A22	0.6	0.6	0.51	0.79	0.79	1	1	1	0.48	0.48	0.39	0.39	0.39	0.88	0.48	0.48	0.48	0.48	0.48	0.48	0.39	--	0.48
A23	0.21	0.21	0.3	0.57	0.57	0.61	0.79	1	0.48	0.48	0.18	0.18	0.18	0.66	0.48	0.48	0.48	0.48	0.48	0.48	0.51	0.61	--
A24	0.82	0.82	0.43	0.88	0.88	0.82	0.82	0.82	0.82	0.43	0.6	1	1	0.82	0.43	1	0.88	0.82	0.6	1	0.43	0.82	0.82
A25	0.12	0.12	0.12	0.69	0.69	0.12	0.52	0.82	0.43	0.43	0.3	0.3	0.3	0.52	0.21	0.3	0.18	0.43	0.6	0.6	0.12	0.12	0.12
A26	0.61	0.61	0.12	0.69	0.69	0.61	0.61	0.82	0.82	0.82	0.69	0.69	0.69	0.61	0.82	1	1	0.82	1	1	0.82	0.61	0.82
A27	0.12	0.12	0.3	0.57	0.57	0.52	0.69	1	0.88	0.39	0.18	0.18	0.18	0.57	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.4	0.57
A28	0.21	0.21	0.12	0.57	0.57	0.61	0.61	0.82	0.82	0.43	0.3	0.3	0.57	0.61	0.43	0.48	0.48	0.43	0.48	0.48	0.43	0.61	0.82
A29	0.21	0.21	0.12	0.57	0.57	0.61	0.61	0.82	0.82	0.43	0.3	0.3	0.18	0.61	0.43	0.48	0.48	0.43	0.48	0.6	0.43	0.61	0.82
A30	0.52	0.52	0.3	0.57	0.57	0.52	0.69	1	1	0.91	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.69	0.52	0.69
A31	0.6	0.6	0.51	0.79	0.79	1	1	1	0.48	0.48	0.39	0.39	0.39	0.88	0.48	0.48	0.48	0.48	0.48	0.48	0.39	0.88	0.48
A32	0.43	0.43	0.43	0.88	0.88	0.82	0.82	0.82	0.31	0.43	0.48	0.48	0.48	0.7	0.31	0.48	0.48	0.31	0.48	0.48	0.43	0.82	0.43
A33	0.21	0.21	0.12	0.57	0.57	0.61	0.61	0.82	0.82	0.43	0.3	0.3	0.57	0.61	0.43	0.48	0.48	0.43	0.48	0.48	0.43	0.61	0.82
A34	0.7	0.7	0.31	0.88	0.88	0.7	0.7	0.7	0.7	0.7	0.88	0.88	0.88	0.7	0.7	0.88	0.88	0.7	0.88	0.88	0.7	0.7	0.7
A35	0.73	0.73	0.34	0.79	0.79	0.73	0.82	0.82	0.82	0.82	0.91	0.91	0.79	0.73	0.82	0.88	0.88	0.82	0.88	0.88	0.73	0.73	0.73
A36	0.52	0.52	0.3	0.57	0.57	0.52	0.69	1	1	0.6	0.18	0.57	0.57	0.69	0.6	0.79	0.79	1	0.39	0.79	0.69	0.52	0.69
A37	0.52	0.52	0.3	0.57	0.57	0.52	0.69	1	1	0.6	0.18	0.57	0.57	0.69	0.6	0.79	0.79	1	0.39	0.79	0.69	0.52	0.69
A38	0.43	0.43	0.43	0.88	0.88	0.82	0.82	0.82	0.43	0.43	0.6	0.6	0.6	0.82	0.43	0.6	0.48	0.43	0.6	0.6	0.43	0.82	0.43
A39	0.43	0.43	0.43	0.88	1	0.82	0.82	0.82	0.43	0.43	0.6	0.6	0.6	0.82	0.43	0.6	0.48	0.43	0.6	0.6	0.43	0.82	0.43
A40	0.43	0.43	0.43	0.88	0.88	0.82	0.82	0.82	0.43	0.43	0.6	0.6	0.48	0.82	0.43	0.48	0.48	0.43	0.48	0.6	0.43	0.82	0.82
A41	0.43	0.43	0.6	0.88	0.88	0.82	1	1	1	0.6	0.48	0.88	0.88	0.88	0.48	0.48	0.48	0.48	0.48	0.48	0.6	0.82	1
A42	0.82	0.82	0.51	0.79	0.79	0.82	1	1	1	0.6	0.39	0.79	0.79	1	0.48	0.88	0.88	1	0.48	0.88	0.6	0.82	1
A43	0.82	0.82	0.43	1	1	0.82	0.82	0.82	0.82	0.82	0.6	1	1	0.82	0.43	1	0.88	0.82	1	1	0.82	0.82	0.82
A44	0.82	0.82	0.51	0.79	0.79	0.82	1	1	0.88	0.88	0.39	0.79	0.79	0.88	0.48	0.88	0.88	0.88	0.88	0.88	1	0.82	0.88
A45	0.61	0.61	0.12	0.57	0.69	0.61	0.61	0.82	0.82	0.82	0.69	0.69	0.69	0.61	0.82	1	0.88	0.82	1	1	0.82	0.61	0.82
A46	0.12	0.12	0.3	0.57	0.57	0.52	0.69	1	1	0.51	0.18	0.57	0.57	0.57	0.18	0.18	0.18	0.18	0.18	0.18	0.3	0.52	0.69
A47	0.61	0.61	0.12	0.57	0.57	0.61	0.61	0.82	0.82	0.82	0.57	0.57	0.57	0.61	0.82	0.88	0.88	0.82	0.88	0.88	0.61	0.61	0.61
A48	0.69	0.69	0.3	0.57	0.57	0.69	0.69	1	0.88	0.88	0.18	0.57	0.57	0.57	0.48	0.88	0.88	0.88	0.88	0.88	0.79	0.57	0.79
A49	0.12	0.12	0.12	0.69	0.69	0.12	0.52	0.82	0.43	0.43	0.3	0.3	0.3	0.52	0.21	0.3	0.18	0.43	0.6	0.6	0.12	0.12	0.12
A50	0.52	0.52	0.12	0.52	0.52	0.52	0.52	0.82	0.82	0.82	0.52	0.52	0.52	0.52	0.52	0.52	0.4	0.52	0.52	0.52	0.52	0.52	0.52
A51	0.12	0.12	0.12	0.52	0.52	0.52	0.52	0.82	0.43	0.43	0.12	0.12	0.12	0.52	0.12	0.12	0	0.12	0.12	0.12	0.12	0.52	0.12
A52	0.52	0.52	0.12	0.57	0.57	0.52	0.52	0.82	0.82	0.43	0.3	0.69	0.69	0.52	0.21	0.57	0.57	0.82	0.88	1	0.52	0.52	0.52

	A24	A25	A26	A27	A28	A29	A30	A31	A32	A33	A34	A35	A36	A37	A38	A39	A40	A41	A42	A43	A44	A45	A46	A47	A48	A49	A50	A51	A52
A1	0.39	0.88	0.39	0.88	0.79	0.79	0.48	0.88	0.79	0.79	0.51	0.48	0.48	0.48	0.57	0.57	0.57	0.57	0.18	0.39	0.39	0.39	0.88	0.39	0.48	0.88	0.48	0.88	0.48
A2	0.18	0.88	0.39	0.88	0.79	0.79	0.48	0.57	0.57	0.79	0.3	0.27	0.48	0.48	0.57	0.57	0.57	0.57	0.18	0.18	0.18	0.39	0.88	0.39	0.48	0.88	0.48	0.88	0.48
A3	0.88	0.88	0.88	0.88	0.88	0.88	0.88	0.7	0.88	0.88	1	0.88	0.88	0.88	0.57	0.57	0.57	0.57	0.88	0.88	0.88	0.88	0.88	0.88	0.7	0.88	0.88	0.88	0.88
A4	0.6	0.31	0.48	0.43	0.6	0.6	0.43	0.43	0.6	0.6	0.6	0.6	0.43	0.43	0.3	0.3	0.3	0.12	0.21	0.48	0.43	0.6	0.43	0.6	0.43	0.48	0.48	0.48	0.6
A5	0.6	0.31	0.48	0.43	0.6	0.6	0.43	0.43	0.6	0.6	0.6	0.6	0.43	0.43	0.3	0.18	0.3	0.12	0.21	0.48	0.43	0.48	0.43	0.6	0.43	0.48	0.48	0.48	0.6
A6	0.39	0.88	0.39	0.48	0.39	0.39	0.48	0.39	0.39	0.39	0.51	0.48	0.48	0.48	0.18	0.18	0.18	0.18	0.18	0.39	0.39	0.39	0.48	0.39	0.48	0.88	0.48	0.48	0.48
A7	0.39	0.48	0.39	0.48	0.39	0.39	0.48	0.21	0.39	0.39	0.51	0.39	0.48	0.48	0.18	0.18	0.18	0.18	0.18	0.39	0.39	0.39	0.48	0.39	0.31	0.48	0.48	0.48	0.48
A8	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0	0.18	0.18	0.3	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0	0.18	0.18	0.18	0.18
A9	0.18	0.57	0.18	0.3	0.18	0.18	0.18	0.52	0.69	0.18	0.3	0.18	0.18	0.18	0.57	0.57	0.57	0.18	0.18	0.18	0.3	0.18	0.18	0.18	0.12	0.57	0.18	0.57	0.18
A10	0.57	0.57	0.18	0.79	0.57	0.57	0.27	0.52	0.57	0.57	0.3	0.18	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.18	0.3	0.18	0.66	0.18	0.12	0.57	0.18	0.57	0.57
A11	0.88	0.7	0.48	0.82	0.88	0.88	0.43	0.82	1	0.88	0.6	0.48	0.82	0.82	0.57	0.57	0.57	0.52	0.61	0.88	0.82	0.48	0.82	0.6	0.82	0.88	0.48	0.88	0.88
A12	0.48	0.7	0.48	0.82	0.88	0.88	0.43	0.82	1	0.88	0.6	0.48	0.43	0.43	0.57	0.57	0.57	0.12	0.21	0.48	0.43	0.48	0.43	0.6	0.43	0.88	0.48	0.88	0.48
A13	0.48	0.7	0.48	0.82	0.6	1	0.43	0.82	1	0.6	0.6	0.6	0.43	0.43	0.57	0.57	0.69	0.12	0.21	0.48	0.43	0.48	0.43	0.6	0.43	0.88	0.48	0.88	0.6
A14	0.39	0.48	0.39	0.6	0.39	0.39	0.6	0.43	0.51	0.39	0.51	0.48	0.48	0.48	0.18	0.18	0.18	0.3	0.18	0.39	0.51	0.39	0.6	0.39	0.43	0.48	0.48	0.48	0.48
A15	0.57	0.79	0.18	1	0.57	0.57	0.6	0.52	0.69	0.57	0.3	0.18	0.88	0.88	0.57	0.57	0.57	0.69	0.69	0.57	0.69	0.18	1	0.18	0.52	0.79	0.48	0.88	0.79
A16	0.18	0.7	0.18	0.82	0.69	0.69	0.43	0.52	0.69	0.69	0.3	0.3	0.43	0.43	0.57	0.57	0.69	0.52	0.12	0.18	0.12	0.18	0.82	0.3	0.12	0.88	0.48	0.88	0.6
A17	0.3	0.82	0.18	0.82	0.69	0.69	0.43	0.52	0.69	0.69	0.3	0.3	0.43	0.43	0.69	0.69	0.69	0.52	0.12	0.3	0.12	0.3	0.82	0.3	0.12	1	0.6	1	0.6
A18	0.18	0.57	0.18	1	0.57	0.57	0.6	0.52	0.69	0.57	0.3	0.18	0.18	0.18	0.57	0.57	0.57	0.69	0.18	0.18	0.3	0.18	1	0.18	0.12	0.79	0.48	0.88	0.39
A19	0.57	0.49	0.18	0.82	0.69	0.69	0.43	0.52	0.69	0.69	0.3	0.3	0.61	0.61	0.57	0.57	0.69	0.52	0.52	0.18	0.12	0.18	0.82	0.3	0.12	0.88	0.48	0.88	1
A20	0.18	0.49	0.18	0.82	0.69	0.57	0.43	0.52	0.69	0.69	0.3	0.3	0.21	0.21	0.57	0.57	0.57	0.52	0.12	0.18	0.12	0.18	0.82	0.3	0.12	0.88	0.48	0.88	0.48
A21	0.57	0.88	0.39	1	0.79	0.79	0.48	0.61	0.57	0.79	0.3	0.27	0.48	0.48	0.57	0.57	0.57	0.57	0.57	0.18	0.18	0.39	0.88	0.48	0.43	0.88	0.48	0.88	0.48
A22	0.39	0.88	0.39	0.6	0.39	0.39	0.48	0.6	0.39	0.39	0.51	0.48	0.48	0.48	0.18	0.18	0.18	0.18	0.18	0.39	0.39	0.39	0.48	0.39	0.6	0.88	0.48	0.48	0.48
A23	0.18	0.88	0.39	0.6	0.39	0.39	0.48	0.61	0.57	0.39	0.3	0.27	0.48	0.48	0.57	0.57	0.18	0.18	0.18	0.18	0.3	0.39	0.48	0.39	0.43	0.88	0.48	0.88	0.48
A24	--	0.7	0.48	0.82	1	1	0.43	0.82	1	1	0.6	0.6	0.43	0.43	0.69	0.57	0.69	0.52	0.61	0.48	0.43	0.48	0.82	0.6	0.43	0.88	0.48	0.88	0.6
A25	0.3	--	0.18	0.43	0.3	0.3	0.43	0.12	0.3	0.3	0.3	0.3	0.21	0.21	0.3	0.3	0.3	0.12	0.12	0.18	0.12	0.3	0.43	0.3	0.12	1	0.48	0.48	0.6
A26	0.69	0.82	--	0.82	1	1	0.82	0.61	0.69	1	0.69	0.79	0.82	0.82	0.69	0.69	0.69	0.52	0.52	0.69	0.52	0.6	0.82	0.6	0.82	1	1	1	1
A27	0.18	0.57	0.18	--	0.18	0.18	0.18	0.52	0.57	0.18	0.3	0.18	0.18	0.18	0.57	0.57	0.57	0.18	0.18	0.18	0.18	0.18	0.18	0.18	0.12	0.57	0.18	0.57	0.18
A28	0.18	0.7	0.48	0.82	--	0.88	0.43	0.61	0.69	1	0.3	0.39	0.43	0.43	0.57	0.57	0.57	0.12	0.12	0.18	0.12	0.48	0.43	0.6	0.43	0.88	0.48	0.88	0.48
A29	0.18	0.7	0.48	0.82	0.6	--	0.43	0.61	0.69	0.6	0.3	0.39	0.43	0.43	0.57	0.57	0.57	0.12	0.12	0.18	0.12	0.48	0.43	0.6	0.43	0.88	0.48	0.88	0.48
A30	0.57	0.57	0.18	1	0.57	0.57	--	0.52	0.69	0.57	0.3	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.69	0.18	0.88	0.18	0.52	0.57	0.39	0.79	0.57
A31	0.39	0.88	0.39	0.48	0.39	0.39	0.48	--	0.79	0.39	0.51	0.48	0.48	0.48	0.57	0.57	0.18	0.18	0.18	0.39	0.39	0.39	0.48	0.39	0.6	0.88	0.48	0.88	0.48
A32	0.48	0.7	0.48	0.43	0.48	0.48	0.31	0.43	--	0.48	0.6	0.48	0.31	0.31	0.57	0.57	0.18	0	0.09	0.48	0.43	0.48	0.31	0.48	0.43	0.88	0.48	0.88	0.48
A33	0.18	0.7	0.48	0.82	0.48	0.88	0.43	0.61	0.69	--	0.3	0.27	0.43	0.43	0.57	0.57	0.57	0.12	0.12	0.18	0.12	0.48	0.43	0.6	0.43	0.88	0.48	0.88	0.48
A34	0.88	0.7	0.48	0.7	0.88	0.88	0.7	0.7	0.88	0.88	--	0.88	0.7	0.7	0.57	0.57	0.57	0.4	0.49	0.88	0.7	0.48	0.7	0.48	0.7	0.88	0.88	0.88	0.88
A35	0.79	0.7	0.39	0.82	0.79	0.79	0.43	0.73	0.91	0.91	0.51	--	0.82	0.82	0.57	0.57	0.57	0.52	0.52	0.79	0.73	0.39	0.82	0.51	0.82	0.88	0.48	0.88	0.88
A36	0.57	0.79	0.18	1	0.57	0.57	0.6	0.52	0.69	0.57	0.3	0.18	--	0.88	0.57	0.57	0.57	0.69	0.69	0.18	0.3	0.18	1	0.18	0.12	0.79	0.48	0.88	0.39
A37	0.57	0.79	0.18	1	0.57	0.57	0.6	0.52	0.69	0.57	0.3	0.18	1	--	0.57	0.57	0.57	0.69	0.69	0.18	0.3	0.18	1	0.3	0.12	0.79	0.48	0.88	0.39
A38	0.48	0.7	0.48	0.43	0.6	0.6	0.43	0.43	0.6	0.6	0.6	0.6	0.43	0.43	--	0.48	0.6	0.43	0.43	0.48	0.43	0.48	0.43	0.6	0.43	0.88	0.48	0.88	0.6
A39	0.6	0.7	0.48	0.43	0.6	0.6	0.43	0.43	0.6	0.6	0.6	0.6	0.43	0.43	0.69	--	0.6	0.43	0.43	0.48	0.43	0.48	0.43	0.6	0.43	0.88	0.48	0.88	0.6
A40	0.48	0.7	0.48	0.43	0.6	0.6	0.43	0.82	1	0.6	0.6	0.6	0.43	0.43	0.57	0.88	--	0.43	0.43	0.48	0.43	0.48	0.43	0.6	0.43	0.88	0.48	0.88	0.48
A41	0.48	0.88	0.48	1	0.88	0.88	0.6	0.82	1	0.88	0.6	0.48	0.48	0.48	0.57	0.79	0.79	--	0.48	0.48	0.6	0.43	0.6	0.43	0.43	0.88	0.48	0.88	0.48
A42	0.39	0.88	0.48	1	0.88	0.88	0.6	0.82	0.91	0.88	0.51	0.48	0.48	0.48	0.57	0.79	0.79	0.91	--	0.39	0.6	0.48	1	0.48	0.43	0.88	0.48	0.88	0.48
A43	1	0.82	0.48	0.82	1	1	0.43	0.82	1	1	0.6	0.6	0.82	0.82	0.69	0.69	0.69	0.52	0.61	--	0.82	0.6	0.82	0.6	0.43	1	0.48	0.88	1
A44	0.79	0.88	0.48	1	0.88	0.88	0.48	0.82	0.79	0.88	0.51	0.48	0.88	0.88	0.57	0.57	0.57	0.57	0.57	0.39	--	0.48	0.88	0.48	0.43	0.88	0.48	0.88	0.88
A45	0.69	0.7	0.88	0.82	1	1	0.82	0.61	0.69	1	0.69	0.79	0.82	0.82	0.69	0.69	0.69	0.52	0.52	0.57	0.52	--	0.82	1	0.82	0.88	0.88	0.88	1
A46	0.18	0.57	0.18	1	0.57	0.57	0.6	0.52	0.69	0.57	0.3	0.18	0.18	0.18	0.57	0.57	0.57	0.57	0.18	0.18	0.3	0.18	--	0.18	0.12	0.57	0.39	0.79	0.18
A47	0.57	0.7	0.57	0.82	0.57	0.57	0.82	0.61	0.69	0.57	0.69	0.66	0.82	0.7	0.57	0.57	0.57	0.52	0.52	0.57	0.52	0.18	0.82	--	0.61	0.88	0.88	0.88	0.88
A48	0.57	0.88	0.39	0.88	0.79	0.79	0.48	0.57	0.57	0.79	0.3	0.18	0.88	0.88	0.57	0.57	0.57	0.57	0.57	0.57	0.57	0.39	0.88	0.39	--	0.88	0.48	0.88	0.88
A49	0.3	0.21	0.18	0.43	0.3	0.3	0.43	0.12	0.3	0.3	0.3	0.3	0.21	0.21	0.3	0.3	0.3	0.12	0.12	0.18	0.12	0.3	0.43	0.3	0.12	--	0.48	0.48	0.6
A50	0.52	0.52	0	0.82	0.52	0.52	0.82	0.52	0.52	0.52	0.12	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.52	0.12	0.82	0.12	0.52	0.52	--	1	0.52
A51	0.12	0.52	0	0.43	0.12	0.12	0.43	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.12	0.43	0.12	0.12	0.52	0.6	--	0.12
A52	0.57	0.49	0.18	0.82	0.69	0.69	0.43	0.52	0.69	0.69	0.3	0.3	0.61	0.61	0.57	0.57	0.69	0.52	0.52	0.18	0.12	0.18	0.82	0.3	0.12	0.88	0.48	0.88	--

Table 13. Discordant matrix obtained by the ELECTRE method

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23	A24	A25	A26
A1	--	0.00	1.00	0.04	0.04	0.00	0.00	0.00	0.00	0.13	1.00	1.00	1.00	0.00	0.97	0.35	0.35	0.10	0.49	0.24	1.00	0.00	0.02	1.00	0.01	1.00
A2	1.00	--	1.00	0.05	0.04	0.00	0.00	0.00	0.00	0.13	1.00	1.00	1.00	0.00	0.98	0.36	0.36	0.10	0.50	0.24	1.00	0.00	0.02	1.00	0.01	1.00
A3	0.01	0.01	--	0.01	0.01	0.00	0.00	0.00	0.01	0.01	0.05	0.02	0.02	0.00	0.03	0.03	0.03	0.02	0.03	0.03	0.02	0.00	0.01	0.03	0.01	0.19
A4	1.00	1.00	1.00	--	0.00	1.00	1.00	0.24	0.94	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A5	1.00	1.00	1.00	1.00	--	1.00	1.00	0.24	0.95	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A6	1.00	1.00	1.00	0.13	0.13	--	0.00	0.00	0.59	1.00	1.00	1.00	1.00	0.01	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.05	1.00
A7	1.00	1.00	1.00	0.22	0.21	1.00	--	0.00	0.63	1.00	1.00	1.00	1.00	0.14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.10	1.00
A8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	--	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	--	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A10	1.00	1.00	1.00	0.59	0.58	0.81	0.76	0.00	0.00	--	1.00	1.00	1.00	0.61	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.98	1.00	1.00	0.64	1.00
A11	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	--	0.00	0.00	0.00	0.00	0.01	0.01	0.00	0.01	0.01	0.00	0.00	0.00	0.01	0.00	1.00
A12	0.62	0.59	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.16	1.00	--	0.08	0.00	0.90	0.46	0.46	0.16	0.52	0.33	1.00	0.00	0.00	1.00	0.00	1.00
A13	0.94	0.91	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.18	1.00	1.00	--	0.00	0.97	0.59	0.59	0.21	0.61	0.42	1.00	0.00	0.00	1.00	0.00	1.00
A14	1.00	1.00	1.00	0.37	0.34	1.00	1.00	0.16	0.65	1.00	1.00	1.00	1.00	--	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A15	1.00	1.00	1.00	0.20	0.20	0.23	0.19	0.00	0.00	0.00	1.00	1.00	1.00	0.18	--	0.78	0.78	0.00	0.53	0.42	1.00	0.27	0.61	1.00	0.08	1.00
A16	1.00	1.00	1.00	0.10	0.10	0.09	0.05	0.00	0.00	0.10	1.00	1.00	1.00	0.07	1.00	--	1.00	0.03	0.64	0.12	1.00	0.12	0.43	1.00	0.00	1.00
A17	1.00	1.00	1.00	0.10	0.10	0.09	0.05	0.00	0.00	0.10	1.00	1.00	1.00	0.07	1.00	0.00	--	0.03	0.64	0.12	1.00	0.12	0.43	1.00	0.00	1.00
A18	1.00	1.00	1.00	0.33	0.33	0.44	0.39	0.00	0.00	0.15	1.00	1.00	1.00	0.33	1.00	1.00	1.00	--	1.00	1.00	1.00	0.54	1.00	1.00	0.26	1.00
A19	1.00	1.00	1.00	0.16	0.16	0.17	0.13	0.00	0.00	0.03	1.00	1.00	1.00	0.13	1.00	1.00	1.00	0.00	--	0.00	1.00	0.21	0.66	1.00	0.00	1.00
A20	1.00	1.00	1.00	0.16	0.16	0.18	0.14	0.00	0.00	0.10	1.00	1.00	1.00	0.14	1.00	1.00	1.00	0.00	1.00	--	1.00	0.22	0.80	1.00	0.00	1.00
A21	0.00	0.00	1.00	0.04	0.04	0.00	0.00	0.00	0.00	0.03	1.00	0.47	0.40	0.00	0.67	0.06	0.07	0.00	0.06	0.03	--	0.00	0.00	1.00	0.01	1.00
A22	1.00	1.00	1.00	0.09	0.09	0.00	0.00	0.00	0.46	1.00	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	--	1.00	1.00	0.04	1.00
A23	1.00	1.00	1.00	0.05	0.05	0.00	0.00	0.00	0.02	0.34	1.00	1.00	1.00	0.00	1.00	1.00	1.00	0.52	1.00	1.00	1.00	0.00	--	1.00	0.01	1.00
A24	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.03	1.00	0.00	0.00	0.00	0.57	0.00	0.01	0.00	0.06	0.00	0.04	0.00	0.00	--	0.00	1.00
A25	1.00	1.00	1.00	0.48	0.46	1.00	1.00	0.00	0.69	1.00	1.00	1.00	1.00	0.53	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	--	1.00
A26	0.00	0.00	1.00	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.24	0.07	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.00	--
A27	1.00	1.00	1.00	0.85	0.84	1.00	1.00	0.00	0.02	1.00	1.00	1.00	1.00	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A28	1.00	1.00	1.00	0.02	0.02	0.00	0.00	0.00	0.00	0.17	1.00	1.00	1.00	0.00	1.00	0.63	0.63	0.19	0.64	0.42	1.00	0.00	0.00	1.00	0.00	1.00
A29	1.00	1.00	1.00	0.02	0.02	0.00	0.00	0.00	0.00	0.19	1.00	1.00	1.00	0.00	1.00	0.77	0.77	0.23	0.71	0.50	1.00	0.00	0.00	1.00	0.00	1.00
A30	1.00	1.00	1.00	0.62	0.62	0.82	0.78	0.00	0.00	0.86	1.00	1.00	1.00	0.65	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.95	1.00	1.00	0.68	1.00
A31	1.00	1.00	1.00	0.05	0.05	0.00	0.00	0.00	0.09	0.45	1.00	1.00	1.00	0.00	1.00	1.00	1.00	0.73	1.00	1.00	1.00	0.00	1.00	1.00	0.01	1.00
A32	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.15	0.52	1.00	1.00	1.00	0.00	1.00	1.00	1.00	0.82	1.00	1.00	1.00	0.00	1.00	1.00	0.01	1.00
A33	1.00	1.00	1.00	0.02	0.02	0.00	0.00	0.00	0.00	0.19	1.00	1.00	1.00	0.00	1.00	0.74	0.74	0.22	0.70	0.48	1.00	0.00	0.00	1.00	0.00	1.00
A34	0.03	0.03	1.00	0.01	0.01	0.01	0.01	0.00	0.01	0.02	0.15	0.04	0.04	0.01	0.05	0.06	0.06	0.03	0.07	0.06	0.05	0.01	0.03	0.06	0.02	0.72
A35	0.13	0.12	1.00	0.06	0.06	0.03	0.00	0.00	0.00	0.00	1.00	0.30	0.29	0.02	0.00	0.00	0.01	0.00	0.00	0.00	0.21	0.04	0.08	0.46	0.00	1.00
A36	1.00	1.00	1.00	0.22	0.22	0.27	0.22	0.00	0.00	0.04	1.00	1.00	1.00	0.20	1.00	1.00	1.00	0.00	1.00	1.00	1.00	0.33	1.00	1.00	0.09	1.00
A37	1.00	1.00	1.00	0.22	0.22	0.27	0.22	0.00	0.00	0.04	1.00	1.00	1.00	0.20	1.00	1.00	1.00	0.00	1.00	1.00	1.00	0.33	1.00	1.00	0.09	1.00
A38	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.41	0.87	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00
A39	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.28	0.69	1.00	1.00	1.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.00	1.00
A40	1.00	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.28	1.00	1.00	1.00	0.00	1.00	0.97	0.97	0.37	0.86	0.70	1.00	0.00	0.00	1.00	0.00	1.00
A41	0.40	0.39	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.15	1.00	0.07	0.12	0.00	0.83	0.38	0.38	0.15	0.46	0.29	1.00	0.00	0.00	1.00	0.01	1.00
A42	0.00	0.00	1.00	0.01	0.01	0.00	0.00	0.00	0.00	0.05	1.00	0.14	0.12	0.00	0.65	0.03	0.04	0.00	0.14	0.01	0.49	0.00	0.00	1.00	0.00	1.00
A43	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.39	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00
A44	0.00	0.00	1.00	0.01	0.01	0.00	0.00	0.00	0.00	0.00	1.00	0.10	0.09	0.00	0.42	0.04	0.05	0.00	0.02	0.02	0.00	0.00	0.00	0.29	0.01	1.00
A45	0.00	0.00	1.00	0.02	0.02	0.00	0.00	0.00	0.00	0.00	0.21	0.07	0.07	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09	0.00	0.07
A46	1.00	1.00	1.00	0.78	0.77	1.00	1.00	0.00	0.00	1.00	1.00	1.00	1.00	0.83	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.94	1.00
A47	0.00	0.00	1.00	0.03	0.03	0.00	0.00	0.00	0.00	0.00	0.42	0.12	0.12	0.00	0.00	0.01	0.01	0.00	0.01	0.00	0.00	0.00	0.00	0.17	0.00	1.00
A48	0.21	0.20	1.00	0.07	0.07	0.04	0.01	0.00	0.00	0.00	1.00	0.39	0.37	0.04	0.02	0.03	0.04	0.01	0.04	0.03	0.34	0.05	0.12	0.63	0.01	1.00
A49	1.00	1.00	1.00	0.48	0.46	1.00	1.00	0.00	0.69	1.00	1.00	1.00	1.00	0.53	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A50	1.00	1.00	1.00	0.33	0.32	0.41	0.37	0.00	0.00	0.00	1.00	1.00	1.00	0.32	1.00	1.00	1.00	0.30	1.00	1.00	1.00	0.47	0.97	1.00	0.27	1.00
A51	1.00	1.00	1.00	0.65	0.83	1.00	1.00	0.00	0.75	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A52	1.00	1.00	1.00	0.16	0.16	0.17	0.13	0.00	0.00	0.03	1.00	1.00	1.00	0.13	1.00	1.00	1.00	0.00	1.00	0.00	1.00	0.21	0.66	1.00	0.00	1.00

	A27	A28	A29	A30	A31	A32	A33	A34	A35	A36	A37	A38	A39	A40	A41	A42	A43	A44	A45	A46	A47	A48	A49	A50	A51	A52
A1	0.00	0.69	0.51	0.33	0.00	0.17	0.54	1.00	1.00	0.31	0.31	0.17	0.19	0.66	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.01	0.69	0.01	0.49
A2	0.00	0.75	0.55	0.33	0.01	0.18	0.58	1.00	1.00	0.32	0.32	0.17	0.19	0.68	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.01	0.69	0.01	0.50
A3	0.00	0.02	0.02	0.01	0.01	0.01	0.02	0.00	0.06	0.02	0.03	0.05	0.04	0.06	0.06	0.02	0.04	0.02	0.23	0.01	0.15	0.02	0.01	0.02	0.01	0.03
A4	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A5	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A6	0.77	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.68	1.00	1.00	0.05	1.00	0.35	1.00
A7	0.82	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.93	1.00	1.00	0.10	1.00	0.42	1.00
A8	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A9	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A10	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.64	1.00	0.37	1.00
A11	0.00	0.00	0.00	0.07	0.00	0.00	0.00	1.00	0.23	0.00	0.00	0.06	0.05	0.09	0.10	0.00	0.02	0.00	1.00	0.00	1.00	0.00	0.00	0.15	0.00	0.01
A12	0.00	0.04	0.05	0.35	0.00	0.00	0.02	1.00	1.00	0.37	0.37	0.09	0.08	0.35	1.00	1.00	1.00	1.00	1.00	0.03	1.00	1.00	0.00	0.68	0.00	0.52
A13	0.00	0.43	0.00	0.36	0.00	0.00	0.00	1.00	1.00	0.43	0.43	0.09	0.09	0.42	1.00	1.00	1.00	1.00	1.00	0.05	1.00	1.00	0.00	0.71	0.00	0.61
A14	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
A15	0.00	0.98	0.93	0.10	0.51	0.54	0.94	1.00	1.00	0.00	0.01	0.40	0.47	0.89	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.08	0.38	0.00	0.53
A16	0.00	1.00	1.00	0.33	0.30	0.39	1.00	1.00	1.00	0.29	0.29	0.28	0.34	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.00	0.76	0.00	0.64
A17	0.00	1.00	1.00	0.33	0.30	0.39	1.00	1.00	1.00	0.29	0.29	0.28	0.34	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.00	0.76	0.00	0.64
A18	0.00	1.00	1.00	0.49	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.71	0.91	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.26	1.00	0.01	1.00
A19	0.00	1.00	1.00	0.30	0.50	0.54	1.00	1.00	1.00	0.00	0.00	0.37	0.45	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.00	0.79	0.00	0.00
A20	0.00	1.00	1.00	0.35	0.57	0.60	1.00	1.00	1.00	0.46	0.46	0.39	0.48	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.00	0.92	0.00	1.00
A21	0.00	0.19	0.17	0.25	0.00	0.12	0.17	1.00	1.00	0.03	0.03	0.14	0.15	0.41	0.75	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.01	0.53	0.01	0.06
A22	0.62	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.71	1.00	1.00	0.04	1.00	0.03	1.00
A23	0.09	1.00	1.00	0.50	0.00	0.35	1.00	1.00	1.00	0.98	0.98	0.23	0.28	1.00	1.00	1.00	1.00	1.00	1.00	0.17	1.00	1.00	0.01	1.00	0.01	1.00
A24	0.00	0.00	0.00	0.24	0.00	0.00	0.00	1.00	1.00	0.03	0.03	0.08	0.07	0.18	0.28	0.01	1.00	1.00	1.00	0.00	1.00	1.00	0.00	0.49	0.00	0.06
A25	0.93	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	0.54	1.00
A26	0.00	0.00	0.00	0.00	0.00	0.04	0.00	1.00	0.00	0.00	0.00	0.08	0.08	0.13	0.14	0.05	0.12	0.05	1.00	0.00	0.08	0.00	0.00	0.00	0.00	0.00
A27	--	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.88	1.00
A28	0.00	--	0.02	0.36	0.00	0.10	0.00	1.00	1.00	0.43	0.43	0.14	0.14	0.65	1.00	1.00	1.00	1.00	1.00	0.04	1.00	1.00	0.00	0.73	0.00	0.64
A29	0.00	1.00	--	0.38	0.00	0.10	1.00	1.00	1.00	0.49	0.49	0.14	0.15	0.77	1.00	1.00	1.00	1.00	1.00	0.05	1.00	1.00	0.00	0.76	0.00	0.71
A30	0.00	1.00	1.00	--	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.68	1.00	0.48	1.00
A31	0.18	1.00	1.00	0.59	--	0.69	1.00	1.00	1.00	1.00	1.00	0.27	0.38	1.00	1.00	1.00	1.00	1.00	1.00	0.25	1.00	1.00	0.01	1.00	0.02	1.00
A32	0.25	1.00	1.00	0.64	1.00	--	1.00	1.00	1.00	1.00	1.00	0.17	0.22	1.00	1.00	1.00	1.00	1.00	1.00	0.32	1.00	1.00	0.01	1.00	0.01	1.00
A33	0.00	1.00	0.26	0.37	0.00	0.10	--	1.00	1.00	0.47	0.47	0.14	0.15	0.74	1.00	1.00	1.00	1.00	1.00	0.05	1.00	1.00	0.00	0.75	0.00	0.70
A34	0.01	0.05	0.04	0.02	0.02	0.02	0.04	--	0.19	0.05	0.06	0.06	0.04	0.07	0.07	0.05	0.08	0.06	0.68	0.02	0.35	0.10	0.02	0.04	0.02	0.07
A35	0.00	0.21	0.20	0.05	0.07	0.15	0.20	1.00	--	0.00	0.00	0.16	0.17	0.31	0.40	0.37	0.64	0.52	1.00	0.00	1.00	0.00	0.00	0.14	0.00	0.00
A36	0.00	1.00	1.00	0.34	0.77	0.75	1.00	1.00	1.00	--	1.00	0.49	0.60	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.09	1.00	0.01	1.00
A37	0.00	1.00	1.00	0.34	0.77	0.75	1.00	1.00	1.00	0.00	--	0.49	0.60	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.09	1.00	0.00	1.00
A38	0.54	1.00	1.00	0.92	1.00	1.00	1.00	1.00	1.00	1.00	1.00	--	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.62	1.00	1.00	0.00	1.00	0.00	1.00
A39	0.39	1.00	1.00	0.77	1.00	1.00	1.00	1.00	1.00	1.00	1.00	0.12	--	1.00	1.00	1.00	1.00	1.00	1.00	0.47	1.00	1.00	0.00	1.00	0.00	1.00
A40	0.07	1.00	1.00	0.44	0.00	0.00	1.00	1.00	1.00	0.64	0.64	0.04	0.00	--	1.00	1.00	1.00	1.00	1.00	0.13	1.00	1.00	0.00	0.82	0.00	0.86
A41	0.00	0.05	0.06	0.34	0.00	0.00	0.04	1.00	1.00	0.34	0.34	0.04	0.02	0.07	--	1.00	1.00	1.00	1.00	0.03	1.00	1.00	0.01	0.64	0.01	0.46
A42	0.00	0.03	0.03	0.26	0.00	0.03	0.02	1.00	1.00	0.09	0.09	0.10	0.09	0.26	0.48	--	1.00	1.00	1.00	0.00	1.00	1.00	0.00	0.53	0.01	0.14
A43	0.00	0.00	0.00	0.19	0.00	0.00	0.00	1.00	1.00	0.00	0.00	0.07	0.06	0.14	0.18	0.00	--	0.00	1.00	0.00	1.00	1.00	0.00	0.38	0.00	0.00
A44	0.00	0.03	0.03	0.19	0.00	0.03	0.02	1.00	1.00	0.01	0.01	0.09	0.08	0.20	0.29	0.04	1.00	--	1.00	0.00	1.00	1.00	0.01	0.40	0.01	0.02
A45	0.00	0.00	0.00	0.00	0.00	0.04	0.00	1.00	0.00	0.00	0.00	0.08	0.07	0.13	0.14	0.05	0.11	0.05	--	0.00	0.00	0.00	0.00	0.00	0.00	0.00
A46	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	--	1.00	1.00	0.94	1.00	0.71	1.00
A47	0.00	0.05	0.05	0.00	0.00	0.07	0.05	1.00	0.01	0.00	0.00	0.10	0.10	0.17	0.20	0.12	0.21	0.14	1.00	0.00	--	0.00	0.00	0.01	0.00	0.01
A48	0.00	0.28	0.27	0.08	0.10	0.19	0.27	1.00	1.00	0.02	0.03	0.18	0.20	0.38	0.50	0.51	0.93	0.77	1.00	0.00	1.00	--	0.01	0.21	0.01	0.04
A49	0.93	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	--	1.00	0.55	1.00
A50	0.00	1.00	1.00	0.00	0.63	0.81	1.00	1.00	1.00	0.92	0.92	0.59	0.70	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.27	--	0.00	1.00
A51	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	--	1.00
A52	0.00	1.00	1.00	0.30	0.50	0.54	1.00	1.00	1.00	0.00	0.00	0.37	0.45	1.00	1.00	1.00	1.00	1.00	1.00	0.00	1.00	1.00	0.00	0.79	0.00	--

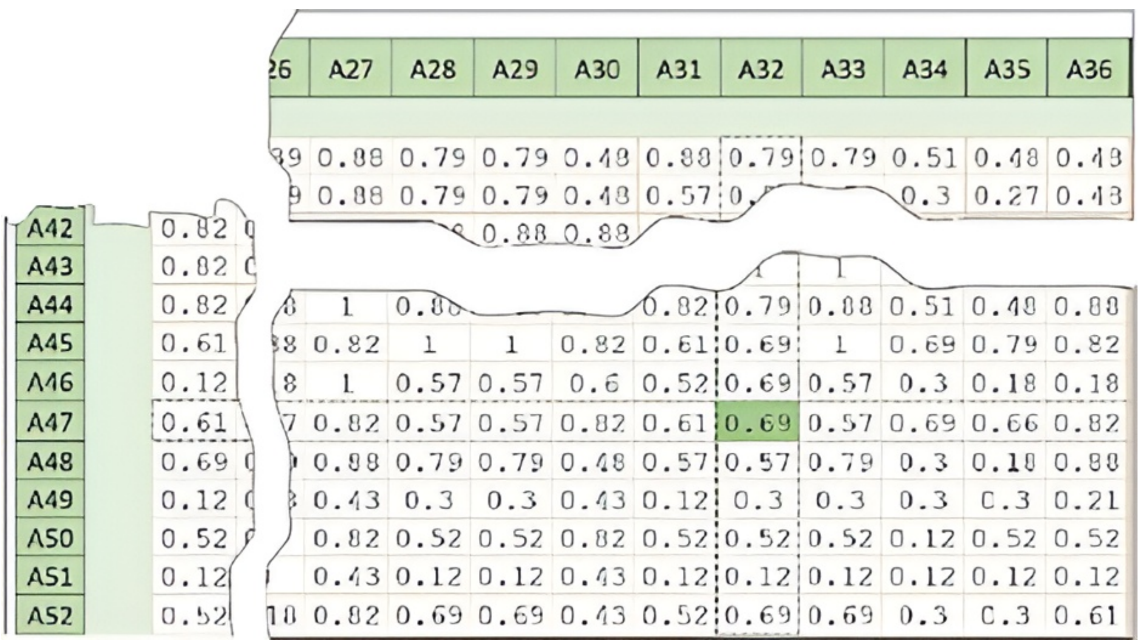

Figure 2. A part of the concordant matrix in the ELECTRE method

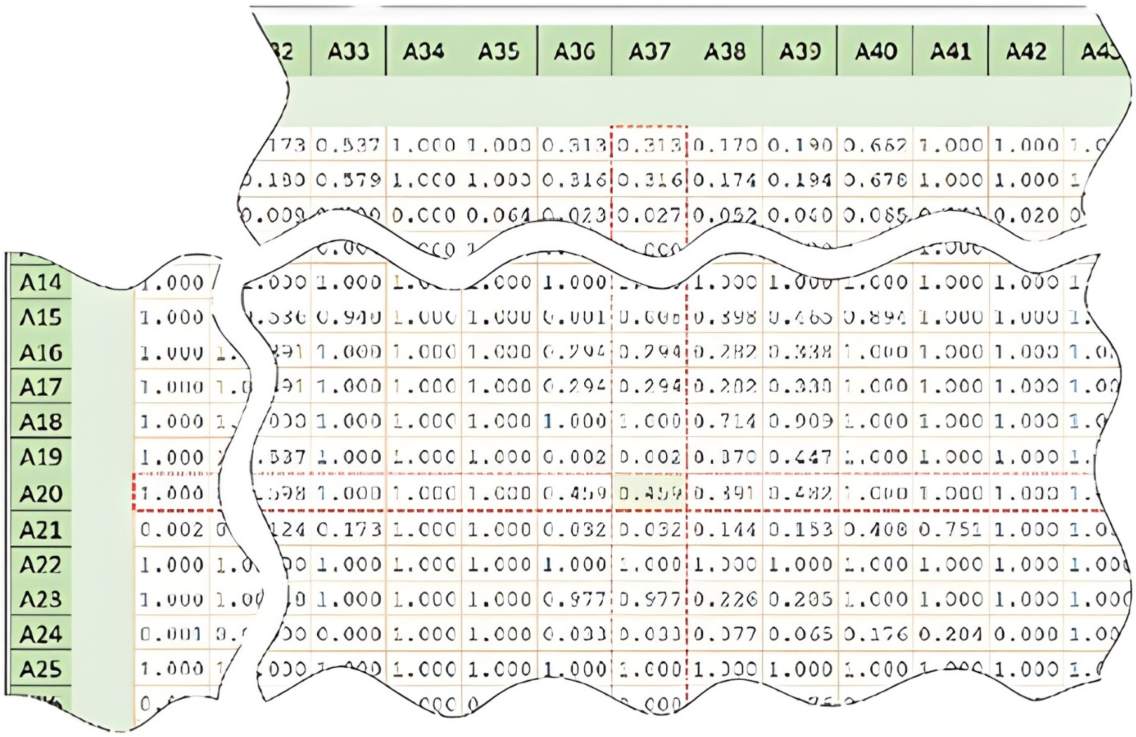

Figure 3 part of the discordant matrix in the ELECTRE method

Step 3: Creation of the effective concordant (H) and effective discordant (G) matrices

At this stage, the effective concordant (H) and effective discordant (G) matrices are created. To construct the effective concordant matrix, a threshold value must first be determined. If an element of the I matrix is greater than or equal to this threshold, the corresponding component in the H matrix is assigned a value of one; otherwise, it takes a value of zero. To determine the threshold value (I'), the average of the values in the I matrix is used:

$$I' = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} I_{kl}}{m(m-1)} \tag{15}$$

The components of the effective concordant matrix are determined as follows:

$if: \mathrm{I_{kl}} \geq \mathrm{I'} \rightarrow \mathrm{H_{kl}} = 1$

$if: I_{kl} < I' \rightarrow \mathrm{H_{kl}} = 0$

In any case, the concordant matrix indicates the superiority of one option over another.

The effective discordant matrix is determined similarly to the effective concordant matrix. In this case, to determine the threshold value (NI′), the average of the values in the NI matrix is used:

$$NI' = \frac{\sum_{i=1}^{m}\sum_{k=1}^{m} NI_{kl}}{m(m-1)} \tag{16}$$

The components of the effective discordant matrix are determined as follows:

$if: NI_{kl} \geq NI' \rightarrow G_{kl} = 0$

$if: NI_{kl} < NI' \rightarrow G_{kl} = 1$

Tables 14 and 15 present the effective concordant and effective discordant matrices, respectively. It should be noted that, due to the extensive nature of the results at this stage, only a portion of each matrix is shown in Figures 4 and 5.

Table 14. Effective concordant matrix obtained by the ELECTRE method

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23	A24	A25	A26
A1	--	1	0	1	1	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0	1	1	0	1	0
A2	1	--	0	1	1	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0	1	1	0	1	0
A3	1	1	--	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A4	0	0	0	--	1	0	0	0	0	0	1	1	1	0	0	1	0	0	1	1	0	0	0	1	0	0
A5	0	0	0	0	--	0	0	0	0	0	1	1	1	0	0	1	0	0	1	1	0	0	0	1	0	0
A6	0	0	0	1	1	--	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0
A7	0	0	0	1	1	0	--	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
A8	0	0	0	1	1	0	1	--	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
A9	0	0	0	1	1	0	1	1	--	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0
A10	0	0	0	1	1	0	1	1	1	--	0	1	1	1	0	1	1	1	1	1	1	0	1	1	1	0
A11	1	1	0	1	1	1	1	1	1	1	--	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A12	0	0	0	1	1	1	1	1	1	0	1	--	1	1	0	0	0	0	0	0	0	1	1	0	1	0
A13	0	0	0	1	1	1	1	1	1	0	1	1	--	1	0	0	0	0	0	1	0	1	1	0	1	0
A14	0	0	0	1	1	0	1	1	1	1	0	0	0	--	0	0	0	1	0	0	0	0	1	0	0	0
A15	0	0	0	1	1	0	1	1	1	1	0	1	1	1	--	1	1	1	1	1	1	0	1	1	1	0
A16	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	--	1	0	1	1	0	0	0	0	1	0
A17	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	--	0	1	1	0	0	0	0	1	0
A18	0	0	0	1	1	0	1	1	1	1	0	1	1	1	0	1	1	--	0	1	0	0	1	0	1	0
A19	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	1	--	1	0	0	0	1	0	0
A20	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	1	0	--	0	0	0	0	0	0
A21	1	1	0	1	1	1	1	1	1	0	0	1	1	1	0	1	1	1	0	1	--	1	1	1	1	0
A22	1	1	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	--	0	0	1	0
A23	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	--	0	1	0
A24	1	1	0	1	1	1	1	1	1	0	1	1	1	1	0	1	1	1	1	1	0	1	1	--	1	0
A25	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	--	0
A26	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	--
A27	0	0	0	1	1	0	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	0	1	0	1	0
A28	0	0	0	1	1	1	1	1	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	1	0
A29	0	0	0	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	1	0	1	1	0	1	0
A30	0	0	0	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	0
A31	1	1	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0
A32	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0
A33	0	0	0	1	1	1	1	1	1	0	0	0	1	1	0	0	0	0	0	0	0	1	1	0	1	0
A34	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A35	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A36	0	0	0	1	1	0	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	0	1	1	1	0
A37	0	0	0	1	1	0	1	1	1	1	0	1	1	1	1	1	1	1	0	1	1	0	1	1	1	0
A38	0	0	0	1	1	1	1	1	0	0	1	1	1	1	0	1	0	0	1	1	0	1	0	0	1	0
A39	0	0	0	1	1	1	1	1	0	0	1	1	1	1	0	1	0	0	1	1	0	1	0	1	1	0
A40	0	0	0	1	1	1	1	1	0	0	1	1	0	1	0	0	0	0	0	1	0	1	1	0	1	0
A41	0	0	1	1	1	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	1	1	1	0	1	0
A42	1	1	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	1	0	1	0
A43	1	1	0	1	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0
A44	1	1	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0
A45	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A46	0	0	0	1	1	0	1	1	1	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0	1	0
A47	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A48	1	1	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0
A49	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0
A50	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A51	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A52	0	0	0	1	1	0	0	1	1	0	0	1	1	0	0	1	1	1	1	1	0	0	0	1	0	0

	A27	A28	A29	A30	A31	A32	A33	A34	A35	A36	A37	A38	A39	A40	A41	A42	A43	A44	A45	A46	A47	A48	A49	A50	A51	A52
A1	1	1	1	0	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	0	0	1	0	1	0
A2	1	1	1	0	1	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	0	0	1	0	1	0
A3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A4	0	1	1	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	1
A5	0	1	1	0	0	1	1	1	1	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1
A6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0
A7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A9	0	0	0	0	0	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0	1	0
A10	1	1	1	0	0	1	1	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	1	0	1	1
A11	1	1	1	0	1	1	1	1	0	1	1	1	1	1	0	1	1	1	0	1	1	1	1	0	1	1
A12	1	1	1	0	1	1	1	1	0	0	0	1	1	1	0	0	0	0	0	0	1	0	1	0	1	0
A13	1	1	1	0	1	1	1	1	1	0	0	1	1	1	0	0	0	0	0	0	1	0	1	0	1	1
A14	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	0	0
A15	1	1	1	1	0	1	1	0	0	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	1	1
A16	1	1	1	0	0	1	1	0	0	0	0	1	1	1	0	0	0	0	0	1	0	0	1	0	1	1
A17	1	1	1	0	0	1	1	0	0	0	0	1	1	1	0	0	0	0	0	1	0	0	1	1	1	1
A18	1	1	1	1	0	1	1	0	0	0	0	1	1	1	1	0	0	0	0	1	0	0	1	0	1	0
A19	1	1	1	0	0	1	1	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	1	0	1	1
A20	1	1	1	0	0	1	1	0	0	0	0	1	1	1	0	0	0	0	0	1	0	0	1	0	1	0
A21	1	1	1	0	1	1	1	0	0	0	0	1	1	1	1	1	0	0	0	1	0	0	1	0	1	0
A22	1	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0
A23	1	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0
A24	1	1	1	0	1	1	1	1	1	0	0	1	1	1	0	1	0	0	0	1	1	0	1	0	1	1
A25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1
A26	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	1	1	1	1	1	1	1	1
A27	--	0	0	0	0	1	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0	1	0
A28	1	--	1	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	1	0	1	0
A29	1	1	--	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	1	0	1	0
A30	1	1	1	--	0	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	0	0	1	0	1	1
A31	0	0	0	0	--	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	1	1	0	1	0
A32	0	0	0	0	0	--	0	1	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0
A33	1	0	1	0	1	1	--	0	0	0	0	1	1	1	0	0	0	0	0	0	1	0	1	0	1	0
A34	1	1	1	1	1	1	1	--	1	1	1	1	1	1	0	0	1	1	0	1	0	1	1	1	1	1
A35	1	1	1	0	1	1	1	0	--	1	1	1	1	1	0	0	1	1	0	1	0	1	1	0	1	1
A36	1	1	1	1	0	1	1	0	0	--	1	1	1	1	1	1	0	0	0	1	0	0	1	0	1	0
A37	1	1	1	1	0	1	1	0	0	1	--	1	1	1	1	1	0	0	0	1	0	0	1	0	1	0
A38	0	1	1	0	0	1	1	1	1	0	0	--	0	1	0	0	0	0	0	0	1	0	1	0	1	1
A39	0	1	1	0	0	1	1	1	1	0	0	1	--	1	0	0	0	0	0	0	1	0	1	0	1	1
A40	0	1	1	0	1	1	1	1	1	0	0	1	1	--	0	0	0	0	0	0	1	0	1	0	1	0
A41	1	1	1	1	1	1	1	1	0	0	0	1	1	1	--	0	0	1	0	1	0	0	1	0	1	0
A42	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	--	0	1	0	1	0	0	1	0	1	0
A43	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	--	1	1	1	1	0	1	0	1	1
A44	1	1	1	0	1	1	1	0	0	1	1	1	1	1	1	1	0	--	0	1	0	0	1	0	1	1
A45	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	--	1	1	1	1	1	1	1
A46	1	1	1	1	0	1	1	0	0	0	0	1	1	1	1	0	0	0	0	--	0	0	1	0	1	0
A47	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0	0	1	0	0	1	--	1	1	1	1	1
A48	1	1	1	0	1	1	1	0	0	1	1	1	1	1	1	1	1	1	0	1	0	--	1	0	1	1
A49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	--	0	0	1
A50	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	--	1	0
A51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	--	0
A52	1	1	1	0	0	1	1	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	1	0	1	--

Table 15. Effective discordant matrix obtained by the ELECTRE method

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23	A24	A25	A26
A1	--	1	0	1	1	1	1	1	1	1	0	0	0	1	0	1	1	1	1	1	0	1	1	0	1	0
A2	0	--	0	1	1	1	1	1	1	1	0	0	0	1	0	1	1	1	1	1	0	1	1	0	1	0
A3	1	1	--	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A4	0	0	0	--	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A5	0	0	0	0	--	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A6	0	0	0	1	1	--	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0
A7	0	0	0	1	1	0	--	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0
A8	0	0	0	0	0	0	0	--	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A9	0	0	0	0	0	0	0	1	--	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A10	0	0	0	1	1	0	0	1	1	--	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A11	1	1	0	1	1	1	1	1	1	1	--	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A12	0	0	0	1	1	1	1	1	1	1	0	--	1	1	0	1	1	1	1	1	0	1	1	0	1	0
A13	0	0	0	1	1	1	1	1	1	1	0	0	--	1	0	0	0	1	0	1	0	1	1	0	1	0
A14	0	0	0	1	1	0	0	1	0	0	0	0	0	--	0	0	0	0	0	0	0	0	0	0	0	0
A15	0	0	0	1	1	1	1	1	1	1	0	0	0	1	--	0	0	1	1	1	0	1	0	0	1	0
A16	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	--	0	1	0	1	0	1	1	0	1	0
A17	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	1	--	1	0	1	0	1	1	0	1	0
A18	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	--	0	0	0	1	0	0	1	0
A19	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	--	1	0	1	0	0	1	0
A20	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	0	--	0	1	0	0	1	0
A21	1	1	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	--	1	1	0	1	0
A22	0	0	0	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	--	0	0	1	0
A23	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	0	0	0	1	--	0	1	0
A24	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	--	1	0
A25	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	--	0
A26	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	--
A27	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A28	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	0	1	0	1	1	0	1	0
A29	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	0	1	0	1	1	0	1	0
A30	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A31	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0
A32	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0
A33	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	0	1	0	1	1	0	1	0
A34	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A35	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A36	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0
A37	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0
A38	0	0	0	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0
A39	0	0	0	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0
A40	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	0	0	0	1	1	0	1	0
A41	1	1	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	0	1	1	0	1	0
A42	1	1	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	0	1	0
A43	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A44	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A45	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A46	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A47	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A48	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	0	1	0
A49	0	0	0	1	1	0	0	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
A50	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	1	0
A51	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A52	0	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	0	1	0	1	0	1	0	0	1	0

	A27	A28	A29	A30	A31	A32	A33	A34	A35	A36	A37	A38	A39	A40	A41	A42	A43	A44	A45	A46	A47	A48	A49	A50	A51	A52
A1	1	0	1	1	1	1	1	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	1
A2	1	0	1	1	1	1	1	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	1
A3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
A7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
A8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
A11	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	0	1	1	1	1	1
A12	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	1	0	1	1
A13	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	1	0	1	0
A14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A15	1	0	0	1	1	1	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	1	1	1
A16	1	0	0	1	1	1	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0
A17	1	0	0	1	1	1	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0
A18	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0
A19	1	0	0	1	1	1	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	1
A20	1	0	0	1	1	0	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0
A21	1	1	1	1	1	1	1	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	1	1	1	1
A22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
A23	1	0	0	1	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0
A24	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	0	0	1	0	0	1	1	1	1
A25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	1	0
A26	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
A27	--	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A28	1	--	1	1	1	1	1	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0
A29	1	0	--	1	1	1	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0
A30	1	0	0	--	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0
A31	1	0	0	0	--	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0
A32	1	0	0	0	0	--	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0
A33	1	0	1	1	1	1	--	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0
A34	1	1	1	1	1	1	1	--	1	1	1	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1
A35	1	1	1	1	1	1	1	0	--	1	1	1	1	1	1	1	0	1	0	1	0	1	1	1	1	1
A36	1	0	0	1	0	0	0	0	0	--	0	1	0	0	0	0	0	0	0	1	0	0	1	0	1	0
A37	1	0	0	1	0	0	0	0	0	1	--	1	0	0	0	0	0	0	0	1	0	0	1	0	1	0
A38	1	0	0	0	0	0	0	0	0	0	0	--	0	0	0	0	0	0	0	0	0	0	1	0	1	0
A39	1	0	0	0	0	0	0	0	0	0	0	1	--	0	0	0	0	0	0	1	0	0	1	0	1	0
A40	1	0	0	1	1	1	0	0	0	0	0	1	1	--	0	0	0	0	0	1	0	0	1	0	1	0
A41	1	1	1	1	1	1	1	0	0	1	1	1	1	1	--	0	0	0	0	1	0	0	1	0	1	1
A42	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	--	0	0	0	1	0	0	1	1	1	1
A43	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	--	1	0	1	0	0	1	1	1	1
A44	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	--	0	1	0	0	1	1	1	1
A45	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	--	1	1	1	1	1	1	1
A46	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	--	0	0	0	0	0	0
A47	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0	1	--	1	1	1	1	1
A48	1	1	1	1	1	1	1	0	0	1	1	1	1	1	1	1	0	0	0	1	0	--	1	1	1	1
A49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	--	0	1	0
A50	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	--	1	0
A51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	--	0
A52	1	0	0	1	1	1	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	--

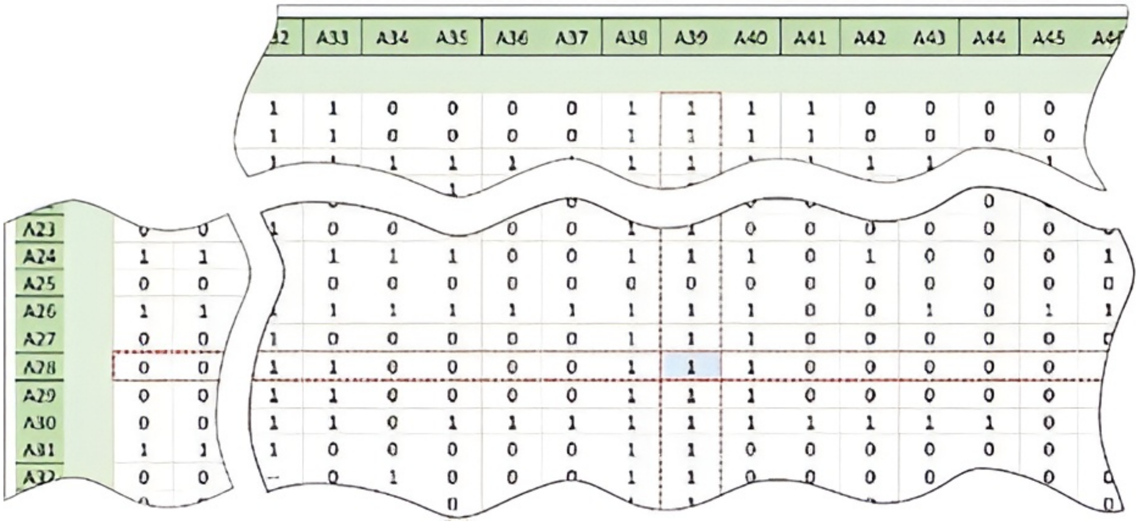

Figure 4. A part of the effective concordant matrix in the ELECTRE method

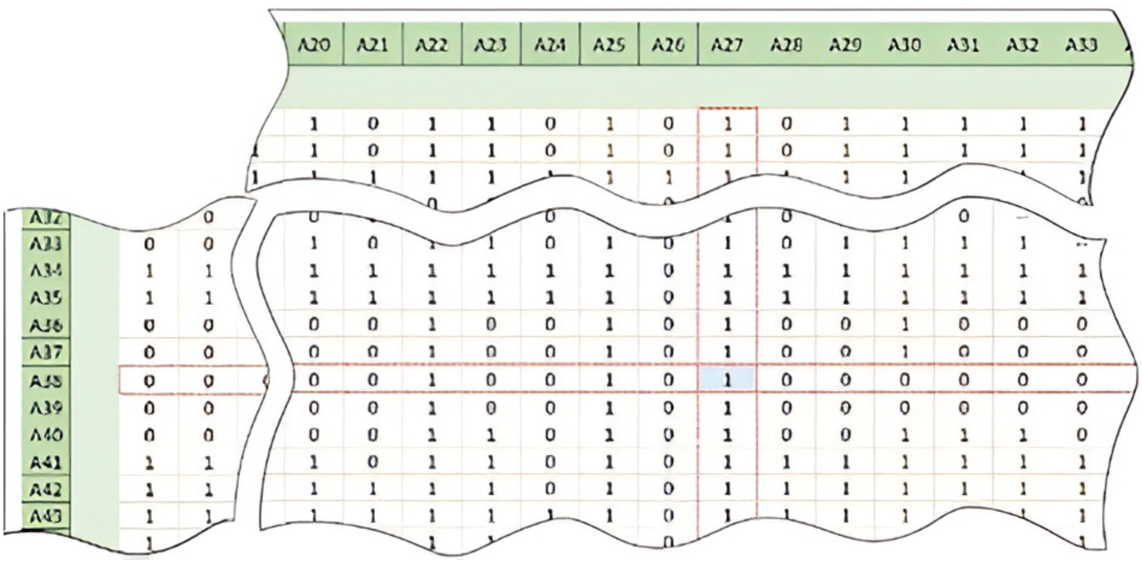

Figure 5. A part of the effective discordant matrix in the ELECTRE method

Step 4: Creation of the effective overall matrices (F)

In the final stage, the overall effective matrix (F) is determined by performing an element-wise multiplication of the effective concordant matrix (H) and the effective discordant matrix (G), as follows:

$$F_{kl} = H_{kl} \times G_{kl} \quad (17)$$

In any case, the matrix F represents the priority of blasting patterns relative to each other. Specifically, if $F_{kl}=1$, it indicates that A_k is superior to A_l. However, this superiority may be influenced by other patterns. Therefore, for A_k to be the superior option, the condition is:

For at least one of l: $F_{kl} = 1$

For the whole of l: $F_{kl} = 0$

Table 16 presents the overall effective matrix, resulting from the multiplication of the effective concordant and discordant matrices from the previous stage, along with the ranking of the blasting patterns. Additionally, a portion of this matrix is shown in magnified form in Figure 6.

Table 16. Effective overall matrix obtained by the ELECTRE method

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23	A24	A25	A26
A1	–	1	0	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	1	0
A2	0	–	0	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	1	0
A3	1	1	–	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A4	0	0	0	–	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A5	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A6	0	0	0	1	1	–	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	1	0
A7	0	0	0	1	1	0	–	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
A8	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A9	0	0	0	0	0	0	0	1	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A10	0	0	0	1	1	0	0	1	1	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A11	1	1	0	1	1	1	1	1	1	1	–	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A12	0	0	0	1	1	1	1	1	1	0	0	–	1	1	0	0	0	0	0	0	0	1	1	0	1	0
A13	0	0	0	1	1	1	1	1	1	0	0	0	–	1	0	0	0	0	0	1	0	1	1	0	1	0
A14	0	0	0	1	1	0	0	1	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0
A15	0	0	0	1	1	0	1	1	1	1	0	0	0	1	–	0	0	1	1	1	0	0	0	0	1	0
A16	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	–	0	0	0	1	0	0	0	0	1	0
A17	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	1	–	0	0	1	0	0	0	0	1	0
A18	0	0	0	1	1	0	1	1	1	1	0	0	0	1	0	0	0	–	0	0	0	0	0	0	1	0
A19	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	1	–	1	0	0	0	0	0	0
A20	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	1	0	–	0	0	0	0	0	0
A21	1	1	0	1	1	1	1	1	1	0	0	1	1	1	0	1	1	1	0	1	–	1	1	0	1	0
A22	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	–	0	0	1	0
A23	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	–	0	1	0
A24	1	1	0	1	1	1	1	1	1	0	0	1	1	1	0	1	1	1	1	1	0	1	1	–	1	0
A25	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0
A26	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	–
A27	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A28	0	0	0	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	1	0
A29	0	0	0	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	1	0	1	1	0	1	0
A30	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A31	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0
A32	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0
A33	0	0	0	1	1	1	1	1	1	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	1	0
A34	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A35	1	1	0	1	1	1	1	1	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A36	0	0	0	1	1	0	1	1	1	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0
A37	0	0	0	1	1	0	1	1	1	1	0	0	0	1	0	0	0	1	0	0	0	0	0	0	1	0
A38	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0
A39	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	0	0	1	0
A40	0	0	0	1	1	1	1	1	0	0	0	0	0	1	0	0	0	0	0	0	0	1	1	0	1	0
A41	0	0	0	1	1	1	1	1	1	1	0	1	1	1	0	0	0	0	0	0	0	1	1	0	1	0
A42	1	1	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	0	1	1	1	1	0	1	0
A43	1	1	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0
A44	1	1	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	1	1	0
A45	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A46	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A47	1	1	0	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	0
A48	1	1	0	1	1	1	1	1	1	1	0	1	1	1	0	1	1	1	1	1	1	1	1	0	1	0
A49	0	0	0	1	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A50	0	0	0	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A51	0	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
A52	0	0	0	1	1	0	0	1	1	0	0	0	0	0	0	0	0	1	0	1	0	0	0	0	0	0

	A27	A28	A29	A30	A31	A32	A33	A34	A35	A36	A37	A38	A39	A40	A41	A42	A43	A44	A45	A46	A47	A48	A49	A50	A51	A52	RANK
A1	1	0	1	0	1	1	1	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0	16
A2	1	0	1	0	1	1	1	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0	19
A3	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
A4	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	50
A5	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	51
A6	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	40
A7	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	43
A8	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	52
A9	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	47
A10	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	35
A11	1	1	1	0	1	1	1	0	0	1	1	1	1	1	0	1	1	1	0	1	0	1	1	0	1	1	6
A12	1	1	1	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0	1	0	17
A13	1	1	1	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0	0	0	0	1	0	1	0	18
A14	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	44
A15	1	0	0	1	0	1	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	1	15
A16	1	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0	27
A17	1	0	0	0	0	1	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0	25
A18	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	1	0	1	0	31
A19	1	0	0	0	0	1	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	1	20
A20	1	0	0	0	0	0	0	0	0	0	0	1	1	0	0	0	0	0	0	1	0	0	1	0	1	0	34
A21	1	1	1	0	1	1	1	0	0	0	0	1	1	1	0	0	0	0	0	1	0	0	1	0	1	0	14
A22	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	38
A23	1	0	0	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	32
A24	1	1	1	0	1	1	1	0	0	0	0	1	1	1	0	1	0	0	0	1	0	0	1	0	1	1	12
A25	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	45
A26	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	1	0	0	1	1	1	1	1	1	1	4
A27	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	46
A28	1	–	1	0	1	1	1	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	21
A29	1	0	–	0	1	1	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	30
A30	1	0	0	–	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	0	1	0	33
A31	0	0	0	0	–	0	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	36
A32	0	0	0	0	0	–	0	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	37
A33	1	0	1	0	1	1	–	0	0	0	0	1	1	0	0	0	0	0	0	0	0	0	1	0	1	0	28
A34	1	1	1	1	1	1	1	–	1	1	1	1	1	1	0	0	1	1	0	1	0	1	1	1	1	1	3
A35	1	1	1	0	1	1	1	0	–	1	1	1	1	1	0	0	0	1	0	1	0	1	1	0	1	1	7
A36	1	0	0	1	0	0	0	0	0	–	0	1	0	0	0	0	0	0	0	1	0	0	1	0	1	0	26
A37	1	0	0	1	0	0	0	0	0	1	–	1	0	0	0	0	0	0	0	1	0	0	1	0	1	0	22
A38	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	0	0	0	0	1	0	1	0	41
A39	0	0	0	0	0	0	0	0	0	0	0	1	–	0	0	0	0	0	0	0	0	0	1	0	1	0	39
A40	0	0	0	0	1	1	0	0	0	0	0	1	1	–	0	0	0	0	0	0	0	0	1	0	1	0	29
A41	1	1	1	1	1	1	1	0	0	0	0	1	1	1	–	0	0	0	0	1	0	0	1	0	1	0	13
A42	1	1	1	1	1	1	1	0	0	0	0	1	1	1	1	–	0	0	0	1	0	0	1	0	1	0	11
A43	1	1	1	0	1	1	1	0	0	1	1	1	1	1	0	1	–	1	0	1	0	0	1	0	1	1	9
A44	1	1	1	0	1	1	1	0	0	1	1	1	1	1	1	1	0	–	0	1	0	0	1	0	1	1	8
A45	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	1	0	–	1	1	1	1	1	1	1	2
A46	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	0	0	0	42
A47	1	1	1	1	1	1	1	0	1	1	1	1	1	1	0	0	1	0	0	1	–	1	1	1	1	1	5
A48	1	1	1	0	1	1	1	0	0	1	1	1	1	1	1	1	0	0	0	1	0	–	1	0	1	1	10
A49	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	0	0	48
A50	1	0	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	1	0	0	0	–	1	0	23
A51	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	–	0	49
A52	1	0	0	0	0	1	0	0	0	1	1	1	1	0	0	0	0	0	0	1	0	0	1	0	1	–	24

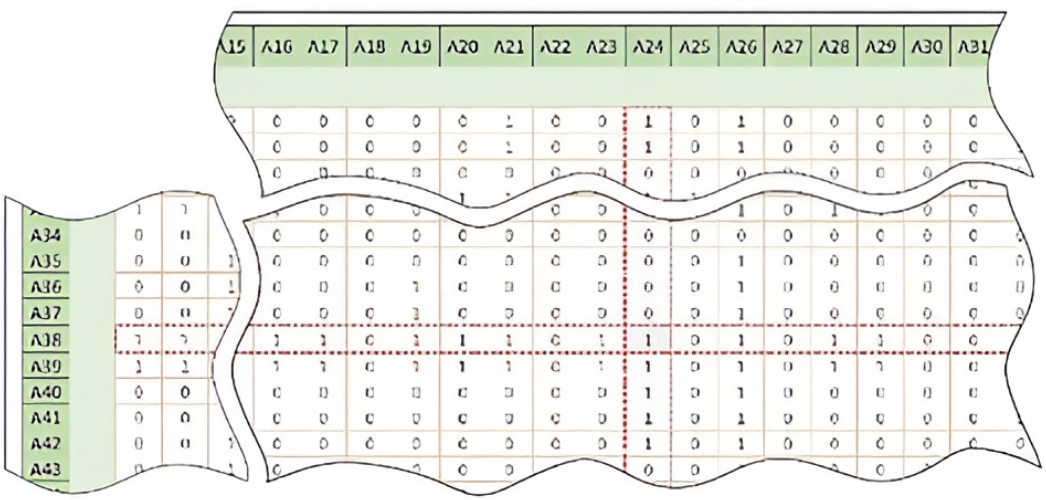

Figure 6. A part of the effective overall matrix in the ELECTRE method

The results obtained from the ELECTRE decision-making model indicate that the outcomes of this method, including the best and worst blasting patterns, are similar to those derived from the TOPSIS decision-making model. Based on the results of this model, Pattern 3 (identified as M_Patt_03) has been determined as the best blasting pattern.

4.3. COCOSO method for MCDM

The COCOSO method is one of the newer multi-criteria decision-making techniques used in this section to evaluate blasting patterns. This method is an integrated model of the Weighted Sum Method (WSM) and the Weighted Product Method (WPM), which can be used to prioritize options based on criteria through both weighted summation and power-based weighting. The foundation of this method is to evaluate NN options based on MM criteria, ultimately determining the best option. Therefore, for decision-making in this method, only the decision matrix and the weight vector of the evaluation criteria are required.

In this method, the steps for implementing the model up to the formation of the decision matrix are similar to the TOPSIS method. Thus, after forming the decision matrix, the steps for performing calculations and ranking the blasting patterns in the COCOSO method are as follows:

Step 1: Normalization of the decision matrix

At this stage, the decision matrix is normalized or dimensionless for both positive and negative criteria based on the following relations:

Normalization of the positive criteria:

$$n_{ij} = \frac{X_{ij} - \min X_{ij}}{\max X_{ij} - \min X_{ij}}$$

Normalization of the negative criteria: (18)

$$n_{ij} = \frac{maxX_{ij} - X_{ij}}{maxX_{ij} - minX_{ij}}$$

In these equations, maxXij and minXij represent the maximum and minimum values of each criterion column, respectively. Accordingly, all elements of the matrix are scaled between 0 and 1. The dimensionless decision matrix (N) obtained from this stage is presented in Table 17.

Step 2: Calculation of WSM and WPM values

This stage involves calculating the two models: weighted sum method (WSM) and weighted product method (WPM). Using the following equations, the weighted sum (S) and weighted product (P) values are calculated for each option

$$S_i = \sum_{j=1}^{n} (W_j \times n_{ij})$$

$$P_i = \sum_{j=1}^{n} (n_{ij})^{W_j} \tag{19}$$

In these equations, Wj represents the weights of the criteria, which are the same as those presented in Table 8. The calculated WSM and WPM parameters from this stage are shown in Table 18.

Step 3: Determination of initial ratings for options

In this stage, the ratings of the options are determined based on the weighted sum (S) and weighted product (P) values from the previous stage. For this purpose, Equation 20 is used. In these equations, K_{ia} represents the arithmetic mean of the S and P ratings, while K_{ib} calculates the relative ratings of S and P in comparison to the best values. The compromise between the S and P models is also determined by K_{ib}:

$$K_{ia} = \frac{P_i + S_i}{\sum(P_i + S_i)} \tag{20}$$

$$K_{ib} = \frac{S_i}{Min\, S_i} + \frac{P_i}{Min\, P_i}$$

$$K_{ic} = \frac{\lambda(S_i) + (1-\lambda)(P_i)}{(\lambda maxS_i + (1-\lambda)maxP_i)}$$

The parameters K_{ia}, K_{ib}, and K_{ic} calculated in this stage are presented in Table 18.

Step 4: Total ratings of options

In the final stage of this method, the total ratings and ranking of the options are determined. The total rating of the options (k) is calculated using the following equation:

$$K_i = (K_{ia} \times K_{ib} \times K_{ic})^{\frac{1}{3}} + \frac{1}{3}(K_{ia} + K_{ib} + K_{ic}) \tag{21}$$

In fact, this equation represents the sum of the geometric mean and the arithmetic mean of the three previous equations. Based on this method, a higher score of k for any option indicates its superiority. The parameter K_i obtained from this section is also presented in Table 18.

The results of this section indicate that, according to the COCOSO method, Pattern 3 (identifier: M_Patt_03) has been determined as the best blasting pattern. Regarding the worst blasting patterns, Patterns 8, 51, and 5 have been identified by this model, respectively.

4.4. VIKOR method for MCDM

The VIKOR method is a decision-making approach that uses a decision matrix to select the optimal option based on a set of criteria. In this section, the method is applied to evaluate the blasting pattern. In this method, options are evaluated according to the criteria, and the optimal option is selected.

In the first step of this method, the decision matrix is formed, similar to the previous methods. In the next step, the decision matrix must be made dimensionless. To achieve this, the following relations are used for normalization, which leads to the determination of the dimensionless matrix (F):

$$f_{ij} = \frac{a_{ij}}{\sqrt[2]{\sum_{i=1}^{m} a_{ij}^2}} \tag{22}$$

$$F = \begin{bmatrix} f_{11} & \cdot & \cdot & \cdot & f_{1n} \\ \vdots & \vdots & \cdot & \vdots & \vdots \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ f_{m1} & \cdot & \cdot & \cdot & f_{mn} \end{bmatrix} \tag{23}$$

The dimensionless decision matrix resulting from this method is presented in Table 19.

Table 17. Dimensionless decision matrix obtained by the COCOSO method

	C1	C2	C3	C4	C5
A1	0.65	0.80	1.00	0.39	0.82
A2	0.65	0.77	1.00	0.39	0.82
A3	1.00	0.80	0.73	0.25	0.89
A4	0.01	0.80	0.38	0.86	0.89
A5	0.00	0.80	0.38	0.85	0.89
A6	0.20	0.80	1.00	0.37	0.81
A7	0.19	0.80	0.73	0.21	0.78
A8	0.20	0.00	0.73	0.22	0.00
A9	0.56	0.07	0.73	0.56	0.21
A10	0.72	0.07	0.73	0.54	0.36
A11	0.83	0.80	0.38	0.69	0.89
A12	0.63	0.80	0.38	0.69	0.89
A13	0.61	0.80	0.38	0.80	0.89
A14	0.07	0.80	0.73	0.58	0.82
A15	0.83	0.53	0.73	0.60	0.62
A16	0.67	0.53	0.38	0.82	0.74
A17	0.67	0.53	0.38	0.99	0.74
A18	0.68	0.47	0.73	0.58	0.54
A19	0.71	0.47	0.38	0.81	0.68
A20	0.68	0.47	0.38	0.78	0.68
A21	0.70	0.73	0.73	0.49	0.83
A22	0.27	0.80	1.00	0.47	0.82
A23	0.55	0.73	0.73	0.53	0.82
A24	0.70	0.80	0.38	0.83	0.89
A25	0.20	0.47	0.38	0.88	0.68
A26	0.94	0.73	0.38	1.00	0.85
A27	0.60	0.13	0.73	0.42	0.28
A28	0.62	0.73	0.38	0.76	0.85
A29	0.61	0.73	0.38	0.80	0.85
A30	0.89	0.20	0.73	0.56	0.29
A31	0.50	0.80	1.00	0.41	0.82
A32	0.45	0.80	0.38	0.55	0.89
A33	0.62	0.73	0.38	0.73	0.85
A34	0.94	0.80	0.38	0.00	0.89
A35	0.85	0.80	0.38	0.76	0.78
A36	0.70	0.53	0.73	0.61	0.62
A37	0.70	0.53	0.73	0.69	0.62
A38	0.28	1.00	0.38	0.83	1.00
A39	0.36	0.87	0.38	0.85	0.95
A40	0.56	0.87	0.38	0.80	0.95
A41	0.63	0.87	0.73	0.57	0.94
A42	0.69	0.87	0.73	0.59	0.87
A43	0.74	0.80	0.38	0.94	0.89
A44	0.73	0.80	0.73	0.49	0.87
A45	0.95	0.73	0.38	0.86	0.85
A46	0.65	0.20	0.73	0.57	0.29
A47	0.94	0.67	0.38	0.68	0.83
A48	0.83	0.73	1.00	0.40	0.77
A49	0.20	0.47	0.38	0.88	0.68
A50	0.90	0.20	0.00	0.98	0.49
A51	0.28	0.20	0.00	0.98	0.49
A52	0.71	0.47	0.38	0.80	0.68

Table 18. The WSM, WPM and k parameters obtained by the COCOSO method

	WSM	WPM	K_ia	K_ib	K_ic	K_i
A1	0.73	4.62	0.02	5.20	0.96	2.53
A2	0.72	4.61	0.02	5.17	0.96	2.52
A3	0.81	4.67	0.02	5.57	0.99	2.68
A4	0.43	3.89	0.02	3.57	0.78	1.81
A5	0.42	3.76	0.02	3.49	0.75	1.77
A6	0.55	4.30	0.02	4.27	0.87	2.13
A7	0.47	4.15	0.02	3.89	0.83	1.97
A8	0.23	2.24	0.01	2.00	0.45	1.02
A9	0.45	4.03	0.02	3.75	0.81	1.90
A10	0.53	4.17	0.02	4.13	0.85	2.06
A11	0.73	4.65	0.02	5.23	0.97	2.55
A12	0.65	4.55	0.02	4.84	0.94	2.38
A13	0.66	4.57	0.02	4.89	0.94	2.40
A14	0.47	4.13	0.02	3.87	0.83	1.96
A15	0.70	4.61	0.02	5.08	0.96	2.48
A16	0.62	4.50	0.02	4.66	0.92	2.31
A17	0.64	4.53	0.02	4.76	0.93	2.35
A18	0.62	4.49	0.02	4.66	0.92	2.30
A19	0.61	4.48	0.02	4.62	0.92	2.29
A20	0.59	4.46	0.02	4.55	0.91	2.26
A21	0.70	4.61	0.02	5.06	0.96	2.47
A22	0.59	4.40	0.02	4.48	0.90	2.23
A23	0.64	4.54	0.02	4.79	0.93	2.36
A24	0.70	4.62	0.02	5.06	0.96	2.48
A25	0.41	4.15	0.02	3.64	0.82	1.87
A26	0.80	4.74	0.02	5.54	1.00	2.68
A27	0.47	4.13	0.02	3.88	0.83	1.96
A28	0.64	4.54	0.02	4.79	0.94	2.36
A29	0.64	4.55	0.02	4.79	0.94	2.36
A30	0.62	4.37	0.02	4.62	0.90	2.28
A31	0.67	4.54	0.02	4.91	0.94	2.41
A32	0.57	4.41	0.02	4.41	0.90	2.20
A33	0.63	4.53	0.02	4.76	0.93	2.35
A34	0.69	3.76	0.02	4.65	0.80	2.22
A35	0.74	4.66	0.02	5.26	0.97	2.56
A36	0.65	4.55	0.02	4.84	0.94	2.38
A37	0.66	4.57	0.02	4.89	0.94	2.41
A38	0.58	4.41	0.02	4.49	0.90	2.23
A39	0.59	4.45	0.02	4.52	0.91	2.25
A40	0.66	4.56	0.02	4.88	0.94	2.40
A41	0.72	4.64	0.02	5.16	0.97	2.52
A42	0.74	4.67	0.02	5.26	0.98	2.56
A43	0.73	4.66	0.02	5.21	0.97	2.54
A44	0.73	4.65	0.02	5.22	0.97	2.54
A45	0.78	4.71	0.02	5.48	0.99	2.65
A46	0.52	4.26	0.02	4.16	0.86	2.08
A47	0.74	4.65	0.02	5.26	0.97	2.56
A48	0.78	4.68	0.02	5.45	0.99	2.63
A49	0.41	4.15	0.02	3.64	0.82	1.87
A50	0.57	3.58	0.02	4.04	0.75	1.96
A51	0.32	3.22	0.01	2.80	0.64	1.44
A52	0.61	4.48	0.02	4.61	0.92	2.29

Table 19. Dimensionless decision matrix obtained by the VIKOR method

	C1	C2	C3	C4	C5
A1	15.0	0.033	0.001	0.624	12.3
A2	15.0	0.035	0.001	0.624	12.5
A3	5.8	0.033	0.002	0.884	6.9
A4	43.1	0.033	0.004	0.089	6.9
A5	43.4	0.033	0.004	0.098	6.9
A6	33.0	0.033	0.001	0.660	12.6
A7	33.4	0.033	0.002	0.962	16.2
A8	33.3	0.133	0.002	0.947	188.9
A9	18.0	0.122	0.002	0.380	123.8
A10	12.9	0.122	0.002	0.399	84.3
A11	9.7	0.033	0.004	0.231	6.9
A12	15.7	0.033	0.004	0.231	6.9
A13	16.1	0.033	0.004	0.130	6.9
A14	39.8	0.033	0.002	0.353	12.3
A15	9.8	0.059	0.002	0.326	35.7
A16	14.3	0.059	0.004	0.115	20.1
A17	14.3	0.059	0.004	0.028	20.1
A18	14.1	0.067	0.002	0.358	49.1
A19	13.2	0.067	0.004	0.124	27.6
A20	14.1	0.067	0.004	0.145	27.6
A21	13.5	0.039	0.002	0.480	11.6
A22	29.8	0.033	0.001	0.511	12.3
A23	18.5	0.039	0.002	0.417	12.3
A24	13.5	0.033	0.004	0.110	6.9
A25	33.1	0.065	0.004	0.076	27.6
A26	7.2	0.039	0.004	0.024	9.6
A27	16.5	0.112	0.002	0.588	106.1
A28	15.9	0.039	0.004	0.161	9.6
A29	16.2	0.039	0.004	0.135	9.6
A30	8.2	0.102	0.002	0.377	101.9
A31	20.3	0.033	0.001	0.601	12.3
A32	22.1	0.033	0.004	0.393	6.9
A33	16.1	0.039	0.004	0.192	9.6
A34	7.2	0.033	0.004	1.445	6.9
A35	9.2	0.033	0.004	0.169	15.5
A36	13.3	0.059	0.002	0.314	35.7
A37	13.3	0.059	0.002	0.232	35.7
A38	29.4	0.019	0.004	0.110	1.5
A39	25.6	0.028	0.004	0.096	3.4
A40	18.0	0.028	0.004	0.130	3.4
A41	15.7	0.028	0.002	0.369	4.2
A42	13.9	0.028	0.002	0.345	8.1
A43	12.3	0.033	0.004	0.046	6.9
A44	12.4	0.033	0.002	0.470	8.4
A45	6.9	0.039	0.004	0.094	9.6
A46	15.1	0.102	0.002	0.372	101.9
A47	7.1	0.045	0.004	0.241	11.6
A48	9.8	0.039	0.001	0.615	17.0
A49	33.2	0.067	0.004	0.076	27.6
A50	7.9	0.102	0.007	0.032	57.3
A51	29.5	0.102	0.007	0.032	57.3
A52	13.2	0.067	0.004	0.130	27.6

In the next step, the positive and negative ideal points are determined. For positive criteria, the positive ideal points are equal to the largest value in the criterion column, and the negative ideal points are equal to the smallest value in the criterion column. For negative criteria, the opposite approach is applied. In any case, increasing positive criteria and decreasing negative criteria improves desirability in this method. Table 20 presents the positive (f_j^+) and negative (f_j^-i) deal points calculated for each criterion.

Table 20. Positive and negative ideal points obtained by the VIKOR method

	C1	C2	C3	C4	C5
f_j^+	5.791	0.019	0.001	0.024	1.529
f_j^-	43.390	0.133	0.007	1.445	188.894

In the next step, the utility (S_i) and regret (R_i) values for each indicator are calculated using the following equations:

$$S_i = \sum_{j=1}^{n}\left(W_j \times \frac{f_j^+ - f_{ij}}{f_j^+ - f_j^-}\right)$$
$$R_i = Max\left(W_j \times \frac{f_j^+ - f_{ij}}{f_j^+ - f_j^-}\right) \tag{24}$$

The utility value represents the relative distance of the ith option from the positive ideal solution, while the regret value represents the maximum discomfort of the ith option from being far from the ideal solution. These values are calculated for each blasting pattern using Excel software, and the results are presented in Table 21.

After calculating the S_i and R_i values, the minimum of each is calculated as S_i^+ and R_i^+, their maximums values as S_i^- and R_i^-, and the VIKOR index Q_i is calculated using the following equations:

$$Q_i = \left(v \times \frac{S_i - {S_i}^+}{{S_i}^- - {S_i}^+}\right) + \left((1-v) \times \frac{R_i - {R_i}^+}{{R_i}^- - {R_i}^+}\right)$$
$${S_i}^+ = Min\{S_i\}$$
$${R_i}^+ = Min\{R_i\}$$
$${S_i}^- = Max\{S_i\}$$
$${R_i}^- = Max\{R_i\} \tag{25}$$

The values of the VIKOR index for each blasting pattern are presented in Table 21. In the final step of this method, the options (blasting patterns) are ranked based on the VIKOR index, from smallest to largest. According to this method, any option with a lower Q_i will have a higher priority. The final ranking results of this method are presented in Table 21. Based on the results obtained from this method, Blasting Pattern 3 (identifier: M_Patt_03) has been determined as the best pattern. Additionally, in terms of the worst blasting patterns, the results of this model have identified Patterns 8, 5, and 4 in that order.

4.5. Copeland method for combined ranking

In the previous sections, blasting patterns were ranked using various multi-criteria decision-making methods: TOPSIS, ELECTRE, COCOSO, and VIKOR. In this section, the Copeland method of result aggregation is used to determine the blasting pattern with the highest rank.

The Copeland method is a modification of the Borda method and is one of the newer strategies for determining the best rank among several options. In prioritizing using this method, in addition to the sum of elements in each row (number of dominances), the sum of elements in each column (number of defeats) is also considered. Accordingly, the superior option is determined through pairwise comparison and evaluation of the superiority of options relative to each other.

Table 22 presents the ranking of blasting patterns resulting from various multi-criteria decision-making methods used in this research. Based on this information, a pairwise comparison matrix of options is formed, where the first row and column indicate the position of the patterns.

After forming the pairwise comparison matrix, at the intersection of the ith row and jth column, if the majority of MCDM methods rank in such a way that they indicate the superiority of pattern A_i over pattern A_j, the letter "M" is placed. Otherwise, if the superiority of pattern A_i over pattern A_j is equal to the superiority of pattern A_j over pattern A_i, or if pattern A_j is superior to pattern A_i, the letter "X" is placed in this cell. The number of pairwise comparisons in the matrix is equal to:

$$\frac{m(m-1)}{2} \tag{26}$$

In this equation, 'm' is the number of options (patterns). In the next step, the number of wins and losses for each option is calculated, and finally, the difference between the number of wins and losses for each option is determined (R). In fact, the ranking criterion in this method is the value of R. Therefore, any option with a higher value has a higher rank.

Table 23 presents the results of all pairwise comparisons, the determination of superiorities, and the prioritization of the options. Additionally, a portion of these results is shown magnified in Figure 7. The final ranking indicates that, based on this method as well, Blasting Pattern 3 is superior to the other blasting patterns examined in this research.

Table 21. The S_i, R_i, and Q_i values and ranking of blasting patterns in the VIKOR method

	S_i	R_i	Q_i	Rank
A1	0.18	0.10	0.11	12
A2	0.19	0.10	0.11	14
A3	0.14	0.07	0.03	2
A4	0.52	0.39	0.83	50
A5	0.53	0.40	0.84	51
A6	0.38	0.29	0.55	44
A7	0.44	0.29	0.61	45
A8	0.71	0.29	0.85	52
A9	0.45	0.19	0.48	43
A10	0.37	0.19	0.41	39
A11	0.18	0.09	0.10	11
A12	0.24	0.11	0.17	20
A13	0.24	0.11	0.18	22
A14	0.45	0.36	0.72	49
A15	0.19	0.08	0.09	7
A16	0.27	0.09	0.18	23
A17	0.27	0.09	0.17	21
A18	0.26	0.09	0.17	19
A19	0.28	0.09	0.19	25
A20	0.29	0.09	0.20	29
A21	0.20	0.08	0.10	10
A22	0.33	0.25	0.46	41
A23	0.24	0.13	0.22	32
A24	0.21	0.09	0.12	18
A25	0.48	0.29	0.64	46
A26	0.15	0.09	0.07	4
A27	0.42	0.17	0.43	40
A28	0.25	0.11	0.18	24
A29	0.25	0.11	0.19	27
A30	0.29	0.16	0.29	34
A31	0.24	0.15	0.23	33
A32	0.33	0.17	0.34	35
A33	0.26	0.11	0.19	28
A34	0.26	0.12	0.21	30
A35	0.17	0.09	0.09	8
A36	0.23	0.08	0.12	17
A37	0.22	0.08	0.12	15
A38	0.35	0.25	0.47	42
A39	0.33	0.21	0.39	38
A40	0.25	0.13	0.21	31
A41	0.19	0.11	0.12	16
A42	0.17	0.09	0.08	6
A43	0.19	0.09	0.11	13
A44	0.17	0.07	0.06	3
A45	0.15	0.09	0.07	5
A46	0.37	0.16	0.35	36
A47	0.18	0.09	0.10	9
A48	0.14	0.05	0.00	1
A49	0.49	0.29	0.65	47
A50	0.38	0.18	0.39	37
A51	0.61	0.25	0.70	48
A52	0.28	0.09	0.19	26

Table 22. Ranking results of MCDM methods of TOPSIS, ELECTRE, COCOSO, and VIKOR

	Pattern ID	TOPSIS	ELECTRE	COCOSO	VIKOR
A1	M_Patt_01	15	16	11	12
A2	M_Patt_02	16	19	13	14
A3	M_Patt_03	1	1	1	2
A4	M_Patt_04	50	50	49	50
A5	M_Patt_05	51	51	50	51
A6	M_Patt_06	44	40	39	44
A7	M_Patt_07	46	43	42	45
A8	M_Patt_08	52	52	52	52
A9	M_Patt_09	45	47	46	43
A10	M_Patt_10	36	35	41	39
A11	M_Patt_11	6	6	8	11
A12	M_Patt_12	22	17	22	20
A13	M_Patt_13	25	18	19	22
A14	M_Patt_14	49	44	45	49
A15	M_Patt_15	14	15	14	7
A16	M_Patt_16	20	27	28	23
A17	M_Patt_17	19	25	26	21
A18	M_Patt_18	33	31	29	19
A19	M_Patt_19	17	20	30	25
A20	M_Patt_20	23	34	33	29
A21	M_Patt_21	12	14	16	10
A22	M_Patt_22	41	38	36	41
A23	M_Patt_23	32	32	25	32
A24	M_Patt_24	11	12	15	18
A25	M_Patt_25	47	45	47	46
A26	M_Patt_26	4	4	2	4
A27	M_Patt_27	42	46	44	40
A28	M_Patt_28	24	21	24	24
A29	M_Patt_29	27	30	23	27
A30	M_Patt_30	38	33	32	34
A31	M_Patt_31	34	36	17	33
A32	M_Patt_32	35	37	38	35
A33	M_Patt_33	26	28	27	28
A34	M_Patt_34	2	3	37	30
A35	M_Patt_35	7	7	5	8
A36	M_Patt_36	29	26	21	17
A37	M_Patt_37	28	22	18	15
A38	M_Patt_38	40	41	35	42
A39	M_Patt_39	37	39	34	38
A40	M_Patt_40	30	29	20	31
A41	M_Patt_41	21	13	12	16
A42	M_Patt_42	13	11	6	6
A43	M_Patt_43	9	9	10	13
A44	M_Patt_44	10	8	9	3
A45	M_Patt_45	3	2	3	5
A46	M_Patt_46	39	42	40	36
A47	M_Patt_47	5	5	7	9
A48	M_Patt_48	8	10	4	1
A49	M_Patt_49	48	48	48	47
A50	M_Patt_50	31	23	43	37
A51	M_Patt_51	43	49	51	48
A52	M_Patt_52	18	24	31	26

Table 23. The results of pairwise comparisons of blasting patterns in the Copeland integrated method

	A1	A2	A3	A4	A5	A6	A7	A8	A9	A10	A11	A12	A13	A14	A15	A16	A17	A18	A19	A20	A21	A22	A23	A24	A25	A26
A1	--	M	X	M	M	M	M	M	M	M	X	M	M	M	X	M	M	M	M	M	X	M	M	X	M	X
A2	X	--	X	M	M	M	M	M	M	M	X	M	M	M	X	M	M	M	M	M	X	M	M	X	M	X
A3	M	M	--	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
A4	X	X	X	--	M	X	X	M	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
A5	X	X	X	X	--	X	X	M	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
A6	X	X	X	M	M	--	M	M	M	X	X	X	X	M	X	X	X	X	X	X	X	X	X	X	M	X
A7	X	X	X	M	M	X	--	M	X	X	X	X	X	M	X	X	X	X	X	X	X	X	X	X	M	X
A8	X	X	X	X	X	X	X	--	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
A9	X	X	X	M	M	X	X	M	--	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	M	X
A10	X	X	X	M	M	M	M	M	M	--	X	X	X	M	X	X	X	X	X	X	X	M	X	X	M	X
A11	M	M	X	M	M	M	M	M	M	M	--	M	M	M	M	M	M	M	M	M	M	M	M	M	M	X
A12	X	X	X	M	M	M	M	M	M	M	X	--	M	M	X	M	M	M	M	M	X	M	M	X	M	X
A13	X	X	X	M	M	M	M	M	M	M	X	X	--	M	X	M	X	M	M	M	X	M	M	X	M	X
A14	X	X	X	M	M	X	X	M	X	X	X	X	X	--	X	X	X	X	X	X	X	X	X	X	X	X
A15	M	M	X	M	M	M	M	M	M	M	X	M	M	M	--	M	M	M	M	M	X	M	M	X	M	X
A16	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	--	X	M	X	M	X	M	M	X	M	X
A17	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	M	--	M	X	M	X	M	M	X	M	X
A18	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	--	X	M	X	M	X	X	M	X
A19	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	X	--	M	X	M	M	X	M	X
A20	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	X	X	--	X	M	X	X	M	X
A21	M	M	X	M	M	M	M	M	M	M	X	M	M	M	X	M	M	M	M	M	--	M	M	X	M	X
A22	X	X	X	M	M	M	M	M	M	X	X	X	X	M	X	X	X	X	X	X	X	--	X	X	M	X
A23	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	X	X	X	X	M	--	X	M	X
A24	X	X	X	M	M	M	M	M	M	M	X	M	M	M	X	M	M	M	M	M	M	M	M	--	M	X
A25	X	X	X	M	M	X	X	M	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	--	X
A26	M	M	X	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	--
A27	X	X	X	M	M	X	X	M	M	X	X	X	X	M	X	X	X	X	X	X	X	X	X	X	M	X
A28	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	M	X	M	X	M	M	X	M	X
A29	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	M	X	M	X	M	M	X	M	X
A30	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	X	X	X	X	M	X	X	M	X
A31	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	X	X	X	X	M	X	X	M	X
A32	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	X	X	X	X	M	X	X	M	X
A33	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	M	X	M	X	M	M	X	M	X
A34	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	X	X	X	X	M	M	X	M	X
A35	M	M	X	M	M	M	M	M	M	M	X	M	M	M	M	M	M	M	M	M	M	M	M	M	M	X
A36	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	M	X	M	X	M	X	M	M	X	M	X
A37	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	M	M	M	X	M	X	M	M	X	M	X
A38	X	X	X	M	M	M	M	M	M	X	X	X	X	M	X	X	X	X	X	X	X	X	X	X	M	X
A39	X	X	X	M	M	M	M	M	M	X	X	X	X	M	X	X	X	X	X	X	X	M	X	X	M	X
A40	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	M	X	X	X	M	M	X	M	X
A41	X	X	X	M	M	M	M	M	M	M	X	M	M	M	X	M	M	M	M	M	X	M	M	X	M	X
A42	M	M	X	M	M	M	M	M	M	M	X	M	M	M	M	M	M	M	M	M	M	M	M	M	M	X
A43	M	M	X	M	M	M	M	M	M	M	X	M	M	M	M	M	M	M	M	M	M	M	M	M	M	X
A44	M	M	X	M	M	M	M	M	M	M	X	M	M	M	M	M	M	M	M	M	M	M	M	M	M	X
A45	M	M	X	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	X
A46	X	X	X	M	M	X	M	M	M	X	X	X	X	M	X	X	X	X	X	X	X	X	X	X	M	X
A47	M	M	X	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	X
A48	M	M	X	M	M	M	M	M	M	M	X	M	M	M	M	M	M	M	M	M	M	M	M	M	M	X
A49	X	X	X	M	M	X	X	M	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
A50	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	X	X	X	X	M	X	X	M	X
A51	X	X	X	M	M	X	X	M	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
A52	X	X	X	M	M	M	M	M	M	M	X	X	X	M	X	X	X	X	X	M	X	M	M	X	M	X

	A27	A28	A29	A30	A31	A32	A33	A34	A35	A36	A37	A38	A39	A40	A41	A42	A43	A44	A45	A46	A47	A48	A49	A50	A51	A52
A1	M	M	M	M	M	M	M	X	X	M	M	M	M	M	M	X	X	X	X	M	X	X	M	M	M	M
A2	M	M	M	M	M	M	M	X	X	M	M	M	M	M	X	X	X	X	X	M	X	X	M	M	M	M
A3	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M
A4	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
A5	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
A6	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	M	X	M	X
A7	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	M	X	M	X
A8	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
A9	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	M	X	M	X
A10	M	X	X	X	X	X	X	X	X	X	X	M	X	X	X	X	X	X	X	X	X	X	M	X	M	X
A11	M	M	M	M	M	M	M	X	X	M	M	M	M	M	M	X	M	M	X	M	X	X	M	M	M	M
A12	M	M	M	M	M	M	M	X	X	X	X	M	M	M	X	X	X	X	X	M	X	X	M	M	M	M
A13	M	M	M	M	M	M	M	X	X	M	X	M	M	M	X	X	X	X	X	M	X	X	M	M	M	M
A14	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X
A15	M	M	M	M	M	M	M	X	X	M	M	M	M	M	X	X	X	X	X	M	X	X	M	M	M	M
A16	M	X	M	M	M	M	M	X	X	X	X	M	M	M	X	X	X	X	X	M	X	X	M	M	M	X
A17	M	X	M	M	M	M	M	X	X	X	X	M	M	M	X	X	X	X	X	M	X	X	M	M	M	X
A18	M	X	X	M	M	M	X	X	X	X	X	M	M	X	X	X	X	X	X	M	X	X	M	X	M	X
A19	M	X	M	M	M	M	M	X	X	X	X	M	M	M	X	X	X	X	X	M	X	X	M	M	M	M
A20	M	X	X	X	M	M	X	X	X	X	X	M	M	X	X	X	X	X	X	M	X	X	M	M	M	X
A21	M	M	M	M	M	M	M	X	X	M	M	M	M	M	X	X	X	X	X	M	X	X	M	M	M	M
A22	M	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	M	X	M	X
A23	M	X	X	M	M	M	X	X	X	X	X	M	M	X	X	X	X	X	X	M	X	X	M	X	M	X
A24	M	M	M	M	M	M	M	X	X	M	M	M	M	M	X	X	X	X	X	M	X	X	M	M	M	M
A25	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	M	X	M	X
A26	M	M	M	M	M	M	M	X	M	M	M	M	M	M	M	M	M	M	X	M	M	M	M	M	M	M
A27	--	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	M	X	M	X
A28	M	--	M	M	M	M	M	X	X	X	X	M	M	M	X	X	X	X	X	M	X	X	M	M	M	M
A29	M	X	--	M	M	M	X	X	X	X	X	M	M	X	X	X	X	X	X	M	X	X	M	M	M	X
A30	M	X	X	--	X	M	X	X	X	X	X	M	M	X	X	X	X	X	X	M	X	X	M	X	M	X
A31	M	X	X	M	--	M	X	X	X	X	X	M	M	X	X	X	X	X	X	M	X	X	M	X	M	X
A32	M	X	X	X	X	--	X	X	X	X	X	M	M	X	X	X	X	X	X	M	X	X	M	X	M	X
A33	M	X	X	M	M	M	--	X	X	X	X	M	M	M	X	X	X	X	X	M	X	X	M	M	M	X
A34	M	X	X	M	M	M	X	--	X	X	X	M	M	M	X	X	X	X	X	M	X	X	M	M	M	X
A35	M	M	M	M	M	M	M	X	--	M	M	M	M	M	M	M	M	M	X	M	X	X	M	M	M	M
A36	M	X	M	M	M	M	M	X	X	--	X	M	M	M	X	X	X	X	X	M	X	X	M	M	M	X
A37	M	X	M	M	M	M	M	X	X	M	--	M	M	M	X	X	X	X	X	M	X	X	M	M	M	M
A38	M	X	X	X	X	X	X	X	X	X	X	--	X	X	X	X	X	X	X	X	X	X	M	X	M	X
A39	M	X	X	X	X	X	X	X	X	X	X	M	--	X	X	X	X	X	X	M	X	X	M	X	M	X
A40	M	X	X	M	M	M	X	X	X	X	X	M	M	--	X	X	X	X	X	M	X	X	M	M	M	X
A41	M	M	M	M	M	M	M	X	X	M	M	M	M	M	--	X	X	X	X	M	X	X	M	M	M	M
A42	M	M	M	M	M	M	M	X	X	M	M	M	M	M	M	--	X	X	X	M	X	X	M	M	M	M
A43	M	M	M	M	M	M	M	X	X	M	M	M	M	M	M	X	--	X	X	M	X	X	M	M	M	M
A44	M	M	M	M	M	M	M	X	X	M	M	M	M	M	M	M	M	--	X	M	X	X	M	M	M	M
A45	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	M	--	M	M	M	M	M	M	M
A46	M	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	--	X	X	M	X	M	X
A47	M	M	M	M	M	M	M	X	X	M	M	M	M	M	M	X	M	M	X	M	--	X	M	M	M	M
A48	M	M	M	M	M	M	M	X	X	M	M	M	M	M	M	M	M	M	X	M	X	--	M	M	M	M
A49	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	--	X	M	X
A50	M	X	X	X	X	X	X	X	X	X	X	M	M	X	X	X	X	X	X	X	X	X	M	--	M	X
A51	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	X	--	X
A52	M	X	M	M	M	M	M	X	X	X	X	M	M	M	X	X	X	X	X	M	X	X	M	M	M	--

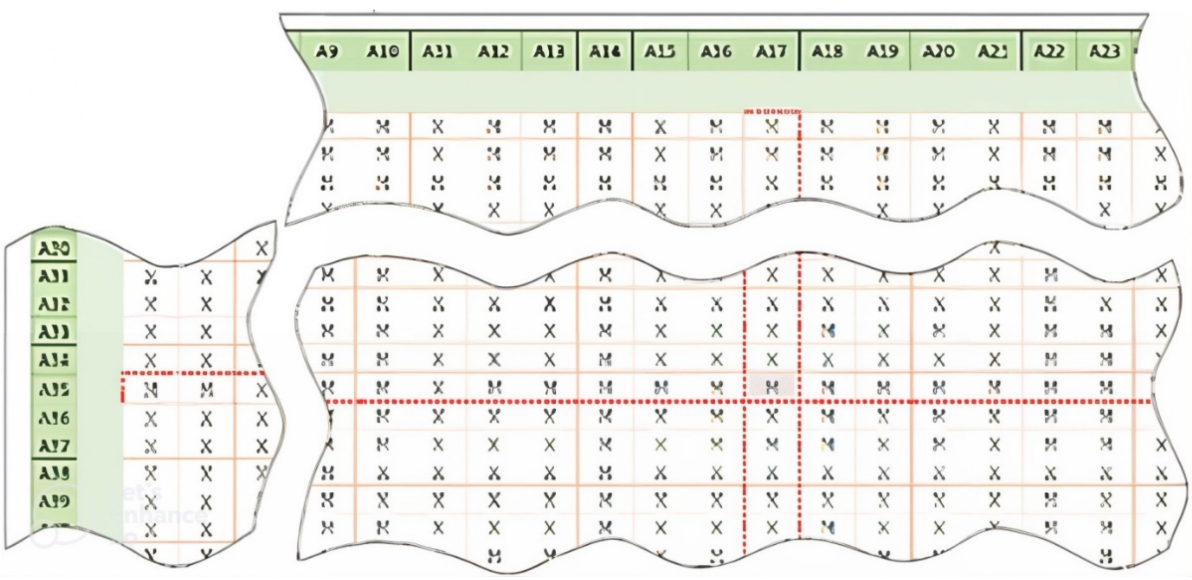

Figure 7. A part of the results of pairwise comparisons of blasting patterns in the Copeland integrated method

In Table 24, the final ranking of the blasting patterns is presented relative to the ranking of patterns based on the indices used in this research. The ranking ranges from 1 to 52, corresponding to the number of blasting patterns examined in this research. Accordingly, from the perspective of each index, the blasting pattern with a lower ranking number holds a higher priority among the options for that index. It should be noted that in this table, the number of indices corresponds to the five indices examined in this research. Additionally, the final score determined by the Copeland method is also presented for all blasting patterns.

Comparing the results reveals that Blasting Pattern 3 has the highest priority. This pattern exhibits more favourable fragmentation compared to the other competing patterns. Additionally, in terms of specific charge and specific drilling, it is ranked among the best and superior patterns. From the perspective of the flyrock index, this blasting pattern also performs well. However, it does not perform as well in the backbreak index. Despite this, the backbreak index carries a lower weight in this evaluation. Therefore, it can be concluded that the results are fully consistent with the evaluation framework.

The Copeland method, based on pairwise comparisons, aggregates preferences by assigning scores to each option (blasting pattern) according to the number of times it is preferred over others, minus the number of times it is less preferred. Based on the results from the Copeland method's pairwise comparison process, Pattern 3 consistently ranked higher than the other patterns across the evaluated criteria, including fragmentation quality, specific charge, specific drilling, and the reduction of backbreak and flyrock.

4.6. Sensitivity analysis

In this section, a sensitivity analysis was conducted to assess the robustness of the final ranking and the stability of the decision-making process. This was achieved by altering the weight of each criterion individually by ±10%, while proportionally adjusting the remaining criteria weights to ensure that the total sum remained equal to 1. This approach allows for the testing of how changes in the weights of individual criteria might affect the final rankings of the alternatives. For this analysis, the TOPSIS method was selected as the representative MCDM technique due to its interpretability and common usage in the literature. TOPSIS is widely used for ranking alternatives in MCDM problems, and its results are easy to understand and explain.

The results of the sensitivity analysis are summarized in Table 25. These results demonstrate that Pattern M_Patt_03 consistently remained the highest-ranked alternative across all tested weight variations. The only significant changes observed were minor shifts in the rankings of the second and third alternatives. These small shifts suggest that, while the rankings of the second and third alternatives are slightly influenced by changes in criteria weights, Pattern M_Patt_03 continues to perform consistently well, confirming its robustness. The stability of the optimal solution, even with changes in the criteria weights, indicates that the chosen blasting pattern is highly reliable and resilient to small variations in expert-assigned weights. This analysis confirms the strength and robustness of the decision-making process, enhancing the credibility of the final recommendation.

Table 24. The final ranking of blasting patterns based on the criteria

	Pattern ID	RANK_Copeland	RANK_C1	RANK_C2	RANK_C3	RANK_C4	RANK_C5
A3	M_Patt_03	1	1	3	2	44	4
A45	M_Patt_45	2	2	5	3	7	7
A26	M_Patt_26	3	4	5	3	1	7
A35	M_Patt_35	4	8	3	3	19	12
A47	M_Patt_47	5	3	6	3	23	8
A48	M_Patt_48	6	11	5	1	41	14
A11	M_Patt_11	7	9	3	3	21	4
A44	M_Patt_44	8	13	3	2	36	6
A42	M_Patt_42	9	19	2	2	26	5
A43	M_Patt_43	10	12	3	3	4	4
A15	M_Patt_15	11	10	7	2	25	17
A21	M_Patt_21	12	17	5	2	37	8
A24	M_Patt_24	13	18	3	3	10	4
A1	M_Patt_01	14	22	3	1	42	9
A41	M_Patt_41	15	24	2	2	29	3
A2	M_Patt_02	16	22	4	1	42	10
A34	M_Patt_34	17	5	3	3	47	4
A12	M_Patt_12	18	25	3	3	21	4
A13	M_Patt_13	19	28	3	3	14	4
A37	M_Patt_37	20	16	7	2	22	17
A17	M_Patt_17	21	21	7	3	2	15
A28	M_Patt_28	22	26	5	3	18	7
A36	M_Patt_36	23	16	7	2	24	17
A19	M_Patt_19	24	15	8	3	13	16
A16	M_Patt_16	25	21	7	3	12	15
A52	M_Patt_52	26	15	8	3	14	16
A33	M_Patt_33	27	27	5	3	20	7
A29	M_Patt_29	28	29	5	3	16	7
A40	M_Patt_40	29	32	2	3	15	2
A18	M_Patt_18	30	20	8	2	28	18
A20	M_Patt_20	31	20	8	3	17	16
A23	M_Patt_23	32	33	5	2	35	9
A31	M_Patt_31	33	34	3	1	40	9
A30	M_Patt_30	34	7	8	2	31	21
A50	M_Patt_50	35	6	8	4	3	19
A32	M_Patt_32	36	35	3	3	33	4
A39	M_Patt_39	37	36	2	3	8	2
A10	M_Patt_10	38	14	9	2	34	20
A46	M_Patt_46	39	23	8	2	30	22
A22	M_Patt_22	40	39	3	1	38	9
A38	M_Patt_38	41	37	1	3	11	1
A6	M_Patt_06	42	40	3	1	43	11
A27	M_Patt_27	43	30	9	2	39	23
A7	M_Patt_07	44	43	3	2	46	13
A9	M_Patt_09	45	31	9	2	32	24
A25	M_Patt_25	46	41	8	3	5	16
A14	M_Patt_14	47	44	3	2	27	9
A49	M_Patt_49	48	41	8	3	5	16
A51	M_Patt_51	49	38	8	4	3	19
A4	M_Patt_04	50	45	3	3	6	4
A5	M_Patt_05	51	46	3	3	9	4
A8	M_Patt_08	52	42	10	2	45	25

Table 25. Top three blasting pattern rankings under ±10% variation in criteria weights (based on TOPSIS method).

Scenario	Top pattern	Top rank	Second pattern	Third pattern
Fragmentation (-10%)	M_Patt_03	1	M_Patt_45	M_Patt_34
Fragmentation (+10%)	M_Patt_03	1	M_Patt_34	M_Patt_45
Specific Charge (-10%)	M_Patt_03	1	M_Patt_34	M_Patt_45
Specific Charge (+10%)	M_Patt_03	1	M_Patt_45	M_Patt_34
Specific Drilling (-10%)	M_Patt_03	1	M_Patt_34	M_Patt_45
Specific Drilling (+10%)	M_Patt_03	1	M_Patt_34	M_Patt_45
Backbreak (-10%)	M_Patt_03	1	M_Patt_34	M_Patt_45
Backbreak (+10%)	M_Patt_03	1	M_Patt_34	M_Patt_45
Flyrock (-10%)	M_Patt_03	1	M_Patt_34	M_Patt_45
Flyrock (+10%)	M_Patt_03	1	M_Patt_34	M_Patt_45

5. Conclusions

Multi-Criteria Decision-Making (MCDM) is a set of classical and modern mathematical methods that can be applied to evaluate and select the best option among available alternatives, based on diverse and sometimes conflicting criteria. In this research, various decision-making methods were employed to achieve an optimal drilling and blasting pattern at the Miduk copper mine.

The primary objectives of this study were to achieve optimal fragmentation, minimize specific charge and drilling, and reduce undesirable phenomena such as backbreak and flyrock resulting from blasting. To meet these goals, 52 blasting patterns implemented in the mine were analyzed and evaluated using MCDM methods, including TOPSIS, ELECTRE, VIKOR, and COCOSO. By forming a decision matrix in each method and ranking the options, the most suitable (optimal) pattern was selected from the available blasting patterns. Furthermore, the Copeland method was used to integrate the results from the decision-making models and achieve a consensus on the ranking of the blasting patterns based on the criteria.

The main innovation of this study lies in the application of advanced MCDM methods for optimizing drilling and blasting patterns, integrating results using the Copeland method, and conducting a comprehensive evaluation of multiple key parameters in a real-world case study. This approach not only enhances design accuracy but also provides a data-driven, practical solution for reducing undesirable blasting phenomena (such as backbreak and flyrock) while optimizing costs.

The results of this study indicated that the blasting pattern with a burden of 6.5 meters, spacing of 8 meters, and a borehole diameter of 150 millimeters (Pattern 3) is the optimal blasting pattern for this mine. This pattern can be used to achieve suitable fragmentation with minimal specific charge and specific drilling, while also reducing backbreak and flyrock during blasting.

However, given the complexity of drilling and blasting operations in mines and the inherent uncertainties associated with these processes, MCDM models are highly effective tools for ranking and selecting the best blasting pattern. By employing multiple decision-making models simultaneously, the results of each model can validate or complement one another, leading to a more accurate ranking of patterns through the integration of results.

Based on the results of this research, the advantages of multi-criteria decision-making (MCDM) methods in the context of blasting pattern optimization in surface mines can be summarized as follows:

Comprehensive Evaluation: MCDM methods enable the simultaneous consideration of multiple, often conflicting criteria—such as cost, efficiency, safety, and environmental impact. This ensures a holistic assessment of engineering alternatives, leading to more balanced and informed decisions.

Structured Decision-Making: MCDM provides a systematic framework for evaluating complex engineering problems. Methods like TOPSIS, ELECTRE, VIKOR, and COCOSO, as used in the Miduk copper mine study, structure the decision-making process by quantifying and ranking options based on diverse criteria, thereby reducing subjectivity.

Handling Uncertainty: Engineering challenges, such as drilling and blasting in mining, often involve uncertainties. MCDM methods are robust in addressing these uncertainties by incorporating both quantitative and qualitative data, allowing for reliable decisions in complex scenarios.

Flexibility and Adaptability: MCDM methods are versatile and can be applied across various engineering domains—from optimizing blasting patterns to selecting materials or designing infrastructure. They adapt to different contexts by allowing customization of criteria and weights based on specific project needs.

Integration of Multiple Models: Using multiple MCDM models concurrently, as demonstrated in the Miduk study with the Copeland method for result integration, enhances decision accuracy. This approach validates outcomes across models, ensuring consistency and reliability when selecting optimal solutions.

Improved Trade-off Analysis: MCDM facilitates the analysis of trade-offs between competing objectives, such as minimizing costs while maximizing safety or performance. This is crucial in engineering, where compromises are often required to achieve practical solutions.

Enhanced Decision Transparency: By providing a clear, mathematical basis for ranking alternatives, MCDM methods improve transparency and traceability in decision-making. This is essential for justifying choices in engineering projects to stakeholders.

Support for Innovation: MCDM encourages the exploration of innovative solutions by systematically comparing new technologies or methods against established ones, thus fostering advancements in engineering practices.

In summary, MCDM methods contribute significantly to science and technology in engineering by providing a robust, flexible, and transparent approach to decision-making. They enable engineers to tackle complex problems, optimize solutions, and drive innovation while effectively managing trade-offs and uncertainties. The significance of our findings in practical blasting lies in the comprehensive evaluation of multiple key parameters, such as rock fragmentation, specific charge, drilling efficiency, backbreak, and flyrock. These parameters are crucial for optimizing both the safety and cost-effectiveness of blasting operations in surface mining. By employing a holistic approach that simultaneously evaluates these factors, our method offers a more robust solution compared to previous studies that typically focus on just one or two parameters.

In conclusion, while this study provides valuable insights into optimizing blasting patterns using MCDM methods, certain limitations must be acknowledged. One of the key aspects of further improving the reliability and credibility of the MCDM model is validating its outcomes using real-world field data. While the model was built based on data from the Miduk copper mine, a more comprehensive validation could be achieved by comparing the predicted results, such as the recommended blasting pattern, with actual performance outcomes. For example, fragmentation levels provide a direct measure of the blasting pattern’s effectiveness in terms of rock breakage, which is crucial for downstream processing efficiency. By comparing the predicted fragmentation outcomes from the MCDM model with actual field data, we can assess how accurately the model reflects real-world conditions. Similarly, vibration measurements are critical in assessing the safety and environmental impacts of blasting operations. Comparing the model's predictions regarding blast-induced vibrations with actual vibration data would allow for a more precise evaluation of the model's ability to minimize undesirable effects like ground displacement and structural damage. An important point to note in this study is the unavailability of historical blasting data (e.g., fragmentation levels, vibration readings) from the Miduk mine, which prevented a direct comparison with actual field data. In this study, due to operational constraints at the time of analysis, complete field validation could not be performed. However, it is worth noting that:

- The recommended pattern (M_Patt_03) is one of the historically applied patterns in the mine (not purely hypothetical).
- Its measured performance indicators (e.g., D80 = 73.7 mm, low specific charge and drilling, moderate flyrock and backbreak) are among the best recorded in the 52 real patterns evaluated.
- Thus, the selection of M_Patt_03 is not only supported by the model but also reflects its historically proven superiority.

Due to the lack of sufficient field data, a full-scale implementation and monitoring are planned as part of future work. Incorporating such data in future studies will not only enhance the robustness of the model but also demonstrate its practical applicability in guiding real-world mining operations. Validation through field data will provide valuable feedback for refining the MCDM model, ensuring it accurately predicts outcomes under varying operational conditions. Expanding the dataset is another key direction for future research; incorporating more diverse blasting patterns would help refine the model and improve its generalizability to different mining contexts.

Conflict of interest

The authors declare that they have no conflict of interest.

References

[1]. Babaeian, M., Ataei, M., Sereshki, F., Sotoudeh, F., & Mohammadi, S. (2019). A new framework for evaluation of rock fragmentation in open pit mines. *Journal of Rock Mechanics and Geotechnical Engineering*, 11(2), 325-336.

[2]. Akkewar, S., & Kant, R. (2022). Environmental Impacts of Blasting in Surface Mining. *NeuroQuantology*, 20(15), 6978.

[3]. Monjezi, M., Mohamadi, H. A., Barati, B., & Khandelwal, M. (2014). Application of soft computing in predicting rock fragmentation to reduce environmental blasting side effects. *Arabian Journal of Geosciences*, 7, 505-511.

[4]. Mohanty, B. (2020). Rock fragmentation by blasting. CRC Press.

[5]. Roy, M. P., Paswan, R. K., Sarim, M. D., Kumar, S. U. R. A. J., Jha, R., & Singh, P. K. (2016). Rock fragmentation by blasting-A review. *Journal of mines, metals and fuels*, 64(9), 424-431.

[6]. Taherkhani, H., & Doostmohammadi, R. (2015). Transportation costs: A tool for evaluating the effect of rock mass mechanical parameters on blasting results in open pit mining. *Journal of Mining Science*, 51, 730-742.

[7]. Zhang, Z. X., Sanchidrián, J. A., Ouchterlony, F., & Luukkanen, S. (2023). Reduction of fragment size from mining to mineral processing: a review. *Rock Mechanics and Rock Engineering*, 56(1), 747-778.

[8]. Monjezi, M., Rezaei, M., & Varjani, A. Y. (2009). Prediction of rock fragmentation due to blasting in Gol-E-Gohar iron mine using fuzzy logic. *International Journal of Rock Mechanics and Mining Sciences*, 46(8), 1273-1280.

[9]. Pugliese, J. M. (1972). Designing blast patterns using empirical formulas. *Information Circular*, 8550.

[10]. Bahadori, M., & Bakhshandeh Amnieh, H. (2023). Numerical analysis of the pile movement resulting from rock blasting in open pit mines. *Journal of Mineral Resources Engineering*, 8(3), 57-73.

[11]. Torano, J., Rodríguez, R., Diego, I., Rivas, J. M., & Casal, M. D. (2006). FEM models including randomness and its application to the blasting vibrations prediction. *Computers and Geotechnics*, 33(1), 15-28.

[12]. Azarafza, M., Feizi-Derakhshi, M. R., & Jeddi, A. (2017). Blasting pattern optimization in open-pit mines by using the genetic algorithm. *Geotechnical Geology*, 13(2), 75-81.

[13]. Guo, J., Zhao, Z., Zhao, P., & Chen, J. (2024). Prediction and optimization of open-pit Mine blasting based on intelligent algorithms. *Applied Sciences*, 14(13), 5609.

[14]. Rezaei, M., Monjezi, M., Matinpoor, F., Bolbanabad, S. M., & Habibi, H. (2023). Simulation of induced flyrock due to open-pit blasting using the PCA-CART hybrid modeling. *Simulation Modelling Practice and Theory*, 129, 102844.

[15]. Tawadrou, A. S., & Katsabani, P. D. (2005). Prediction of surface blast patterns in limestone quarries using artificial neural networks. *Fragblast*, 9(4), 233-242.

[16]. Bakhshandeh Amnieh, H., Aref Mand, E., & Porghasemi Saghand, M. (2022). Controlling backbreak and improving technical and economic parameters in Mishdovan Iron Ore Mine. *Journal of Mineral Resources Engineering*, 7(1), 99-111.

[17]. Monjezi, M., Rezaei, M., & Yazdian, A. (2010). Prediction of backbreak in open-pit blasting using fuzzy set theory. *Expert Systems with Applications*, 37(3), 2637-2643.

[18]. Rad, H. N., Hasanipanah, M., Rezaei, M., & Eghlim, A. L. (2018). Developing a least squares support vector machine for estimating the blast-induced flyrock. *Engineering with Computers*, 34(4), 709-717.

[19]. Rezaei, M., Monjezi, M., & Varjani, A. Y. (2011). Development of a fuzzy model to predict flyrock in surface mining. *Safety science*, 49(2), 298-305.

[20]. Zhou, J., Koopialipoor, M., Murlidhar, B. R., Fatemi, S. A., Tahir, M. M., Jahed Armaghani, D., & Li, C. (2020). Use of intelligent methods to design effective pattern parameters of mine blasting to minimize flyrock distance. *Natural Resources Research*, 29, 625-639.

[21]. Dehghani, H., & Beiromvand, H. (2019). Blasting pattern design for decreasing the ground vibration using genetic algorithm. *Journal of Mineral Resources Engineering*, 4(2), 39-55.

[22]. Scott, A. (2020). 'Blastability'and blast design. *Rock Fragmentation by Blasting*, 27-36.

[23]. AyalaCarcedo, F. (2017). *Drilling and blasting of rocks*. Routledge.

[24]. Noorian-Bidgoli, M. (2014). Strength and deformability of fractured rocks. Doctoral dissertation, KTH Royal Institute of Technology, Sweden.

[25]. Paswan, R.K., Priyadarshi, V., Kushwaha, S., Rana, V.S., Roy, M.P. & Singh, P.K. (20022). Uncontrollable and Controllable Parameters of Rock Blasting-A Review. *The Indiln Mining & Engineering Journl*, 12-44.

[26]. Taherkhani, H., & Doostmohammadi, R. (2015). Transportation costs: A tool for evaluating the effect of rock mass mechanical parameters on blasting results in open pit mining. *Journal of Mining Science*, 51, 730-742.

[27]. Leng, Z., Fan, Y., Gao, Q., & Hu, Y. (2020). Evaluation and optimization of blasting approaches to reducing oversize boulders and toes in open-pit mine. *International Journal of Mining Science and Technology*, 30(3), 373-380.

[28]. Dai, Y., Khandelwal, M., Qiu, Y., Zhou, J., Monjezi, M., & Yang, P. (2022). A hybrid metaheuristic approach using random forest and particle swarm optimization to study and evaluate backbreak in open-pit blasting. *Neural Computing and Applications*, 1-16.

[29]. Babaeian, M., Sereshki, F., Ataei, M., Nehring, M., & Mohammadi, S. (2023). Application of soft computing, statistical and multi-criteria decision-making methods to develop a predictive equation for prediction of flyrock distance in open-pit mining. *Mining*, 3(2), 304-333.

[30]. Nikkhah, A., Vakylabad, A. B., Hassanzadeh, A., Niedoba, T., & Surowiak, A. (2022). An evaluation on the impact of ore fragmented by blasting on mining performance. *Minerals*, 12(2), 258.

[31]. Ghanizadeh Zarghami, A., Shahriar, K., Goshtasbi, K., & Akbari Dehkharghani, A. (2020). A new technical and economic model to calculate specific charge and specific drilling using hole diameter, bench height, uniaxial compressive strength, and joint set orientation. *Journal of Mining and Environment*, 11(4), 991-1005.

[32]. Tosun, A., & Konak, G. (2015). Determination of specific charge minimizing total unit cost of open pit quarry blasting operations. *Arabian Journal of Geosciences*, 8, 6409-6423.

[33]. Abd Elwahab, A., Topal, E., & Jang, H. D. (2023). Review of machine learning application in mine blasting. *Arabian Journal of Geosciences*, 16(2), 133.

[34]. Munagala, V., Thudumu, S., Logothetis, I., Bhandari, S., Vasa, R., & Mouzakis, K. (2024). A comprehensive survey on machine learning applications for drilling and blasting in surface mining. *Machine Learning with Applications*, 15, 100517.

[35]. Roy, M. P., Paswan, R. K., Sarim, M. D., Kumar, S. U. R. A. J., Jha, R., & Singh, P. K. (2016). Rock fragmentation by blasting-A review. *Journal of mines, metals and fuels*, 64(9), 424-431.

[36]. Zhou, J., Zhang, Y., & Qiu, Y. (2024). State-of-the-art review of machine learning and optimization algorithms applications in environmental effects of blasting. *Artificial Intelligence Review*, 57(1), 5.

[37]. Mohammadi, D., Mikaeil, R., & Abdollahei Sharif, J. (2020). Investigating and Ranking Blasting Patterns to Reduce Ground Vibration using Soft Computing Approaches and MCDM Technique. *Journal of Mining and Environment*, 11(3), 881-897.

[38]. Triantaphyllou, E., & Triantaphyllou, E. (2000). Multi-criteria decision making methods. Springer Us, 5-21.

[39]. Aruldoss, M., Lakshmi, T. M., & Venkatesan, V. P. (2013). A survey on multi criteria decision making methods and its applications. *American Journal of Information Systems*, 1(1), 31-43.

[40]. Thakkar, J. J. (2021). Multi-criteria decision making. Singapore: Springer, 336, 1-365.

[41]. Rahimdel, M. J., Aryafar, A., & Tavakkoli, E. (2020). Selection of the most proper drilling and blasting pattern by using MADM methods (A case study: Sangan Iron Ore Mine, Iran). *Rudarsko-geološko-naftni zbornik*, 35(3).

[42]. Rahimdel, M. J., Aryafar, A., & Tavakkoli, E. (2020). Selection of the most proper drilling and blasting pattern by using MADM methods (A case study: Sangan Iron Ore Mine, Iran). *Rudarsko-geološko-naftni zbornik*, 35(3).

[43]. Sitorus, F., Cilliers, J. J., & Brito-Parada, P. R. (2019). Multi-criteria decision making for the choice problem in mining and mineral processing: Applications and trends. *Expert Systems With Applications*, 121, 393-417.

[44]. Kazakidis, V. N., Mayer, Z., & Scoble, M. J. (2004). Decision making using the analytic hierarchy process in mining engineering. *Mining Technology*, 113(1), 30-42.

[45]. Wu, Z., Li, Q., Kong, D., Chen, G., & Luo, D. (2020). The ANP-Fuzzy-TOPSIS model for the optimization of the scheme of large-section blasting. *Arabian Journal of Geosciences*, 13, 1-9.

[46]. Pouresmaieli, M., Ataei, M., Nouri Qarahasanlou, A., & Barabadi, A. (2024). Multi-

criteria decision-making methods for sustainable decision-making in the mining industry (A comprehensive study). *Journal of Mining and Environment*, 15(2), 683-706.

[47]. Marimuthu, R., Sankaranarayanan, B., & Karuppiah, K. (2023). Prioritizing the factors affecting the occupational health and safety of workers in the mining industry using the SWARA technique. *Journal of Current Science and Technology*, 13(1), 59-73.

[48]. Monjezi, M., Dehghani, H., Singh, T. N., Sayadi, A. R., & Gholinejad, A. (2012). Application of TOPSIS method for selecting the most appropriate blast design. *Arabian journal of geosciences*, 5(1), 95.

[49]. Gul, M., Ak, M. F., & Guneri, A. F. (2019). Pythagorean fuzzy VIKOR-based approach for safety risk assessment in mine industry. *Journal of Safety Research*, 69, 135-153.

[50]. Yari, M., Monjezi, M., Bagherpour, R., & Jamali, S. (2014). Developing a mathematical assessment model for blasting patterns management: Sungun copper mine. *Journal of Central South University*, 21, 4344-4351.

[51]. Wang, Q., Cheng, T., Lu, Y., Liu, H., Zhang, R., & Huang, J. (2024). Underground mine safety and health: A hybrid MEREC–CoCoSo system for the selection of best sensor. *Sensors*, 24(4), 1285.

[52]. Favardin, P., Lepelley, D., & Serais, J. (2002). Borda rule, Copeland method and strategic manipulation. *Review of Economic Design*, 7, 213-228.

نشریه مهندسی معدن و محیط زیست
نشانی نشریه: www.jme.shahroodut.ac.ir
انجمن مهندسی معدن ایران
دانشگاه صنعتی شاهرود

دستیابی به الگوی بهینه انفجار با استفاده از مدل‌های تصمیم‌گیری چندمعیاره (مطالعه موردی: معدن مس میدوک)

مجید نوریان بیدگلی*، و محمدرضا رضایی

گروه مهندسی معدن، دانشکده مهندسی، دانشگاه کاشان، کاشان، ایران

اطلاعات مقاله

تاریخ ارسال: ۲۰۲۵/۰۷/۰۸
تاریخ داوری: ۲۰۲۵/۰۷/۲۱
تاریخ پذیرش: ۲۰۲۵/۰۹/۰۷

DOI: 10.22044/jme.2025.16486.3223

کلمات کلیدی

تصمیم‌گیری چندمعیاره (MCDM)
الگوی آتشباری
حفاری
بهینه‌سازی
معدن مس میدوک

چکیده

حفاری و انفجار از عملیات اصلی در معادن روباز است که با هدف بهینه‌سازی خردایش سنگ، کاهش اثرات منفی مانند؛ عقب‌زدگی و پرتاب سنگ، و کاهش هزینه‌ها، در عین حال افزایش بهره‌وری و به حداقل رساندن اثرات محیط‌زیستی انجام می‌شوند. این مطالعه بر بهینه‌سازی الگوهای حفاری و انفجار در معدن مس میدوک با استفاده از روش‌های تصمیم‌گیری چندمعیاره (MCDM) تمرکز دارد. اهداف اصلی این مطالعه، دستیابی به خردایش بهینه، به حداقل رساندن هزینه‌های خرج ویژه و حفاری و کاهش پدیده‌های نامطلوب مانند؛ عقب‌زدگی و پرتاب سنگ ناشی از انفجار در این معدن است. بر این اساس، ۵۲ الگوی انفجار اجرا شده در معدن مذکور با استفاده از تکنیک‌های مختلف MCDM، از جمله؛ TOPSIS، ELECTRE، VIKOR و COCOSO ارزیابی شدند. بدین منظور، با ساخت ماتریس‌های تصمیم‌گیری و رتبه‌بندی گزینه‌ها در هر روش، مناسب‌ترین الگوی انفجار تعیین گردیدند. در ادامه از روش Copeland، به منظور ادغام نتایج حاصل از مدل‌های تصمیم‌گیری و ایجاد اجماع در مورد رتبه‌بندی نهایی الگوهای انفجار بر اساس معیارها استفاده شد. نوآوری این مطالعه در کاربرد تکنیک‌های پیشرفته MCDM برای بهینه‌سازی الگوهای حفاری و انفجار، و همچنین ادغام نتایج برای افزایش دقت فرآیند تصمیم‌گیری نهفته است. نتایج نشان می‌دهد الگوی انفجار بهینه (M_Patt_03) دارای بارسنگ ۶/۵ متر، فاصله‌داری ۸ متر و قطر چال ۱۵۰ میلی‌متر است که در آن بهترین تعادل بین خردایش، خرج ویژه و هزینه‌های حفاری ایجاد می‌شود، همچنین عقب‌زدگی و پرتاب سنگ به حداقل می‌رسد. این مطالعه اثربخشی روش‌های MCDM را در بهینه‌سازی فرآیندهای پیچیده مهندسی در معدنکاری سطحی نشان می‌دهد و چارچوبی جامع برای ارزیابی همزمان چندین معیار و امکان تصمیم‌گیری آگاهانه‌تر و متعادل‌تر فراهم می‌نماید.

نویسنده مسئول مکاتبات: noriyan@kashanu.ac.ir (م. نوریان بیدگلی)