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An Integrated FDM–GA Framework for Automated Back Analysis in Tunneling–Part I: Methodology, Implementation, and Verification

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Abstract

Back analysis of tunnel excavation plays a fundamental role in calibrating geomechanical parameters using field monitoring data. However, conventional direct back analysis procedures remain computationally demanding and highly dependent on operator supervision. This study presents an integrated Finite Difference Method–Genetic Algorithm (FDM–GA) framework for automated tunnel back analysis, implemented entirely within the FLAC environment using the embedded FISH programming language. The proposed approach eliminates the need for external optimization software and data transfer between numerical and artificial intelligence platforms. A simplified genetic algorithm is coupled directly with finite difference simulations to iteratively minimize the discrepancy between measured and computed tunnel convergences. The framework incorporates constrained parameter optimization, automated handling of non-convergent models, and a robust convergence-based stopping criterion that avoids predefined error thresholds. Verification is performed using two synthetic plane-strain tunnel models representing stiff cohesive soil and dense granular material. Six unknown parameters (ρ, E, ν, c, φ, and K0) are back-calculated using only three convergence measurements. Results from multiple independent runs demonstrate stable convergence toward very small error values (on the order of 10-6–10-5) and consistent reproduction of synthetic monitoring data. The method successfully narrows broad initial parameter ranges and produces multiple acceptable parameter sets, explicitly acknowledging the non-uniqueness inherent in back analysis problems. The developed FDM–GA framework provides an efficient, self-contained, and adaptable tool for practical tunnel back analysis applications.

1. Introduction

The construction of geomechanical structures, including tunnels, underground caverns, vertical shafts, and slopes, presents noteworthy challenges due to the complex geological and mechanical properties of natural rocks. While numerous tests exist for assessing rock mass mechanical characteristics, obtaining accurate and reliable values for crucial parameters such as elastic modulus, strength properties, and groundwater level is often a daunting task. Moreover, a precise understanding of the initial stress state, shaped by gravitational forces and tectonic activities, is vital for effective analysis. It is essential to recognize that in rock engineering, the actual performance of rock structures frequently diverges from the predictions made during the design phase, even when utilizing advanced computational simulations. This underscores the importance of rigorous testing and analysis in achieving successful outcomes in geomechanical projects [1].

To effectively bridge discrepancies between the actual and expected behavior of underground and surface excavations, it is essential to employ field instrumentation. This step not only validates the input data used in the initial design but also ensures the stability of structures. Quantitative interpretation of these measurements is vital, and back analysis stands out as a powerful method for

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this purpose [2]. In back analysis, the input comprises critical monitored parameters such as deformations and stresses. The resulting outputs are integral rock mass mechanical characteristics, which include the elastic modulus, Poisson's ratio, strength parameters (such as friction angle and cohesion), and the state of in-situ stresses. In contrast, forward analyses, widely utilized in the design phase of rock excavations, calculate project stresses, strains, and displacements based on inputs like external forces (including the state of in-situ stresses) and rock mass mechanical characteristics. However, back analysis leverages real-time instrumentation results, like deformations and stress metrics, to inform the relevant inputs. The outputs yield invaluable insights into the rock mass's mechanical properties and initial stress conditions, making back analysis an indispensable tool in ensuring the safety and integrity of excavation projects [3].

When predicting the response of rock mass, it is essential to select a robust mechanical model, whether elastic, elastoplastic, rigid-plastic, or viscoelastic. By deriving the model's mechanical parameters from thorough laboratory and in-situ testing programs, a solid foundation for stress and strain analyses is established. The outcomes of forward analyses, which give the values of different displacements and strain components along with stress states around excavations, establish a one-to-one, clear, and direct relationship between input parameters and output calculations, as demonstrated in Figure 1. Conversely, back analyses rely on instrumentation data, such as the deformations and stresses recorded during construction, to inform the process. This method, depicted in Figure 1, may alter perceptions of reliability, as it requires the assumption of a mechanical model to derive the necessary parameters. It is crucial to recognize that the results from back analysis are entirely dependent on this chosen model. Unlike the straightforward input-output framework in forward analysis, back analysis introduces complexities since the mechanical model of the rock mass influences the relationship between input and output. This nuance makes it imperative to choose the appropriate model to ensure accurate analyses [4].

In the field of geotechnical engineering, addressing back-analysis problems can be highly effective through different methods, with the inverse and direct approaches standing out as the most widely utilized [6]. The inverse procedure reinterprets the problem as the reverse of the forward formulation, employing the same fundamental equations. In contrast, the direct approach leverages an iterative optimization technique to enhance the trial values of unknown variables, focusing on minimizing the gaps between observed and calculated results. This method rigorously quantifies the differences between actual data and outcomes from numerical analysis, as illustrated by the following equation [2]:

$$error = \frac{\sum_{i=1}^{N}(u_i^m - u_i^c)^2}{\sum_{i=1}^{N} u_i^m} \quad (1)$$

Where are:

error – trial model's error,

u_i^m – measured values,

u_i^c – computed values,

N – the count of measurement points.

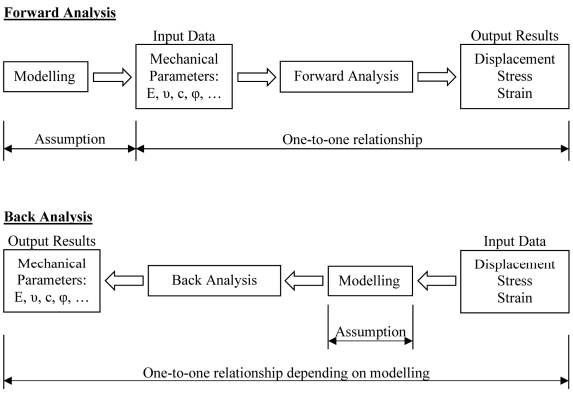


Figure 1. Comparison between the forward and back analysis procedures [5]

The advantage of the direct procedure is substantial. It bypasses the need to invert complex governing mathematical equations, making it highly adaptable to a range of non-linear issues. The error function described in **Equation (1)** embodies a quintessential non-linear complex function involving unidentified variables. In many scenarios, particularly in non-linear elastoplastic cases, deriving the gradient of this function analytically can be nearly impossible. As a solution, a minimization process that circumvents the need for analytical gradient calculations is utilized. To effectively tackle this challenge, mathematical programming offers powerful algorithms classified as direct search techniques. These iterative methods meticulously minimize the function by continuously assessing the error function, as illustrated in **Equation (1)**. Each cycle precisely calculates stress and strain components for the geotechnical issue based on trial values for

the unknown parameters, ensuring accurate and reliable outcomes [5].

The algorithms used to minimize the error function include traditional mathematical programming techniques such as the Rosenbrock algorithm [7], the Conjugate Gradient method [8], the Powell method [9], and the Simplex algorithm [10]. These techniques typically require extensive computations due to numerous iterations. Deng et al. effectively applied the principle of virtual work combined with an iterative process to accurately back-calculate the initial stress state and the stress relaxation factor at the support installation point for tunneling [11]. They also successfully used this method to evaluate surface subsidence in mining applications. Feng et al. developed an innovative displacement back analysis method that combines neural networks with numerical analysis techniques to determine ground mechanical parameters [12]. A similar approach for the displacement back analysis of rock slopes was employed by Deng and Lee [13]. The concept of Intelligent Back Analysis (IBA) was first introduced by Shang et al. [14]. They developed a Precedent-Type Analysis (PTA) method that leverages results from laboratory physical modeling, field tests, and in-situ monitoring from previous tunneling projects as reference cases for future projects. These results help refine displacement back analysis outcomes and assist in determining in-situ stress states and the Young's modulus of the rock mass. Li et al. effectively used this technique to back-calculate critical parameters for two water tunnels in China [15]. Additionally, Feng et al. introduced a robust deformation back analysis method that combines hybrid intelligence techniques, including numerical analysis, Support Vector Machines (SVMs), and a genetic algorithm [16, 17]. This method proved successful at the Baihetan hydropower station in China, where the rock mass was heavily jointed and highly stressed [18]. It has also been applied to accurately identify mechanical and creep parameters of rock masses [19]. Miranda et al. convincingly demonstrated that evolution strategies (ES) [20] outperform classical gradient-based optimization algorithms, particularly in terms of robustness and convergence in challenging elastoplastic scenarios [21, 22]. Yang et al. successfully implemented a genetic algorithm for back analysis of a synthetic, deep, simply-shaped tunnel using a no-tension Drucker-Prager model [23]. Guan et al. combined a forward procedure based on complex variable solutions with a backward approach utilizing classical optimization methods and a traversal strategy [24]. This robust method back-calculated the horizontal in-situ stress and ground deformation modulus from measured tunnel convergences in mountain tunnels. Their results show that this approach requires minimal computational resources and effectively avoids trapping in local minima. To address the complexities and uncertainties of rock masses, Khamesi et al. proposed a novel back analysis procedure [25], integrating a fuzzy system with three techniques: nearest neighbor clustering with gradient descent training, particle swarm optimization, and the imperialistic algorithm. Zhao et al. used a least squares support vector machine (LSSVM)-based response surface to model the nonlinear relationship between geomechanical parameters and monitored data [26]. They optimized the parameters using an MS Excel solver, demonstrating the superiority of their method in a simple circular tunnel. An et al. presented a robust back-analysis method for segment-lined tunnels that uses stress and displacement data, employing a differential evolution algorithm to determine key parameters such as elastic modulus, cohesion, and friction angle [27]. To handle discontinuous geomechanical problems, Ma et al. integrated a particle swarm optimization (PSO) algorithm with the 3DEC/UDEC distinct element code, enabling precise calculation of the mechanical properties of discrete block contact surfaces [28]. Li et al. applied an optimized particle swarm optimization (OPSO) method combined with a support vector machine (SVM) to analyze the geomechanical properties of rock masses based on displacement monitoring data [29]. Chang et al. used a differential evolution (DE) algorithm [30] to determine optimal hyperparameters for precise analysis [31], which were then employed in a multi-output support vector regression (MSVR) model [32] to back-calculate rock mass properties from monitoring data. In a recent study, Zhu et al. combined the extreme gradient boosting (XGBoost) algorithm [33] with the regressor chain (RC) technique [34], successfully applying the XGBoost-RC method to analyze deformation monitoring data from the Changning Expressway tunnel [35].

Recent research indicates a clear shift toward data-driven and hybrid intelligent–numerical approaches in tunnel back analysis. Li et al. used DEM-based back analysis calibrated with in-situ measurements to assess tunnel resilience [36], while Yao et al. (2026) employed a Sparrow Search Algorithm–BP neural network for efficient inversion of surrounding rock parameters [37].

Case histories remain important; Allouache et al. conducted a geomechanical back assessment of a twin-tunnel collapse [38], and Thirukumaran combined field observations with numerical back analysis to interpret stress-induced cavern failure [39]. The coupling of machine learning (ML) and optimization is increasingly prominent: Wang et al. integrated Auto ML with multi-objective optimization for mechanized twin tunnels [40], and Shi et al. proposed a GPSO-BP framework to improve inversion stability [41]. Additionally, Domínguez et al. calibrated 3D TBM excavation models through back analysis [42], and Lu et al. revisited analytical displacement-based solutions [43]. Overall, recent literature emphasizes intelligent inversion, 3D model calibration with monitoring data, and the application of back analysis to resilience and failure assessment [36-43].

The reviewed studies clearly demonstrate that various optimization algorithms, particularly those grounded in artificial intelligence (AI) techniques, have been effectively employed to tackle the error minimization problem inherent in direct back analysis procedures. While these approaches have proven successful, they are not without significant challenges. A primary challenge in minimizing **Equation (1)** using AI-based techniques lies in the necessity of calculating trial error values from **Equation (1)** through numerical modeling software such as FLAC/FLAC3D, UDEC/3DEC, PLAXIS, or ABAQUS. Furthermore, another software tool is essential to execute the AI algorithm based on the data generated by these numerical models. Given that most AI-driven optimization algorithms involve iterative processes of training, testing, and evaluation, user involvement is critical throughout the entire minimization procedure. Users must run numerical models, calculate trial error values, input data into the AI optimization algorithms, prepare features for the next round of numerical modeling, and continuously generate new data to inform subsequent optimization iterations. Additionally, users must vigilantly monitor for stopping conditions, which may manifest as convergence indicators or predefined thresholds. Figure 2 exemplifies this standard process in AI-based optimization. The persistent requirement for user supervision positions back analysis as a costly and time-consuming task, often resulting in its neglect in practical contexts. To effectively address this complex issue and improve analysis efficiency, Deng and Lee decisively replaced the finite element method with an error back-propagation neural network [13]. They employed a genetic algorithm (GA) as an optimization strategy for the displacement back analysis of the permanent shiplock slope at the Three Gorges Project in Central China. Jeon and Yang adopted a unique approach by utilizing the FISH programming language within FLAC to execute three simple direct search techniques: univariate search, pattern search, and alternating univariate methods [44]. They addressed a theoretical displacement back analysis problem, successfully calculating the elastic modulus (E) and the horizontal to vertical stress ratio (K_0) for a circular tunnel in an isotropic-elastic medium. Although this method has been applied to other practical examples, it is specifically designed to calculate these two essential parameters in the isotropic-elastic medium. The simple techniques previously employed are inadequate for managing multiple variables, non-linear relationships, and constrained optimization problems, especially in full-scale back analysis applications. Gutierrez et al. and Vardakos et al. introduced a superior methodology by employing a differential evolution genetic algorithm (DEGA) specifically designed for back-analyzing tunnel responses. This algorithm was effectively implemented using the FISH programming language within FLAC [45, 46]. By applying this technique, they successfully determined critical geotechnical parameters for two types of tunnels: deep and shallow circular tunnels, along with a horseshoe-shaped highway tunnel. The monitoring data for the circular tunnels were synthetically generated through forward numerical simulations, while actual instrumentation data were utilized for the highway tunnel. This method calculated essential parameters such as the elastic modulus (E), Poisson's ratio (υ), in-situ horizontal (σ_x) and vertical (σ_y) stresses, cohesion (c), and friction angle (φ) for the deep circular tunnel. For the shallow circular tunnel, the calculations included E, the horizontal-to-vertical in-situ stress ratio (K_0), c, and φ, while the unit weight (γ) and υ were maintained as constants. In the highway tunnel analysis, the unknown variables identified were γ, E, υ, c, and φ [45]. While the algorithm yielded accurate reproductions of the synthetic monitoring data, substantial discrepancies emerged between the actual measured displacements and the calculated results in the calibrated model of the actual example. The methodology faces significant limitations, compounded by its poor performance on real monitoring data. During genetic algorithm computations, problem constraints, namely, the

pre-defined ranges for the unknown variables, were often violated, requiring extra checks to eliminate out-of-range values. Moreover, the presented work failed to establish a clear stopping condition for the algorithm. Computations for circular tunnels were arbitrarily halted after 70 generations, while highway tunnel analyses were curtailed after only 50 generations, without justification. Furthermore, no final error values were reported, creating uncertainty regarding the accuracy of the outcomes. Identifying a critical shortcoming, the work provided only a single solution for the analyzed cases, which directly contradicts the established principle that a direct one-to-one relationship among input data and output results is nonexistent in back analysis problems [4].

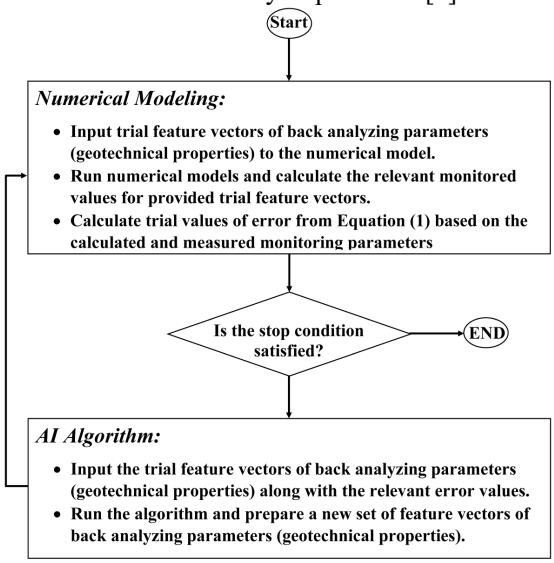


Figure 2. The flowchart of a common direct back analysis process with an AI-based optimization algorithm

This paper presents an integrated modeling algorithm that combines the Finite Difference Method (FDM) and Genetic Algorithm (GA) to automate the direct back analysis process, reducing the need for continuous operator supervision. The Itasca FLAC 8.00 software was used for numerical modeling and to conduct a simplified genetic algorithm. The implementation relied solely on the FISH programming language, which is embedded in FLAC, to perform the genetic algorithm calculations without the use of any additional computational software. This technique features a straightforward programmable algorithm and includes a robust stopping condition to determine when the algorithm has converged to a satisfactory solution. Furthermore, it demonstrates the capability to generate multiple distinct solutions within a specified error threshold, acknowledging that back analysis problems often do not have a unique answer. The paper also discusses and addresses the technical challenges associated with implementing Genetic Algorithms using FISH. To validate the proposed algorithm, two sets of synthetic tunnel monitoring data were tested.

The main features of the proposed back analysis framework, which enhance its efficiency compared with previously developed procedures, can be summarized as follows:

1. All numerical simulations and parameter optimization processes are implemented within the same software environment, FLAC. This integration eliminates the need for data transfer between the numerical modeling platform and an external optimization program. Consequently, the workflow becomes more streamlined, reduces the requirement for continuous expert supervision, and improves overall efficiency from a human-resource perspective, making the approach more attractive for industrial applications.
2. The developed algorithm does not rely on a predefined stopping condition to terminate the search for the optimal solution. This feature is particularly advantageous because, given the variability and uncertainty in recorded instrumentation data, it is often difficult to define a reliable acceptance threshold to determine when a trial model sufficiently matches the measured responses to qualify as a valid back analysis solution.
3. The incorporation of random selection of initial parameter values allows the algorithm to produce different solutions in different runs. This characteristic is consistent with the inherent non-uniqueness of the solutions of back analysis problems and supports the principle that multiple parameter sets may provide acceptable fits to the observed field data.
4. The algorithm performs the optimization within predefined parameter ranges. This ensures that all optimized parameters remain within physically meaningful and user-defined bounds, thereby eliminating the need for additional post-processing checks or corrective refinements.

The proposed framework is flexible and can be adapted to various types of instrumentation data and different sets of unknown parameters, making it suitable for a wide range of tunnel back analysis applications.

2. Integrated FDM–GA Modeling

The genetic algorithm (GA) is one of the most widely used and well-known algorithms and search techniques based on the principles of genetics and natural selection [47]. It has demonstrated its effectiveness in various applications [48-54]. Genetic algorithms are commonly used to generate better solutions for optimization and search challenges by employing biologically inspired mechanisms such as mutation, crossover, and selection [47]. In order to carry out a direct back analysis process, the genetic algorithm (GA) was selected due to its high efficiency in optimizing complex multidimensional problems under given constraints. The Itasca FLAC 8.00 finite difference code was selected for numerical modeling because of its widespread use in simulating continuum geotechnical problems and its high-performance embedded FISH programming language. However, because FISH is not a full-fledged programming language, a simplified genetic algorithm was used to make the programming more manageable.

2.1 Simplified Genetic Algorithm

To carry out the simplified genetic algorithm process, a matrix of $[v_{ij}]_{m\times n}$ is required. This matrix consists of m parameter vectors of $(v_j)_n$, where n represents the number of unknown geotechnical parameters to be estimated using GA. Each component of the $(v_j)_n$ vector is called a gene in GA terminology. Within GA, each vector is referred to as a chromosome. The total of m chromosomes forms the population. Therefore, the population matrix comprises m chromosomes, each containing n genes. The genes of the $(v_j)_n$ chromosome consist of ordered values of unknown geotechnical parameters, with each component representing a specific parameter. Initially, the value of each gene is chosen randomly. However, this random selection process can be conducted within the constraints predefined by the problem. It is important to note that, depending on the nature of each geotechnical formation, a reasonable range can be defined for its properties. The random selection of parameters can be adjusted according to these predefined ranges, allowing for the straightforward application of the problem's constraints [47, 55]

After assigning random, but in-range, values to all genes of the $[v_{ij}]_{m\times n}$ population matrix, numerical models are run for each $(v_j)_n$ chromosome. The relevant error values for the chromosomes are then calculated, resulting in an $(e_j)_m$ error vector. The $((e_j)_m)^T$ is added as a new column to the $[v_{ij}]_{m\times n}$ population matrix (Figure 3), creating a new matrix of $\{[v_{ij}]_{m\times n} \mid ((e_j)_m)^T\}$ [55]. An ascending sorting process is then applied to the $((e_j)_m)^T$ column of the obtained matrix, and the sorting is extended to the remaining chromosomes. This process results in the same matrix of property vectors (Figure 3), but the chromosomes are now sorted based on the calculated error values, with the upper chromosomes exhibiting less error than the lower ones [47, 55].

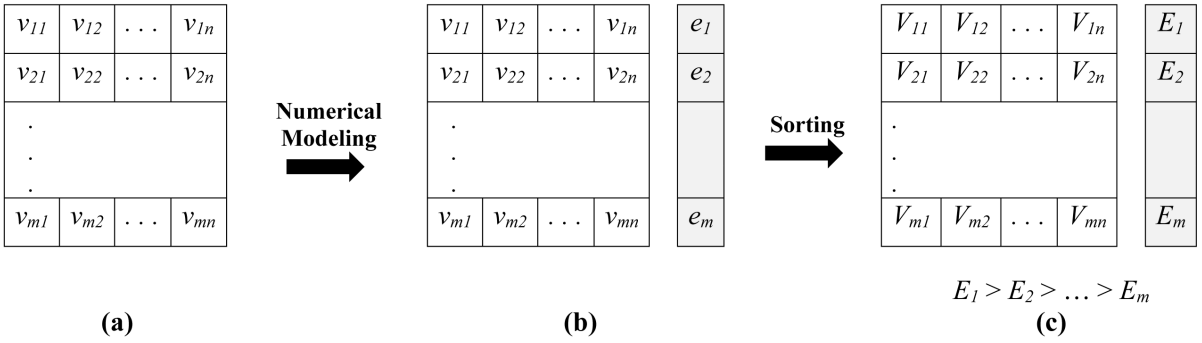


Figure 3. (a) Initial population matrix; (b) Error vector calculated for the chromosomes of the initial population matrix, and (c) The ascending sorted population matrix based on error values

In any genetic algorithm process, there is a survival rate (SR) ($0 < SR < 1$), which determines the proportion of the top best chromosomes that will survive to create the next generation population. Therefore, out of the total m chromosomes, the first ($m \times SR$) are selected to form the next population matrix (Figure 4). The remaining $m - (m \times SR)$ chromosomes of the new generation matrix are created through a crossover process. Two chromosomes from the surviving ones are randomly selected as parents for each new chromosome. A random inheritance number (IN) ($1 < IN < (n - 1)$) is assigned to each new chromosome. The new chromosome inherits the

1st, 2nd, …, IN^{th} genes from the first parent and the $(IN + 1)^{th}$, $(IN + 2)^{th}$, …, n^{th} genes from the second one. This process is repeated for all $m - (m \times SR)$ new chromosomes, and precautions are taken to avoid the repetition of chromosomes in the newly generated population [47, 55].[47, 55]

To enhance the diversity of results in the GA optimization process and to overcome local extremums, a mutation process is incorporated into the genetic algorithm. A mutation rate (*MR*) is determined within the range of 0 to 1. A random number (also between 0 and 1) is selected for each newly generated chromosome during the crossover process. If this random number is less than the specified *MR* value, the relevant chromosome is replaced with a new one, and its genes are selected randomly within the given ranges, similar to the initial chromosome selection process (Figure 4). Following the mutation process, a new population is formed, with each chromosome providing a property vector for error calculation. These vectors are then used in the numerical model to compute the relevant error values. After completing the error calculation process, a new GA iteration begins by sorting chromosomes based on their errors and performing crossover and mutation steps until a stop condition is met. The stop condition could be defined as achieving a minimum error below a specified threshold or having the minimum error unchanged over a certain number of GA iterations.

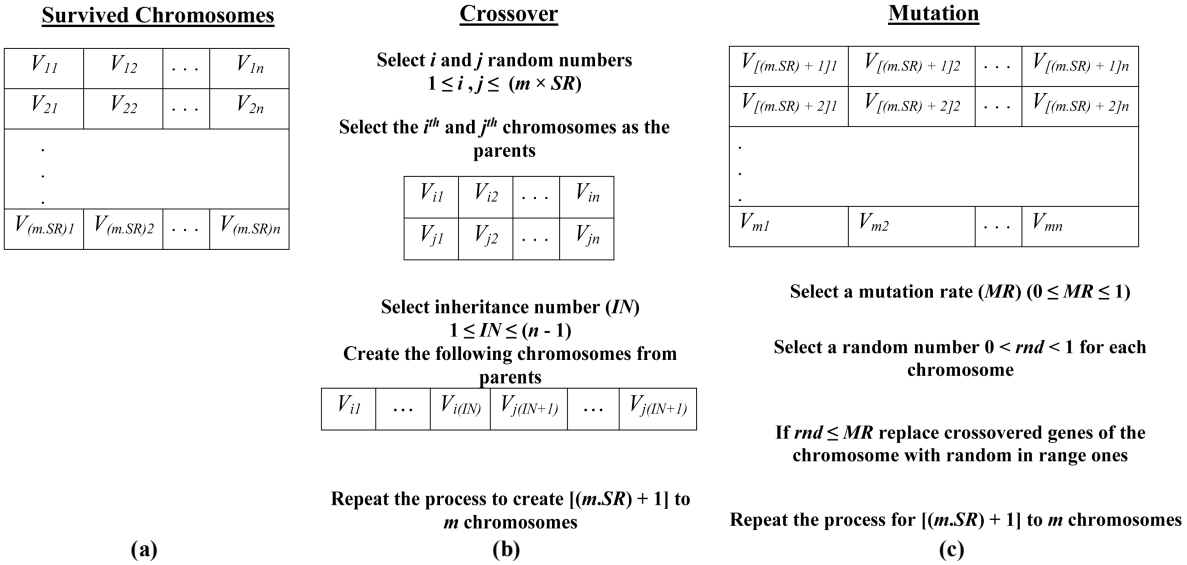


Figure 4. (a) The survived chromosomes from the previous generation; (b) Crossover process to create new chromosomes, and (c) Mutation on the new chromosomes

2.2. FDM–GA model

In order to create a computational code without the need to transfer data between numerical modeling software and computational software, the simplified genetic algorithm was implemented in the ITASCA FLAC 8.00 numerical code. The FLAC has a built-in programming language called FISH, which can be used to create user-defined variables, generate complicated grids, conduct servo-controlled numerical tests, and more. The FISH provides access to basic mathematical functions, loop structures, if-then logic, and an unlimited number of tables with various manipulation tools. Therefore, using FISH, it is possible to implement and control simplified to moderate computational algorithms as well. The main goal of establishing an integrated FDM–GA computational code is to minimize or eliminate the need for operator supervision during the routine back analysis runtime. This will be very helpful for consultant companies in saving the time and efforts of their experts and human resources, and will significantly increase their efficiency.

The approach to the GA mentioned above is explained in two steps. First, the general flowchart of the computational model is outlined. Since FISH is not a comprehensive programming language, coding computational algorithms such as GA, even in their simplest forms, may pose significant challenges. These challenges and their technical solutions will be addressed in the second part of this section.

2.2.1 General Computational Algorithm

The flowchart of the FDM-GA code (Figure 5) is based on the common AI-based back analysis flowchart in Figure 2.

1. **Setting General Numerical Model Configuration:** The process begins with setting the general configuration of the numerical program. Since the GA will need to perform a large number of computational iterations, certain default numerical computational limits of FLAC must be readjusted initially. The limits and the necessary actions required will be addressed in the next section.
2. **Input Numerical Modeling Parameters:** The geometric characteristics of the monitored structure, including its shape and dimensions, must be determined. Additionally, the support properties of the structures need to be specified. Because 2D numerical models only include beam elements to simulate the effects of steel sets and shotcrete, some calculations may be necessary to determine the properties of equivalent beam elements for steel set or composite shotcrete-steel set support systems [56]. Furthermore, if a 2D numerical model is used for error calculations, the support installation distance to the tunnel face and the pre-support displacement of the tunnel wall must be determined using LDP (longitudinal deformation profile) curves [57-59] or 3D numerical models. The structure of the integrated code will depend on the selected geotechnical parameters for back analysis. Therefore, the relevant parameters and their reasonable value ranges should be chosen.[56-59]
3. **Input GA Parameters:** This includes the main parameters of the simplified genetic algorithm. It covers the population number of chromosomes (m), the number of genes for each chromosome (n), which is selected during the analysis of geotechnical parameters, the surviving chromosome rate for the next generation (SR), and the mutation rate (MR).
4. **Input Monitoring Parameters:** Including the precise location of measuring points, the accurate measured values, and the distance of the tunnel face when the instrument was installed. It is important to note that the distance of the tunnel face during installation, as pre-installation deformations, cannot be captured by the instrument and should be subtracted when calculating errors with the numerical model. If there are multiple instrument units used in the structure monitoring program, consider using reasonable averaging to obtain more representative values of the measured parameters.
5. **Build the Basic Numerical Model:** To create the basic numerical model, one should start by using the prepared data from the previous steps. At this stage, only the numerical grid will be generated, and the boundary conditions will be defined. It is crucial to ensure that the numerical grid is arranged to have grid points at the exact locations where deformation measurements have been taken. It is also important to have a denser mesh around the measurement points to increase accuracy in deformation and error calculations. Note that at this stage, the model properties cannot be input into the numerical model because they will be input and optimized by the genetic algorithm. If the parameters for back analysis include ground density and the ratio of in-situ horizontal to vertical stress, the initial stress state cannot be established at this stage as well.
6. **Generate Initial Random Population Matrix:** The back analysis process begins by creating a chromosome with a sequence of genes, forming a vector with geotechnical parameters. A population matrix $[v_{ij}]_{m\times n}$ is then set up using an assembly of m populations of chromosomes. Random values within predetermined ranges (step 2) are assigned to the genes/components of the population matrix.
7. **Run Numerical Models and Calculate Error Values:** The population matrix from the previous step is fed into the numerical model. Each chromosome in the matrix provides the necessary data for completing and running a numerical model. The initial equilibrium state is calculated based on the density and the ratio of horizontal to vertical in-situ stress values (if they are included in the back analysis parameters). The model undergoes excavation, and in 2D models, the pre-support stress relaxation phase is carried out before the structure support is activated. The model must run until the deformations before the installation of the monitoring instrument occur in the model. At this point, the model deformations are reset to zero, and the model runs to the final equilibrium. The model deformations at the end of the process are equivalent but may not be the same as the measured deformations. After completing the numerical model calculations, the error values can be calculated and stored in a vector. For each iteration of the genetic algorithm (GA), the numerical modeling is repeated m times. Therefore, it is helpful to establish a loop from 1 to m and perform the repetitive modeling computations inside the loop. At the end of this step, the $(e_j)_m$ vector will

be obtained, and the $\{[v_{ij}]_{m\times n} \mid ((e_j)_m)^T\}$ matrix will be established.

8. **Check the Stop Condition:** A stop condition is needed to confirm if the outcome of the back analysis process is satisfactory for stopping or not. Different criteria can be selected as the stop condition, such as reducing the error values below a predefined threshold or having the error values unchanged over predefined GA iterations. Since the range of resulting error values is unknown at the beginning of the back analysis process, the last one may be more practical.

9. **Run GA Calculations:** If the stop condition is not met, then optimization of the input parameters is necessary. To start, a sorted matrix $\{[V_{ij}]_{m\times n} \mid ((E_j)_m)^T\}$ is obtain from the matrix $\{[v_{ij}]_{m\times n} \mid ((e_j)_m)^T\}$. Then the surviving chromosomes are selected and crossover and mutation calculations are performed to establish a new generation of the $[v_{ij}]_{m\times n}$ population matrix. This matrix is fed into the numerical model, and steps 7, 8, and 9 are repeated until the stop condition is met.

2.2.2 Technical Challenges and Solutions

The FISH programming language enhances FLAC's capabilities by introducing new user-defined variables and plots, implementing additional constitutive models, conducting automated sensitivity analyses, running servo-controlled numerical tests, and more [60]. However, while FISH is powerful for enhancing FLAC models, it is not as robust as full-scale programming languages. Using FISH to implement algorithms such as GA, even in simplified versions, can be challenging. Additionally, performing a GA iteration requires calculating the error value for m times (the chromosome population count), and a genetic algorithm may need thousands of iterations to converge to an optimized solution. FLAC and similar numerical codes are not designed to handle multiple calculations while preserving the results of previous models to set parameters for subsequent ones. However, utilizing a single computational code to perform numerical simulations and GA calculations for back analyzing geotechnical properties can eliminate the need for separate codes, leading to increased efficiency and substantial time and cost savings. Despite these challenges, it is worthwhile to address and resolve the technical issues associated with using GA with FISH. Some major challenges are listed below:[60]

1. **FLAC Runtime Defaults:** The "SOLVE" command has a default limit of 1440 minutes (24 hours) for elapsed time and 10,000,000 steps. However, completing a GA optimization typically requires more time and steps than these defaults. To prevent interruptions, one must adjust these values in the general program configuration (step 1 in the previous section) using the "SET" commands.

2. **Generating Random Values:** The two built-in FISH commands, "grand" and "urand", can generate random numbers between 0.0 and 1.0 from normal or uniform distributions, respectively. The uniform distribution will be used to generate random values in the GA. However, additional calculations are needed to scale the random number to the specified range.

3. **Using Tables Instead of Arrays to Represent Matrices:** In FISH functions, matrices can be represented by "ARRAYs." However, there are no built-in tools for manipulating "ARRAYs." Instead, FISH provides several built-in commands for controlling and managing "TABLEs." In the FISH structure, a "TABLE" is an unlimited collection of number pairs. Using multiple "TABLEs" to represent a data matrix can facilitate the implementation of the genetic algorithm (GA) steps. Therefore, separate "TABLEs" were used for each geotechnical property during back analysis. In each table, the first pair (x component) represents the error value, which is filled after the relevant numerical model run. The second pair (y component) represents the geotechnical property, selected randomly during the first GA iteration and calculated during subsequent iterations through crossover and mutation steps. The FISH command "table(n,x) = y" is used to insert an (x,y) pair into the appropriate position in table n. This command automatically generates sorted tables based on error values, which are necessary for selecting the surviving chromosomes for creating the next generation of the population.

4. **Discarding Unconverging Models:** When performing numerical models in a GA process, the input parameters are initially selected randomly and may, through mutation, result in random values from the previous generation. This can cause some models to undergo plastic material flow, preventing the unbalanced force ratio from converging within the specified limits when the "SOLVE" command is used. This condition leads to the failure of the genetic algorithm (GA). To address this, initial numerical models should be run with reasonable input parameters to determine the average number of computational steps needed to reach equilibrium. To ensure plastic flow in yielding models, this average step number can be multiplied by a factor

of at least 5 (or more). Instead of the "SOLVE" command, the "SOLVE STEP" command can be employed to set an upper limit for computational steps. If a model fails to reach equilibrium within this limit, it has experienced plastic flow. Such models will exhibit significant deformations and very high error values. These models will be automatically discarded when selecting sorted chromosomes based on errors for the next generation.

5. **Running a Sequence of Models in the GA Process:** In FLAC, a typical numerical model begins with the "new" command. When this command is issued, the existing grid is no longer defined, and problem variables are reset to zero or their default values. However, in a genetic algorithm (GA) process, a sequence of thousands of models is run, and the results need to be preserved for future GA steps. Using the "new" command would discard all existing parameters, tables, and relevant values. Instead of using the "new" command, in a GA process, a new model starts by retrieving the excavated model parts, initializing stress, displacement, and velocity components to zero, and resetting the values of input geotechnical parameters. This is possible because a GA is performed on a specific structure (e.g., a known tunnel) with the same geometry, and there is no need to make geometric changes or alter the numerical grid during the performance of the GA. It is important to reset the inertial mass of gridpoints. This can be done by setting a very low-density value and issuing a "step 1" command.
6. **Running FALC in the DOS Environment:** When running FLAC in its graphical user interface (GUI), a warning window is displayed on the screen if the number of timesteps exceeds the set threshold. Although the program continues to run in the background, the user needs to close the window. Additionally, based on the authors' experience, running a large number of models in a sequence where structural elements are repeatedly activated and deleted may cause the GUI to hang. To resolve both problems, running FLAC in its original DOS environment is recommended.

The following implementation section provides some further discussion about the problems mentioned.

3. Implementation and Verification of the FDM–GA Approach

The effectiveness and applicability of the proposed FDM-GA method in back analyzing tunnel responses are first validated through numerical models of a hypothetical simple tunnel shape, utilizing synthetic monitoring data. These synthetic data represent the model's response at specific points and are generated through forward analysis with predefined geotechnical input parameters. The goal of the back analysis is to determine the input parameters that would produce the synthetic monitoring data using the proposed method. In Part II, the applicability of the proposed FDM–GA back analysis framework to real tunneling conditions is demonstrated through its implementation on the Pardis Highway tunnel in northern Tehran, Iran. Additionally, the influence of ground stress relaxation parameters on the back analysis results will be examined using this real-case application.

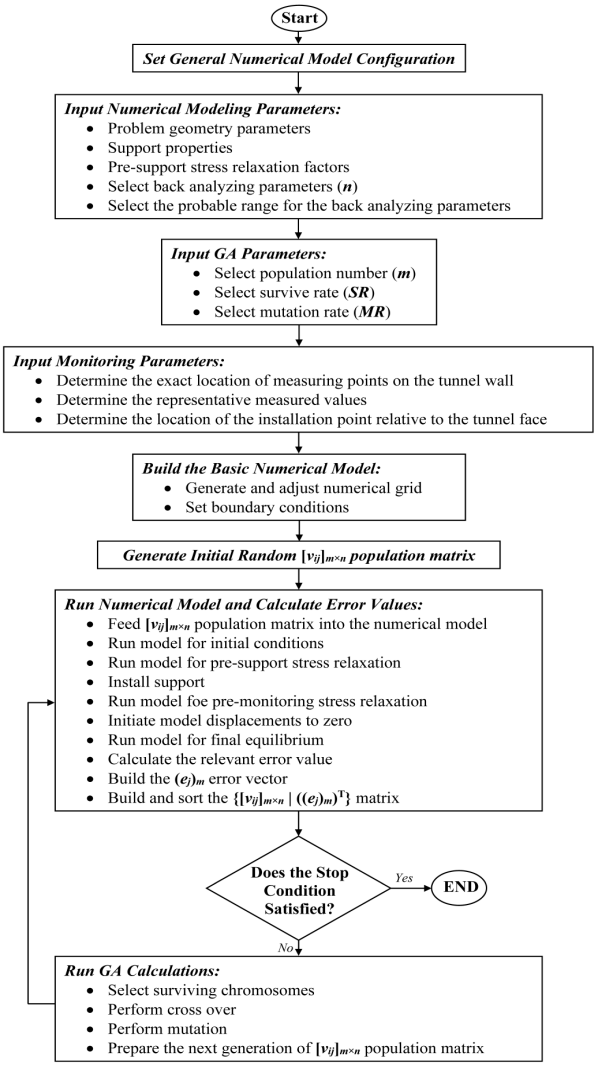


Figure 5. The flowchart of the general FDM-GA computational algorithm

The verification tunnel is modeled as constructed in Mohr-Coulomb elastoplastic soil, with plane strain conditions applied. The algorithm is independent of the constitutive model and can be

coupled with any suitable material model. In this study, the Mohr–Coulomb model was chosen for practical reasons. Its simplicity ensures high computational efficiency, which is essential in a GA-based optimization framework where numerous trial models are evaluated. This significantly reduces overall convergence time. Second, the Mohr–Coulomb parameters (c and φ) are widely used in engineering practice and are familiar to geotechnical professionals, facilitating interpretation and practical implementation. Finally, given the inherent non-uniqueness of back analysis solutions, selecting the most representative parameter set often involves comparison with documented values from similar projects within the same geotechnical context (discussed in Part II). Since most published case studies report shear strength in terms of Mohr–Coulomb parameters, adopting this model allows straightforward comparison and more reliable judgment among alternative solutions. The plane strain assumption is also a common numerical simplification that provides faster computation compared to three-dimensional models, enabling quicker solutions for various geotechnical situations. Nonetheless, the proposed method is adaptable to three-dimensional models. The verification problems involve a 10-meter-diameter circular tunnel excavated at a depth of 50 meters (from the tunnel axis to the ground surface). Considering symmetry, only half of the tunnel was modeled in both forward and back analyses (Figure 6). The model geometry consists of a 30 m × 80 m mesh. Due to symmetry, the left vertical boundary is defined as a plane of symmetry. The distances from the tunnel axis to the top, right, and bottom boundaries are 50 m, 30 m, and 30 m, respectively. The X signs on the left and right boundaries indicate that the velocity in the x direction is fixed to zero along these boundaries. Similarly, the Y signs on the bottom boundary indicate zero velocity in the y direction. The top boundary represents the ground surface; thus, no displacement or velocity constraints are applied there. Two cases were considered for ground geotechnical characteristics: Case 1 involves a stiff cohesive soil, and Case 2 involves a dense granular material. These cases were chosen to evaluate the proposed method under different geotechnical conditions. Table 1 shows the input parameters for both cases [61]. To keep the tunnel models simple, no support was installed on the inner walls. The hypothetical instrumentation system includes only three-point convergence pins installed at the tunnel crown and walls (Figure 7). The reasons for this simple instrumentation system are: 1) to assess whether the approach can produce reasonable results with limited monitoring data, 2) because convergence measurements are low-cost and common in tunnel projects, and 3) because this system matches the instrumentation used in the real case discussed in Part II [61].

Performing forward modeling using the initial geotechnical parameters in Table 1, the hypothetical measured convergencies between top-left (TL_m), top-right (TR_m), and left-right (LR_m) pins were calculated as shown in Table 2. Again, due to the symmetry of the model, the TL_m and TR_m values are the same.

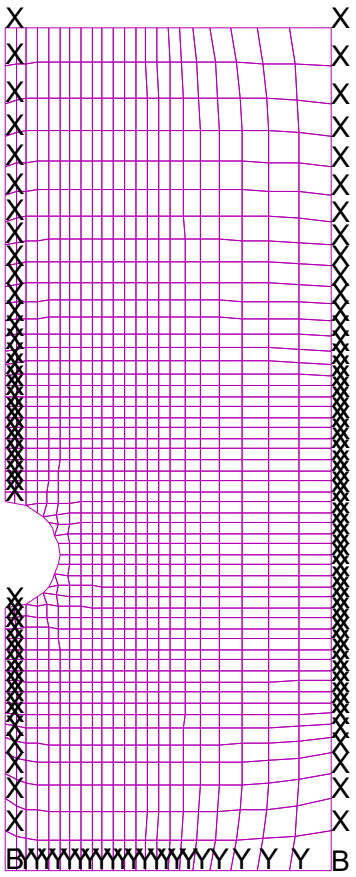


Figure 6. The geometry of verification models

Table 1. The values and range of parameters used in verification models [61]

		Case 1: Stiff Cohesive Soil	Case 2: Dense Granular Soil
Density - ρ (kg/m^3)	Value	2000	2000
	Range	1700 – 2200	1700 – 2200
Elastic Modulus - E (MPa)	Value	175	90
	Range	50 – 200	50 – 200
Poisson's Ratio - υ	Value	0.425	0.325
	Range	0.4 – 0.5	0.3 – 0.4
Cohesion - c (kPa)	Value	400	50
	Range	250 – 750	0 – 150
Friction Angle - φ (°)	Value	5	45
	Range	0 – 10	33 – 50
In-situ Horizontal to Vertical Stress Ratio - K_0	Value	0.91	0.36
	Range	0.2 – 1.0	0.2 – 1.0

Table 2. Synthetic monitoring data for verification models

	TL_m (cm)	TR_m (cm)	LR_m (cm)
Case 1: Stiff Cohesive Soil	4.6272	4.6272	5.6988
Case 2: Dense Granular Soil	10.116	10.116	5.3546

TL_m: Measured convergence along the Top-Left direction
TR_m: Measured convergence along the Top-Right direction
LR_m: Measured convergence along the Left-Right direction

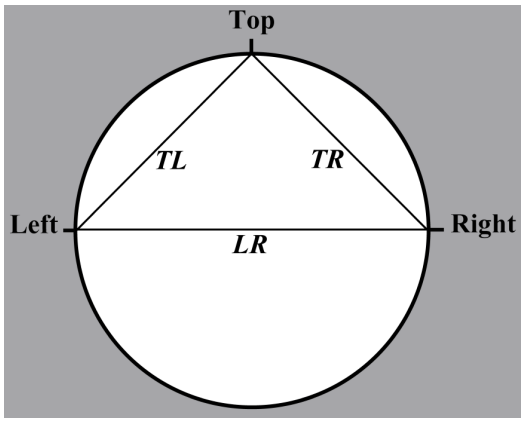


Figure7. Hypothetic instrumentation system for verification models

When solving back analysis problems, it is important to note that there is no unique answer. However, having a reasonable range for the unknown parameters of the analysis can be very helpful in narrowing down potential answers. These ranges can be determined by examining ground formations and checking the typical reported values in published textbooks and papers, conducting rock mechanical and physical laboratory tests, referring to rock mass classification systems, and reviewing similar project reports accomplished in the same formations. The reference for selecting geotechnical properties for dense granular and stiff soil materials [61] was consulted to determine the ranges of parameters shown in Table 1, which are used in the analyses to verify the developed method. The verification problems considered all six parameters in Table 1 as the unknown variables. In other words, the proposed method aims to estimate six unknown parameters: density (ρ), elastic modulus (E), Poisson's ratio (υ), cohesion (c), friction angle (φ), and in-situ horizontal to vertical stress ratio (K_0), using only the three measured values of TL_m, TR_m, and LR_m. The K_0 values in Table 1 were calculated using the following equation:

$$K_0 = 1 - \sin \varphi \quad (2)$$

Where are:

K_0 – in-situ horizontal to vertical stress ratio,

φ – friction angle.

However, the validity of the equation will be discussed based on the obtained results in the next paragraphs.

In the back analysis process of verification models, there are six unknown variables. Therefore, the genetic algorithm (GA) involves six-component vectors (n = 6) as chromosomes. According to Holland, for a problem with n variables, the recommended population number (m) is $2n$ – $4n$ (12 to 24 for n = 6) [62]. However, for highly non-linear objective functions like the ground response for tunnel excavation, a higher population number might be necessary. In this case, the population number considered is 40 (m = 40), and 40% of the best chromosomes with the smallest error values are transferred to the next generation as parents (SR = 0.4). The mutation rate is recommended to be in the range of 0.4 to 1.0 [63], and for the verification GA analyses, the

mutation rate is set at $MR = 0.4$. The stop condition is configured so that the algorithm will halt if the error value (**Equation 1**) remains unchanged for the last 200 iterations. This approach eliminates the need to set a predetermined error value at the start of a back analysis process, the magnitude of which is initially unknown, or a predefined number of iterations to reach an optimal result, which can vary significantly across different optimization problems. The commonly recommended optimal ranges for GA parameters reported in the established genetic algorithm literature were adopted, rather than conducting an additional parameter-tuning process. The objective was to assess whether these generally accepted values are adequate for the present back analysis problem and potentially transferable to similar real case studies, without introducing a separate and computationally demanding meta-optimization stage.

After feeding the prepared code with a range of unknown parameters, GA parameters, and input monitoring data (Figure 5), the code was run five times for each case. The estimated geotechnical parameters and relevant error values obtained were recorded and summarized in Table 3. The lowest obtained error values, along with the best obtained estimation of the unknown parameters, are denoted with ***bold-italic*** fonts and will be discussed in the next paragraphs. As shown in the first column of the table, the error values resulting from the GA optimization process are not only minimal, but they also have the same order across different models. This indicates that the stop condition used to halt the GA optimization procedure is effective in converging the models to very small errors with consistent amounts. The difference between the estimated values for geotechnical parameters of different models in both Case 1 and Case 2 demonstrates that the algorithm can produce different solutions for a single set of monitoring data, all with small error values. This guarantees that there is no one-to-one relationship between the estimated geotechnical parameters and the input monitoring data, as previously emphasized by [5].

Table 3. Summary of ground geotechnical parameters along with the relevant error values for Case 1 and Case 2 soil materials

	Model No.	Error	TL_c (cm)	LR_c (cm)	ρ (kg/m^3)	E (MPa)	υ	C (kPa)	φ (°)	K_0
Case 1: Stiff Cohesive Soil	1	1.679×10^{-6}	4.620	5.693	1968	194.4	0.4714	***383.4***	***4.063***	0.9942
	2	***1.370×10^{-6}***	4.626	5.689	***2007***	184.3	0.4530	367.0	6.371	0.8906
	3	2.311×10^{-6}	4.621	5.708	1976	***176.0***	0.4308	350.5	8.437	***0.9199***
	4	4.702×10^{-6}	4.614	5.693	2045	166.2	0.4026	502.2	1.398	0.9922
	5	2.703×10^{-5}	4.629	5.654	2052	177.1	***0.4325***	482.3	0.3839	0.9527
	Ave. Value:		4.622	5.6874	2009.6	179.6	0.4381	417.1	4.131	0.9499
	True Value:		4.6272	5.6988	2000	175	0.425	400	5	0.91
Case 2: Dense Granular Soil	1	9.788×10^{-6}	10.08	5.366	2130	86.28	0.3432	83.48	42.80	0.3925
	2	2.274×10^{-6}	10.13	5.350	***1996***	***89.18***	0.3894	63.29	42.33	0.3177
	3	***1.062×10^{-6}***	10.13	5.356	1970	78.74	***0.3236***	92.69	40.50	0.3869
	4	7.958×10^{-6}	10.10	5.322	1837	79.00	0.3324	***49.82***	***45.12***	***0.3631***
	5	1.990×10^{-4}	10.27	5.375	2087	98.29	0.3202	28.89	49.91	0.4152
	Ave. Value:		10.142	5.3538	2004	86.30	0.3418	63.63	44.13	0.3751
	True Value:		10.116	5.3546	2000	90	0.325	50	45	0.36

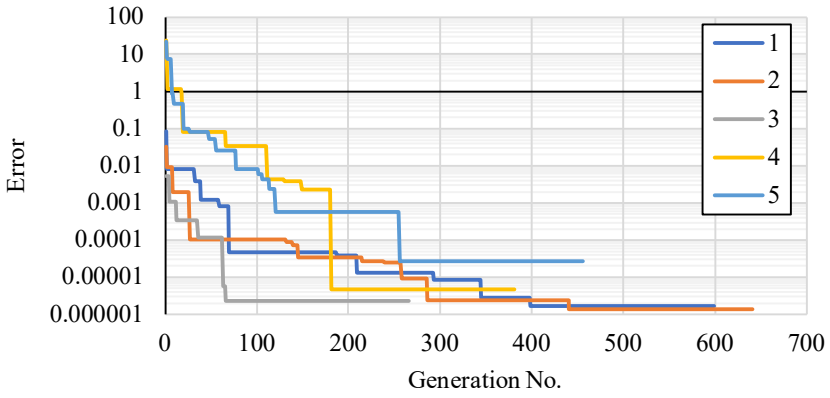
TL_c: Calculated convergence along the Top-Left direction
LR_c: Calculated convergence along the Left-Right direction
ρ: Density
E: Modulus of elasticity
υ: Poisson's ratio
c: Cohesion
φ: Friction angle
K_0: in-situ horizontal to vertical stress ratio

Figure 8 illustrates the evolution of error in models over the course of the genetic algorithm for both stiff cohesive soil and dense granular material. To have a better view of the error, the y-axis is displayed in logarithmic scale. In all models, the error value approaches nearly zero. However, because of the random nature of the first generation from which the genetic algorithm starts, and also the mutation process, each model follows a unique path, resulting in different geotechnical parameters. However, as Table 3 reveals, all models lead to very small error values between the synthetic monitoring data and the relevant calculated ones. In Figure 8, it is evident that the considered stop condition is sufficient to ensure that the model's error converges to a very small value. To better understand the difference between the synthetic tunnel convergence data and the output of the models obtained using the geotechnical parameters listed in Table 3, the measured synthetic convergencies (TL_m and LR_m in Table 2) are plotted alongside the calculated ones (TL_c and LR_c) in Figure 9. It is important to note that due to the symmetry of the models, the values

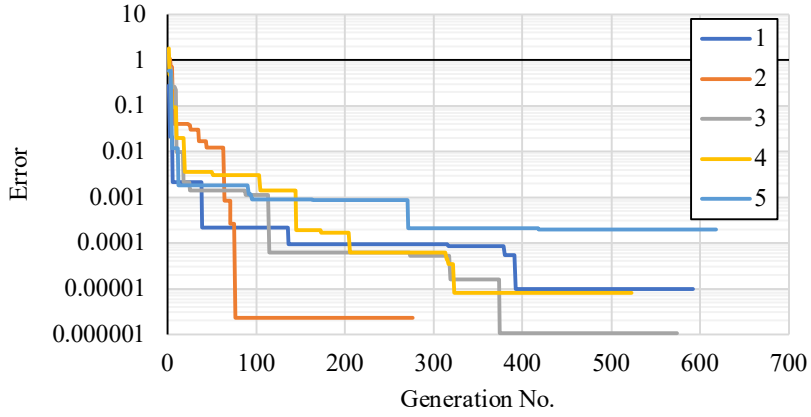
of TL_m and TR_m, as well as TL_c and TR_c, are the same. For clarity, only the graphs of model no. 3 in Case 1 and model no. 5 in Case 2 (i.e., the models with the largest error values for each case) are compared, and the difference in the rest of the models is even smaller due to their smaller error values. Figure 9 illustrates that the difference between the graphs produced by true geotechnical properties and those resulting from the estimated ones is negligible. While there are significant differences between some estimated and true values of geotechnical parameters for the mentioned models, the estimated sets can still yield small and acceptable error values, as evident in Table 3 and Figure 9.

In Figure 10, the relationships between the actual geotechnical parameters, the input range provided to the FDM–GA algorithm, and the estimated values for both Case 1 and Case 2 are plotted visually. The yellow diamond represents the **true** values, the black strip is the considered range of parameters, and the colored bands represent the estimated values by FDM–GA. Each model from Table 3 is shown in a unique color to demonstrate its efficiency in estimating the target value (e.g., a white band is used for model 1). Dashed lines are used when the estimated values are very close to each other. Upon comparing the true and estimated values on the initial parameter ranges, the outcome of the FDM–GA algorithm can be summarized as follows:

1. All estimated geotechnical parameter values fall within the predetermined ranges. This indicates that the proposed FDM–GA algorithm is successful in optimizing the error function (**Equation 1**) within constraints. While the results of the five models for each case do not directly provide the true values, they do significantly narrow down the initial range of geotechnical parameters. It is important to note that the small difference between the largest and smallest values, and the considered range for υ and φ (in Case 1), is primarily due to the relatively small range considered for these parameters.



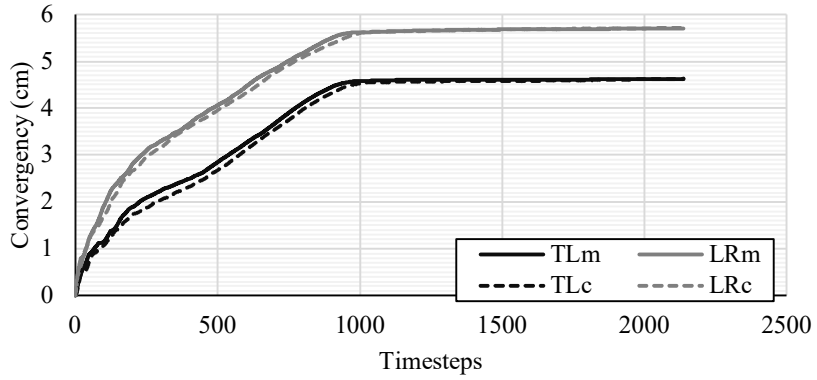
(a)



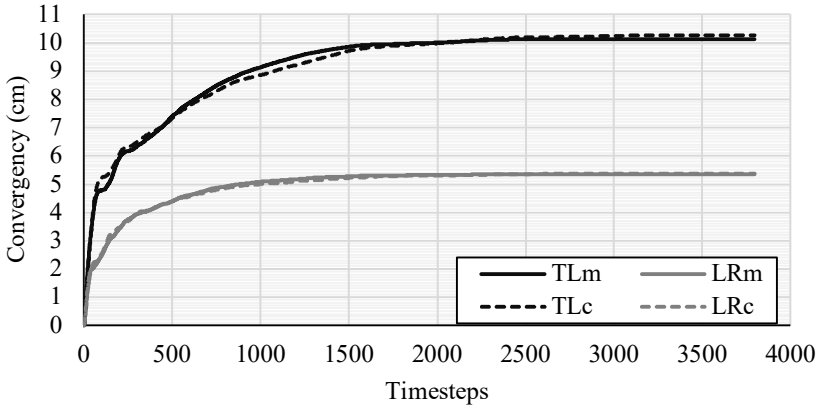
(b)

Figure 8. The evolution of error value during GA progress for the different models of (a) Case 1: stiff cohesive soil and (b) Case 2: dense granular soil

2. It is important to note that when *error* values converge to very low levels, as shown in Table 3, the lower *error* value does not necessarily mean that the model provides a better estimation of unknown parameters. Looking at Table 3, model no. 2 (*error* = 1.370×10^{-6}) and model no. 3 (*error* = 1.062×10^{-6}) resulted in the smallest *error* values for Case 1 and Case 2, respectively. However, model no. 2 was only successful in providing the best estimation of the ρ value in Case 1, while model no. 3 only gave the best estimation of the υ value in Case 2. Surprisingly, model no. 5, which has the highest error in Case 1, still provided the best estimation of the υ value. In Case 2, model no. 4 was able to provide the best estimations of c, φ, and K_0 values simultaneously. However, its error was higher than those of models no. 2 and 3. Therefore, it is clear that the outcome models of the back analysis process cannot be simply judged based on the associated error values. Evaluating which back-analysis result from the optimization outputs most accurately reflects actual ground behavior relies on prior knowledge. This includes experience from similar projects in the same geological formations and reference values from the literature. By using this background information, one can define a reasonable range for the unknown parameters. The most plausible back analysis result is the one that has a parameter set most consistent with these expected ranges. This process is further illustrated and discussed through the real case example in the next part.



(a)



(b)

Figure 9. The histories of synthetically measured and calculated convergencies between top-left (*TL*) and left-right (*LR*) pins for (a) Case 1: stiff cohesive soil and (b) Case 2: dense granular soil

3. It should be noted that the reported average values for *error*, *TL*, and *LR* are the average values of the relevant five optimized models. These values are not related to the model with the calculated average geotechnical properties. When a model with the calculated average properties in Table 3 is run, the following results are obtained for Case 1: error = 3.054×10^{-4}, *TL* = 4.573 cm, and *LR* = 5.568 cm. For Case 2, the averaged parameters yield the following results: *error* = 1.032×10^{-3}, *TL* = 9.780 cm, and *LR* = 5.234 cm. A comparison of the error and convergence values calculated for the five optimized models reveals a significant increase in errors. Referring to Table 3, these

averaged geotechnical property values provide the best-estimated collection of all parameters. This further demonstrates that the lowest error in the obtained optimized model does not necessarily indicate the best match of the estimated values with the true ones.

4. The plots obtained for both cases reveal that the true values (yellow diamonds in Figure 10) fall within the colored bands. This indicates that the true geotechnical parameters fall somewhere between the smallest and largest estimated values. Consequently, the possible range of input geotechnical properties is limited, which in turn restricts the likely mechanisms to occur under real conditions. The set of obtained geotechnical properties that minimizes the error function can be considered the "General Solution" for the back analysis problem.

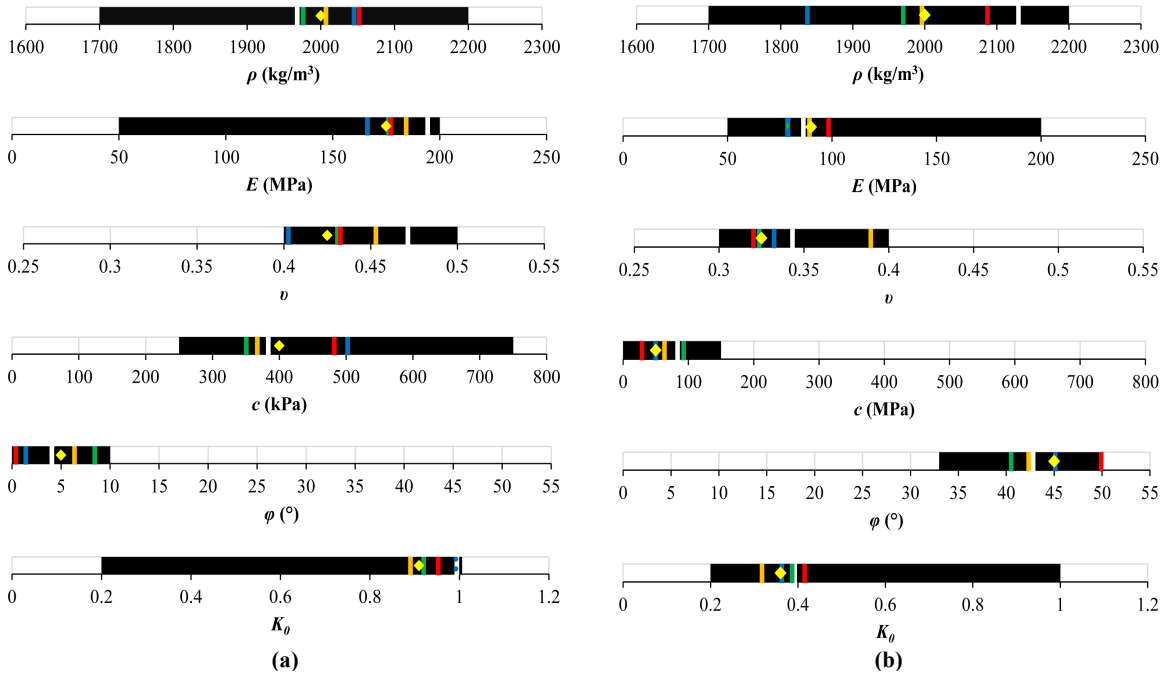


Figure 10. The true values (yellow diamonds), considered ranges (black strips), and estimated values (colored bands) for different geotechnical properties of (a) Case 1: stiff cohesive soil and (b) Case 2: dense granular soil

5. After analyzing all geotechnical parameters in both cases, it seems that there is no significant correlation between the estimated values and the actual ones. While some estimates are very close to the true values for each parameter, the varying colors of the closest bands to the diamonds indicate that no single model is capable of accurately estimating all geotechnical parameters. However, it is worth noting that the true values are spread throughout the estimated values of all models, even within the five restricted calculated models.

6. In some previous studies, the values of ρ and υ were assumed to be constant during back analysis efforts [15, 45]. However, the results indicate that these values also vary among different models. Given that the change in these parameters could potentially influence other geotechnical properties in the final optimized models, it is equally important to treat ρ and υ as unknown parameters in the back analysis process, similar to other geotechnical properties.

In the back analysis problem, obtaining the "General Solution" using various algorithms, such as the proposed FDM–GA, is possible. However, finding the "Specific Solution" – a unique set of geotechnical properties that exactly match the actual values – can be quite challenging. If the "General Solution" is obtained correctly for a practical back analysis problem, the "Specific Solution" will lie within the estimated values of different parameters. Yet, extracting the correct values requires additional information, similar to solving a differential equation where obtaining a general solution is well-known, but finding the Specific solution requires extra information. For a back analysis problem, the additional information to extract the "Specific Solution" from unlimited "General Solutions" comes from the a priori knowledge of prevailing geological conditions, the geotechnical nature of the ground formations, and performing in-situ and laboratory tests, etc. Due to a lack of such information for the considered

verification problems, a simple averaging method was tried here to obtain the "Specific Solution" or the true values. It should be noted that this method was used only to test if reasonable results could be obtained for cases 1 and 2, and does not make technical sense. The average values were calculated and reported in Table 3. The calculated averages closely align with the true values, and at least in these verification problems, simple averaging can provide acceptable results for the "Specific Solution". However, it is unlikely that such a simple approach can be used in real complicated problems.

4. Exploring the Correlations between the Back Analyzed Values

As discussed in the previous section, any optimization algorithm aimed at minimizing the discrepancy between measured instrumentation data and corresponding model-calculated values may yield a set of parameter combinations that constitute the "General Solution" to a back analysis problem. However, identifying the true "Specific Solution" can be particularly challenging when the number of unknown parameters to be determined is large. In such cases, exploring intercorrelations among the parameters within the General Solution set can be valuable. If meaningful correlations are identified, the dimensionality of the problem can be effectively reduced, thereby narrowing the range of unknowns and facilitating the determination of the "Specific Solution" with less reliance on a priori knowledge of geological conditions or extensive in-situ and laboratory testing. The presence of such correlations is plausible, considering that all parameter sets within the "General Solution" reproduce the behavior of the same structure under identical geotechnical conditions. Consequently, any variation in a given parameter across different solutions must be offset by compensatory changes in other parameters to maintain a comparable level of accuracy in minimizing the error value.

In the presented verification study, two soil types are considered: stiff cohesive soil (Case 1) and dense granular soil (Case 2). Each case involves six unknown geotechnical parameters: density (ρ), elastic modulus (E), Poisson's ratio (υ), cohesion (c), internal friction angle (φ), and the coefficient of earth pressure at rest (K_0). For both cases, five parameter sets yielding minimal error values were identified using the proposed FDM–GA algorithm. Correlations between pairs of these parameters were analyzed, and the corresponding coefficients of correlation (R^2) are summarized in Table 4. Direct and inverse correlations are denoted as (dir) and (inv), respectively. The upper and lower entries in each cell of the table represent R^2 values for Cases 1 and 2, respectively. In Case 1, notable correlations include $\rho - c$, $\rho - \varphi$, $E - \upsilon$, and $c - \varphi$. The direct correlation between ρ and c is expected, as higher densities typically correspond to increased cohesion in cohesive soils. However, the inverse $\rho - \varphi$ correlation appears counterintuitive, especially given the limited variation in φ values for stiff cohesive soils, as observed in Tables 1 and 3. The strong direct correlation between E and υ (R^2 = 0.9888) is noteworthy and can be attributed to the fact that both are elastic parameters. Additionally, the inverse $c - \varphi$ correlation is reasonable from a mechanical standpoint: in Mohr-Coulomb materials, maintaining constant shear strength often requires a trade-off between cohesion and friction angle. For Case 2 (dense granular soil), significant correlations were observed between $E - \varphi$, $\upsilon - K_0$, and $c - \varphi$. These differ from those identified in Case 1, highlighting that parameter correlations are influenced by the specific structural and geotechnical context of the back analysis problem and cannot be generalized. While the physical interpretation of the $E - \varphi$ and $\upsilon - K_0$ correlations is not straightforward, the recurrence of a strong $c - \varphi$ correlation in both cases with the same R^2 values is consistent with the Mohr-Coulomb failure criterion, which governs shear strength in both soil types.

While combining the data from both cohesive (Case 1) and granular (Case 2) soils may lead to improved correlation coefficients compared to those reported separately in Table 4, such an approach may not be physically meaningful due to the fundamentally different mechanical behaviors of these soil types. However, for certain parameter pairs, specifically $c - \varphi$ and $K_0 - \varphi$, the existence of general theoretical relationships justifies the consideration of all data points across both cases when investigating potential correlations. In particular, when shear strength is governed by the Mohr-Coulomb failure criterion, the relationship applies uniformly to both cohesive and granular soils and is expressed as:

$$\tau = c + \sigma_n \tan \varphi \quad (3)$$

Where are:

τ – shear stress along the failure plane,
c – cohesion,
σ_n – normal stress on the failure plane,
φ – friction angle.

This implies that, in order to achieve a consistent mechanical response, such as the deformations observed around the perimeter of a circular tunnel, an increase in cohesion (c) must be offset by a corresponding decrease in friction angle (φ), and vice versa. This inverse relationship is supported by the high coefficients of correlation ($R^2 = 0.8650$ for Case 1 and $R^2 = 0.8625$ for Case 2), confirming the physical rationale. When the $c - \varphi$ data from both cases are combined, the correlation improves further, reaching an R^2 value of 0.9753, as indicated by the third entry in Table 4. Figure 11a presents the separately obtained correlations for each case, along with the trendline fitted to the combined dataset. As shown, the generalized trendline aligns well with the data points from both soil types, suggesting a robust relationship.

Table 4. The obtained correlation coefficients between the different pairs of back analysis parameters

	E (MPa)	υ	c (kPa)	φ (°)	K_0
ρ (kg/m^3)	0.4542 (inv) 0.4393 (dir)	0.4512 (inv) 0.0002 (-)	0.7869 (dir) 0.0064 (dir)	0.6098 (inv) 0.0297 (dir)	0.0181 (dir) 0.2029 (dir)
E (MPa)		0.9888 (dir) 0.0086 (dir)	0.3754 (inv) 0.4164 (inv)	0.0807 (dir) 0.5157 (dir)	0.0022 (inv) 0.0682 (dir)
υ			0.4342 (inv) 0.0158 (dir)	0.1148 (dir) 0.1461 (inv)	0.0228 (inv) 0.7806 (inv)
c (kPa)				0.8650 (inv) 0.8625 (inv) 0.9753 (inv)	0.3292 (dir) 0.0187 (inv)
φ (°)					0.4151 (inv) 0.2293 (dir) 0.9648 (inv)

ρ: Density
E: Modulus of elasticity
υ: Poisson's ratio
c: Cohesion
φ: Friction angle
K_0: in-situ horizontal to vertical stress ratio

Another generalized trend can be explored using **Equation (2)**, which is commonly employed to estimate the at-rest earth pressure coefficient (K_0) in soil deposits. Individually, the $K_0 - \varphi$ correlations in the two cases yield modest R^2 values. Furthermore, while Case 1 demonstrates an inverse correlation, consistent with the theoretical expectation from **Equation (2)**, the data from Case 2 exhibit a direct correlation, which contradicts the expected behavior. However, fitting a trendline to the combined dataset yields an inverse relationship with an excellent R^2 value of 0.9648. Figure 11b illustrates both the individual trendlines and the overall fitted line for the combined data. It is worth noting that Figure 11b plots K_0 against $\sin\varphi$, which introduces slight discrepancies in R^2 values compared to those reported in Table 4. The resulting generalized trendline is described by the following equation:

$$K_0 = 1.0931 - 1.071 \sin\varphi \quad (4)$$

Where are:

K_0 – in-situ horizontal to vertical stress ratio,

φ – friction angle.

The derived equation closely aligns with **Equation (2)**, further validating the correlation. In conclusion, since the calibrated values obtained through back analysis reflect the behavior of the same structure under identical geotechnical conditions, the emergence of intercorrelations among these parameters is both reasonable and expected. Identifying such correlations can significantly reduce the reliance on supplementary geotechnical information when deriving the final "Specific Solution" in a back analysis problem. For instance, in the presented verification cases, if independent geotechnical investigations provide a reliable estimate of the friction angle (φ), the corresponding values for cohesion (c) and the earth pressure coefficient (K_0) can be inferred from the established correlations with acceptable accuracy. Nevertheless, it is important to emphasize that these correlations are project-specific and cannot be generalized across different sites, owing to variations in structural configurations and geotechnical conditions.

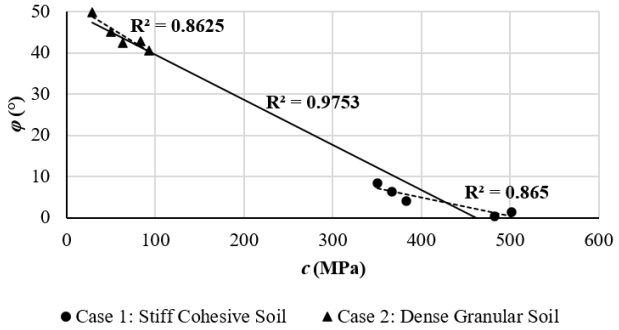
5. Conclusions

This study presented and verified a fully integrated FDM–GA framework for automated direct back analysis in tunneling. The principal

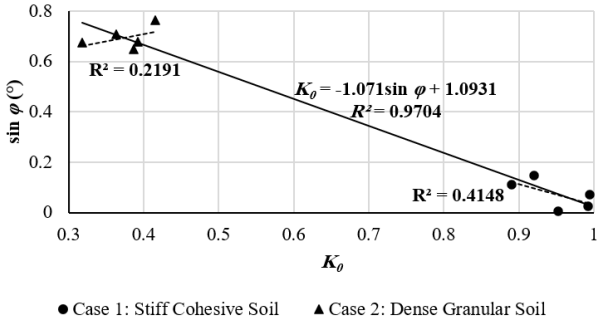
findings and contributions can be summarized as follows:

- The genetic algorithm and numerical modeling procedures are implemented entirely within the FLAC platform using FISH. This eliminates the need for external optimization software, avoids data-transfer loops, and significantly reduces operator intervention compared to conventional AI-based back analysis workflows.
- The proposed framework introduces a convergence-based stopping condition (unchanged minimum error over successive generations), removing the requirement for arbitrarily predefined iteration limits or target error thresholds. This enhances reliability and practical applicability under real monitoring uncertainty.
- The estimated geotechnical properties demonstrate stable convergence and satisfactory accuracy, indicating that the selected GA parameters (SR = 0.4 and MR = 0.4) are appropriate for this class of GA-based tunnel back-analysis problems.
- All optimized parameters remain within predefined physically meaningful bounds throughout the optimization process. This avoids post-processing corrections required in some previous GA-based approaches and improves numerical stability.
- A systematic procedure based on step-limited equilibrium calculations enables automatic detection and penalization of yielding or non-convergent models, ensuring uninterrupted GA progression.
- Multiple independent runs produce different parameter sets with similarly small error values (10^-6–10^-5 range), confirming the absence of a one-to-one mapping between monitoring data and geotechnical parameters. The framework therefore identifies a general solution domain rather than a single deterministic answer, which is consistent with the theoretical nature of inverse geomechanical problems.
- Successful estimation of six unknown parameters using only three convergence measurements demonstrates the robustness of the optimization scheme, even under sparse instrumentation conditions.

Overall, the developed FDM–GA methodology provides a computationally efficient, automation-oriented, and industry-applicable solution for tunnel back analysis. Part I establishes the methodological basis required for practical application. Part II will apply the FDM–GA method to the Pardis Tunnel and investigate the influence of stress relaxation ratio and stress relaxation rate on the back analysis results.



(a)



(b)

Figure 11. (a) Cohesion (c) – friction angle (φ) and (b) earth pressure coefficient (K_0) – sin φ correlations for separate data points of Case 1 and Case 2 along with the fitted trendlines for all data points

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چارچوب یکپارچه FDM–GA برای تحلیل برگشتی خودکار در تونل‌سازی – بخش اول: توصیف روش، پیاده‌سازی و اعتبارسنجی

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کلمات کلیدی

آنالیز برگشتی
روش تفاضل محدود (FDM)
الگوریتم ژنتیک (GA)
زبان برنامه‌نویسی FISH
مدل‌سازی عددی

چکیده

تحلیل برگشتی پروژه‌های حفاری تونل، نقشی اساسی در کالیبراسیون پارامترهای ژئومکانیکی، بر اساس داده‌های ابزاربندی ایفا می‌کند. روش‌های متداول تحلیل برگشتی مستقیم، همچنان از نظر محاسباتی پرهزینه بوده و به‌شدت به نظارت اپراتور وابسته‌اند. در این مطالعه، یک چارچوب یکپارچه مبتنی بر روش تفاضل محدود–الگوریتم ژنتیک (FDM–GA) برای تحلیل برگشتی خودکار تونل‌ها ارائه شده است؛ که به‌طور کامل در محیط FLAC و با استفاده از زبان برنامه‌نویسی داخلی FISH، پیاده‌سازی شده است. روی‌کرد پیشنهادی، نیاز به استفاده از نرم‌افزارهای بهینه‌سازی جانبی و انتقال داده میان بسترهای عددی و هوش مصنوعی را حذف می‌کند. در این چارچوب، یک الگوریتم ژنتیک ساده‌شده، به‌طور مستقیم، با شبیه‌سازی‌های تفاضل محدود کوپل شده است تا به‌صورت تکرارشونده، اختلاف میان همگرایی‌های اندازه‌گیری‌شده و همگرایی‌های محاسبه‌شده تونل را حداقل کند. این چارچوب، شامل بهینه‌سازی پارامترها با قیود مشخص، مدیریت خودکار مدل‌های غیرهمگرا، و یک معیار توقف مبتنی بر همگرایی پایدار است، که از به‌کارگیری آستانه‌های خطای از پیش تعریف‌شده، اجتناب می‌کند. اعتبارسنجی این روش، با استفاده از دو مدل مصنوعی تونل شبیه‌سازی شده در حالت کرنش صفحه‌ای، در دو نوع مصالح خاک چسبنده سخت و خاک دانه‌ای متراکم، انجام شد. شش پارامتر مجهول (ρ، E، ν، c، φ و K_0) تنها با استفاده از سه اندازه‌گیری همگرایی، تخمین زده شدند. نتایج حاصل از چندین اجرای مستقل، همگرایی پایدار به مقادیر خطای بسیار کوچک (در حدود 10^{-6} تا 10^{-5}) و بازتولید دقیق داده‌های ابزاربندی مصنوعی را نشان دادند. این روش، با موفقیت، بازه‌های پیشنهادی اولیه گسترده برای پارامترها را محدود کرده و مجموعه‌های متعددی از پارامترهای ورودی قابل قبول ارائه می‌دهد که به‌طور صریح، با غیر یکتایی ذاتی مسائل تحلیل برگشتی، انطباق دارد. چارچوب توسعه‌یافته FDM–GA، ابزاری کارآمد و انعطاف‌پذیر برای کاربردهای عملی تحلیل برگشتی، در تونل‌سازی فراهم می‌آورد.

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