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A Fractal Perspective on Rock Physicomechanical Variations Using the Box-Counting Technique: Experimental Evidence and Statistical Analysis

Sasan Ghorbani*

Department of Mining Engineering, Hamedan University of Technology, Hamedan, Iran

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Abstract

Understanding the link between rock microstructure and macroscopic physicomechanical behavior remains a central challenge in rock mechanics due to the intrinsic heterogeneity and discontinuity of geomaterials. This study presents an experimental investigation into the relationships between fractal dimension, determined using the box-counting method on thin-section microphotographs, and key physicomechanical properties of rocks through statistical analysis. A total of 40 rock samples representing a wide range of lithologies were collected from different mines in Iran. Standardized laboratory tests were conducted to determine Brazilian tensile strength, Schmidt rebound hardness, density, and porosity. High-resolution microphotographs were obtained from polished thin sections under controlled optical conditions and processed to extract fractal dimensions using ImageJ software. Regression models were then developed to examine the relationships between fractal metrics and measured properties. The results reveal clear and statistically significant relationships between fractal dimension and all examined properties. Among the investigated parameters, porosity exhibited the strongest correlation with fractal dimension, indicating that fractal geometry is particularly sensitive to the spatial distribution and configuration of void spaces within the rock fabric. Brazilian tensile strength, Schmidt rebound hardness, and density also exhibited systematic relationships with fractal dimension, confirming that microstructural irregularity plays a decisive role in governing both physical and mechanical behavior. This study shows that thin-section fractal analysis is a practical and cost-effective method for quantifying rock heterogeneity and relating petrographic features to engineering properties. Coupling fractal geometry with regression modelling provides a reliable framework for indirectly predicting physicomechanical parameters and linking microstructural characteristics to macroscopic rock behavior.

1. Introduction

Rock masses are inherently heterogeneous and discontinuous geomaterials whose mechanical behavior is governed by complex interactions among mineral composition, grain morphology, microcrack networks, and pore structures across multiple length scales [1–6]. These intrinsic characteristics exert a decisive influence on the physicomechanical properties of rocks, including compressive and tensile strength, elastic modulus, wave velocity, and failure mode. Accurate characterization of such properties is fundamental to the safe and economical design of rock engineering systems such as underground excavations, slope stability, tunneling, and energy resource development [7]. However, the pronounced variability of rock behavior, even within nominally similar lithologies, continues to pose a significant challenge to both rock mechanics research and engineering practice.

Traditional approaches to rock characterization rely primarily on laboratory-based physical and mechanical testing combined with empirical or

Corresponding author: s.ghorbani@hut.ac.ir (S. Ghorbani)

semi-empirical correlations. While these methods provide valuable macroscopic parameters, they often fail to adequately reflect the influence of microstructural complexity on mechanical response. In particular, classical descriptors such as porosity, density, or average grain size are insufficient to capture the irregular, non-Euclidean geometry of rock textures and fracture patterns. Consequently, there remains a disconnect between observable microstructural features and their quantitative contribution to rock deformability and strength, especially under complex loading conditions [8].

Fractal geometry offers a mathematically rigorous framework for describing such scale-invariant behavior, enabling the quantification of structural complexity beyond the limitations of conventional geometric measures. The fractal dimension, as a key parameter in this framework, provides an effective indicator of spatial irregularity, crack density, and surface roughness, which are directly linked to stress concentration, crack initiation, and propagation mechanisms in rock materials.

Among the various techniques proposed for fractal analysis, the box-counting method has been widely adopted in rock mechanics due to its simplicity, numerical stability, and compatibility with digital image processing. By analysing thin-section images, fracture maps, or failure surfaces, the box-counting method allows the extraction of fractal dimensions that reflect the geometric organization of microstructural features [9]. Recent studies have demonstrated the sensitivity of fractal dimension to lithological variations, damage evolution, and loading-induced microcrack growth [7,10]. Nevertheless, the translation of these geometric descriptors into reliable predictors of physicomechanical properties remains an open research problem.

Previous studies have increasingly employed fractal geometry as a quantitative tool to describe the inherent heterogeneity and multiscale complexity of geomaterials. Shi and Xiao [11] conducted a comprehensive study on the effect of fractal dimensions on UCS of rocks using the digital texture model, the Voronoi polygon model, and the Weibull distribution model. Their results revealed that an increase in the fractal dimension of the rocks corresponded to a marked enhancement in their UCS. Importantly, the magnitude of this strength improvement was significantly more pronounced in the three-dimensional models compared to the two-dimensional ones, highlighting the critical role of spatial complexity in governing rock mechanical behavior. Huang et al. [12] conducted a systematic investigation into the interrelationships between uniaxial compressive strength and Young's modulus of mudstone, sandstone, limestone, and basalt with the fractal dimensions extracted from scanning electron microscopy (SEM) images, alongside the evaluation of rock homogeneity. Their results demonstrated the pronounced inverse correlations between the fractal dimension and both the compressive strength and Young's modulus. Additionally, an enhancement in rock homogeneity was found to correspond with an increase in fractal dimension, indicating a more uniform distribution of microstructural features. Tan et al. [13] conducted a comprehensive study on sandstone and diabase to elucidate the relationship between rock brittleness and fractal dimension, aiming to advance the understanding of failure mechanisms and mechanical behaviors of rocks. Their results demonstrated a clear inverse correlation; whereby higher macroscopic brittleness is associated with a lower fractal dimension at the microscopic scale. The concept of fractal dimension has also been discussed in the field of drilling. In this regard, Deng et al. [14] investigated the relationship between drilling time and fractal dimension of rocks. Their results showed that as the fractal dimension increases, the drilling time in rock also increases linearly. Wang et al. [15] investigated the relationship between fractal dimension and peak strength and elastic modulus of shale samples. Their results clearly showed that peak strength and elastic modulus could be predicted by linear and power relationships based on fractal dimension, respectively. They also showed that a higher fractal dimension indicated more damage to samples under uniaxial compressive loading. In a study, Gori and Mari [16] investigated the relationship between fractal dimension and internal friction angle of various granular materials. Their results indicated a direct linear relationship between them. Huang et al. [17] examined the relationships between fractal dimension with UCS and modulus of elasticity of sandstone and limestone. They showed that there are linear and power relationships between fractal dimension with UCS and modulus of elasticity, respectively. Also, Gupta and Sharma [18] analysed the correlations between fractal dimension and UCS, P-wave, and S-wave velocity of quartzites. They observed that as the fractal dimension increased, the P-wave velocity and S-wave velocity decreased and the UCS increased. Pola et al. [19] demonstrated that with increasing the fractal dimension of volcanic rocks,

the UCS and tensile strength increased. Zhang et al. [20] depicted the reasonable relationships between fractal dimensions with uniaxial compressive strength, elastic modulus, P-wave velocity, and resistivity of sandstone. Zou et al. [21] presented a clear logarithmic relationship between the porosity and the fractal dimension of the pores of coal samples with a high value of R^2 (0.90).

Recently, several studies have also investigated the relationships between the engineering properties of rocks and their fractal dimensions using various machine learning (ML) algorithms. Ma et al. [22] classified the tunnel surrounding rock using fractal and machine learning methods integrating Bayesian optimization (BO) with random forest (RF), gradient boosting decision tree (GBDT), support vector machine (SVM), and adaptive boosting (AdaBoost) algorithms. Also, Lu and Liu [23] assessed the microscopic pore structure of the sandstones based on nuclear magnetic resonance (NMR) and mercury injection capillary pressure (MICP) data, combined with fractal theory and the self-organizing map (SOM) neural network algorithm. Zhong et al. [24] successfully proposed a crack fractal dimension prediction model based on the random forest algorithm. Wu et al. [25] investigated the effect of pore characteristics and fractal dimension of limestone on permeability. Their results show that the fractal dimension is strongly correlated with permeability.

Existing investigations into the relationships between fractal characteristics and rock mechanical behavior have largely focused on isolated parameters or specific rock types, often yielding inconsistent or non-generalizable results. Moreover, many studies lack a systematic statistical framework capable of distinguishing meaningful correlations from coincidental trends. The absence of comprehensive regression-based models limits the practical applicability of fractal analysis in engineering design, where quantitative prediction and uncertainty assessment are essential. In particular, the combined influence of fractal dimension on multiple physicomechanical properties has not been sufficiently addressed through integrated experimental and analytical approaches.

The present experimental investigation aims to systematically evaluate variations in key physicomechanical properties of rocks including Brazilian tensile strength, porosity, density, and Schmidt rebound hardness as a function of their fractal dimension obtained using the box-counting method. High-resolution imaging techniques are employed to capture microstructural features, while standardized laboratory tests are conducted to determine physical and mechanical parameters. Regression analysis is then applied to establish quantitative relationships between fractal dimensions and measured rock properties, enabling an assessment of the predictive capability of fractal descriptors. By bridging microstructural geometry and macroscopic mechanical behavior, this study seeks to advance the understanding of structure-property relationships in rock materials and to provide a robust analytical basis for incorporating fractal concepts into rock mechanics and engineering applications. It should be noted that this study represents one of the first systematic attempts to quantify fractal dimensions from thin-section microphotographs and relate them to rock physicomechanical properties. While previous studies have explored the relationships between rock properties and fractal dimensions, often relying on Scanning Electron Microscopy (SEM) images, there remains a gap in systematically evaluating these correlations using fractal dimensions derived from microphotographs of rock thin sections. This study addresses this gap by uniquely employing high-resolution microphotographs and the box-counting method to quantify fractal dimensions, subsequently linking them to key physicomechanical properties. By establishing these quantitative relationships, this research aims to provide a more accessible and potentially more representative microstructural analysis for rock mechanics applications, complementing existing methodologies.

2. Materials and methods

2.1. Sample collection and preparation

In this study, a total of 40 rock samples were collected from different mining sites across Iran, representing a wide range of lithologies. The rock types included granite, limestone, travertine, marble, granodiorite, quartz monzonite, tuff, rhyolite, syenogranite, andesite, diorite, muscovite granite, and mylonite granite.

The sample selection strategy aimed to maximize variability in both microstructural and physicomechanical characteristics rather than to focus on a single rock type. Considering that the study investigates relationships between one independent variable (fractal dimension) and several response variables through regression analysis, the adopted sample size was considered adequate for identifying statistically meaningful

trends and evaluating the predictive capability of the proposed relationships.

After collection, all samples were carefully transported to the laboratory to prevent mechanical damage. Specimens were then prepared in accordance with the standards recommended by the International Society for Rock Mechanics (ISRM). Cylindrical and/or prismatic specimens were produced with precise control over geometry, surface flatness, and parallelism to ensure the reliability and reproducibility of rock mechanical testing.

In the next step, for each rock sample, representative thin sections were prepared following standard petrographic procedures. Thin sections were cut, ground, and polished to a standard thickness of approximately 30 μm. These sections were used to obtain high-quality microphotographs using a polarization microscope with crossed polarized light (XPL), enabling detailed observation of mineralogical composition, grain size distribution, grain boundaries, and microstructural features.

The acquired microphotographs served as the primary input data for fractal dimension analysis. Image acquisition conditions, including magnification, illumination, and resolution, were kept consistent across all samples to ensure comparability of the results. The microstructural images were subsequently processed and analysed to quantify the fractal characteristics of the rock textures.

2.2. Physicomechanical tests

In this study, four key physical and mechanical properties of rocks, including Brazilian tensile strength (BTS), Schmidt rebound hardness (SRH), density, and porosity were measured. Rock hardness was determined using the Schmidt rebound hardness test in accordance with the recommendations of the International Society for Rock Mechanics [26]. An L-type Schmidt hammer with an impact energy of 0.735 Nm was employed throughout the testing program. All tests were performed on cylindrical core specimens with a diameter of 54 mm to ensure uniform testing conditions. For each specimen, 20 rebound measurements were obtained from independent single impacts, with a minimum spacing of one plunger diameter between adjacent impact points. To minimize the influence of surface heterogeneities and outlier values, the rebound number for each sample was calculated as the mean of the ten highest recorded readings.

The tensile strength of the rock samples was determined using the Brazilian tensile strength test in accordance with ISRM suggested methods [27]. For this purpose, at least three core specimens with a diameter of 54 mm and a thickness of 27 mm were prepared and tested for each rock type.

In addition to mechanical testing, the physical properties of the rock samples were evaluated. Density and porosity were measured following ISRM-recommended procedures [27]. The inclusion of these physical parameters provides a more comprehensive characterization of the rock materials and supports a more robust interpretation of their mechanical behavior. The average values of the physicomechanical parameters conducted on the studied samples are presented in Table 1.

2.3. Fractal analysis of microphotographs using the box-counting method

The box-counting technique has been widely employed to quantify the fractal dimension (D_b) of pore surface irregularities and fracture geometries, owing to its methodological simplicity and its suitability for analysing both self-similar and non-self-similar structures. Using this approach, the fractal dimension is determined based on the scaling relationship between the number of covering boxes and the box size [1,28,29]. The fractal dimension can be derived from Eq. (1).

$$\mathrm{Log}(N(\delta)) = D_b \log(\delta) + \alpha \quad (1)$$

Where are:

$N(\delta)$ - the number of contour boxes,

δ the box sizes,

α a proportionality constant.

The FracLac module of the ImageJ software was used to measure the fractal dimension (D_b) of each sample. Figure 1 illustrates the workflow adopted for estimating fractal dimensions using the box-counting method. For each rock specimen, four microphotographs were acquired from different representative regions. These images were subsequently converted into binary format by applying an appropriate grayscale threshold using the "*Fractal Box Counter*" module in ImageJ software. The fractal dimension corresponding to each processed image was denoted as D_i [29,30]. The step-by-step process for calculating the fractal dimension of each rock sample is schematically shown in Figure 2.

To enhance the robustness of the analysis, each binary image was subjected to 12 box-counting

scans using the FracLac plugin, incorporating different scanning orientations and box sizes. The representative fractal dimension for each image (D_i) was calculated as the average value derived from its corresponding scans (Eq. (2)). Finally, the fractal dimension of each rock sample (D_b) was determined by averaging the fractal dimensions of the four images (Eq. (3)). The calculated fractal dimensions for all 40 rock samples are summarized in Figure 3. Also, the methodological workflow of this research is shown in Figure 4.

$$D_i = \frac{\sum_{j=1}^{j=12} D_j}{12} \quad (2)$$

$$D_b = \frac{\sum_{i=1}^{i=4} D_i}{4} \quad (3)$$

Table 1. The results of the physicomechanical tests of the studied samples

Sample No.	Rock name	Parameter			
		BTS (MPa)	SRH	Porosity (%)	Density (kN/m^3)
1	Limestone	5.1±0.35	40±5	4.76±0.22	23.5±1.2
2	Limestone	7.1±0.42	52±3	4.22±0.12	24.7±2.1
3	Limestone	6.9±0.28	50±7	5.89±0.3	24.3±3
4	Limestone	5.3±0.54	39±6	4.93±0.32	25.1±3.1
5	Limestone	6±0.9	35±2	5.36±0.41	23.7±3.2
6	Limestone	5.8±0.53	39±1	2.48±0.11	24.2±2.4
7	Limestone	5.2±0.67	39±3	6.08±0.36	23.9±2.1
8	Limestone	7.9±0.33	51±5	3.5±0.40	25.4±2
9	Travertine	5.4±0.87	39±4	6.88±0.41	24.3±1.8
10	Limestone	8.3±1.12	46±4	3.55±0.6	25.4±1.9
11	Marble	7.2±0.91	40±3	5.48±0.51	23.4±1.3
12	Limestone	5.5±0.70	41±1	7.02±0.44	24.5±3
13	Marble	3.9±0.54	42±3	6.07±0.6	25.5±2.5
14	Marble	6±0.56	39±4	7.9±0.23	23.8±2.7
15	Travertine	4.7±0.28	46±3	4.62±0.31	23.4±1.5
16	Travertine	10.2±1.58	57±2	2.1±0.07	25.3±1.6
17	Travertine	4.5±0.73	43±1	6.3±0.44	22.8±1.9
18	Granodiorite	11.2±2.08	58±3	0.55±0.04	26.4±2.1
19	Quartz monzonite	11.9±2	58±3	1.38±0.23	26.1±2.4
20	Tuff	9.8±1.75	59±2	3.56±0.12	25.4±2.7
21	Rhyolite	12.8±1.34	57±3	2.4±0.1	27.4±2.9
22	Tuff	7.9±0.89	49±3	4.64±0.25	24.2±1.7
23	Granite	12.8±2.13	63±4	1.11±0.20	27.4±2
24	Granite	13.3±2.5	63±3	1.36±0.05	26±1.3
25	Granite	12.6±1.78	65±5	1.44±0.1	26.4±1.7
26	Granite	14±1.9	77±5	0.51±0.06	28.6±3.1
27	Granite	11.9±1.85	65±4	1.01±0.03	26.7±3.5
28	Syenogranite	12.3±1.47	65±5	1.08±0.2	26.2±4.1
29	Granite	8.5±1	60±4	1.47±0.3	26.3±4.3
30	Granite	11.7±2.63	72±6	1.62±0.07	25.7±3.5
31	Andesite	8.9±1.54	60±3	1.99±0.1	26±4.6
32	Granite	10.4±2	57±2	1.08±0.1	26.2±3.2
33	Monzonite	10.5±0.87	61±4	1.22±0.24	25.7±4
34	Granite	9.6±1	59±6	1.62±0.19	27.1±2.4
35	Diorite	11±0.5	57±3	0.88±0.08	27.5±5.1
36	Muscovite granite	12.5±1.47	55±5	1.48±0.2	26.2±3.3
37	Granite	11.7±2.3	72±2	0.98±0.09	28.6±3.7
38	Mylonite granite	13.7±1.96	70±4	1.14±0.15	28.1±3.6
39	Granite	12.7±2	79±4	0.88±0.05	28±2.8
40	Tuff	11.4±2.57	64±3	2.57±0.43	25.3±3.8

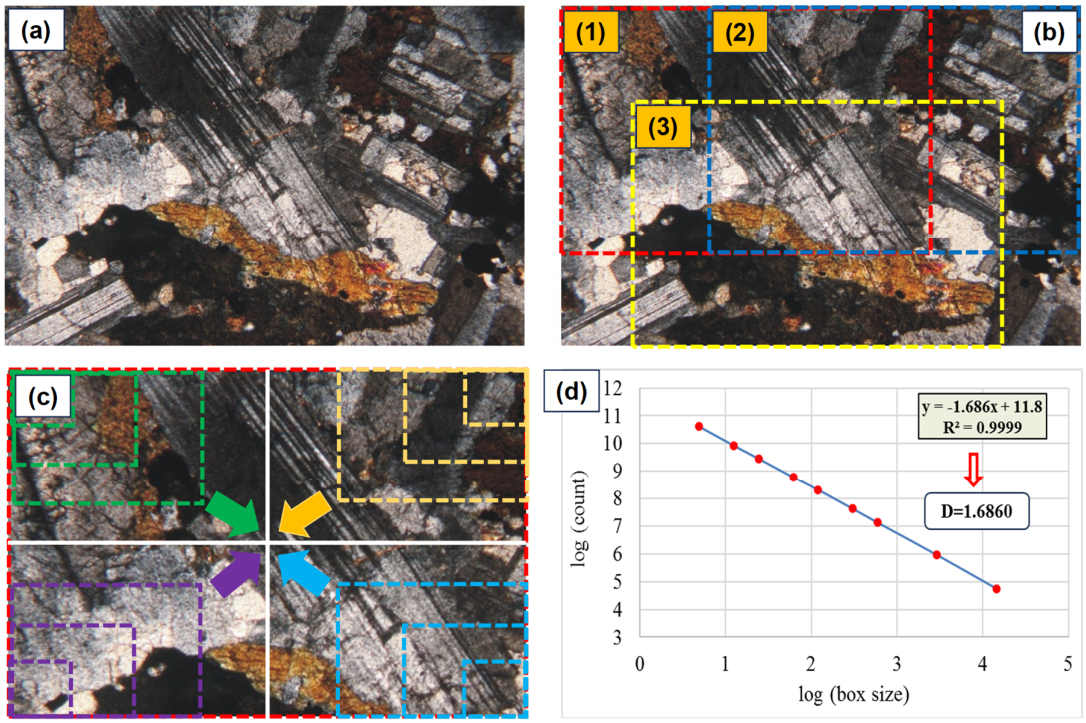


Figure 1. Fractal dimension (D_b) calculation process based on a microphotograph (a) representative microphotograph; (b) selection of three subregions; (c) box-counting analysis in four directions for each subregion; (d) resulting fractal dimension of sample 38 obtained using the ImageJ software

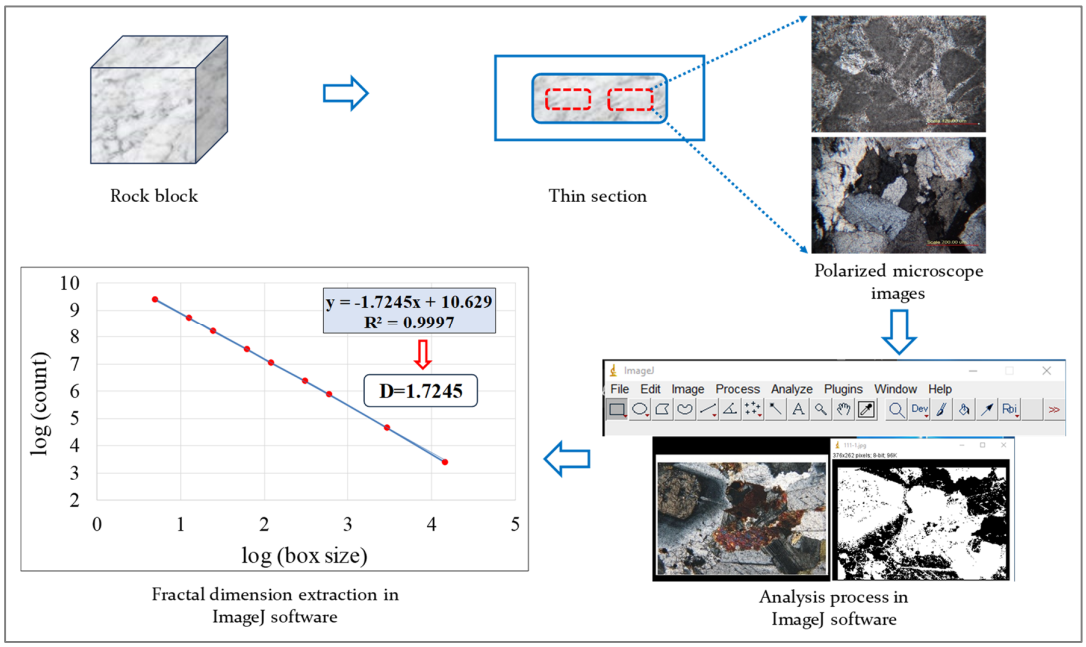


Figure 2. Schematic of calculating the fractal dimension of each microphotograph

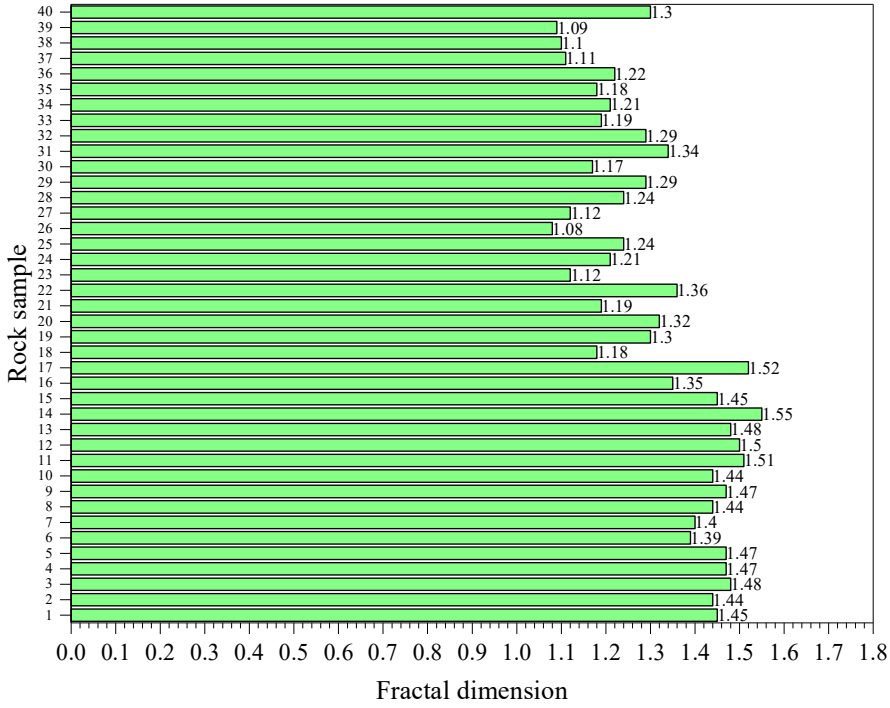


Figure 3. Average values of the fractal dimension of each sample

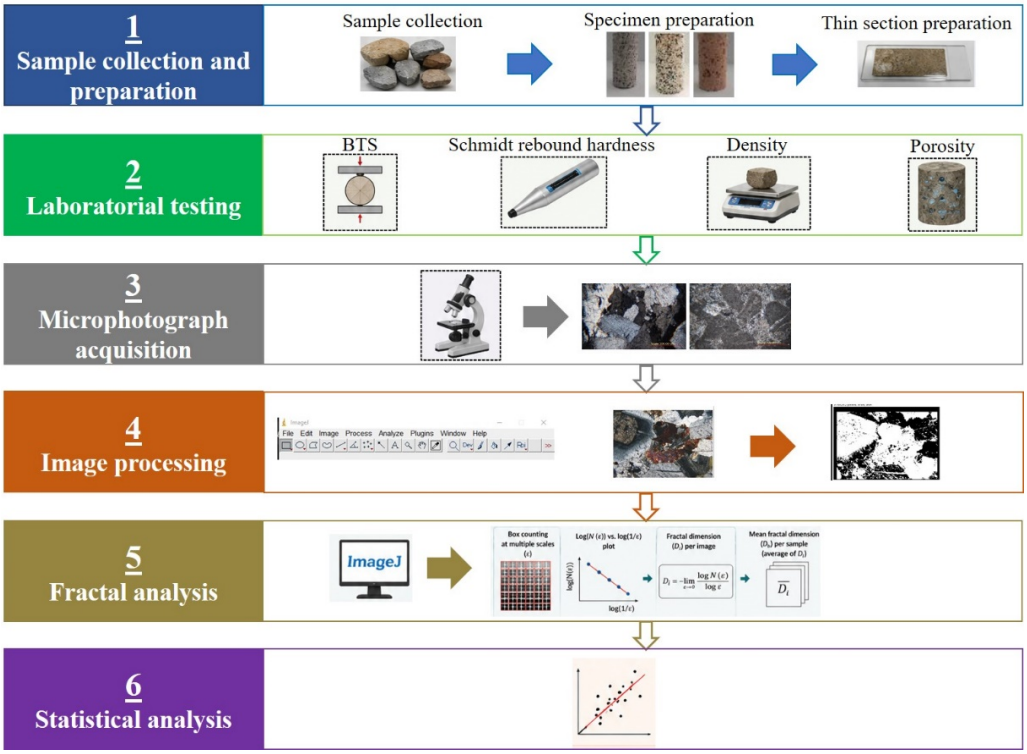


Figure 4. Methodological workflow of the study

3. Results

The regression analysis performed between the fractal dimension of the rock microstructures and their measured physical and mechanical properties provides meaningful insight into how micro-scale textural complexity governs macro-scale behavior. Fractal dimension, as a quantitative descriptor of geometrical irregularity and structural heterogeneity, proved to be a sensitive indicator of the internal organization of the rock fabric. The

results of regression analyses are shown in Figures 5 to 8. The observed trends demonstrate that increasing fractal dimension is systematically associated with reductions in Brazilian tensile strength, Schmidt rebound hardness, and density, while porosity exhibits an increasing trend. These relationships highlight the fundamental role of microstructural complexity in controlling the engineering performance of the rocks.

An increase in fractal dimension reflects a higher degree of geometric irregularity, microcrack density, grain boundary tortuosity, and pore space complexity within the thin-section images. Such complexity indicates that the rock fabric deviates from a compact and well-interlocked crystalline arrangement toward a more fragmented and discontinuous structure. This microstructural disorder directly weakens the load-bearing skeleton of the rock. As the internal framework becomes more irregular and discontinuous, stress transfer between grains becomes less efficient, and stress concentration points increase. Consequently, the rock becomes more susceptible to tensile failure, which explains the inverse relationship observed between fractal dimension and BTS.

The reduction in Brazilian tensile strength with increasing fractal dimension can therefore be interpreted as the mechanical manifestation of microstructural heterogeneity. Rocks with lower fractal dimensions tend to possess more uniform grain distributions, smoother grain boundaries, and fewer microvoids, allowing tensile stresses to be distributed more evenly across the material. In contrast, rocks with higher fractal dimensions contain numerous microdefects and irregular interfaces that act as stress raisers, facilitating crack initiation and propagation under tensile loading. This behavior is consistent with fracture mechanics principles, where crack growth is strongly influenced by the presence and distribution of microstructural discontinuities.

A similar interpretation applies to the decreasing trend observed for Schmidt rebound hardness. Schmidt hardness is primarily influenced by surface compactness, grain interlocking, and resistance to local deformation. Rocks characterized by lower fractal dimensions exhibit more compact and cohesive microstructures, resulting in greater resistance to impact and higher rebound values. Conversely, higher fractal dimensions imply increased micro-porosity and irregular grain contacts, which dissipate impact energy through localized deformation and microcrack activation. This leads to lower rebound readings and reflects a reduced capacity of the rock surface to resist mechanical disturbance.

The observed decrease in density with increasing fractal dimension further supports the interpretation of fractal dimension as a proxy for internal structural compactness. Density is closely related to the volume fraction of solid mineral matter relative to void spaces. As the fractal dimension increases, the microstructure becomes more irregular and pore spaces become more abundant and complex, effectively reducing the overall solid fraction. The regression trend between density and fractal dimension therefore confirms that rocks with more complex micro-textures contain higher volumes of voids and discontinuities, leading to lower bulk density values.

In contrast to the mechanical properties, porosity shows a direct relationship with fractal dimension. This is an expected and significant finding because fractal dimension captures the irregularity and distribution of pore spaces and microcracks within the rock matrix. A higher fractal dimension indicates a more intricate pore network with greater surface irregularity and connectivity. Such complexity not only increases the total pore volume but also affects the geometry of the pore spaces, making them more tortuous and interconnected. This explains why porosity increases as the fractal dimension rises, reinforcing the notion that fractal analysis effectively quantifies the internal void structure of rocks.

The combined interpretation of these trends reveals that fractal dimension serves as an integrative parameter linking microstructural features to both physical and mechanical behavior. Rather than analysing porosity, density, tensile strength, and hardness independently, fractal dimension provides a single quantitative measure that encapsulates the complexity of the rock fabric. This makes it a powerful tool for predicting engineering properties based solely on microstructural image analysis, reducing reliance on extensive mechanical testing.

From a geological perspective, the results also suggest that rocks formed under conditions promoting homogeneous crystal growth and minimal post-formation alteration are likely to exhibit lower fractal dimensions and superior mechanical performance. In contrast, rocks subjected to weathering, alteration, or tectonic stresses develop microcracks, irregular grain boundaries, and complex pore systems, all of which increase the fractal dimension and degrade mechanical integrity. Therefore, fractal dimension

may also serve as an indicator of the rock's geological history and degree of structural degradation.

The regression relationships demonstrated in Figures 5 to 8 emphasize the predictive capability of fractal analysis in rock engineering and geomechanics. The consistent trends across multiple independent properties indicate that the fractal dimension is not coincidentally correlated with a single parameter but is fundamentally related to the intrinsic fabric of the rock. This supports the validity of using fractal geometry as a scientific framework for describing natural materials whose structures are inherently irregular and non-Euclidean.

Moreover, the inverse correlations with tensile strength and hardness, combined with the direct correlation with porosity, suggest that fractal dimension can be employed as a rapid, non-destructive index for assessing rock quality. Thin-section image analysis is considerably less time-consuming than laboratory mechanical testing, and when combined with fractal analysis, it can provide reliable estimates of engineering behavior. This approach can be particularly valuable in preliminary site investigations, quarry assessments, and rock mass classification studies.

In summary, the results clearly demonstrate that increasing fractal dimension corresponds to increasing microstructural complexity, which in turn leads to higher porosity, lower density, and reduced mechanical strength and hardness. These findings confirm that the fractal dimension is a meaningful quantitative descriptor of rock fabric and a strong predictor of engineering performance. The observed relationships provide a robust scientific basis for integrating fractal geometry into the analysis and characterization of rock materials in both geological and engineering contexts.

An additional observation derived from Figures 5 to 8 is that, among the four examined properties, porosity exhibits the strongest relationship with fractal dimension based on R^2 and mean absolute error (MAE) values. While Brazilian tensile strength, Schmidt hardness, and density all show systematic trends with fractal dimension, the correlation with porosity is more pronounced, indicating that fractal geometry is especially sensitive to the spatial configuration and distribution of void spaces within the rock fabric.

This stronger relationship can be interpreted by considering the fundamental nature of fractal dimension as a descriptor of geometrical irregularity. In thin-section microphotographs, fractal analysis primarily captures the complexity of boundaries, discontinuities, pore edges, and grain interfaces. Among these features, pore spaces contribute most significantly to irregularity and boundary complexity. As porosity increases, the number of voids, their shapes, and their interconnections introduce intricate patterns that directly elevate the fractal dimension. Therefore, fractal dimension responds more immediately and more strongly to changes in porosity than to variations in mechanical strength or density, which are indirect consequences of these microstructural features.

In contrast, parameters such as BTS and Schmidt hardness are influenced not only by porosity but also by mineralogical composition, grain bonding strength, cementation, and microcrack orientation. These additional controlling factors introduce variability that weakens the direct statistical link between fractal dimension and mechanical properties. Density, similarly, is affected by both porosity and the intrinsic density of constituent minerals, which may vary independently of microstructural complexity. As a result, although meaningful correlations exist, they are inherently less direct than the relationship observed with porosity.

The dominant correlation between fractal dimension and porosity suggests that fractal analysis is particularly effective in quantifying the geometry of the pore network rather than solely predicting mechanical performance. Since porosity itself plays a central role in governing density, strength, permeability, and durability, this finding reinforces the importance of porosity as an intermediary variable linking microstructure to engineering behavior. In other words, fractal dimension primarily captures pore structure complexity, and through porosity, indirectly influences other physical and mechanical characteristics.

This observation also has methodological implications. It indicates that fractal dimension derived from thin-section imagery can be used as a reliable proxy for estimating porosity, especially in cases where direct porosity measurement is difficult or time-consuming. The strong statistical relationship suggests that image-based fractal analysis may serve as an efficient alternative for rapid porosity assessment in rock characterization studies.

Overall, the stronger association between fractal dimension and porosity confirms that pore geometry is the principal microstructural feature influencing fractal behavior in the studied rocks. This insight enhances the interpretive power of

fractal analysis and clarifies the mechanism through which microstructural complexity affects broader physical and mechanical properties.

Figures 9 to 12 show the comparisons of the measured and predicted physicomechanical results obtained by fractal dimension. The predicted values show good agreement with the measured data, indicating the reliability of the proposed regression models.

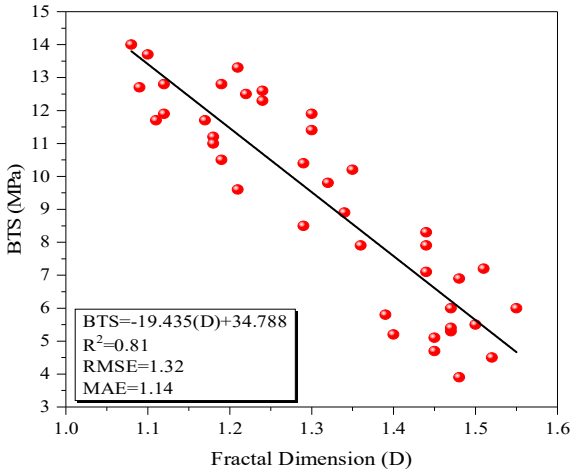


Figure 5. Relationship between BTS and D of rocks

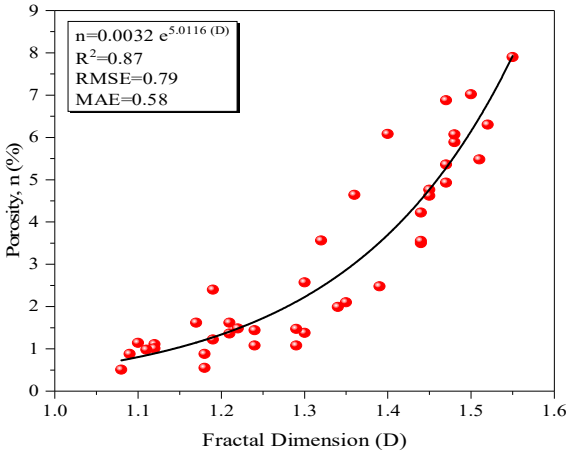


Figure 6. Relationship between porosity and D of rocks

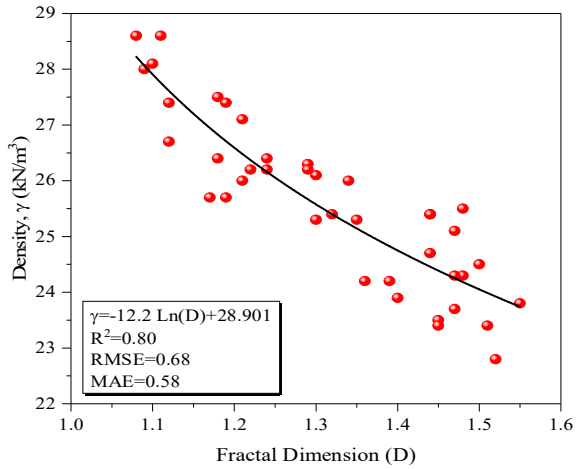


Figure 7. Relationship between density and D of rocks

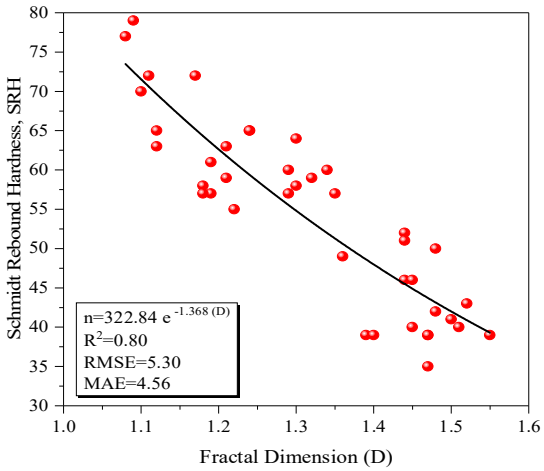


Figure 8. Relationship between SRH and D of rocks

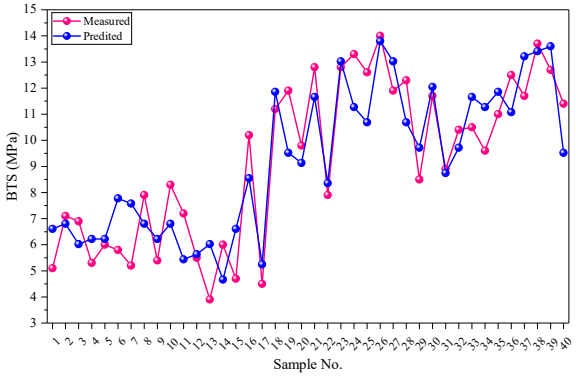


Figure 9. Measured vs predicted values of BTS

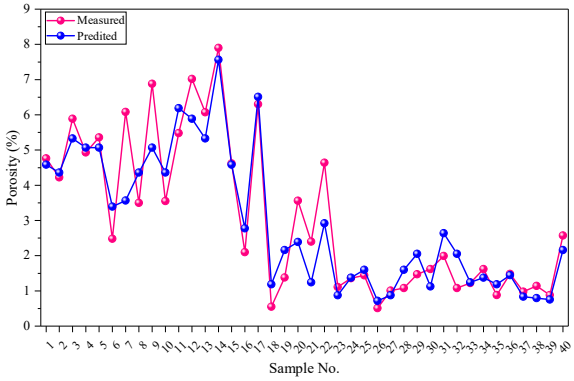


Figure 10. Measured vs predicted values of porosity

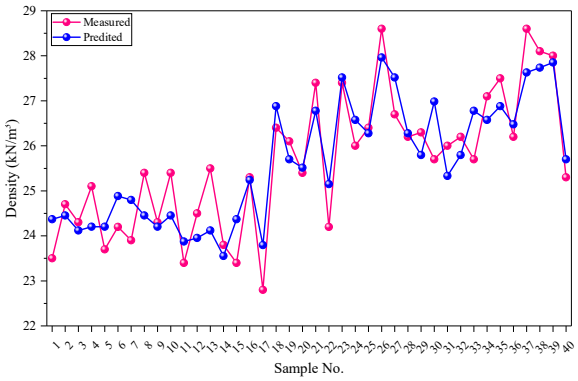


Figure 11. Measured vs predicted values of density

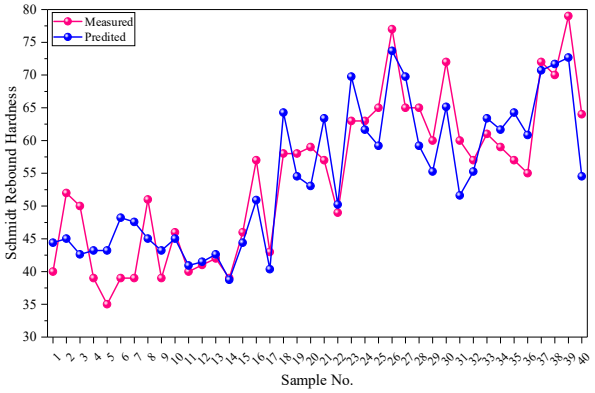


Figure 12. Measured vs predicted values of Schmidt hardness

The comparative evaluation of sedimentary and igneous rock samples provides a deeper understanding of how pore architecture governs the observed relationship between porosity and fractal dimension. Microphotographic evidence, supported by the data in Table 2, reveals a clear textural distinction between limestones and the igneous rocks (andesite and diorite). In the sedimentary rocks, pores are an inherent and dominant component of the rock fabric, whereas in the igneous samples the pore fraction is minimal due to their mode of formation. This contrast offers a compelling explanation for why porosity emerges as the most influential parameter associated with fractal dimension.

These pore spaces in sedimentary rocks are irregular in shape, variably distributed, and often interconnected, producing highly complex internal boundaries. When such features are captured in microphotographs, they contribute significantly to the geometric irregularity quantified by fractal analysis. The abundance and diversity of pore geometries increase edge complexity and boundary roughness, which directly elevates the calculated fractal dimension. Therefore, in sedimentary rocks, the fractal dimension becomes a sensitive measure of pore distribution and morphology.

In contrast, igneous rocks such as andesite and diorite originate from the cooling and solidification of magma, a process that promotes crystalline interlocking and compact mineral packing. The resulting texture is typically dense, cohesive, and characterized by tightly intergrown mineral grains with very limited void space. Any pores that do exist are generally isolated, microscopic, and far less geometrically complex than those observed in sedimentary rocks. Consequently, microphotographs of igneous samples display smoother grain boundaries and fewer discontinuities, which translates into lower fractal dimension values. In these rocks, the contribution of pore geometry to overall structural irregularity is minor, and fractal dimension is influenced more by grain boundaries than by void spaces.

This fundamental difference in pore abundance explains why porosity plays a more prominent role in controlling fractal dimension across the studied samples. In limestones, where pores are widespread and texturally significant, variations in porosity strongly alter the microstructural complexity and are therefore directly reflected in fractal measurements. In andesite and diorite, however, the limited presence of pores reduces their influence on geometric complexity, weakening the sensitivity of fractal dimension to porosity changes.

The findings also suggest that fractal dimension is not merely a generalized descriptor of rock texture but is particularly responsive to the presence and arrangement of void spaces. Since pores introduce irregular edges, tortuous boundaries, and discontinuities into the rock matrix, their prevalence becomes the dominant factor shaping fractal behavior. This explains why, among the various physical and mechanical properties examined, porosity demonstrates the strongest statistical relationship with fractal dimension.

Moreover, the geological origin of the rocks provides an important contextual framework for interpreting these results. The sedimentary environment promotes heterogeneity and pore development through compaction, cementation, and dissolution processes, while the igneous environment favors homogeneity and structural compactness through crystallization. These contrasting formation mechanisms are directly reflected in the microstructural patterns captured by fractal analysis. Thus, fractal dimension can also

be viewed as an indirect indicator of the rock's genetic history and textural evolution.

The microphotographic justification strengthens the reliability of the statistical relationships by linking numerical findings to observable textural features. Rather than relying solely on regression trends, the study demonstrates visually how pore abundance and geometry differ between rock groups and how these differences influence fractal outcomes. This integration of image-based observation with quantitative analysis enhances the interpretive strength of the results.

Overall, the stronger association between porosity and fractal dimension can be attributed to the dominant presence of pores in sedimentary textures compared to their scarcity in igneous rocks. The percentage and geometry of pores act as primary contributors to microstructural irregularity, which is precisely what fractal dimension quantifies. This explains why porosity emerges as the key parameter governing fractal behavior and reinforces the value of fractal analysis as a tool for characterizing pore-driven textural complexity in rock materials.

While the results of this study demonstrate consistent and interpretable relationships between fractal dimension and physicomechanical properties, the robustness of the proposed framework should be considered within the context of the available dataset and methodological assumptions. The derived regression models are based on a limited number of samples and specific lithological conditions; therefore, their broader applicability requires further validation. The findings should be regarded as indicative rather than definitive, highlighting the potential of fractal analysis as a complementary tool for rock characterization rather than a standalone predictive solution.

4. Discussion

The fractal dimension of rocks serves as a quantitative descriptor of the geometric complexity, roughness, and heterogeneity of microstructural features, including pore systems, microcracks, and grain boundaries. Generally, fractal geometry has been widely recognized as an effective framework for characterizing natural geological materials that deviate from ideal Euclidean geometries. An increase in fractal dimension reflects a transition toward multiscale, self-similar, and highly irregular microstructures, which exert a first-order control on stress transmission, damage evolution, and mechanical failure in rocks [1].

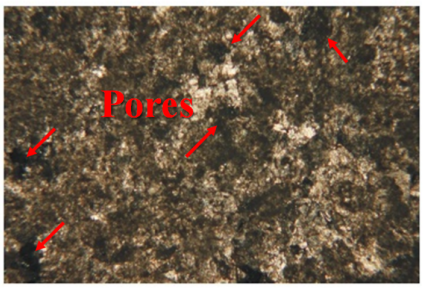
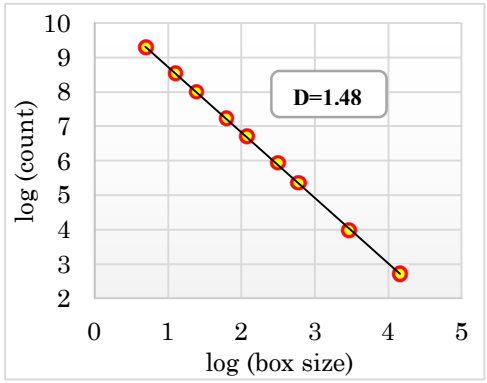
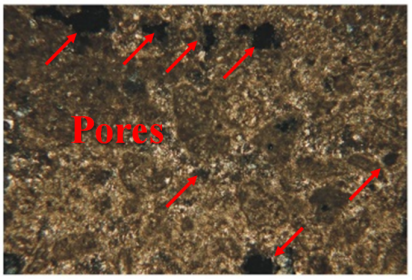
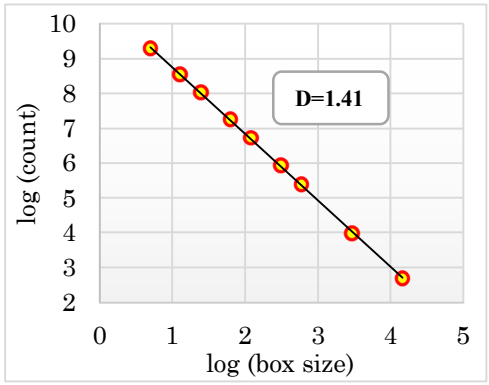
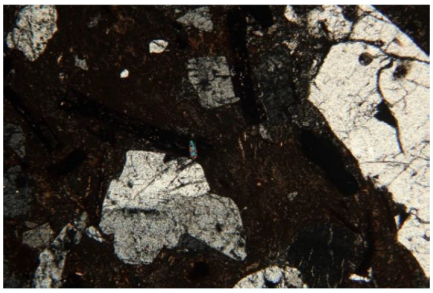
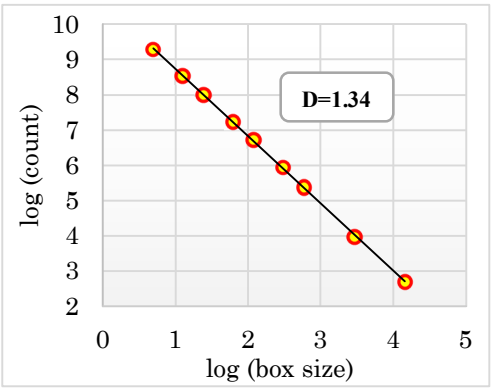
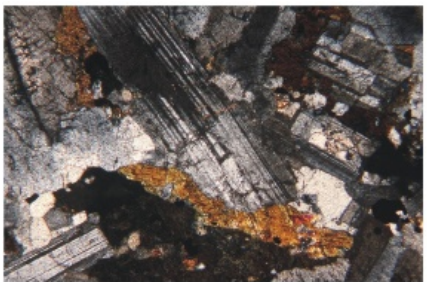
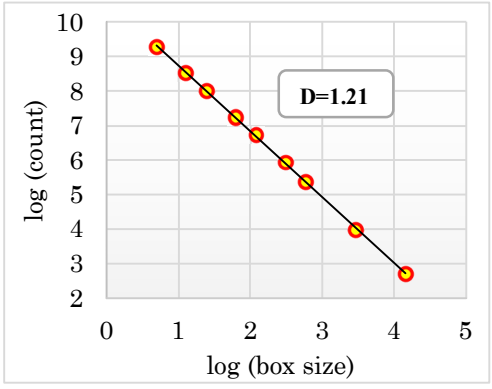
In rocks exhibiting higher fractal dimensions, porosity is not merely a volumetric parameter but a reflection of the fractal nature of void geometry and connectivity. Pores and microcracks are characterized by irregular, elongated, and branched morphologies that display scale invariance across several orders of magnitude. Such fractal pore networks significantly increase the effective void space and internal surface area, even for modest increases in total porosity.

An elevated fractal dimension is commonly associated with increased roughness and tortuosity of grain boundaries, weakened cementation, and the proliferation of intergranular microfractures. These features promote the development of a connected discontinuity network, enhancing effective porosity and permeability at the microscale. As a result, the rock matrix becomes increasingly discontinuous and mechanically fragmented.

Fractal pore structures reduce the effective load-bearing cross-sectional area of the rock matrix. A decrease in this property leads to localized stress amplification, accelerating microcrack propagation and reducing the strength.

In rocks with high fractal dimensions, stress distribution under external loading becomes highly non-uniform. Sharp pore edges, crack tips, and branching microfractures act as dominant stress concentration sites, consistent with fractal damage mechanics theories. With increasing fractal dimension, the density of these stress concentrators increases, and the interaction between neighboring microcracks intensifies, facilitating early crack initiation and coalescence under lower applied stresses. A summary of the factors affecting the relationships between some physicomechanical properties of rocks and their fractal dimension is shown in Figure 13.

Table 2. Comparison of some of the calculated fractal dimensions of the studied samples based on their microphotographs

Sample	Microphotograph	Fractal dimension obtained from ImageJ program
1: Limestone		
6: Limestone		
31: Andesite		
35: Diorite		

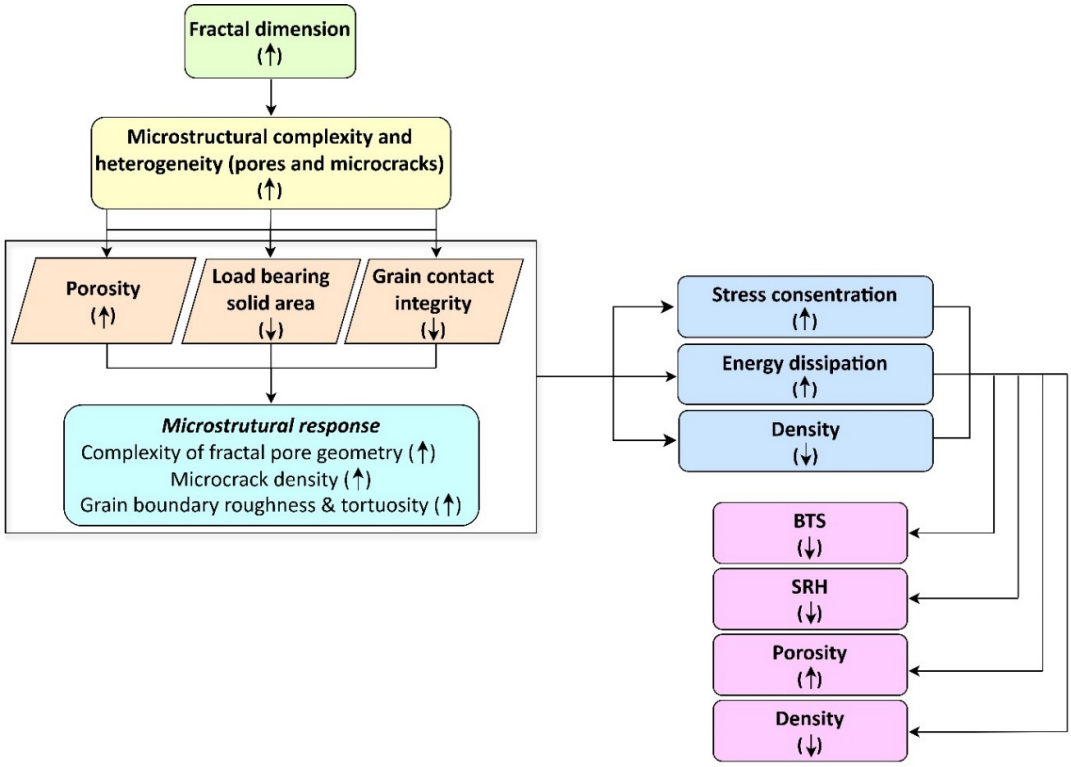


Figure 13. A microscopic perspective on the factors affecting the relationship between physicomechanical properties and the fractal dimension of rocks

Generally, recent developments in geomaterials research have extensively employed machine learning (ML) techniques and deep learning algorithms to predict rock properties and engineering behavior. These models are capable of capturing complex nonlinear relationships and often provide high predictive accuracy. However, they typically require large datasets and operate as black-box systems with limited physical interpretability. In contrast, the fractal-based approach adopted in this study provides a physically meaningful and interpretable framework by directly linking microstructural complexity to macroscopic behavior. Unlike ML models, the proposed method does not rely on extensive training data and instead utilizes fractal dimension as an intrinsic descriptor of rock heterogeneity. Therefore, fractal analysis can serve as a complementary approach to ML, particularly in data-scarce conditions or where interpretability is essential.

Although the developed relationships exhibited consistent statistical performance across the investigated samples, their transferability beyond the present dataset requires further validation. The studied rocks encompass a range of lithologies; however, geological factors such as mineralogical variability, weathering intensity, alteration history, and scale-dependent heterogeneity may influence the applicability of the proposed models in other settings. Therefore, the presented equations should be interpreted as physically based empirical relationships whose robustness is supported within the investigated dataset, while their broader generalization should be verified using independent datasets and field-scale observations.

5. Conclusions

This study established a systematic link between microstructural geometric complexity and the macroscopic physicochemical behavior of diverse rock types through fractal characterization using thin-section microphotographs and box-counting method. By integrating petrographic imaging with standardized laboratory testing and regression modeling, the investigation demonstrated that fractal dimension serves as a meaningful quantitative descriptor that captures the inherent heterogeneity of rock textures beyond the capabilities of conventional geometric or petrographic indices. The main results of this research are as follows:

- The results confirmed that measurable variations in Brazilian tensile strength, Schmidt rebound hardness, density, and porosity are

consistently associated with changes in fractal dimension across a broad spectrum of lithologies. The regression analyses revealed statistically significant relationships between fractal dimensions and these properties, indicating that the degree of spatial irregularity and textural complexity at the microscopic scale exerts a direct influence on tensile strength, surface hardness response, and physical characteristics of rocks. In particular, the fractal dimension was shown to reflect the distribution and organization of grains, pores, and microstructural discontinuities that govern stress concentration and crack initiation mechanisms.

- Unlike many previous studies relying on SEM imagery or fracture surface analysis, this research demonstrated that thin-section optical microphotographs obtained under controlled petrographic conditions provide reliable and reproducible input data for fractal evaluation. This approach offers a practical, accessible, and cost-effective alternative for extracting fractal descriptors while preserving the mineralogical and textural context of the rock fabric. The consistency of imaging conditions and the diversity of lithologies enhanced the robustness and generality of the derived correlations.
- The integration of fractal analysis and regression modeling transformed fractal geometry from a purely descriptive tool into a predictive framework for rock characterization. The established relationships suggest that fractal dimension can be used as an indirect predictor of key physicomechanical parameters, supporting rapid preliminary assessments in situations where extensive mechanical testing is impractical. This capability has significant implications for rock engineering applications, including site characterization, material classification, and the estimation of mechanical performance in early design stages.
- Among the four evaluated properties, porosity demonstrated the most pronounced association with fractal dimension, as evidenced by the highest coefficient of determination obtained from the regression analyses. Although Brazilian tensile strength, Schmidt rebound hardness, and density exhibited consistent and interpretable trends with increasing fractal dimension, the strength of correlation was notably greater for porosity. This finding indicates that fractal geometry is particularly responsive to the spatial arrangement, connectivity, and distribution of void spaces within the rock fabric, which are directly captured in thin-section imagery and effectively quantified through the box-counting approach.

Overall, the findings highlight the potential of fractal dimension as a unifying parameter linking microstructure to mechanical response in rocks. The study contributes to bridging the long-standing gap between petrographic observations and engineering properties by offering a quantitative pathway grounded in geometric complexity. Future research may extend this framework to three-dimensional imaging, larger datasets, and additional mechanical indices to further refine the predictive capacity of fractal descriptors in rock mechanics and geotechnical engineering.

Despite the promising results obtained in this study, several limitations should be acknowledged. The dataset, although comprising diverse lithologies, is relatively limited in size, which may affect the generalizability of the derived relationships. In addition, the fractal dimension was calculated from two-dimensional thin-section microphotographs, which may not fully represent the three-dimensional complexity of pore networks and microcrack systems in rocks. The box-counting method is also sensitive to image processing parameters, including threshold selection and resolution, which may introduce minor uncertainties in fractal estimation.

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نگاهی فراکتالی به تغییرات فیزیکی-مکانیکی سنگ با استفاده از روش شمارش جعبه‌ای: شواهد تجربی و تحلیل آماری

ساسان قربانی*

گروه مهندسی معدن، دانشگاه صنعتی همدان، همدان، ایران

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کلمات کلیدی

بُعد فراکتالی
تخلخل
ریزتصاویر
تحلیل آماری

چکیده

درک ارتباط میان ریزساختار سنگ و رفتار فیزیکی-مکانیکی آن در مقیاس ماکروسکوپی، به دلیل ناهمگنی و ناپیوستگی ذاتی مواد زمین‌شناسی، همچنان یکی از چالش‌های اساسی در مکانیک سنگ به شمار می‌رود. در این مطالعه، ارتباط میان بُعد فراکتالی، که با استفاده از روش شمارش جعبه‌ای بر روی تصاویر میکروسکوپی مقاطع نازک تعیین شده است، و ویژگی‌های کلیدی فیزیکی-مکانیکی سنگ‌ها از طریق تحلیل آماری مورد بررسی تجربی قرار گرفته است. در مجموع، ۴۰ نمونه‌سنگ با دامنه متنوعی از سنگ‌شناسی، از معادن مختلف ایران جمع‌آوری شد. آزمایش‌های استاندارد آزمایشگاهی به‌منظور تعیین مقاومت کششی برزیلی، سختی بازگشت چکش اشمیت، چگالی و تخلخل انجام گرفت. همچنین، تصاویر میکروسکوپی با وضوح بالا از مقاطع نازک صیقلی، تحت شرایط نوری کنترل‌شده تهیه و با استفاده از نرم‌افزار ImageJ پردازش شدند تا ابعاد فراکتالی استخراج شوند. سپس، مدل‌های رگرسیونی به‌منظور بررسی روابط میان شاخص‌های فراکتالی و ویژگی‌های اندازه‌گیری‌شده توسعه یافتند. نتایج، وجود روابط مشخص و از نظر آماری معنادار میان بُعد فراکتالی و تمامی ویژگی‌های مورد بررسی را نشان دادند. در میان پارامترهای مورد مطالعه، تخلخل قوی‌ترین همبستگی را با بُعد فراکتالی نشان داد که بیانگر حساسیت بالای هندسه فراکتالی نسبت به توزیع فضایی و آرایش فضاهای خالی در بافت سنگ است. مقاومت کششی برزیلی، سختی بازگشت چکش اشمیت و چگالی نیز روابط منظمی با بُعد فراکتالی نشان دادند که تأیید می‌کند بی‌نظمی ریزساختاری نقش تعیین‌کننده‌ای در کنترل رفتار فیزیکی و مکانیکی سنگ ایفا می‌کند. این مطالعه نشان می‌دهد که تحلیل فراکتالی مقاطع نازک، روشی کاربردی و مقرون‌به‌صرفه برای کمی‌سازی ناهمگنی سنگ و برقراری ارتباط میان ویژگی‌های پتروگرافی و خواص مهندسی آن است. تلفیق هندسه فراکتالی با مدل‌سازی رگرسیونی، چارچوبی قابل‌اعتماد برای پیش‌بینی غیرمستقیم پارامترهای فیزیکی-مکانیکی و برقراری ارتباط میان ویژگی‌های ریزساختاری و رفتار سنگ در مقیاس ماکروسکوپی فراهم می‌کند.

* نویسنده مسئول مکاتبات: s.ghorbani@hut.ac.ir (س. قربانی)