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An investigation of the particle size effect on coal flotation kinetics using multivariable regression

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Abstract

An attempt has been made in this paper to investigate the effect of particle size distribution on coal flotation kinetics. The effect of particle size (Ps) on kinetics constant (k) and maximum theoretical flotation recovery (RI) was investigated while other operational parameters were kept constant. The relationship between flotation kinetics constant and theoretical flotation recovery with particle size was estimated with nonlinear equations. Analysis of variance showed that the effect of particle size on the kinetics constant was statistically significant at 95% confidence level. However, it was not significant on maximum theoretical flotation recovery (RI). Different regression methods were conducted in order to model the effect of coal particle size on flotation kinetics. Results indicated that the quadric regression method gave better prediction of the cumulative recovery for different particle size fractions. The correlation coefficient (R^2) values of this model were 0.99, 0.996, 0.98, 0.98 and 0.97 for average of particle sizes of 37.5 µm, 112.5 µm, 225 µm, 400 µm and 625 µm respectively.

Keywords: *Coal flotation, Particle size, Flotation kinetics, Multivariable regression*.

1. Introduction

Froth flotation is a physicochemical method which is widely used in mineral processing technologies for the separation of finely ground valuable minerals from a mixture with gangue minerals initially present in a pulp. Since the cumulative recovery of a component in the concentrate is proportional to flotation time, the flotation process can be considered as a time-rate recovery process [1, 2]. Therefore, mathematical flotation models that incorporate both a recovery and a rate function can completely describe flotation time-recovery profiles. They provide an excellent tool to evaluate flotation tests. The general rate equation for flotation, demonstrated in Eq.1 [3].

$$\frac{dC_p(t)}{dt} = -k(t)C_p^m(t)C_b^n(t) \tag{1}$$

where Cp(t) and Cb(t) are the concentrations of the particles and bubbles at time t, respectively. The exponents, m and n are the respective orders for particles and bubbles, and k(t) is a pseudo rate constant that depends on various parameters governing the flotation process, and may vary with time.

There has been a great deal of discussion over the actual order of the flotation process [4-9]. Batch flotation test data in the literature support the first-order rate equation under reasonable operating conditions [10-18]. A classical first order rate equation of the form:

$$R = RI[1-\exp(-kt)] \tag{2}$$

is proposed, where R is the cumulative recovery after time t, k is the first order rate constant (time-l), t is the cumulative flotation time and RI is the maximum theoretical flotation recovery. In the derivation of this equation, it has been assumed that the only independent variable has been the concentration of floatable material, and that everything else has remained constant including size and size distribution, bubble concentration, reagent concentrations, cell operation, etc [18-20].

RI (ultimate recovery) and k (first order rate constant) are obtained from adjustment of the model for an experimental recovery–time curve. They can be effectively used to evaluate variables affecting flotation process. Kinetics constant is complex and many authors have argued that the value of the kinetics constant is a function of hydrodynamic, chemical and operational parameters such as inducement time, aeration, reagent concentration, particle size, prior treatment, design of the flotation cell, etc [17, 21-24].

Numerous researchers have studied the kinetics aspects of froth flotation paying special attention to particle size [3, 12, 24-29]. However, the form of the relationship between flotation rate and particle size is not yet clear. The most recent theoretical analyses suggest that,

$$k \propto Ps^{m} \tag{3}$$

where k is a suitable measure of the flotation rate, Ps is particle size (diameter), and m is the number between 1.5 and 2 [20,24].

Regression analysis is one of the most used statistical tools by engineers and scientists. This study has examined the effects of particle size distribution on the coal flotation kinetics parameters (RI and k). Different regression methods were conducted to modeling the effect of coal particle size on flotation kinetics.

2. Experimental

A coal sample of about 50 kg was collected from the feed of the Zarand Coal Washing plant in Iran. The sample was screened at 0.85 mm for kinetics test. The collected sample was subsampled by coning and quartering to obtain a representative sample. The size distribution and ash analysis of the coal sample is provided in Table 1.

Kinetics test were carried out in a Denver laboratory flotation machine. The impeller speed, and solids content were kept constant at 1000 rpm and 10% (by weight), respectively. The collector used in these tests was gas oil (2.5 kg/t) and the frother was pine oil (230 g/t).

In flotation kinetics test, the pulp was first agitated in the flotation cell for 2 min, after which the required dosage of flotation reagents was added, and the slurry was conditioned for 5 min, Air was then introduced, and froth samples were collected after 10, 20, 30, 45, 60, 90, 120, 150, 210 and 270 seconds. After the final froth sample was collected, the machine was stopped. Size-wise and ash analysis for the five fractions, namely (-75, -150+75, -300+150, -500+300 and -850+500 microns) was conducted for froth products and the tailings.

Table 1. Size analysis and size-wise ash content of the sample

Size(micron)	Wt(%)	Ash(%)
-850+500	20.7	24.8
-500+300	13.2	22.4
-300+150	22.4	20.8
-150+75	15.9	21.1
-75	27.8	32.6
Total feed	100	25.2

Using the results of yield and ash percentages of the concentrates collected at different time intervals along with tailing weight, the cumulative recoveries of non-ash materials for each fraction were calculated as follows:

$$Re = Y(100 - A_c)/(100 - A_f) \tag{4}$$

Where Re is the none ash recovery, Y is the percentage yield of the concentrate and Ac and A_f are the percentage ash contents of the concentrates and the feed materials, respectively.

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3- Results and discussion

3-1. Effect of particles size on flotation kinetics parameters

The recovery vs. time data for various size fractions were subjected to a curve fitting procedure using the first-order model. Figure 1 showes cumulative recovery of various size fractions of coal against flotation time based on the laboratory flotation test results. For each size fractions, kinetics parameters (RI and k) are summarized in Table 2.

The effect of particle size (Ps) on kinetics constant (k) and maximum theoretical flotation recovery (RI) was investigated when other operational parameters were kept constant. Particle size was considered as the independent variable (factor) and kinetics constant and maximum theoretical flotation recovery was selected as response.

Several empirical models were driven to describe the response as a function of individual

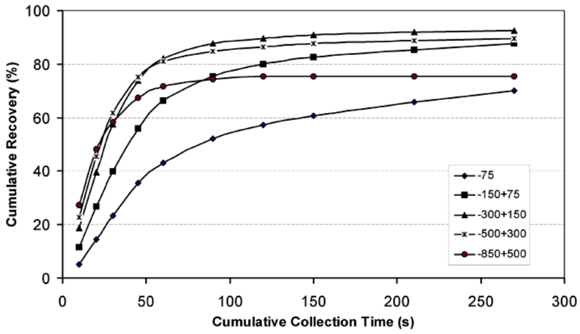

Figure 1. Cumulative recovery of different fractions of coal particles against flotation time

Table 2. Kinetics parameters of the first-order model for various size fractions

Size(micron)	k(min^{-1})	RI(%)
-850+500	2.93	75.4
-500+300	2.24	88.9
-300+150	1.87	92.9
-150+75	1.26	87.5
-75	0.86	70.5

parameters, In this case, by using least square method, the empirical models (Eq. 5 and 6) were proposed to relate kinetics constant (k) and maximum theoretical flotation recovery (RI), to particle size (Ps).

$$k = -3.02 * 10^{-6} * Ps^2 + 0.0055 * Ps + 0.6827 \quad (5)$$

$$RI = -8.8904 * (\ln(Ps))^2 + 92.65 * \ln(Ps) - 149.25 \quad (6)$$

analysis of errors was conducted to assess the predictability of empirical models. Figure 2 illustrates linear plots of the calculated kinetics constant (k) and maximum theoretical flotation recovery (RI) against observed values, the correlation coefficient (R^2) values for kinetics constant and maximum theoretical flotation recovery were 0.989 and 0.999 respectively which indicates that these parameters can be predicted reasonably well.

Normal distribution of residuals of model is illustrated in Figure 3. As depicted in this figure, residuals of models followed a normal distribution describing that the predictability of the acceptable models are good.

Analysis of variance was conducted to asses the effect of particle size for kinetics constant and maximum theoretical flotation recovery statistically in Table 3. P-value for each term is estimated by a comparison between observed F-value and standard F-value. Depending on the statistical significance level, if P-value falls below 0.05, the term would be statistically significant [30].

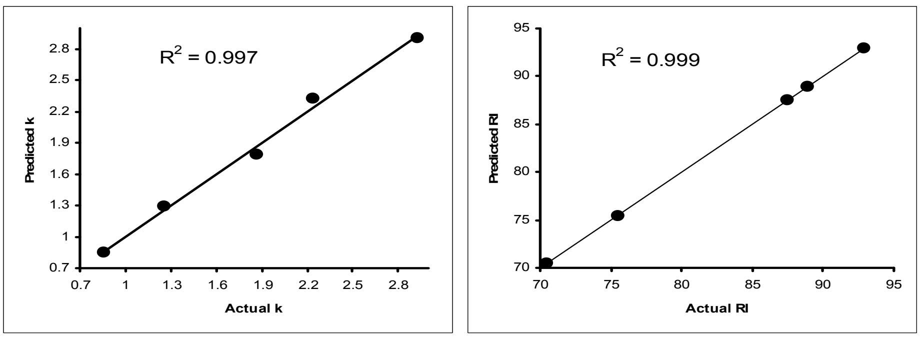

Figure 2. Predicted responses against actual data

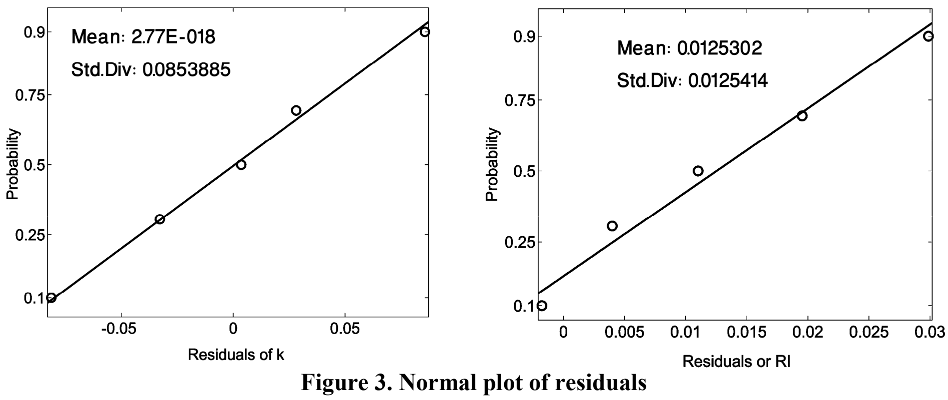

Figure 3. Normal plot of residuals

Table 3. Analysis of variance, the effect of Ps vs. k and RI

Source	Sum of squares	DF	Mean square	F-value	P-value Prob > F
k vs. Ps	207602.7	1	207602.7	6.392	0.035
RI vs. Ps	107060.8	1	107060.8	3.292	0.107

$$R = 2.758 + 0.6787 * T + 0.1719 * Ps - 0.00179 * T^2 - 0.0002 * Ps^2 \quad (9)$$

$$R = -26.704 + 1.397 * T + 0.365 * Ps - 0.00019 * T^2 - 0.00086 * Ps^2 + 0.000017 * T^3 + 0.00000062 * Ps^3 \quad (10)$$

$$R = -33.868 + 2.152 * T + 0.407 * Ps - 0.021 * T^2 - 0.00131 * Ps^2 + 0.00009 * T^3 + 1.76 * 10^{-6} * Ps^3 - 1.3 * 10^{-7} * T^4 - 8.8 * 10^{-10} * Ps^4 \quad (11)$$

Analysis of variance shows that the effect of particle size is statistically significant on kinetics constant however it is not significant on the RI at the same level.

3-2. Modeling of the cumulative recovery by multivariable regression

Multivariate regression (MVR) is a widely used classical method for regression in many fields of chemistry and industrial process control [31]. The MVR problem can be stated as follows: Features are measured for m variables, x_j (j = 1 to m), and for a variable, y, with the goal of establishing a linear (or first-order) relationship between them. The relationship can be mathematically represented as:

$$y = b_1x_1 + b_2x_2 + \ldots + b_jx_j + \ldots + b_mx_m + e \quad (7)$$

where, x_j's are the independent variables, y is the dependent variable, b_j's are sensitivities (regression coefficients) and e is the error or residual. Different techniques based on multivariate regression were conducted to describe the cumulative recovery as a function of particle size and time.

By the simple linear regression method, the equation correlating time and particle size with the cumulative recovery can be developed as following:

$$R = 35.339 + 0.2001 * T + 0.0250 * Ps \quad (8)$$

Based on the experiment data, the quadratic regression equation will be as Eq. 9.

In addition, cubic regression equation, Eq.10, and quadric regression equation, Eq.11, are given above.

For each model, goodness of fit may be demonstrated by low RMSE and/or high R_2. Prediction accuracy is demonstrated by high good estimate (GE). Goodness of fit and prediction accuracy for various models are presented in Table 4.

As can be seen in Table 4, the quadric model showed the best performance with the lowest RMSE, the highest R^2, and the highest number of GE. This indicates that the quadric model provided the most accurate predictions.

Table 4. Comparisons of the goodness of fit and accuracy

Model	RMSE[a]	R^{2b}	Good estimate(%)[c]
Linear	16.8	0.521	10
Quadratic	8.91	0.865	24
Cubic	4.89	0.96	36
Quadric	2.95	0.985	68

[a] RMSE = square-root of mean square error which is defined as the mean of the squared deviations between the actual values of the observations and the predicted values of the corresponding observations.
[b] R^2 = 1 - residual sum of squares/corrected sum of squares.
[c] Good estimate = the percentage of predicted values that are within the 5% tolerance of the corresponding actual values.

The distribution of difference between cumulative recovery predicted from Eq. (10) and actual determined amounts of cumulative recovery is shown in Figure 4. The difference between predicted value and actual data, followed a normal distribution therefore there is no significant deviation between the predicted data and the experiment data.

3.3. Modeling the effect of particle size on cumulative recovery with quadric regression equation

The typical response surface and contours for cumulative recovery of different particle size are shown in Figure 5. This figure shows the effect of particle size on cumulative recovery. The flotation recovery increases initially, reaches a maximum and decreases afterwards with increasing particle size. This is due to the combined effect of the collision, and attachment/detachment sub-processes, dominant in small and large sizes, respectively.

In order to check the validity of the quartic regression model, the cumulative recoveries derived from Eq. (10) for individual size fractions (average particle size) are plotted against the corresponding cumulative recovery (Figure 6). The correlation coefficient (R^2) values of the model were 0.99, 0.996, 0.98, 0.98 and 0.97 for 37.5, 112.5, 225, 400 μm and 625 μm average particle sizes respectively. It was observed that cumulative recoveries of different particle size and time using quartic regression model could be predicted reasonably well.

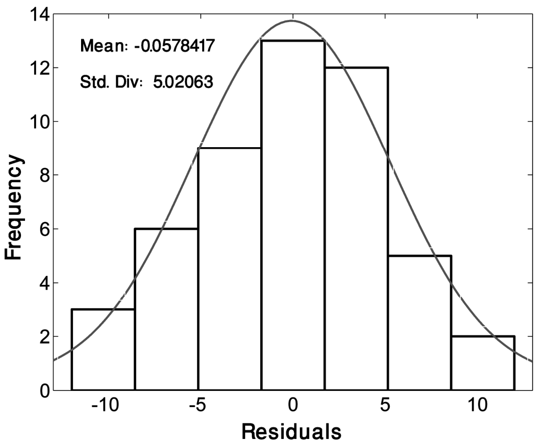

Figure 4. Distribution of residuals between predicted value and actual data of cumulative recovery

4. Conclusion

1- The relationship between kinetics constant (k) and maximum theoretical flotation recovery (RI) with particle size (Ps) were estimated through the following Equations:

$$k = -3.02 * 10^{-6} * Ps^2 + 0.0055 * Ps + 0.6827$$

$$RI = -8.8904 * (\ln(Ps))^2 + 92.65 * \ln(Ps) - 149.25$$

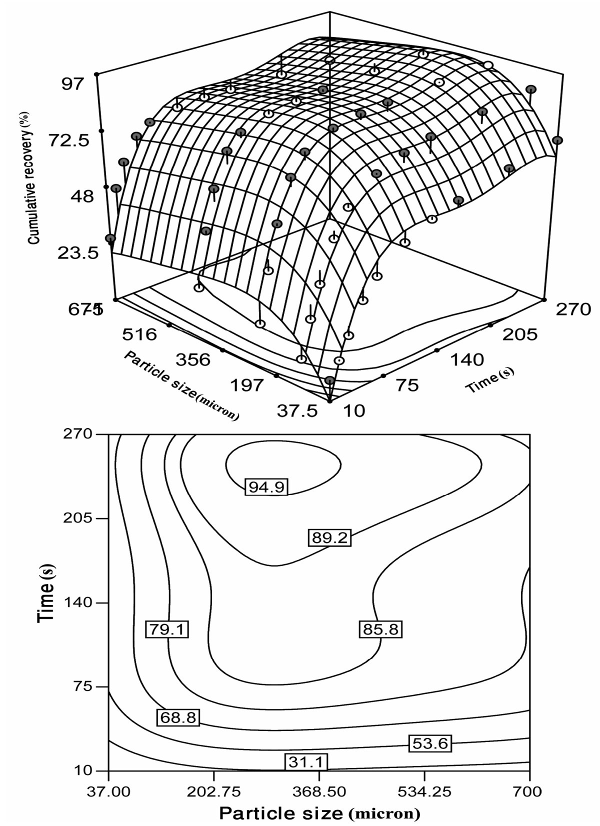

Figure 5. Response surface and contours for the cumulative recovery

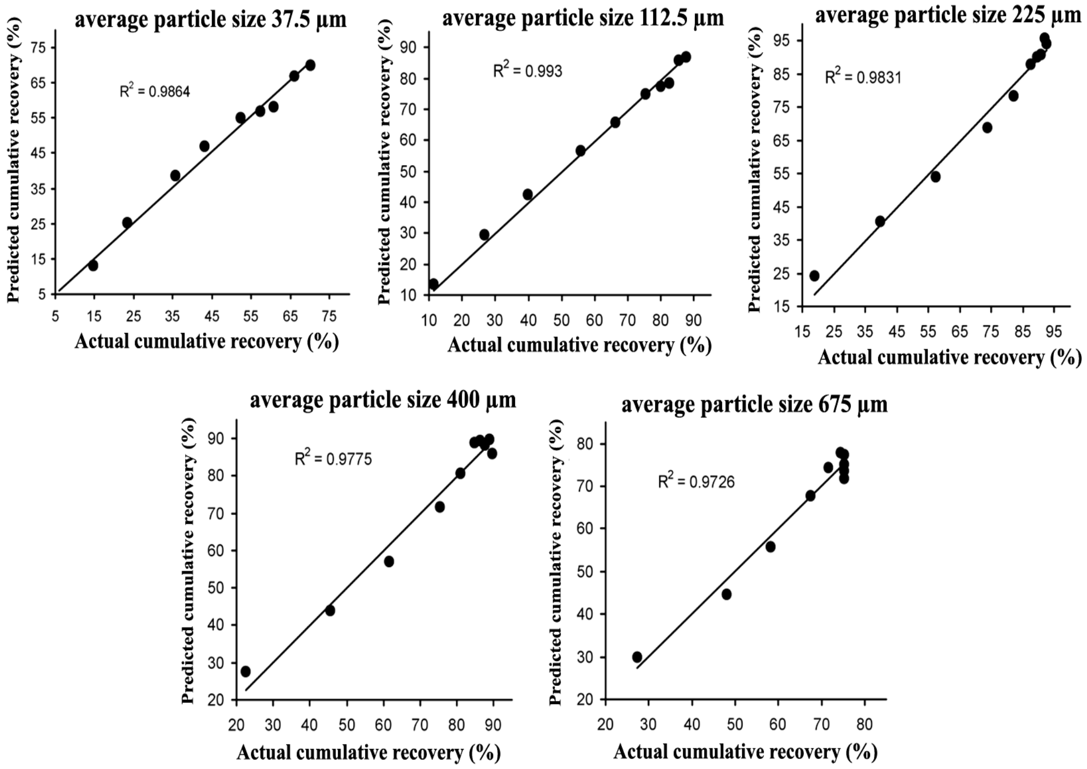

Figure 6. Comparison between actual and predicted cumulative recoveries for individual size fractions

The distribution of residuals of models followed a normal distribution indicating that the predictability of models is good.

2- Analysis of variance shows that varying of particle size is statistically significant on kinetics constant however it has not significant effect on maximum theoretical flotation recovery (RI) at 95% confidence level.

3- Different techniques were conducted to describe the cumulative recovery as a function of particle size and time. The quadric model showed the best results with the lowest RMSE, the highest R2, and the highest number of GE. This indicates that the quadric model provided the most accurate predictions.

4- The correlation coefficient (R^2) values of the quadric regression model were 0.99, 0.996, 0.98, 0.98 and 0.97 for 37.5, 112.5, 225, 400 and 625 μm average particle sizes respectively. It was observed that cumulative recoveries of different particle size and time using quadric regression model could be predicted reasonably well.

Notation

A_c: percentage ash content of feed
A_f: percentage ash content of concentrate
$C_b(t)$: babble concentration, p mL^{-1}
$C_p(t)$: particle concentration, p mL^{-1}
k: first order rate constant, min^{-1}
Ps: particle size, micron
R: cumulative recovery after t (%)
Re: non ash recovery (%)
RI: maximum theoretical flotation recovery (%)
T: time, s
Y: percentage yield of the concentrate

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